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Stiffness Characteristics of a Basic Nonlinear Air Spring Model

Abdullah S. Alsuwaiyan

Department of Mechanical Engineering, College of Engineering, Qassim University, Buraydah, Saudi Arabia
Email: a.alsuwaiyan@qu.edu.sa

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Abstract

This study predicts the characteristics of a compressible polytropic air spring model. A second-order nonlinear autonomous air spring model is presented. The proposed model is based on the assumption that polytropic processes occur. Isothermal and isentropic compression and expansion of the air within the spring chambers are the two scenarios that are taken into consideration. In these situations, the air inside the spring chambers compresses and expands, resulting in nonlinear spring restoring forces. The MATLAB/Simulink software environment is used to build a numerical simulation model for the dynamic behavior of the air spring. To quantify the values of the stiffnesses of the proposed models, a numerical solution is run over time for various values of the design parameters. The isentropic process case has a higher dynamic air spring stiffness than the isothermal process case, according to the results. The size of the air spring chamber and the area of the air spring piston influence the air spring stiffness in both situations. It is demonstrated that the stiffness of the air spring increases linearly with increasing piston area and decreases nonlinearly with increasing air chamber length. As long as the ratio of the vibration's amplitude to the air spring's chamber length is small, there is good agreement in both scenarios between the linearized model and the full nonlinear model. This implies that linear modeling is a reasonable approximation of the complete nonlinear model in this particular scenario.

Keywords

Air Spring, Dynamic Stiffness, State Space, Polytropic Modeling, Isentropic Process, Isothermal Process

1. Introduction

Air springs are mechanical devices that use confined air to accomplish specific tasks. Their history dates back to 1847 when John Lewis received the first US patent for rubberized air springs [1]. They have been employed in numerous applications ever since. Spring actuators found in packaging machinery, clutch systems, conveyor belts, scissor lifts, and other devices are a few examples. In addition, they serve as vibration insulators in textile looms, commercial laundry machines, centrifuges, and other devices. The automotive industry saw the greatest impact from air springs because of their use in suspension systems. In car suspension systems, air springs lessen undesired vibrations and absorb motion shocks. As a result, the uncomfortable ride conditions caused by longitudinal vibrations are reduced.

Despite the fact that air springs are sold commercially, not enough research has been done on their dynamic properties. One of the most important design factors for air springs is their vertical stiffness. Numerous studies on that have been conducted utilizing intricate techniques and frameworks.

For the purpose of designing automobiles with non-adiabatic air spring suspensions, the dynamic behavior of an air spring suspension was examined experimentally and numerically [2].

The air suspension system, comprising an auxiliary tank, an air spring, and a pipe connecting the two, was modeled both linearly and non-linearly by Nieto *et al.* and other comparable studies [3] [4] [5]. For a reasonable operating range of the suspension, the obtained solutions from both the non-linear and linear models in this study correlate well with experimental measurements of the stiffness, damping factor, and transmissibility.

Kat *et al.* [6] studied the air springs model of the interconnected rolling diaphragm type numerically and experimentally. Complete vehicle multi-body dynamic simulations can make use of this model. The mass transfer and flow effects in pipes that connect three air springs are taken into account by the model. In that work, it was reported that even when bump stops are struck, the behavior of the air springs in the air suspension unit is accurately described by the air spring model based on the correlation that was found between the predicted and measured data.

Sayyaadi *et al.* [7] unveiled a new model of suspension system components with 70 DOFs for rail vehicles. Considering the effects of thermodynamics, the nonlinear air spring model was fully developed. The actual test data was used to fine-tune the model's parameters. This study demonstrates that the suggested model can be applied to improve the comfort index and ride quality.

In order to simulate and design an air spring that is coupled to an auxiliary volume for a suspension seat, Holtz *et al.* [8] presented a simplified model of an air spring with an auxiliary volume derived from first principles. This paper demonstrated the effectiveness of air springs and an auxiliary volume for seat suspension. In contrast to the results of tests conducted with large diameter flow restrictions, the simulation model of the air-spring and auxiliary volume showed about 27% lower transmissibility amplitude and 21% lower system natural frequency. However, the model did follow the trend predicted in the literature.

A variety of techniques were used to produce models that faithfully capture the dynamics of air springs. By considering the thermodynamic, heat transfer, and fluid mechanics components in the models, the accuracy of the models was increased [9].

The dynamics and modeling of air vibration isolators are covered in a number of books and research articles. One can see references [10]-[16] as examples.

In this work, a basic air spring is modeled using the conservation of mass/energy principles and Newton's second law of motion in order to assess its vibration amplitude and stiffness. Two instances are examined. In the first, the temperature of air inside the spring chambers remains constant during operation, *i.e.*, air undergoes isothermal processes as the spring compresses and expands. In the second, heat transfer to or from the air does not take place during spring operation, *i.e.*, air undergoes adiabatic processes as the spring compresses and expands. Furthermore, the model under consideration is derived with the primitive variables changing solely in terms of time and amplitude. The findings of this study should assist air spring designers in reducing the likelihood that their products will have an impact on the structural dynamics of the applications in which they are employed.

2. Mathematical Model

The basics of fluid dynamics and thermodynamics are employed to develop the dynamic equations of the air spring model considered in this work and schematically depicted in **Figure 1**.

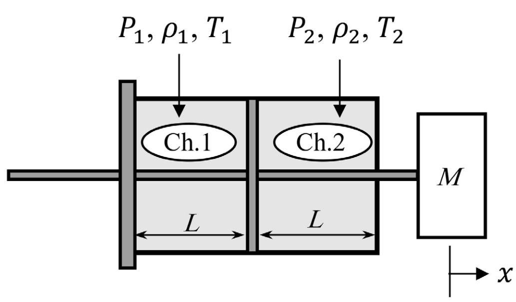

Figure 1. Basic schematic of an air spring.

The air spring system's fluid mechanics are explained by the conservation of mass and the differential conservation of energy. Constitutive equations are required when modeling the restoring forces of the air spring pneumatic system in order to account for the thermodynamic variables [17]. The equation of state is the first constitutive relation:

$$PV = mRT \tag{1}$$

where P is the pressure, V is the volume of the air inside the spring chamber, m is the mass of air, R is the gas constant which is 287 $m^2/s^2 \cdot K$ for air, and T is the air temperature. The second constitutive equation results from assuming the Polytropic process inside the pneumatic chamber of the air spring assembly depicted in **Figure 1**:

$$P\rho^{-n} = P\left(\frac{V}{m}\right)^{n} = \text{constant} \tag{2}$$

where ρ is the air density and n is the polytropic index which depends on the particular process under consideration. The gas in chamber 2 of **Figure 1** is compressed $x > 0$ and $P_2 > P_0$ to produce a restoring force in chamber 2 and this reduces P_1 such that ($P_1 < P_0$), where P_0 is the initial pressure in the chambers when $x = 0$, and P_1 and P_2 are the gas pressures at chambers 1 and 2, respectively. A net pressure force in the negative x direction will result from this. With reference to **Figure 1**, the air spring arrangement's force balance is obtained by applying Newton's second law in such a way that:

$$M\ddot{x} + \left(P_2 - P_1\right)A = 0 \tag{3}$$

where A is the piston's effective area and M is the payload mass. The pressures can be expressed using the conservation of mass in terms of the displacement x as follows [18]:

The constant mass of the gas in chambers 1 and 2 ensures that:

$$m_1 = \rho_1 A\left(L + x\right) = \rho_0 AL \tag{4}$$

and

$$m_2 = \rho_2 A\left(L - x\right) = \rho_0 AL \tag{5}$$

where, when $x = 0$, L is the length of each chamber. As a result, each chamber's gas density is determined by:

$$\rho_1 = \rho_0\left(\frac{1}{1 + x/L}\right) \tag{6}$$

and

$$\rho_2 = \rho_0\left(\frac{1}{1 - x/L}\right) \tag{7}$$

To relate pressure and density, the polytropic relation of Equation (2) is used as follows:

$$P_1\rho_1^{-n} = P_0\rho_0^{-n} \tag{8}$$

and

$$P_2\rho_2^{-n} = P_0\rho_0^{-n} \tag{9}$$

Substituting Equations (8) and (9) into Equations (6) and (7), the following relations for the chamber pressures in terms of the amplitude displacement, x are obtained:

$$P_1 = P_0\left(\frac{1}{1 + \left(x/L\right)}\right)^{n} \tag{10}$$

and

$$P_2 = P_0\left(\frac{1}{1 - \left(x/L\right)}\right)^{n} \tag{11}$$

Entering Equations (10) and (11) into Equation (3), one gets the following equation of motion:

$$M\ddot{x} + AP_0\left[\left(\frac{1}{1-(x/L)}\right)^n - \left(\frac{1}{1+(x/L)}\right)^n\right] = 0 \quad (12)$$

Note that when the piston is at the center of the cylindrical chamber where $x = 0$, the thermodynamic variables P_0, ρ_0, T_0 have known values at rest. Additionally, the following differential equation relating the pressure to the piston's velocity is obtained by differentiating Equation (2):

$$\dot{P}V^n + nP\dot{V}V^{n-1} = 0 \quad (13)$$

where V is the air volume inside the spring chamber. The differential equation that results from rearranging Equation (13) is as follows

$$\dot{P} = -nP\dot{V}/V \quad (14)$$

Writing differential equations for the pressures inside each chamber using this equation and noting that $V_1 = A(L+x)$ and $V_2 = A(L-x)$ gives the following results:

$$\dot{P}_1 = -nP_1\dot{x}/(L+x) \quad (15)$$

$$\dot{P}_2 = nP_2\dot{x}/(L-x) \quad (16)$$

Differentiating the net force on the piston gives the nonlinear air spring stiffness, K_x:

$$K_x = \frac{\mathrm{d}}{\mathrm{d}x}\left[(P_2 - P_1)A\right] = A\left(\frac{\mathrm{d}P_2}{\mathrm{d}x} - \frac{\mathrm{d}P_1}{\mathrm{d}x}\right) \quad (17)$$

Equation (2) is differentiated to determine how the pressure changes in relation to displacement, x. This results in the following:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(PV^n\right) = PnV^{n-1}\frac{\mathrm{d}V}{\mathrm{d}x} + V^n\frac{\mathrm{d}P}{\mathrm{d}x} = 0 \quad (18)$$

Solving for $\mathrm{d}P_1/\mathrm{d}x$, $\mathrm{d}P_2/\mathrm{d}x$ in Equation (18) and noting that $\mathrm{d}V_1/\mathrm{d}x = A$, and $\mathrm{d}V_2/\mathrm{d}x = -A$ produces:

$$\frac{\mathrm{d}P_1}{\mathrm{d}x} = -nP_1A/V_1 \quad (19)$$

$$\frac{\mathrm{d}P_2}{\mathrm{d}x} = nP_2A/V_2 \quad (20)$$

Consequently, the following equation provides the air spring stiffness:

$$K_x = nA\left[\frac{P_1}{L+x} + \frac{P_2}{L-x}\right] \quad (21)$$

The stiffness of the air spring can alternatively be obtained using the formulation of the restoring force of the air spring in Equation (12). Equation (12) gives the air spring restoring force as:

$$F_s = AP_0\left[\left(\frac{1}{1-x/L}\right)^n - \left(\frac{1}{1+x/L}\right)^n\right] \quad (22)$$

The stiffness of the air spring is then:

$$K_x = \frac{\mathrm{d}F_s}{\mathrm{d}x} = \frac{nAP_0}{L}\left[\left(\frac{1}{1-x/L}\right)^{n+1} + \left(\frac{1}{1+x/L}\right)^{n+1}\right] \tag{23}$$

The evaluation of the air spring's displacement, x, the pressures inside its air chambers, and its nonlinear spring stiffness can be evaluated by solving the differential Equations (12), (15), and (16) in conjunction with Equation (21) or (23).

The normalized nonlinear air spring stiffness is defined as:

$$\bar{K}_x = K_x / P_0 L \tag{24}$$

where Equation (21) or (23), can be used to obtain the air spring stiffness, K_x.

Equation (23) is expanded in power series about the equilibrium position $x = 0$ in order to obtain an expression for the linear spring stiffness. This gives:

$$K_{xs} = \frac{2nAP_0}{L} + \frac{n\left(2+3n+n^2\right)AP_0}{3L^3}x^2 + O\left(x^4\right) \tag{25}$$

The linear stiffness is

$$K_{xl} = \frac{2nAP_0}{L} \tag{26}$$

And the normalized linear stiffness is

$$\bar{K}_{xl} = K_{xl} / P_0 L = \frac{2nA}{L^2} \tag{27}$$

In this work, two scenarios are considered. In the first, it is assumed that the air inside the spring's chambers goes through an isothermal process, meaning that its temperature doesn't change. This is appropriate for low frequencies, where the air expands and compresses over an extended period of time. In the second scenario, it is assumed that the air inside the chambers experiences an isentropic process, meaning that no heat is transferred to or from the surrounding environment. This is well for high frequencies, where air expansion and compression occur quickly. The polytropic index for the first scenario is equal to unity, or $n = 1$. For the second, it is equal to the specific heat ratio, or, $n = \gamma = 1.4$ for air [17].

3. Results and Discussion

The pressure rates Equations (15) and (16), as well as the equation of motion (12), are expressed in state space form, where the state variables are: $x, \dot{x}, P_1, P_2$ and $x_1 = x$, $x_2 = \dot{x}$ as follows:

$$\dot{x}_1 = x_2 \tag{28}$$

$$\dot{x}_2 = \frac{A}{M}P_0\left[\left(\frac{1}{1+x_1/L}\right)^n - \left(\frac{1}{1-x_1/L}\right)^n\right] \tag{29}$$

$$\dot{P}_1 = -nP_1x_2/\left(L+x_1\right) \tag{30}$$

$$\dot{P}_2 = nP_2x_2/(L - x_1) \quad (31)$$

Initial conditions:

$$x_1(0) = x_0,\ \ x_2(0) = 0,\ \ P_1(0) = P_0\left(\frac{1}{1-(x_0/L)}\right),\ \ P_2(0) = P_0\left(\frac{1}{1+(x_0/L)}\right).$$

A nonlinear system of first-order differential equations is represented by Equations (28)-(31). Through the use of a model program written in the MATLAB/Simulink software environment, the numerical solution to this system is obtained. It is carried out for various values of the chamber length, *L*, and piston effective area, *A*, of the spring.

Figure 2(a) shows the air spring restoring force for both isothermal and isentropic scenarios obtained using Equation (22) and full simulation of the nonlinear system. It's evident that the force from the spring assuming an isentropic process is greater than that assuming an isothermal process. It is also evident that the nonlinearity in the air spring becomes more significant as the magnitude of the ratio of the displacement to the chamber length of the spring, *x*/*L*, increases. Figure 2(b) shows the normalized spring restoring force and stiffness for both scenarios obtained using Equations (22), (23) and full simulation of the nonlinear system. It is clear that isentropic model spring is stiffer than the isothermal model one. The agreement between Equations (22) and (23) and simulation is also clear.

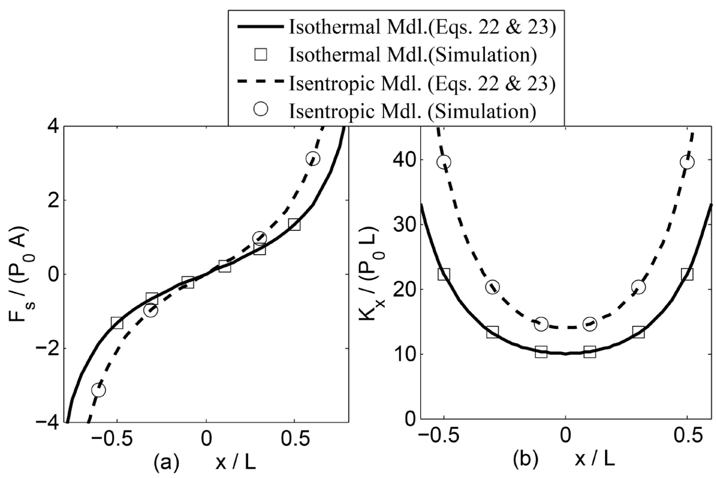

Figure 2. (a) Spring normalized restoring force, (b) Normalized spring stiffness for $L = 0.1$ m, $A = 0.05$ m^2.

Figure 3 shows a sample of the results for the air spring's pressures and displacement for some chosen design parameter values. Since the damping effects are not taken into account in the current analysis, it is clear that the oscillations are not decaying.

Figure 4 shows the normalized spring stiffness calculated for various piston areas while maintaining a constant air chamber length. This figure shows that, as would be expected, the air spring stiffens with increasing piston area. It is also evident that the air spring stiffness has a linear relationship with the spring's effective area. Additionally, it is evident that the spring exhibits greater stiffness when an isentropic process is assumed as opposed to an isothermal process.

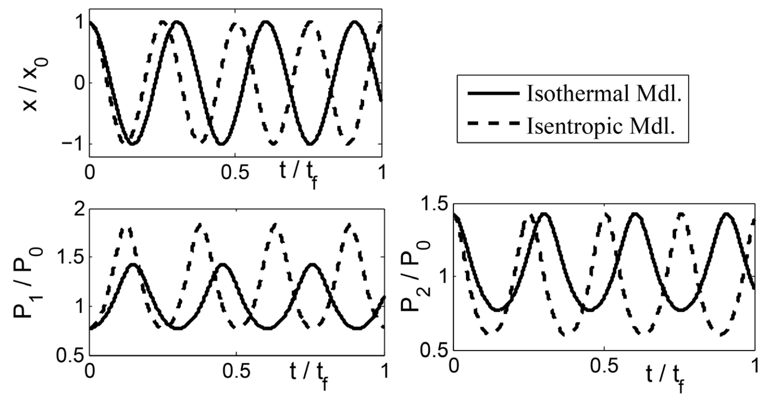

Figure 3. Normalized Amplitude of Oscillation and Pressures for $L = 0.1$ m, $A = 0.05$ m^2 and $x(0) = 0.03$ m.

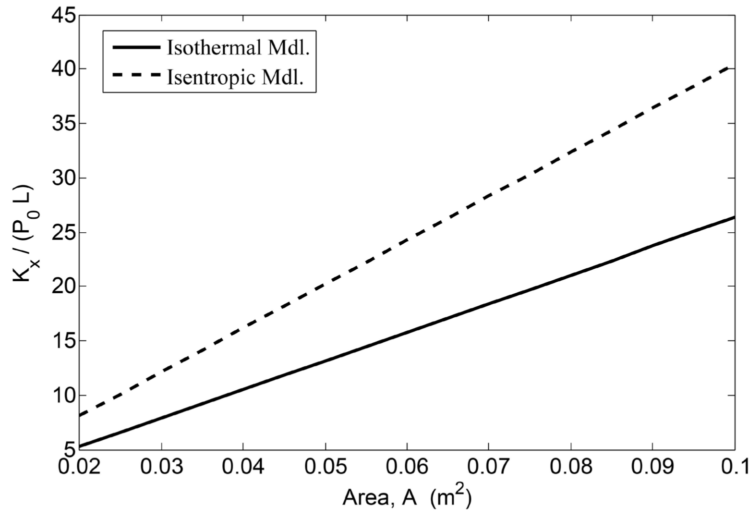

Figure 4. Normalized stiffness for different piston areas, $M = 2$ Mg, $L = 0.1$ m, and $x(0) = 0.03$ m.

The normalized stiffness is computed for a range of chamber lengths while holding the piston area constant. **Figure 5** illustrates this. This figure shows that the air spring becomes less stiff as the chamber length increases. It is also evident that there is a nonlinear relationship between the air spring stiffness and chamber length. Once more, it is evident that assuming an isentropic process results in a spring with greater stiffness than one assuming an isothermal process.

Figure 6 shows the normalized nonlinear and the linear stiffnesses for the isothermal and isentropic process cases for various vibration amplitude to chamber length ratios. It is evident that as long as the air spring's vibration amplitude is small in relation to the spring's chamber length, the linearized stiffness can be regarded as a reasonable approximation of the full nonlinear stiffness.

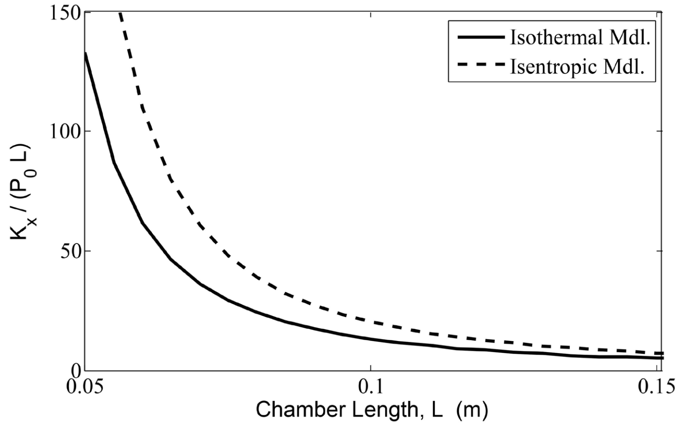

Figure 5. Normalized stiffness for different chamber lengths, $M = 2$ Mg, $A = 0.05$ m^2, and $x(0) = 0.03$ m.

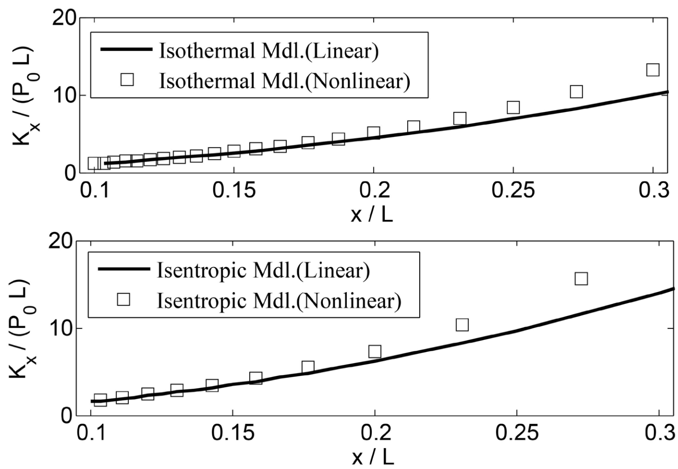

Figure 6. Normalized linear and nonlinear air spring stiffness for different x/L ratio, $x_0 = 0.03$, $A = 0.05$ m^2.

4. Summary and Conclusions

This work presents a theoretical nonlinear model for assessing the dynamics of an air spring. The model is built using the thermodynamic and fluid mechanics properties of the air inside the spring's champers, as well as the system dynamics derived from Newton's second law of motion. In this model, a one-dimensional approach is assumed. MATLAB/Simulink software is used to write a computer program that integrates the derived governing equations. The dynamic air spring stiffness is calculated for both isothermal and isentropic processes of the air inside the spring's champers. It is noted that the isentropic process case exhibits greater stiffness than the isothermal process case. Additionally, it is noted that an increase in the air spring's effective piston area causes the spring to stiffen. Furthermore, it is noted that when the air spring's chamber length is shortened, the air spring becomes stiffer. It is also noted that as the ratio of the vibration's amplitude to the air springs' chamber length is small, the nonlinearity in the air spring models taken into consideration in this work is negligible. If this is the case, then the air spring's linearized model can be regarded as a reliable approximation of the complete nonlinear model.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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