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The Role of Returns to Scale in the Relationship between Exchange Rate and Economic Growth

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Abstract

This paper aims to understand how the returns to scale can boost or deprive the effects of exchange rate depreciation on economic growth. A simple mathematical model was developed to describe the problem of the exporting firm in this context of internationalization. Theoretical results show that exchange rate depreciations can boost the growth of production of firms with technologies with decreasing returns to scale, while in firms that have increasing returns of scale, the effect is null or negative. In addition, two important exchange rates were found for the firm's decision, namely the minimum operating exchange rate (MOE) and the reversal exchange rate (RVER). Therefore, the present model helps understanding the role that the returns to scale plays in the relationship between exchange rate and economic growth. Finally, the current study presents a few limitations that can be addressed in future research. For instance, the following statements can integrate them: 1) allow that firms to influence the exchange rate and place it above the minimum operational level; 2) considering new players in the markets considered and 3) take into account more input and outputs variables.

Keywords

Exchange Rate, Firm Internationalization, Returns to Scale, Economic Growth

1. Introduction

Many papers highlight the impacts of the exchange rate on economic growth. See, for example, contributions of Ocampo (2005), Eichengreen (2007), Rodrik (2008), Araujo (2013), Vieira and Damasceno (2016) and Missio et al. (2017).

However, none of them highlights the role of returns to scale on the transmission of these effects. As we will argue later, taking into account this variable is crucial to better understand economic growth.

The main purpose of this study is to develop a simple mathematical model that describes the problem of the commodity exporter and input importer, which produces a single final good that is sold in the international market. In a few words, the main point is making a comparative statics exercise in which the prices of two commodities (one final good and one input) are multiplied by the same factor, the exchange rate.

We depart from a standard profit maximization model and we assumed that the firm is a price-taker in the following markets: foreign exchange, final goods, imported and domestic inputs. Afterwards, output is aggregated so we can infer the impact of an exchange rate depreciation is able on aggregate output.

This paper is partitioned in five sections besides this introduction. The next section presents a brief literature review in respect to the relationship between exchange rate and economic growth. Section 3 shows a simple extension of a standard profit maximization model of an exporter firm that purchases inputs both domestically and abroad. In Section 4, the aggregate analysis is displayed. Numeric simulations are carried out in Section 5 using the theoretical results of the model presented in Section 4. Finally, some concluding remarks are given.

2. Literature Review

Eichengreen (2007) models exchange rate as a volatile asset that can affect trade and investment, affecting growth. He formulates hypotheses about exchange rate competitiveness, volatility and the possibility of to be used as a political instrument. He also shows that an overappreciated effective real exchange rate encourages the allocation of resources to the tradable manufactured goods sector, changing production and boosting a higher level of income. In the case of elastic external demand, this allocation of resources may not decrease prices, unlike the situation with non-tradable goods, where demand is purely internal and relatively inelastic. To boost economic growth, policy makers should maintain a stable real exchange rate and at competitive levels (devalued exchange rate), without excessive volatility.

Subsequently, Eichengreen (2007) shows that the exchange rate is influenced by policy, as its volatility increases when changes occur in the conduct of monetary and fiscal policies. Thus, an appropriate policy of the real exchange rate would then be one that seeks to favor as many people as possible, being an important facilitating condition for economic growth. Development experience, in general, shows that keeping the real exchange rate at competitive levels can be critical to initial growth and it also shows that high levels of exchange rate volatility can harm exports and investments (Eichengreen, 2007). For him, the exchange rate policy can be useful to boost growth, encouraging the redistribution of resources in manufacturing and reaping immediate productivity gains.

Rodrik's (2008) goal is to show that currency devaluation (a high real exchange rate) stimulates economic growth. However, the author admits that so far there is no consideration that associates exchange rate overvaluation with low economic growth. According to the author, the existing exchange rate theories show that low economic growth occurs mainly due to macroeconomic instability in the country, such as: the scarcity of foreign currency, corruption, current account deficits, among others. Rodrik (2008) says that this is especially true in developing countries.

Undervaluation is associated with rapid growth and the real exchange rate is important in the economic shift from developing to developed countries. Such a finding is robust when using different measures of the real exchange rate and different estimation techniques, focusing on institutional/contractual weaknesses and market failures. Exchange rate depreciation has been shown to cause economic growth, not the other way around. The author analyzes the real exchange rate as a political variable in the same way as Eichengreen (2007), where the equilibrium level of the exchange rate can be affected by the instruments that governments have at their disposal. It was concluded that countries with undervalued real exchange rates have greater growth, and that where the exchange rate is overvalued, the stimulus for economic growth is lower. The undervaluation of the real exchange rate is more prevalent in developing countries. This, according to the author, is due to institutional weakness (imperfection of contracts and their poor execution, waiting problems, corruption, lack of property rights) and more severe market failures in this group of countries (widespread valuable information, industry coordination externalities, difficulty in financing projects due to asymmetric information and wage premiums).

Vieira and Damasceno (2016) investigated the importance of exchange rate misalignment and exchange rate volatility for the growth of the Brazilian economy in the period from 1995 to 2011. Exchange rate misalignment is equivalent to deviations from the exchange rate in relation to its equilibrium level, which is taken by an internal (full employment) and an external balance (sustainable current account balance). To investigate the role of exchange misalignment, a series was constructed with the effective real exchange rate and the estimate of the equilibrium real effective exchange rate for the period 1994T3-2011T4 (from the third quarter of 1994 to the fourth quarter of 2011), considering the top twenty trading partners. Exchange rate misalignment becomes relevant for the explanation of economic growth, since systematic evidence was presented that exchange rate undervaluation stimulates economic growth and overvaluation discourages. Thus, greater exchange rate volatility discourages economic growth and less volatility stimulates. From the results obtained it is recommended, especially for developing economies, to maintain the real exchange rate at a competitive level (undervalued exchange rate) and with low volatility.

Sekkat (2016) seeks to clarify the impact of exchange rate misalignment on export diversification. Rodrik (2008) suggested that the undervaluation of the

currency promotes diversification of exports from an institutionally weak country, as the currency would thus increase the cost of importing some production inputs. However, Sekkat proposes a different conclusion: that the overvalued national currency reduces the company's production costs and may also favor the diversification of exports.

In search of economic growth, many developing countries try to diversify their exports and this can be done with the real misalignment of the exchange rate (Sekkat, 2016). The article investigated whether the impact of exchange rate misalignment on export diversification depends on the level of financial development or the quality of the institutions of the exporting economy. Although support was found for the effect of underestimating the share of manufactured goods in total exports, support for an exchange misalignment impact on export diversification within manufactures was not found.

Bresser-Pereira (2012) established that for economic growth, the exchange rate must be relatively devalued, as, while increasing domestic savings, it drives export-oriented investments. Locatelli and Da Silva (1991) sought to calculate the bilateral real exchange rate based on analyses of the real exchange rate and incentives and disincentives to imports, measuring the evolution of export costs and the estimation of exchange rate lags (appreciation of the national currency in relation to strong currencies that can discourage exports). It was concluded that the parity index varies enormously, according to the variables included in the analysis, and there is no natural tendency for the exchange rate to self-correct to a stable value.

The argument of floating exchange rates by Friedman (1953) and Johnson (1969) is that floating exchange rates would provide a more efficient international adjustment system, autonomy in relation to the use of monetary policy instruments and national independence, and therefore would support a freer trade world. Nurkse's (1944) counter-argument consists in showing that floating exchange rates are equivalent to volatile and unstable rates, a source of disturbance and instability, rather than an efficient adjustment mechanism. The exchange rate is determined by analyzing the relative prices of domestic, imported and exported goods (Dornbusch, 1990), with changes in exchange rates affecting only the last two. Dornbusch (1990) calls attention to the prices of exported goods, since with a depreciation of the exchange rate, it tends to increase at a relative price of foreign goods. Diaz-Alejandro (1966) was the first to draw attention to the fact that short-term devaluation can reduce economic activity, in addition to having inflationary effects. In the short run, a currency devaluation reduces real wages in terms of tradable products, thereby reducing purchasing power and the demand for consumer goods. In the long run, production and employment are expanding. Dornbusch (1990) also states that the exchange rate undervaluation generates investment in the domestic country. When an industry is affected by foreign competition and strong overvaluation, it can, for example, close and reopen in a country whose wages are lower.

Razin and Collins (1997) constructed an indicator of misalignment of the real exchange rate for a large sample of countries, making a regression analysis to say whether the misalignment is related to the country's growth. The misalignment of the real exchange rate (RVER) occurs when the RVER is different from the equilibrium RVER, which is found when the economy is in internal (full employment) and external (sustainable current account position) equilibrium. They also used pooled data and regression estimates with small country samples to determine the misalignment in all types of economies, making sure that there are no important linearities between exchange misalignment and economic growth, only very strong depreciations that are associated with higher economic growth. The misalignment of the RVER can influence the economy's investment, affecting the accumulation of capital. In addition, it can affect the production of goods and the competitiveness of the sector in relation to international competitors. Finally, the authors argue that a volatile environment also negatively affects economic growth.

Krueger (1983) argued that exchange rate misalignment is detrimental to economic growth and, through the "Washington Consensus" approach, the exchange rate will come from an internal (full employment and low inflation) and an external (sustainable current account balance) macroeconomic balance.

Cushman (1983) analyzes the effects of exchange rate risk on the volume and prices of trade, extending a structure that uses econometric tests with bilateral trade flows between the United States and five other countries. The author maintains that the long-term expectation of a depreciation in the real exchange rate increases the volume of trade, while an increase in uncertainty regarding this variable reduces this volume.

Weder (1999) presented an open economy version of the optimum growth model of two sectors in Benhabib and Farmer (1996) with specific production externalities. In the open economy version, indeterminacy is obtained not only at lower returns to scale than in the case of the closed economy, but also at insignificant levels. It was concluded that rising returns in relative prices of goods prevent the reallocation of resources over time to obtain capital gains and smooth consumption.

Jones (1999) analyzes economic growth based on "new ideas". At the technological level, one person's use of ideas does not prevent another person from using it at the same time. The idea behind each product only needs to be created once, so that afterwards, taking into account the fixed cost of creating an idea, the total production of any first idea is characterized by increasing returns to scale. The reasoning is that the growth rate of the economy is proportional to the total amount of research carried out in the economy. An increase in the population increases the number of researchers, which stimulates economic growth by generating ideas.

The purpose of the study by Chao and Yu (1991) was to analyze the effects on well-being when the industries of a growing economy, under import quotas,

have a variable production scale. The economic growth of the small open economy improves well-being when industries are endowed with constant returns to scale, especially if an industry experiencing technical progress has higher returns to scale than a static industry. On the other hand, if the static industry presents returns to scale greater than the sector that experiences technical progress, the growth of the economy can be immiserating (when growth results in a worsening of well-being).

In this brief literature review, studies were considered that presented methodologies and tests of how the exchange rate can influence the performance of an economy. Although the reviewed articles are of great contribution, none discussed the situation for a firm specifically importing inputs and exporting products to the international market and the role of returns to scale, which will be done in the next topic. This new modeling becomes essential to shed light on the understanding of the role that returns to scale plays in the relation between exchange rate and economic growth.

3. The Internationalization Problem of the Firm

Two problems of the firm will be presented, namely, the minimization of costs with imported inputs and the maximization of profit in the international market. Based on this, its effects on a firm that operates in competitive markets, in final goods and inputs and that sells its products in the international market will be analyzed. After the analysis, a new definition of currency misalignment will be presented and, later, some numerical simulations will be presented. Finally, last considerations are presented.

For the model that will be presented below, suppose the existence of a representative firm that uses two factors of production, x_1 and x_2, being x_1 the amount of factor 1 used, which is purchased domestically at a price w_1, and x_2 the amount of factor 2 used, which is imported at a price Sw_2, being S the nominal exchange rate[1]. Thus, the firm's total cost is divided between the two inputs, expressed as follows:

$$C\left(x_1, x_2\right) = w_1 x_1 + S w_2 x_2 \tag{1.1}$$

Equation (1.1) shows that, as the firm is an importer of inputs, changes in the exchange rate will cause changes in its division of resources among factors of production. This consideration differs from that made by Dong, Kouvelis and Su (2014), because even if they consider a global company, they did not use the exchange rate in the firm's production cost.

Assume that the firm has a production function of the Cobb-Douglas type, so it wanted to minimize Equation (1.1) subject to a certain level of production, as described below:

$$w_1 x_1 + S w_2 x_2 \quad \text{subject to} \quad y = x_1^{\alpha} x_2^{\beta} \tag{1.2}$$

Let y be the amount of goods the firm wants to produce, $\alpha > 0$ the sensitivi-

[1]The nominal exchange rate, S, is defined by the ratio of domestic currency to foreign currency.

ty of production to changes in input 1, $\beta > 0$ the sensitivity of production to changes in input 2 and $\alpha + \beta \neq 1$. Naturally, the sum $\alpha + \beta$ gives the returns to scale.

The solution to this first optimization problem is presented below:

$$x_1 = \left(\frac{\beta w_1}{\alpha S w_2} \right)^{-\frac{\beta}{\alpha+\theta}} y^{\frac{1}{\alpha+\beta}} \tag{1.3}$$

$$x_2 = \left(\frac{\beta w_1}{\alpha S w_2} \right)^{\frac{\alpha}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}} \tag{1.4}$$

Equations (1.3) and (1.4) represent the demand for input 1 and input 2, respectively. In addition, they show that the exchange rate must be taken into account in the quantities that cause the cost of producing y units of final goods to be minimized. As w_2 increases in relation to w_1, the company will reduce the use of input 2 and increase the use of input 1. Note that in the event of a currency depreciation, $\frac{\mathrm{d}S}{\mathrm{d}t} > 0$, the price of the imported input would increase, the quantity used of it would decrease and that of the domestic input would increase, since the firm, in this situation, avoids wasting resources by reallocating the purchase of the imported input towards the domestic one. In an exchange rate appreciation, exactly the opposite occurs and these two antagonistic situations can be seen due to the negative exponent of the first term of x_1.

Replacing Equations (1.3) and (1.4) into Equation (1.1), we arrive at the minimum total cost function of an input importing firm— $C(w_1, w_2, S, y)$ —which measures the minimum cost of producing y units of product, given the prices of the factors of production, the exchange rate:

$$C(S, y) = S^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}} \tag{1.5}$$

If the firm's domestic input is also sourced internationally, then the cost function is just $C(S, y) = S y^{\frac{1}{\alpha+\beta}}$, which means this cost function is just the traditional cost function multiplied by the exchange rate.

The effect of exchange rate depreciation on the total cost is always positive but the increase is always smaller, that is, the cost function is concave in the exchange rate (its inclination becomes flatter as the exchange rate increases), as can be seen by Equations (1.6) and (1.7) below:

$$\frac{\mathrm{d}C(S, y)}{\mathrm{d}S} = \left(\frac{\beta}{\alpha+\beta} \right) y^{\frac{1}{\alpha+\beta}} \frac{1}{S^{\frac{\alpha}{\alpha+\beta}}} > 0 \tag{1.6}$$

$$\frac{\mathrm{d}^2 C(S, y)}{\mathrm{d}S^2} = -\left(\frac{\alpha}{\alpha+\beta} \right) \left(\frac{\beta}{\alpha+\beta} \right) \frac{C(S, y)}{S^2} < 0 \tag{1.7}$$

Equation (1.7) shows that the increase in cost decreases as the exchange rate depreciates. Thus an exchange rate depreciation will always increase the cost to

produce y units, but at a decreasing rate, given the level of production. Consequently, an exchange rate appreciation tends to lower the cost of production, but each time to a lesser extent. This result is similar to Sekkat's (2016), in which the overvalued currency reduces the firm production costs.

Being aware of the minimum total cost function for the proposed model, the average total cost (ATC) function is obtained by dividing Equation (1.5) by production, y, which can be expressed as follows:

$$CTM = S^{\frac{\beta}{\alpha+\beta}} y^{\frac{1-\alpha-\beta}{\alpha+\beta}} \tag{1.8}$$

Therefore, for the proposed model, the ATC will behave differently depending on the returns to scale, although the exchange rate effect on the ATC is identical for any of the situations. This means that, for a fixed level of output, a depreciation of the nominal exchange rate leads to different effects on the average total cost. While in one case the cost grows at an increasing rate, in the other the cost grows at a decreasing rate.

Figure 1 illustrates the case of firms with decreasing returns to scale for Equation (1.8). Figure 1 shows that, for these firms, the ATC is strictly increasing for the exchange rate and for the product. Therefore, Figure 1 displays the increasing of ATC when the exchange rate is increasing to any given amount of output. Also, this increasing of ATC is always lower than the increasing of exchange rate. As we will argue later, this is an essential condition to depreciating exchange rate boost economic growth.

Figure 2, on the other hand, simulates the ATC for companies with increasing returns to scale, that is, $\alpha+\beta>1$. Unlike the first case, the increase in production reduces the average total cost for any level of exchange rate.

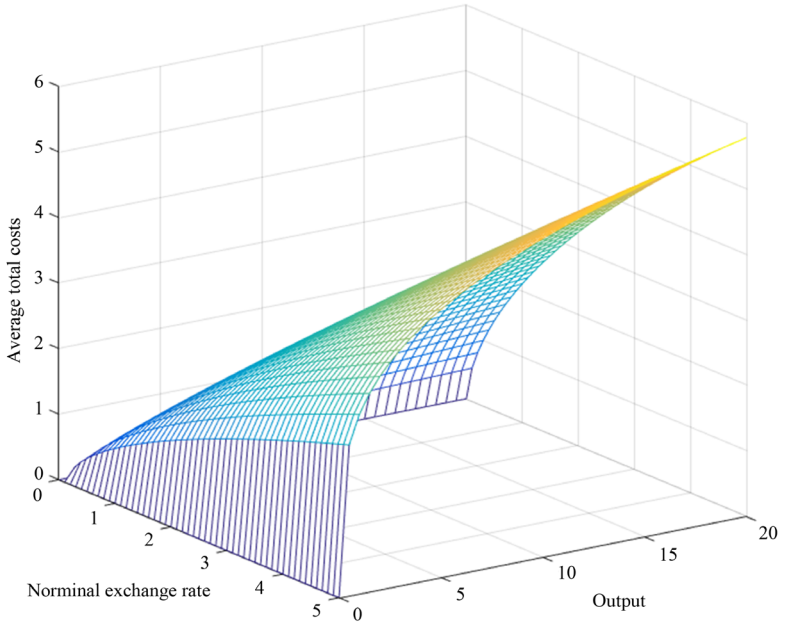

Figure 1. Behavior of the average total cost for firm with decreasing returns to scale.

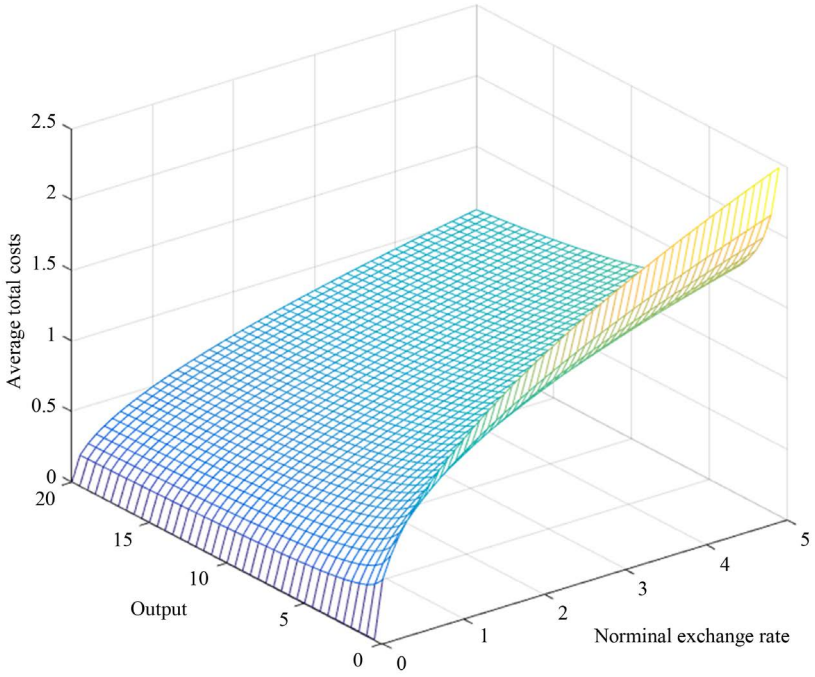

Figure 2. Average total cost of firm with increasing returns to scale.

Therefore, the effects of returns to scale on ATC are opposite for firms in different stages of maturity. Thus, at small scale of production, there is increasing returns to scale, so that, as the product increases, the average cost tends to decrease. Then, with production at a higher level, there is the presence of decreasing returns to scale.

Since profit corresponds to the difference between total revenue and total cost, defined in Equation (1.5), the profit function of the firm is obtained:

$$\pi(y) = Spy - S^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}} \tag{1.9}$$

Due to the model's assumptions, the exchange rate is present in the total revenue function and in the company's total cost function. If the exchange rate was only in revenue, that is, if the firm did not import inputs, exchange rate depreciations would always be desired by the company. If the exchange rate were present only in cost, that is, if the firm sold only on the domestic market and imported input, the exchange rate depreciation would cause its profit to worsen. Thus, the maximization of the firm's profit is stated according to the equation below:

$$\pi(y) = \max_{y \in R_+} \left(Spy - S^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}} \right) \tag{2.0}$$

If the firm's domestic input is also sourced internationally, then the firm's maximization problem is reduced to a standard one, with no imported inputs multiplied by the exchange rate,

$\pi(y) = \max_{y \in R_+} \left(Spy - Sy^{\frac{1}{\alpha+\beta}} \right) = S \left[\max_{y \in R_+} \left(py - y^{\frac{1}{\alpha+\beta}} \right) \right]$, which imply the trivial solution and that firm should be completely insensitive to the nominal exchange rate.

If we define $\eta = \left[p(\alpha+\beta) \right]^{\frac{\alpha+\beta}{1-\alpha-\beta}} > 0$, we have the following expression for the optimal level of production, obtained from the profit maximization equation:

$$\tilde{y} = \eta S^{\frac{\alpha}{1-\alpha-\beta}} \tag{2.1}$$

Equation (2.1) shows the level of production that maximizes company profit (company's supply curve as a function of the nominal exchange rate). From Equation (2.1), this level of production depends not only on the price of the final good, p, but also on the level of the exchange rate, *S*. It is also important to see the effects of exchange rate variations on the level of production. Therefore, it is necessary to derive the expression (2.1) in relation to the exchange rate to find the following:

$$\frac{\mathrm{d}\tilde{y}}{\mathrm{d}S} = \eta \left(\frac{\alpha}{1-\alpha-\beta} \right) S^{\frac{\beta-1}{1-\alpha-\beta}} \tag{2.2}$$

Equation (2.2) implicitly shows the conditions for an exchange rate depreciation to cause an increase in production. The criterion for knowing the effect of depreciation on the product is in the term $\left(\frac{\alpha}{1-\alpha-\beta} \right)$, because the other two are necessarily positive. As can be seen, the ratio can be positive or negative, depending on the returns to scale of the analyzed company. In view of the above, it is possible to summarize the condition for an exchange rate depreciation to cause an increase in production as follows:

$$\frac{\mathrm{d}\tilde{y}}{\mathrm{d}S} > 0 \Leftrightarrow \alpha + \beta < 1 \tag{2.3}$$

Condition (2.3) reveals that an exchange rate depreciation will cause an increase in production if, and only if, the firm is endowed with technologies with decreasing returns to scale. It is known that a firm's scale yields change as the production scale changes (Varian, 2006). Thus, depending on the scale that the firm is in, an exchange rate depreciation may create stimuli to increase production or destroy these stimuli and, therefore, may encourage economic growth, or not.

In order to maintain the consistency of the model's propositions with the literature, the following hypothesis is proposed: for economies still developing, an exchange rate depreciation may, in fact, create incentives for the country's growth. This phenomenon may occur due to a lack of investments in research and innovation (Klenow and Li, 2020), a fact that would reflect the operation in the region of decreasing returns of scale for most companies in developing economies.

Thus, as the countries of this group normally invest less than enough in research and development, a large part of the existing companies are at first stagnant and, sooner or later, reach maturity, when they start operations with decreasing returns to scale, where exchange rate depreciation starts to favor its production and, consequently, economic growth. Eichengreen (2007) argued that an undervalued real exchange rate encourages a change in a firm's production by allocating resources towards the tradable manufactured goods sector, which stimulates the country's income level. According to our model, this proposition is true for companies with diminishing returns to scale, exporters of commodities and importers of inputs.

Dornbusch (1990), stated that a period of currency devaluation would be necessary to generate reinvestment in the country. According to the present model, a policy inducing such outcome would be appropriate if the return of the dominant scale in the economy were the decreasing one, which can be the case of economies that are not yet developed. Vieira and Damasceno (2016) suggested achieving an undervalued real exchange rate to stimulate economic growth. According to the proposed model, this exchange policy suggestion is reasonable only in economies whose firms have diminishing returns to scale. Otherwise, such a policy may trigger the opposite and unwanted effect. Rodrik (2008) analyzed that, in an institutionally weak country, the undervalued national currency promotes export diversification, since the cost of importing some production inputs would increase. Such an argument is compatible with the model. As it is an aggregate model, it was not possible to find out what happens across different industries, only with the final produced quantity.

When production is small, fixed costs dominate variable costs, so that increasing the use of inputs will generate a greater increase in production than in total cost (because the average cost is reduced); subsequently, as production is large, variable costs dominate fixed costs, so that increasing the use of inputs will generate a greater increase in total cost than in production (because the average total cost increases). These conclusions are compatible with Equation (1.8) of the model presented. To see this, it is enough to set a level for the exchange rate and consider that when production is small, the firm obtains increasing returns to scale (as showed by **Figure 2**) and, after a certain level of production, the firm starts to operate in a range of decreasing returns to scale (as showed by **Figure 1**).

Also, according to Equation (2.2), if the firm uses only the imported input ($\alpha = 0$), an exchange rate depreciation will not cause an increase in production. If the technology is of increasing returns to scale, $\alpha + \beta > 1$, a currency depreciation will cause a reduction in the level of production, being $\frac{\mathrm{d}\tilde{y}}{\mathrm{d}S} < 0$. The model does not apply in the case of constant returns to scale, given that there would be indeterminacy in Equation (2.2).

There is a vast literature that mentions the existence of an equilibrium ex-

change [see more in Vieira and Damasceno (2016), Eichengreen (2007), Razin and Collins (1997), Sekkat (2016), Clark and MacDonald (1999) and etc.] and, in the model presented, we intend to relate it, subsequently, with two other exchange rates that are important for the decision-making of the firm, namely, "reversal exchange rate" (RVER), which consists of the level of exchange rate from which the loss begins to decrease as the exchange rate depreciates, and the "minimum operating exchange rate" (MOE), which consists of the minimum level that the exchange rate must be so that the firm does not incur losses, given the level of production chosen, $\tilde{y}$. In this sense, the RVER represents the exchange rate that minimizes the firm's profit, given the optimal level of production. Consider that $\tilde{y}$ be the level of production that solves the problem (2.0). Therefore, the firm's profit will be:

$$\pi(S) = Sp\tilde{y} - S^{\frac{\beta}{\alpha+\beta}}\tilde{y}^{\frac{1}{\alpha+\beta}} \tag{2.4}$$

To find the RVER it is necessary to derive the expression (2.4) in relation to the exchange rate and equal to zero. Thus, it is obtained:

$$\underline{S} = \left[\frac{\beta}{p(\alpha+\beta)}\right]^{\frac{\alpha+\beta}{\alpha}} \tilde{y}^{\frac{1-\alpha-\beta}{\alpha}} \tag{2.5}$$

Now, it is necessary to check the sign of function $\frac{\pi^2(\underline{S})}{\mathrm{d}S^2}$. If it is positive, the RVER will be a minimum point. Thus:

$$\frac{\pi^2(\overline{S})}{\mathrm{d}S^2} = \tilde{y}^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha}{\alpha+\beta}\right)\left(\frac{\beta}{\alpha+\beta}\right)\frac{1}{\underline{S}^{\frac{2\alpha+\beta}{\alpha+\beta}}} > 0 \tag{2.6}$$

Equation (2.5) shows the exchange rate level at which any exchange rate depreciation raises profit (reduces loss). Naturally, function (2.4) is strictly increasing on the exchange rate, for all $S > \underline{S}$. This result is important because it shows, in a generalized way, how the exchange rate affects the profit of exporting companies and also shows that both firms with increasing and decreasing returns to scale face the same behavior of profit due to exchange variations.

As already stated, an important level of exchange rate for the firm to know is the "minimum operating exchange rate" (MOE), which consists of the lowest exchange rate so that the firm has no loss, given the level of production chosen. The MOE is defined as follows:

$$\hat{S} = \frac{1}{p}\tilde{y}^{\frac{1-\alpha-\beta}{\alpha+\beta}} \tag{2.7}$$

The expression (2.7) above is found by equating Equation (2.4) to zero and shows the minimum level that the exchange rate must have so that the firm does not incur losses, given the level of production chosen, $\tilde{y}$. On one hand, expression (2.7) shows that if the firm has decreasing returns to scale ($\alpha+\beta<1$), the greater the product that will maximize the firm's profit, $\tilde{y}$, and the higher the

exchange rate level required for the company to start making profit will be. On the other hand, if the firm has increasing returns to scale ($\alpha+\beta>1$), the greater the product that maximizes the firm's profit will be and the lower the exchange rate will be for the company to start making profit. The model does not apply in the case of constant returns to scale.

Figure 3 illustrates profit as a function of the exchange rate, given the level of production that solves the problem (2.0), for firms with decreasing and increasing scale yields. Note that, regardless of the firm's scale returns, the behavior of the profit function is the same: for highly appreciated exchange rates it is negative and, from RVER, the company begins to reduce the loss.

The Aggregate Analysis

Before moving on to the aggregate analysis, it is essential to add the production of each firm individually to arrive at the product of the country under consideration. To do so, suppose that there are infinite firms within the continuous [0, 1], so that the GDP will be given by $\int_0^1 \tilde{y}(i)\mathrm{d}i$. Thus, the GDP of the country considered will be:

$$Y=\int_0^1 \tilde{y}(i)\mathrm{d}i=\int_0^1 \eta S^{\frac{\alpha}{1-\alpha-\beta}}\mathrm{d}i=\eta S^{\frac{\alpha}{1-\alpha-\beta}} \tag{2.8}$$

To clarify, we can differentiate Equation (2.8) with respect to time to find the relation between economic growth and exchange rate:

$$\hat{Y}=\frac{\alpha}{1-\alpha-\beta}\hat{S} \tag{2.9}$$

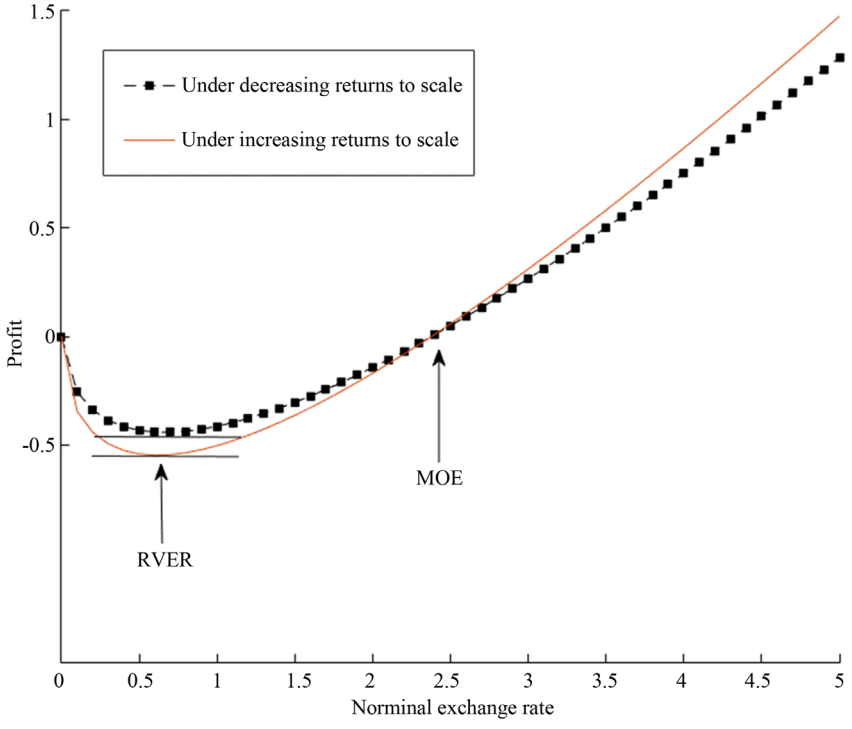

Figure 3. Profit in domestic currency based on the nominal exchange rate.

Equation (2.9) exhibited carefully how the returns to scale can boost economic growth when exchange rate increase. Also, Equation (2.9) stated that only countries whose firms have decreasing returns to scale can trigger the economy through exchange rate depreciation.

We assume that if the relative participation in the aggregate output of firms that have decreasing returns to scale is higher than the participation of other firms, then the impacts of exchange rate depreciation on the aggregate output will tend to be positive; while in the opposite case, the effect of the exchange rate on the aggregate output will be null or negative.

Figure 4 shows the effect of exchange rate depreciation on aggregate output of firms with increasing and decreasing returns to scale. At this point, it is opportune to graphically illustrate the inferences that are obtained from Equations (2.0) and (2.1). As can be best seen in Figure 4, exchange rate depreciation causes an increase in production when most companies in a country have decreasing returns to scale, while it causes a reduction in Product when most companies in a country have increasing returns to scale.

In order to better understand the firm's decisions, it is necessary to compare the equilibrium exchange rate in the foreign exchange market MOE to the exchange rates described by the Equations (2.5) and (2.7). Suppose that the equilibrium exchange rate of the foreign exchange market (or simply market exchange rate) denoted by S^*, be such that $S^* < \underline{S} < \hat{S}$. In this scenario, the firm, which has already solved its problem (1.9), finds itself in a situation of complete disinterest in executing its production plan, since the market exchange rate is more

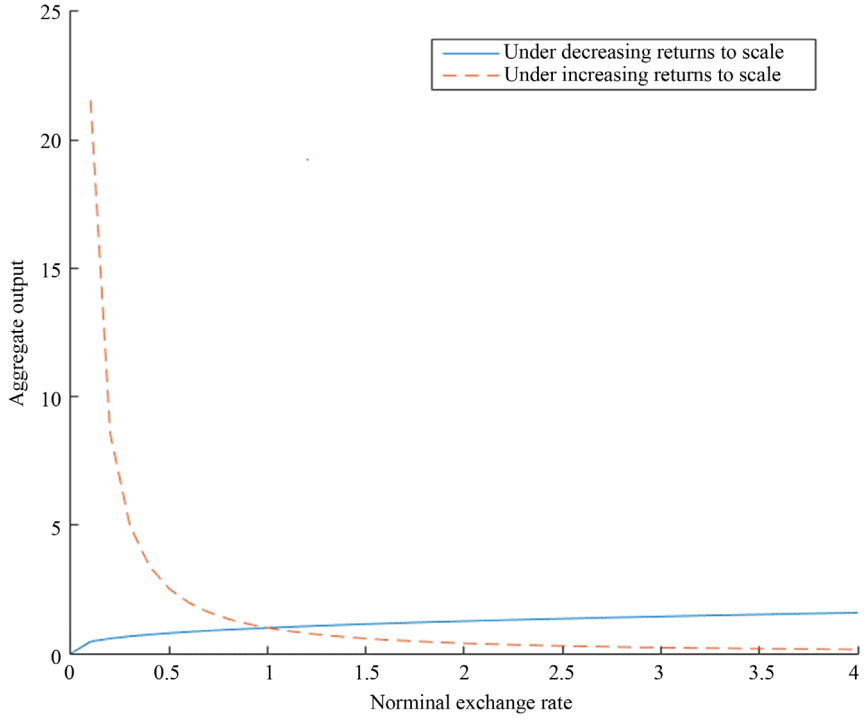

Figure 4. Aggregate output as a function of the nominal exchange rate.

appreciated than that which is capable of reversing the multiplication of the loss. In this situation, if the firm has no fixed cost, then it will have no incentive to produce in the short term.

If the equilibrium exchange rate depreciates and the foreign exchange market reaches a level that $\underline{S} < S^* < \hat{S}$ be satisfied, firms will be able to start thinking about executing their production plan, once the market exchange rate has approached the minimum operational level. Finally, if the market exchange rate reaches a level that is higher than the MOE, $\underline{S} < \hat{S} < S^*$, then companies must execute their production plan, $\tilde{y}$.

Furthermore, within the model, there is no mechanism that induces the equilibrium exchange rate of the foreign exchange market to become equal to the exchange rate presented by Equation (2.7). Likewise, Locatelli and Da Silva (1991) stated that there is no natural tendency for the exchange rate to self-correct for a stable value. This is because the exchange rate was considered exogenous, so that changes in demand for the factors or the quantity produced by the company do not change the market exchange rate. Future work can relax this hypothesis to see if existing firms, capable of influencing the exchange rate, will be able to bring it close to the optimum level that satisfies their own profit goals.

Thus, it is possible to measure the degree of misalignment of the current exchange rate in relation to the MOE exchange rate described by Equation (2.7) which indicates a measure of deviation from the exchange rate in relation to its minimum level for firms to start operating without economic losses. Therefore, currency misalignment is treated as follows:

$$DS = \left(\frac{S - \hat{S}}{\hat{S}} \right) \times 100 \tag{2.9}$$

According to Equation (2.9), the foreign exchange misalignment will be positive when the current exchange rate (which is the equilibrium of the foreign exchange market) is higher than the minimum operational exchange rate (super-depreciated exchange rate) and, in this case, there will be an increase in profit. The increase in profit stems from the fact that, starting from the optimum level, any exchange rate depreciation will cause an increase in total revenue greater than the increase in costs. This is due to the fact that the total revenue is linearly increasing in the exchange rate and the total cost is concave in the exchange rate. In this way, any foreign exchange depreciation from this level increases the total revenue more than the total cost. Conversely, when foreign exchange misalignment is negative (current exchange rate lower than the minimum operating exchange rate, which is the case of overvaluing the exchange rate), there will be a loss, because at low exchange rate levels, the cost is higher than total revenue, given the level of production.

This measure is useful for companies to get a sense of how much they could increase profit, without necessarily changing the quantity produced, while still employing their current production plan, and only with changes in the exchange

rate. Razin and Collins (1997) found evidence that misalignment of the real exchange rate influences investment for the economy and can also affect production and competitiveness in relation to international competitors. Thus, firms have incentives to influence the level of the exchange rate, a subject that can be explored in future work.

According to Vieira and Damasceno (2016), exchange rate misalignment is relevant because it stimulates economic growth based on exchange rate undervaluation (as well as higher exchange rate volatility), and overvaluation discourages it. Based on this result, many economists recommended policies intended to keep the real interest rate at an artificially low level, especially developing countries. However, it was shown in Equation (2.2) that this occurs in a specific context, that is, in economies with decreasing returns to scale firms.

Krueger (1983) argues that exchange misalignment harms economic growth, unlike Razin and Collins (1997), who found, based on an indicator of real exchange rate misalignment, that there are no important linearities between exchange misalignment and economic growth, except for the event of strong undervaluations that economic growth would become higher. According to the model presented here, production and the stimuli that come from them may indeed be encouraged by exchange rate variations. However, they depend on the types of returns of scale that the technologies of the companies will present.

In this topic, the effects of exchange rate variations on production were verified by taking into account the role of returns to scale. In addition, some definitions were presented for specific values of the nominal exchange rate that are of interest to the firm, namely the Minimum Operating Exchange Rate (MOE) and the Reversal Exchange Rate (RVER). Future work may be carried out by generalizing the mathematical model to introduce imperfect markets, a greater number of inputs and outputs and other economic agents relevant to the considered context.

4. Numeric Simulations

Here, computer simulation graphs of the main equations of the model described in the previous section will be presented. For the current nominal exchange rate, the annual average commercial exchange rate for sale between the real and the US dollar was used from 2010 to 2019 plus a zero mean random walk and a standard deviation of 0.5. The simulations were performed using MatLab software.

Figure 5 shows a computational simulation for the company production level that maximizes its profit according to the returns to scale, over time, illustrating what is concluded from Equation (2.0) of the previous section with the condition of Equation (2.2). The time was simulated for 10 years ahead of the year 2020, in order to show how the production level of companies will behave in the near future. We used the average of the current exchange rate with a random walk, the normalized price in one unit and the returns to scale with decreasing returns and

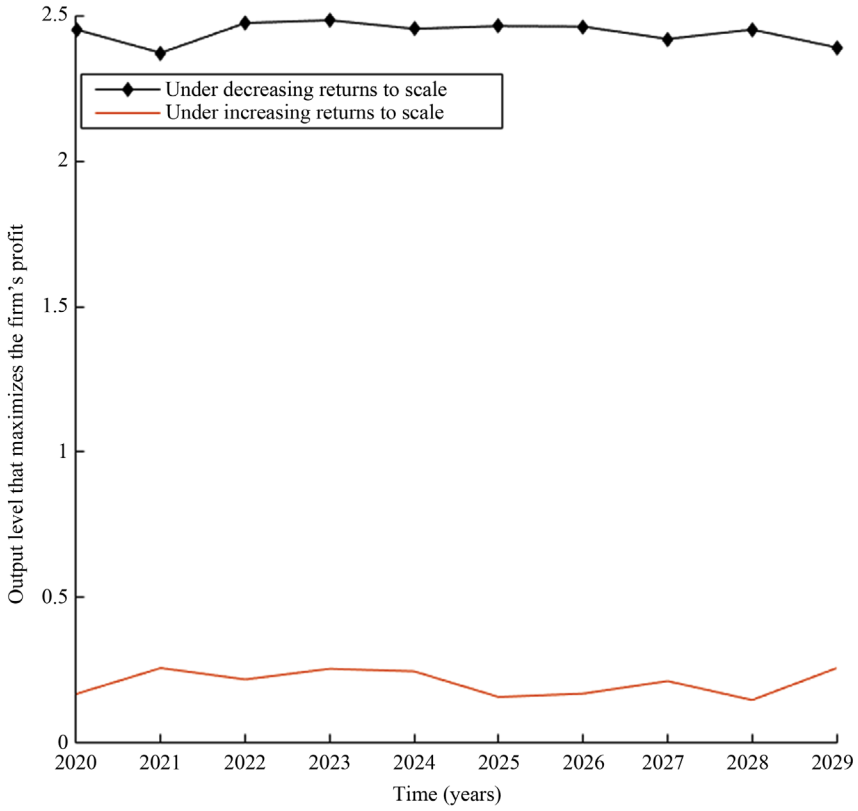

Figure 5. Production level that maximizes the company profit over time according to returns to scale.

with increasing returns to scale. As described by the referred equation, the impact of exchange variation is different on the firm's production, depending on the scale returns it has. This graph reveals that for decreasing returns, the level of production of the firm that maximizes its profit is higher than for firms with increasing returns to scale. In addition, it is possible to verify that the sensitivity of the firm's production level to returns to scale is similar, significantly altering only the product level that maximizes its profit. Thus, firms with decreasing returns to scale are expected to produce a higher level of output than firms with increasing returns to scale, *coeteris paribus*.

Figure 6 shows a computational simulation for the impact of exchange rate depreciation on the product, based on increments in scale returns, illustrating what is inferred from Equation (2.1) in the previous section. To carry out the simulation, the average of the current exchange rate was considered as described at the beginning of this section, the normalized price in one unit and the sum of the scale yield parameters varying between 0.2 and 1.8. This range was chosen to vary the returns to scale for two reasons: 1) to illustrate in a single graph the impacts of exchange rate depreciation for different sized returns (increasing and decreasing) firms and 2) to keep the range of returns in values reasonable.

The vertical axis of **Figure 6** shows the derivate of aggregate output with respect to the exchange rate. It measures how an increase in exchange rate affects

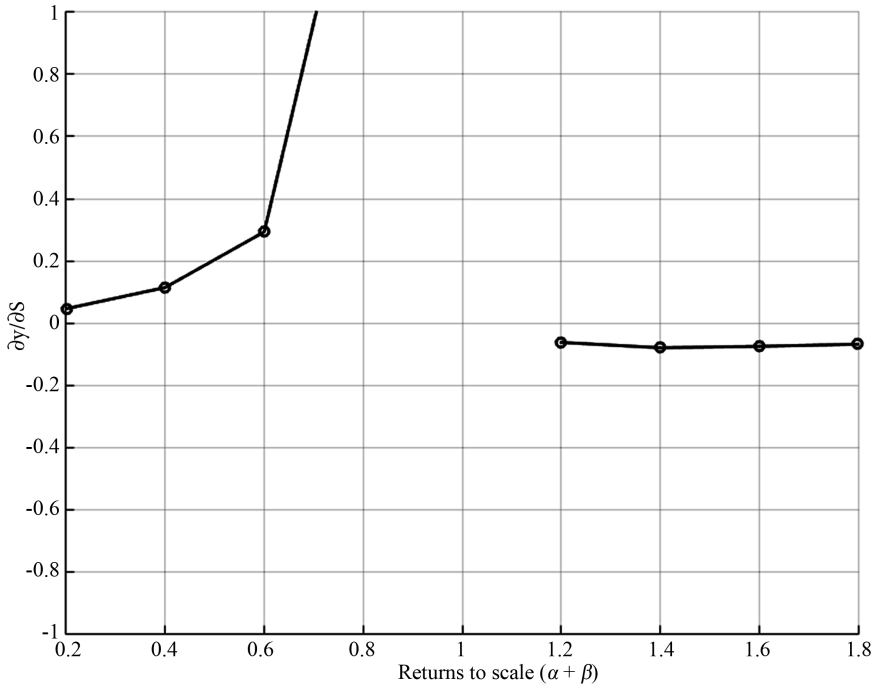

Figure 6. Impact of exchange depreciation on aggregate output from increments on returns to scale.

the aggregate output. The horizontal axis of **Figure 6** shows the returns to scale, measured as the sum of alpha and beta values from production function. **Figure 6** shows an increasing impact of the exchange rate on aggregate output when returns to scale are decreasing, while a decreasing or ineffective impact when returns to scale are increasing.

Figure 6 also shows that an exchange rate depreciation stimulates an increase in production for firms with decreasing income, while discouraging production for firms with increasing income. If firms in an economy are, for the most part, owners of technologies whose returns to scale are decreasing, then it is expected that exchange rate depreciation will, in fact, lead to economic growth, as advocated by Cushman (1983), Dornbusch (1990), Eichengreen (2007), Rodrik (2008) and Vieira and Damasceno (2016). On the other hand, in countries where research and innovation are already common, exchange rate depreciation will not stimulate economic growth. Therefore, mature and developed countries, whose practices in innovation and research are already widespread and stimulated, need to find other channels to stimulate growth.

5. Concluding Remarks

The present study aimed to understand how a representative commodity exporter and input importer responds in perfect competition markets for final goods, inputs and foreign exchange, when exchange rate varies. To achieve this goal, a simple mathematical model was developed to describes the problem of the exporting firm in this context of internationalization. We also investigate the level

of production that maximizes the firm's profit. Then, we aggregate the output of each firm to verify the impacts of exchange rate variations on the levels of aggregate output, take into consideration the returns to scale. In addition, two important exchange rates were found for the firm's decision, namely the minimum operating exchange rate (MOE) and the reversal exchange rate (RVER). Therefore, the present model helps understanding the role that the returns to scale plays in the relationship between exchange rate and economic growth.

The theoretical results obtained indicate that an exchange rate depreciation generates economic growth only for firms with technologies with decreasing returns to scale. If production technology presents increasing returns to scale, exchange rate depreciation reduces the level of production. The model does not apply for constant returns to scale. The hypothesis was formulated that developing countries inevitably lack research and innovation, which causes most of their firms to age and reach the range of production whose returns are decreasing in scale, which would justify the use of the exchange rate to stimulate economic growth. The graphs presented in the numerical simulation illustrate the results of this study.

Finally, the current study presents a few limitations that can be addressed in future research. For instance, the following statements can integrate them: 1) allow that firms to influence the exchange rate and place it above the minimum operational level; 2) considering new players in the markets considered and iii) take into account more input and outputs variables.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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