Theoretical Economics Letters, 2020, 10, 292-298
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

The Hybrid New Keynesian Phillips Curve with Multiple Lags of Inflation

Carl M. Campbell III

Department of Economics, Northern Illinois University, DeKalb, IL, USA
Email: carlcamp@niu.edu

How to cite this paper: Campbell III, C. M. (2020). The Hybrid New Keynesian Phillips Curve with Multiple Lags of Inflation. *Theoretical Economics Letters, 10,* 292-298.
https://doi.org/10.4236/tel.2020.102019

Received: February 22, 2020
Accepted: April 12, 2020
Published: April 15, 2020

Abstract

In deriving the hybrid new Keynesian Phillips curve (HNKPC) in Galí and Gertler (1999) and Holmberg (2006), it is assumed that backward-looking firms index their prices to the average prices newly set last period plus last period's inflation rate, resulting in a Phillips curve equation that relates current inflation to a demand variable, expected future inflation, and last period's inflation. The present study generalizes the derivation of the HNKPC to allow firms to index prices to multiple lags of inflation, resulting in a HNKPC in which current inflation depends on multiple lags of inflation instead of only one lag of inflation, providing theoretical justification for empirical specifications of the HNKPC that include more than one lag of inflation.

Keywords

Phillips Curve, Inflation, Indexation

1. Introduction

The hybrid new Keynesian Phillips curve (HNKPC) is generally expressed as an equation that relates current inflation to a real demand variable (usually either the output gap or real marginal cost), next period's inflation, and last period's inflation. A notable example of the HNKPC is the model of Galí and Gertler (1999). The HNKPC assumes Calvo (1983) price setting, in which a fraction of firms (randomly chosen) are able to reset their prices. Of the firms that can reset prices in a given period, a fraction set their price at a level that maximizes the present value of expected profits, while the remainder index their prices to the average price chosen by firms that reset prices in the previous period, adjusted for last period's inflation rate. The former are considered forward-looking firms, and the latter are considered backward-looking firms. Microfoundations for the

HNKPC are derived in Holmberg (2006), which presents a derivation of the HNKPC with one lag of inflation.

This study extends the previous literature by deriving an expression for the HNKPC in which firms index prices to an arbitrarily large number of lags of inflation. As a result, inflation depends on multiple lags of inflation, instead of just last period's inflation rate, allowing for a more flexible specification in modeling inflation dynamics. Mavroeidis, Plagborg-Møller, and Stock (2014) estimate the HNKPC with both one lag and four lags of inflation (although they do not derive microfoundations for a specification that includes multiple lags of inflation), and they find significant differences in the estimated coefficients between these specifications, suggesting that specifications limiting the lag structure to one lag may miss important inflation dynamics. In addition, many empirical estimates of the Phillips curve (both purely backward looking and hybrid specifications) include multiple lags of inflation. Examples of studies that include multiple lags of inflation in estimating the Phillips curve include King and Watson (1994), Fuhrer (1995, 1997), Akerlof et al. (1996), Duca (1996), Gordon (1997), Murphy (2014), and Coibion and Gorodnichenko (2015).

Because there is evidence that incorporating more than one lag of inflation in the HNKPC affects the estimated coefficients and because previous estimates of the Phillips curve often include multiple lags of inflation, it is beneficial to provide microfoundations for a specification that includes multiple lags of inflation. The equation derived in the present study provides theoretical justification for empirical estimates of the HNKPC that include more than one lag of inflation.

2. Model of the HNKPC with Multiple Lags of Inflation

In Holmberg's (2006) derivation of the HNKPC, it is assumed that backward-looking firms index their prices to the average price newly set last period plus last period's inflation rate. This study extends Holmberg's analysis by assuming that backward-looking firms index their prices to the average price newly set last period plus a weighted average of past inflation rates, allowing for an arbitrarily large number of lags of inflation. The derivation in this study is valid both when the demand variable is the output gap and when the demand variable is real marginal cost.

Following Holmberg (2006), let p_t^* represent the price (in natural logs) set by firms that are able to adjust their prices in the current period and let θ represent the proportion of firms that are not able to reset their prices. Then, the average price (p_t) is

$$p_t = (1-\theta) p_t^* + \theta p_{t-1}. \tag{1}$$

By subtracting p_{t-1} from both sides of (1), the inflation rate can be expressed as

$$\pi_t = p_t - p_{t-1} = (1-\theta)\left(p_t^* - p_{t-1}\right). \tag{2}$$

Let ω represent the proportion of firms resetting prices that index their prices to lagged inflation (i.e., backward-looking firms), p_t^b represent the price set by backward-looking firms, and p_t^f represent the price set by forward-looking firms. Then,

$$p_t^* = \omega p_t^b + (1-\omega) p_t^f . \tag{3}$$

Solving (1) for p_t^* and substituting this expression into (3) yields

$$\frac{1}{1-\theta} p_t - \frac{\theta}{1-\theta} p_{t-1} = \omega p_t^b + (1-\omega) p_t^f . \tag{4}$$

Subtracting p_t from both sides of (4) and multiplying by $1-\theta$ results in the relationship,

$$\theta \pi_t = \omega(1-\theta)\left(p_t^b - p_t\right) + (1-\omega)(1-\theta)\left(p_t^f - p_t\right). \tag{5}$$

Let β represent the discount factor and $p_{t+j}^{*,e}$ represent firms' expectation of the optimal price in period $t + j$ when they set prices in period t. Then forward-looking firms set their prices equal to

$$p_t^f = (1-\beta\theta) \sum_{j=0}^{\infty} \beta^j \theta^j E_t p_{t+j}^* , \tag{6}$$

which can be expressed as,

$$p_t^f = (1-\beta\theta) p_t^* + \beta\theta E_t p_{t+1}^f \text{ }^1. \tag{7}$$

In Roberts (1995: p. 977), a firm's optimal price can be expressed as $p_t^* = p_t + \kappa y_t$, where y_t represents the output gap[2]. A similar relationship is provided in Galí and Gertler (1999), in which a firm's optimal price equals its nominal marginal cost (mc_t^n). The relationship between nominal marginal cost and real marginal cost (mc_t) is $mc_t^n = p_t + mc_t$, so that $p_t^* = p_t + mc_t$. This is the same equation as in Roberts, but with $y_t = mc_t$ and $\kappa = 1$. Thus, y_t can represent either real marginal cost or the output gap. The price set by forward-looking firms is therefore equal to

$$p_t^f = (1-\beta\theta)(p_t + \kappa y_t) + \beta\theta E_t p_{t+1}^f , \tag{8}$$

and the difference between p_t^f and p_t is

$$p_t^f - p_t = (1-\beta\theta)\kappa y_t - \beta\theta p_t + \beta\theta E_t p_{t+1}^f . \tag{9}$$

In Galí and Gertler (1999) and Holmberg (2006), when backward-looking firms set prices, they set them equal to the average price set last period by firms that adjusted prices, plus last period's inflation rate. In the present study, when these firms set prices, they set them equal to the average price set last period by firms that adjusted prices plus a weighted average of lagged inflation. Thus, the price set by backward-looking firms can be expressed as

$$p_t^b = p_{t-1}^* + \sum_{i=1}^{T} \lambda_i \pi_{t-i} \quad \text{with} \quad \sum_{i=1}^{T} \lambda_i = 1, \tag{10}$$

[1]See, for example, Equations (7.56) and (7.57) on p. 328 of Romer (2019).

[2]The equation in Roberts (1995) is $p_t^* = \hat{p}_t + \beta y_t + \varepsilon_t$, where p_t and κ in the present study correspond to $\hat{p}_t$ and β in Roberts and where the error term in Roberts is ignored.

where the λ's represent the weight given to each lag of inflation in the indexation process.

Combining (1) and (10) yields

$$\begin{aligned} p_t^b - p_t &= p_{t-1}^* + \sum_{i=1}^{T} \lambda_i \pi_{t-i} - (1-\theta) p_t^* - \theta p_{t-1} \\ &= \theta\left(p_{t-1}^* - p_{t-2}\right) + \sum_{i=1}^{T} \lambda_i \pi_{t-i} - (1-\theta)\left(p_t^* - p_{t-1}\right). \end{aligned} \quad [3] \tag{11}$$

Since $\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$ and $\pi_{t-1} = (1-\theta)\left(p_{t-1}^* - p_{t-2}\right)$,

$$p_t^b - p_t = -\pi_t + \frac{\theta}{1-\theta}\pi_{t-1} + \sum_{i=1}^{T} \lambda_i \pi_{t-i}. \tag{12}$$

By solving (5) for p_t^f, the price set by forward-looking firms is

$$p_t^f = \frac{\theta}{(1-\theta)(1-\omega)}\pi_t - \frac{\omega}{1-\omega}\left(p_t^b - p_t\right) + p_t, \tag{13}$$

which, shifted one period forward, implies that

$$E_t p_{t+1}^f = \frac{\theta}{(1-\theta)(1-\omega)} E_t \pi_{t+1} - \frac{\omega}{1-\omega}\left(E_t p_{t+1}^b - E_t p_{t+1}\right) + E_t p_{t+1}. \tag{14}$$

Substituting (12) shifted one period forward into (14) yields

$$\begin{aligned} E_t p_{t+1}^f = {} & \frac{\theta}{(1-\theta)(1-\omega)} E_t \pi_{t+1} - \frac{\omega}{1-\omega}\left[-E_t \pi_{t+1} + \left(\frac{\theta}{1-\theta} + \lambda_1\right)\pi_t \right. \\ & \left. + \sum_{i=1}^{T-1} \lambda_{i+1} \pi_{t-i} \right] + E_t p_{t+1}, \end{aligned} \tag{15}$$

which can be expressed as,

$$\begin{aligned} E_t p_{t+1}^f = {} & \frac{\theta + (1-\theta)\omega}{(1-\theta)(1-\omega)} E_t \pi_{t+1} - \frac{\omega}{1-\omega}\left(\frac{\theta}{1-\theta} + \lambda_1\right)\pi_t \\ & - \frac{\omega}{1-\omega} \sum_{i=1}^{T-1} \lambda_{i+1} \pi_{t-i} + E_t p_{t+1}. \end{aligned} \tag{16}$$

By substituting (16) into (9), the difference between p_t^f and p_t is

$$\begin{aligned} p_t^f - p_t = {} & (1-\beta\theta)\kappa y_t - \beta\theta p_t + \beta\theta\left[\frac{\theta + (1-\theta)\omega}{(1-\theta)(1-\omega)} E_t \pi_{t+1} \right. \\ & \left. - \frac{\omega}{1-\omega}\left(\frac{\theta}{1-\theta} + \lambda_1\right)\pi_t - \frac{\omega}{1-\omega} \sum_{i=1}^{T-1} \lambda_{i+1} \pi_{t-i} + E_t p_{t+1} \right] \end{aligned}$$

$$\begin{aligned} p_t^f - p_t = {} & (1-\beta\theta)\kappa y_t + \beta\theta\left[\frac{\theta + (1-\theta)\omega}{(1-\theta)(1-\omega)} E_t \pi_{t+1} \right. \\ & \left. - \frac{\omega}{1-\omega}\left(\frac{\theta}{1-\theta} + \lambda_1\right)\pi_t - \frac{\omega}{1-\omega} \sum_{i=1}^{T-1} \lambda_{i+1} \pi_{t-i} + E_t \pi_{t+1} \right] \end{aligned}$$

[3]The relationship, $p_{t-1}^* - p_{t-1} = \theta\left(p_{t-1}^* - p_{t-2}\right)$, can be derived by lagging (1) by one period.

$$p_t^f - p_t = (1-\beta\theta)\kappa y_t + \beta\theta\left[\frac{1}{(1-\theta)(1-\omega)}E_t\pi_{t+1} - \frac{\omega}{1-\omega}\left(\frac{\theta}{1-\theta}+\lambda_1\right)\pi_t - \frac{\omega}{1-\omega}\sum_{i=1}^{T-1}\lambda_{i+1}\pi_{t-i}\right]. \quad (17)$$

Substituting (12) and (17) into (5) results in the relationship,

$$\theta\pi_t = \omega(1-\theta)\left(-\pi_t + \frac{\theta}{1-\theta}\pi_{t-1} + \sum_{i=1}^{T}\lambda_i\pi_{t-i}\right) + (1-\omega)(1-\theta)\left\{(1-\beta\theta)\kappa y_t + \beta\theta\left[\frac{1}{(1-\theta)(1-\omega)}E_t\pi_{t+1} - \frac{\omega}{1-\omega}\left(\frac{\theta}{1-\theta}+\lambda_1\right)\pi_t - \frac{\omega}{1-\omega}\sum_{i=1}^{T-1}\lambda_{i+1}\pi_{t-i}\right]\right\}$$

$$\theta\pi_t = -(1-\theta)\omega\pi_t + \theta\omega\pi_{t-1} + (1-\theta)\omega\sum_{i=1}^{T}\lambda_i\pi_{t-i} + (1-\omega)(1-\theta)(1-\beta\theta)\kappa y_t + \beta\theta E_t\pi_{t+1} - \beta\theta\omega\left(\theta+(1-\theta)\lambda_1\right)\pi_t - \beta\theta\omega(1-\theta)\sum_{i=1}^{T-1}\lambda_{i+1}\pi_{t-i}$$

$$\left[\theta+\omega(1-\theta)+\beta\theta\omega\left(\theta+(1-\theta)\lambda_1\right)\right]\pi_t = (1-\omega)(1-\theta)(1-\beta\theta)\kappa y_t + \beta\theta E_t\pi_{t+1} + \theta\omega\pi_{t-1} + \omega(1-\theta)\sum_{i=1}^{T}(\lambda_i - \beta\theta\lambda_{i+1})\pi_{t-i}. \quad (18)$$

Dividing both sides of (18) by the coefficient on π_t yields the HNKPC curve equation,

$$\pi_t = \frac{\beta\theta E_t\pi_{t+1} + \theta\omega\pi_{t-1} + \omega(1-\theta)\sum_{i=1}^{T}(\lambda_i - \beta\theta\lambda_{i+1})\pi_{t-i} + (1-\omega)(1-\theta)(1-\beta\theta)\kappa y_t}{\theta+\omega\left[1-\theta+\beta\theta\left(\theta+(1-\theta)\lambda_1\right)\right]} \quad (19)$$

in which y_t can represent either the output gap or real marginal cost[4].

When wages are indexed to only one lag of inflation, so that $\lambda_1 = 1$ and $\lambda_2 = \lambda_3 = \lambda_4 = \cdots = \lambda_\infty = 0$, the HNKPC simplifies to the formulation in Galí and Gertler (1999) and Holmberg (2006) in which

$$\pi_t = \frac{\beta\theta E_t\pi_{t+1} + \omega\pi_{t-1} + (1-\omega)(1-\theta)(1-\beta\theta)\kappa y_t}{\theta+\omega\left[1-\theta(1-\beta)\right]}. \quad (20)$$

Thus, when prices are indexed to one lag of inflation, the model yields the same equation as in Galí and Gertler (1999) and Holmberg (2006). However, the model in the present study extends the previous literature by allowing firms to index their prices to more than one lag of inflation, instead of just a single lag, resulting in a HNKPC that can include any number of lags of inflation.

3. Conclusion

In recent years, there has been an increased emphasis on developing micro-foundations for macroeconomic models. While some empirical estimates of the

[4]In the term involving the summation sign, $\lambda_{T+1} = 0$, so the last term of the summation is $\omega(1-\theta)\lambda_T\pi_{t-T}$.

Phillips curve include multiple lags of inflation, they are included on an ad hoc basis. This study provides microeconomic justification for a HNKPC with more than one lag of inflation, and it derives an equation that researchers can use in empirically estimating the HNKPC with multiple lags of inflation. In addition, dynamic stochastic general equilibrium (DSGE) models, such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), generally assume that a fraction of wages and prices that are reset are indexed to last period's inflation rate. The present study derives an equation in which these variables can be indexed to multiple lags of inflation, possibly enabling these DSGE models to fit macroeconomic data more closely.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Akerlof, G. A., Dickens, W. T., Perry, G. L., Gordon, R. J., & Mankiw, N. G. (1996). The Macroeconomics of Low Inflation. *Brookings Papers on Economic Activity, 1,* 1-76. https://doi.org/10.2307/2534646

Calvo, G. A. (1983). Staggered Contracts in a Utility-Maximizing Framework. *Journal of Monetary Economics, 12,* 383-398. https://doi.org/10.1016/0304-3932(83)90060-0

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy, 113,* 1-45. https://doi.org/10.1086/426038

Coibion, O., & Gorodnichendo, Y. (2015). Is the Phillips Curve Alive and Well After All? Inflation Expectations and the Missing Disinflation. *American Economic Journal: Macroeconomics, 7,* 197-232. https://doi.org/10.1257/mac.20130306

Duca, J. V. (1996). Inflation, Unemployment, and Duration. *Economics Letters, 52,* 293-298. https://doi.org/10.1016/S0165-1765(96)00853-1

Fuhrer, J. C. (1995). The Phillips Curve Is Alive and Well. *New England Economic Review,* March/April 1995, 41-56.

Fuhrer, J. C. (1997). The (Un)importance of Forward-Looking Behavior in Price Specifications. *Journal of Money, Credit, and Banking, 29,* 338-350. https://doi.org/10.2307/2953698

Galí, J., & Gertler, M. (1999). Inflation Dynamics: A Structural Econometric Analysis. *Journal of Monetary Economics, 44,* 195-222. https://doi.org/10.1016/S0304-3932(99)00023-9

Gordon, R. J. (1997). The Time-Varying NAIRU and Its Implications for Economic Policy. *Journal of Economic Perspectives, 11,* 11-32. https://doi.org/10.1257/jep.11.1.11

Holmberg, K. (2006). *Derivation and Estimation of a New Keynesian Phillips Curve in a Small Open Economy.* Sveriges Riksbank Working Paper Series #197. https://doi.org/10.2139/ssrn.985902

King, R. G., & Watson, M. W. (1994). The Post-War U.S. Phillips Curve: A Revisionist Econometric History. *Carnegie-Rochester Conference Series on Public Policy, 41,* 157-219. https://doi.org/10.1016/0167-2231(94)00018-2

Mavroeidis, S., Plagborg-Møller, M., & Stock, J. H. (2014). Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve. *Journal of Economic Literature, 52,* 124-188. https://doi.org/10.1257/jel.52.1.124

Murphy, R. G. (2014). Explaining Inflation in the Aftermath of the Great Recession. *Journal of Macroeconomics, 40,* 228-244. https://doi.org/10.1016/j.jmacro.2014.01.002

Roberts, J. M. (1995). New Keynesian Economics and the Phillips Curve. *Journal of Money, Credit, and Banking, 27,* 975-984. https://doi.org/10.2307/2077783

Romer, D. (2019). *Advanced Macroeconomics* (5th ed.). New York: McGraw-Hill Education.

Smets, F., & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review, 97,* 586-606. https://doi.org/10.1257/aer.97.3.586