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Cluster-Cluster Potentials for the Carbon Nucleus

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Abstract

Two types of $\alpha-\alpha$ potentials are given in the present paper. The two potentials have Gaussian radial dependences. Such shapes of radial functions are suitable for using in the unitary scheme model. The first potential is given in the form of an attractive force and the second is given in the form of a superposition of repulsive and attractive forces. The two potentials are used to calculate the binding energy of the carbon nucleus ^{12}C. For this purpose, we expand the ground-state wave function of carbon in a series of the bases of the unitary scheme model and apply the variational method. To calculate the necessary matrix elements required to obtain the binding energy of carbon, we factorized the unitary scheme model bases in the form of products of two wave functions: the first function represents the set of the A-4 nucleons and the second function represents the set of the last four nucleons by using the well-known four-body fractional parentage coefficients. Good results are obtained for the binding energy of ^{12}C by using the two potentials.

Keywords

Unitary Scheme Model, Four-Particle Fractional Parentage Coefficients, Cluster-Cluster Potentials, Carbon Nucleus

1. Introduction

The ground states of nuclei with Z = N = magic number: ^{4}He, ^{16}C, etc. have spherical shapes with total angular momentum equals zero, spin angular momentum zero, and isotopic spin equals zero. In such nuclei, the nucleons are clustered into groups with similar features. This can be recognized from some phenomena of nuclear behavior. A conventional description of a nucleus is one in which there is a homogeneous distribution of protons and neutrons. However, even at the inception of nuclear structure, it was known that conglomerates

of nucleons (nuclear clustering) were extremely important in determining the structure of light nuclei. The first and simplest nuclear model to consider such nuclear characteristics is the α-particle model [1]. The fact that two protons and two neutrons in the α-particle could strongly interact played an important role in calculating the binding energy of this nucleus. Thus, the nuclear ground states would be expected to favor such quasi-α particle configurations and, consequently, exhibit large spatial symmetry.

In general, phenomenological cluster-cluster potentials are of interest for applications to α-cluster models [1] [2] [3] [4] [5], for the correlation of experimental $\alpha-\alpha$ scattering and for comparison with the results of theoretical studies of the interaction. These studies make use of the nucleon-nucleon forces and the resonating group structure method with the α-particles constrained to be in the ground state.

Doma and El-Zebidy [5] obtained two Gaussian cluster-cluster potentials which are suitable for the calculations of the ground states of the four lithium isotopes, namely: ^{6}Li, ^{7}Li, ^{8}Li and ^{9}Li, by constructing the suitable clusters for these nuclei. One of these potentials is attractive and the other is a superposition of attractive and repulsive forces. The authors in [5] used the bases of the unitary scheme model (USM) [5]-[10] with the number of quanta of excitations N up to 6, 7, 8 and 9 for the nuclei ^{6}Li, ^{7}Li, ^{8}Li and ^{9}Li, respectively. The USM achieved great success in describing the ground and the excited state characteristics of light nuclei [10]-[30].

Furthermore, Abdel-Khalek [31] proceeded as in the work of the lithium nuclei [5] to study the structure of the ground-state of the three beryllium isotopes: ^{8}Be, ^{9}Be and ^{10}Be by employing the same two different types of cluster-cluster potentials.

All the carbon in the universe is created in red giant stars by two alpha particles (^{4}He nuclei) fusing to create a short-lived ^{8}Be nucleus, which then captures a third alpha particle to form ^{12}C. But exactly how this reaction occurs initially puzzled physicists, whose early understanding of ^{12}C suggested that it would proceed much too slowly to account for the known abundance of carbon in the universe. However, the precise arrangement of the protons and neutrons in the ^{12}C nucleus remains a matter of much debate. While some physicists feel that ^{12}C is the best thought of as 12 interacting nucleons, others believe that the nucleus can be modelled as three alpha particles that are bound together. The rational for the latter model is that alpha particles are extremely stable and so are likely to endure within the ^{12}C nucleus. Accordingly, the aim of the present paper is to investigate the behavior of the ground state of ^{12}C as three α particles interacting according to an $\alpha-\alpha$ potential. Hence, we introduce in the present paper two $\alpha-\alpha$ potentials for the carbon nucleus. The first of which has one attractive Gaussian force and the second consists of repulsive and attractive Gaussian-forces. The methods of calculating the four-body orbital and spin-isospin fractional parentage coefficients (FPC) [7] [8] [13] [16] [17] [18] are used to construct the

ground-state nuclear wave functions of ^{12}C in the form of sums of products of two wave functions. The first represents the set of the four-particles and the second represents the set of the other eight particles added to them the coordinates, the spins, and the isotopic spins of the line joining the center of masses of the two systems. In these calculations, we have used the bases of the USM [7] [8] [9] [10].

2. The Unitary Scheme Model Hamiltonian

The Hamiltonian $\mathscr{H}$ of a given nucleus consisting of A nucleons, interacting according to two-body interactions, can be written in the following form [7]-[18]:

$$\mathscr{H} = \frac{1}{2m}\sum_{i=1}^{A} \boldsymbol{p}_i^2 + \sum_{i<j}^{A} V_{i,j} \tag{2.1}$$

where $\boldsymbol{p}_i$ is the nucleon momentum, m is its mass, and $V_{i,j}$ is the nucleon-nucleon interaction. The center of mass kinetic energy is given by:

$$T_{CM} = \frac{1}{2mA}\left(\sum_{i=1}^{A} \boldsymbol{p}_i\right)^2. \tag{2.2}$$

From the translational invariance properties of the Hamiltonian $\mathscr{H}$, one can easily separate the center of mass motion from this Hamiltonian and as a result the Hamiltonian corresponding to the internal motion of the nucleons is given by:

$$H = \mathscr{H} - T_{CM} \tag{2.3}$$

Adding and subtracting a harmonic oscillator potential referred to the center of mass of the nucleus, the Hamiltonian corresponding to the internal motion can be written in the form:

$$H = H + \frac{1}{2}\sum_{i=1}^{A} m\omega^2 \left(\boldsymbol{r}_i - \boldsymbol{R}\right)^2 - \frac{1}{2}\sum_{i=1}^{A} m\omega^2 \left(\boldsymbol{r}_i - \boldsymbol{R}\right)^2 = H^{(0)} + V' \tag{2.4}$$

The unperturbed term in (2.4), $H^{(0)}$, is given by:

$$H^{(0)} = \frac{1}{A}\sum_{1=i<j}^{A}\left[\frac{1}{2m}\left(\boldsymbol{p}_i - \boldsymbol{p}_j\right)^2 + \frac{1}{2}m\omega^2\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right)^2\right], \tag{2.5}$$

and is known as the USM Hamiltonian. The second term in (2.4)

$$V' = \sum_{1=i<j}^{A}\left[V\left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right) - \frac{m\omega^2}{2A}\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right)^2\right]. \tag{2.6}$$

is the residual two-body interaction.

Schrodinger's wave equation for the Hamiltonian $H^{(0)}$ is

$$H^{(0)}\psi_N^{(0)} = E_N^{(0)}\psi_N^{(0)} \tag{2.7}$$

The wave functions $\psi_N^{(0)}$ constitutes the bases of the USM states. The methods of classifying the states in the USM by using the group theoretical techniques were pro posed by Kretzschmar [6], and later by Vanagas [7] and Doma *et al.* [8] [9] [10] [11]. The energy eigenvalues and the total antisymmetric wave functions

of a nuclear state characterizing such Hamiltonian are given by [19] [22]-[29]

$$E_N^{(0)} = \left\{ N + \frac{3}{2}(A-1) \right\} \hbar\omega , \tag{2.8}$$

$$\left| A\Gamma \right\rangle \equiv \left| AN\{\rho\}(\nu)[f]\gamma LM_L SM_S TM_T \right\rangle \tag{2.9}$$

In Equation (2.9), the number Γ stands for the set of all orbital and spin-isospin quantum numbers characterizing the USM nuclear wave function. From the group structures of the function (2.9), the number of quanta of excitation N is an irreducible representation (irrep) of the unitary group $U_{3(A-1)}$. Also, the representation $\{\rho\} = \{\rho_1, \rho_2, \rho_3\}$, where $\rho_1 \geq \rho_2 \geq \rho_3 \geq 0$ and $\rho_1 + \rho_2 + \rho_3 = N$, is related to Elliot symbol $(\lambda\mu)$ [6] [7] by the relations $\lambda = \rho_1 - \rho_2$, and $\mu = \rho_2 - \rho_3$. Furthermore, the representation $\{\rho\}$ is an irrep of the unitary group U_{A-1} and the unitary unimodular subgroup of three dimensions SU_3, simultaneously. Moreover, the representation (ν) is an irrep of the orthogonal group O_{A-1}, $[f]$ is an irrep of the symmetric group S_A, and γ is a repetition quantum number of the irreps in the chain of groups which classify the nuclear state. The numbers L and M_L stand for the orbital angular momentum and its z-component and are also irreps of the rotational groups SO_3 and SO_2, respectively. Finally, S, M_S are the spin, its z-component and T, M_T are the isospin, its z-component which are irreps of the direct product of the groups $SU_2 \times SU_2$. The basic functions (2.9) transform according to the following chain of groups [7]:

$$\begin{array}{lll} & & SU_4 \supset SU_2 \times SU_2 \\ & U_{4^A} \supset U_{4A} \supset & \times \\ & & U_A \supset O_A \supset S_A \\ \Im \supset & & \\ & \times & U_{A-1} \supset O_{A-1} \supset S_A \\ & U_{3(A-1)} \supset & \times \\ & & SU_3 \supset SO_3 \supset SO_2 \end{array} \tag{2.10}$$

Wave functions having definite total angular momentum quantum number J, its z-projection M_J, and isotopic spin T, and its z-projection M_T are constructed from the functions (2.9) as follows [28] [29]

$$\left| AJM_J TM_T \right\rangle = \sum_{M_L + M_S = M_J} \left(LM_L, SM_S \middle| JM_J \right) \left| AN\{\rho\}(\nu)[f]\alpha LM_L SM_S TM_T \right\rangle , \tag{2.11}$$

where $\left(LM_L, SM_S \middle| JM_J \right)$ are the Clebsch-Gordan coefficients of the rotational group SO_3.

The fractional parentage coefficients (FPC) of separation of four particles from the nuclear wave function are given in terms of the FPC of separation of two particles as follows [3] [7]

$$\begin{aligned} \left\langle A\Gamma_A \middle| A-4\Gamma_{A-4}, 4\Gamma_4 \right\rangle = & \sum_{\Gamma_{A-2}, \Gamma_2} \left\langle A\Gamma_A \middle| A-2\Gamma_{A-2}, 2\Gamma_2 \right\rangle \\ & \times \left\langle A-2\Gamma_{A-2} \middle| A-4\Gamma_{A-4}, 2\Gamma_2 \right\rangle \left\langle 4\Gamma_4 \middle| 2\Gamma_2, 2\Gamma_2 \right\rangle \\ & \times U\left(L_{A-4} L_2 L L_2; L_{A-2} L_4 \right) \Lambda_{[f]}. \end{aligned} \tag{2.12}$$

The first three factors on the right-hand side of Equation (2.12) are two-particles FPC for the sets of A, $A-2$, and 4-particles, respectively. U is the Racah coefficient and $\Lambda_{[f]}$ is defined by [3]

$$\Lambda_{[f]}=\left\{6\frac{\tau_{A,A-3}\tau_{A-1,A-3}\tau_{A-1,A-2}\tau_{A,A-2}}{\left(1+\tau_{A,A-3}\right)\left(1+\tau_{A-1,A-3}\right)\left(1+\tau_{A-1,A-2}\right)\left(1+\tau_{A,A-2}\right)}\right\}^{1/2}, \tag{2.13}$$

where $\tau_{i,j}$ is the axial distance between the numbers i and j in the Young tableau. Γ_A is the set of all orbital, spin, and isotopic spin quantum numbers of the system consisting of A nucleons and similarly for the other systems.

3. The Cluster-Cluster Potentials

Our calculations depend on using two types of $\alpha-\alpha$ potentials having Gaussian-radial dependence. The first consists of one attractive term of the form [5]

$$V_{\alpha\alpha}^{(1)}=V_{a1}\exp\left\{\frac{-R^2}{R_{a1}^2}\right\} \tag{3.1}$$

where V_{a1} and R_{a1} are the depth and the range parameters of the potential, respectively. The second potential consists of a sum of attractive and repulsive forces of the form [5]

$$V_{\alpha\alpha}^{(2)}=V_{a2}\exp\left(\frac{-R^2}{R_{a2}^2}\right)+V_{r2}\exp\left(\frac{-R^2}{R_{r2}^2}\right) \tag{3.2}$$

where V_{a2} and V_{r2} are the depth parameters of the attractive, and the repulsive terms, respectively. R_{a2} and R_{r2} are the corresponding range parameters, respectively. In Equations (3.1), and (3.2) R is the inter-alpha distance defined by

$$R=\left|\boldsymbol{R}\right|=\frac{\left(\boldsymbol{R}_{\alpha}^{(1)}-\boldsymbol{R}_{\alpha}^{(2)}\right)}{\sqrt{2}}. \tag{3.3}$$

Here, $\boldsymbol{R}_{\alpha}^{(1)}$ and $\boldsymbol{R}_{\alpha}^{(2)}$ are the radius vectors of the centers of masses of the two clusters.

4. The Four-Particle Matrix Elements

The four-particle matrix elements of the potential $V_{\alpha\alpha}^{(1)}$, given by Equation (3.1), or the potential $V_{\alpha\alpha}^{(2)}$, given by Equation (3.2), with respect to the USM bases, Equation (2.8), are given by [5]

$$\begin{aligned}\left\langle A\Gamma_A\left|V_{\alpha\alpha}^{(1)}\right|A\Gamma_A'\right\rangle=&\sum_{\Gamma_{A-4},\Gamma_4,\Gamma_A'}\frac{A(A-1)(A-2)(A-3)}{4!}\\&\times\left\langle A\Gamma_A\left|A-4\Gamma_{A-4},4\Gamma_A\right\rangle\left\langle A\Gamma_A'\right|A-4\Gamma_{A-4},4\Gamma_4'\right\rangle\\&\times\begin{Bmatrix}L_{A-4}&S_{A-4}&J_{A-4}\\L_4&0&J_4\\L&S&J\end{Bmatrix}\begin{Bmatrix}L_{A-4}&S_{A-4}'&J_{A-4}\\L_4'&0&J_4\\L'&S'&J'\end{Bmatrix}\\&\times\left\langle A-4\Gamma_{A-4},4\Gamma_4\left|V_{\alpha-\alpha}^{1(2)}\right|A-4\Gamma_{A-4},4\Gamma_4'\right\rangle\end{aligned} \tag{4.1}$$

In Equation (4.1), $\begin{Bmatrix} L_1 S_1 J_1 \\ L_2 S_2 J_2 \\ L_{12} S_{12} J_{12} \end{Bmatrix}$ are the normalized $9-j$ symbols.

For the evaluation of the two-particle operator $\sum_{1=i<j}^{A}\left[-\frac{m\omega^2}{2A}\left(\boldsymbol{r}_i-\boldsymbol{r}_j\right)^2\right]$, we the two-particle FPC [13] [14] [15]. Accordingly, the Hamiltonian matrix H, can be constructed as function of the oscillator parameter $\hbar\omega$ and the potential parameters. Diagonalizing this matrix with respect to the oscillator parameter $\hbar\omega$, which is allowed to vary in a wide range of values $8 \le \hbar\omega \le 30$ MeV in order to obtain the best ground-state energy eigenvalue for ^{12}C. Hence, the ground-state energy eigenvalue and eigenfunction are then obtained for this nucleus.

5. Results and Discussions

By varying the six parameters of the two potentials, V_{a1}, R_{a1}, V_{a2}, R_{a2}, V_{r2} and R_{r2}, it was found that the suitable ranges of variations which may produce good results for the binding energy of the carbon nucleus ^{12}C are: 1) the attractive depth parameter V_{a1} of the first potential is varied in the range $-40.0 \le V_{a1} \le -20.0$ MeV with a step 0.5 MeV, and the corresponding attractive range parameter R_{a1} is varied in the range $1.0 \le R_{a1} \le 4.0$ fm with a step 0.001 fm; 2) the attractive depth parameter of the second type of potentials V_{a2} is allowed to vary in the same energy range as for V_{a1} with step 1.0 MeV and the corresponding attractive range parameter R_{a2} is allowed to vary in the same range as for R_{a1} with a step 0.0005 fm; 3) the repulsive depth parameter V_{r2} of the second type of potentials is varied in the range $10.0 \le V_{r2} \le 50$ MeV with a step 1.0 MeV, and the corresponding repulsive range parameter R_{r2} is varied in the range $2.0 \le R_{r2} \le 5.0$ fm with a step 0.05 fm. Several potentials resulting from the above ranges gave binding energy for the carbon nucleus in reasonable agreement with the corresponding experimental value 92.15 MeV.

In **Figure 1**, we present the variation of the attractive depth parameter V_{a1} with respect to the corresponding attractive range parameter R_{a1} for potentials of the first type which produced acceptable value for the binding energy of ^{12}C. Among these parameters the resulting best $\alpha-\alpha$ potential for ^{12}C is given by

$$V_{\alpha\alpha}^{(1)} = -32.41\exp\left\{\frac{-R^2}{(2.812)^2}\right\} \tag{5.1}$$

This potential produced binding energy of ^{12}C equals 88.79 MeV.

In **Figure 2**, we present the variation of the attractive range parameter R_{a2} with respect to the repulsive depth parameter V_{r2} for the second type of potentials. Similarly, the variation of the repulsive range parameter R_{r2} with respect to the repulsive depth parameter V_{r2} is shown in **Figure 3**. In **Figure 4**, we present the variation of the attractive range parameter R_{a2} with respect to the attractive depth parameter V_{a2} for the second type of potentials. Also, in **Figure 5**, we present the variation of the repulsive range parameter R_{r2} with respect to the re-

pulsive depth parameter V_{r2} for the second type of potentials. In Figure 6, we present the variation of the repulsive range parameter R_{r2} with respect to the attractive range parameter R_{a2} for the second type of potentials. Finally, in Figure 7, we present the variation of the repulsive range parameter R_{r2} with respect to the attractive depth parameter V_{a2} for the second type of potentials.

As seen from the figures it is always possible to find a potential of the form given by either of Equation (3.1) or Equation (3.2), in the considered ranges of values of the potential parameters, which gives rise to good agreement between the calculated value of the binding energy of ^{12}C and the corresponding experimental value. Among all the resulting potentials of the second type of potentials, the following potential gives the best agreement

$$V_{\alpha\alpha}^{(2)} = -33.27\exp\left(-\frac{R^2}{(3.02)^2}\right) + 26.41\exp\left(-\frac{R^2}{(2.72)^2}\right) \tag{5.2}$$

This potential produces the value 91.98 MeV for the binding energy of ^{12}C.

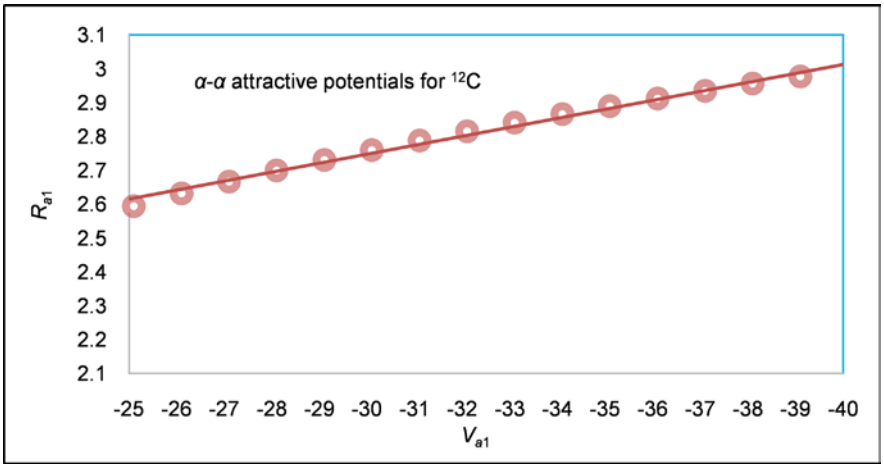

Figure 1. The dependence of the attractive range parameter (R_{a1}) on the corresponding attractive depth parameter (V_{a1}) for the first potential.

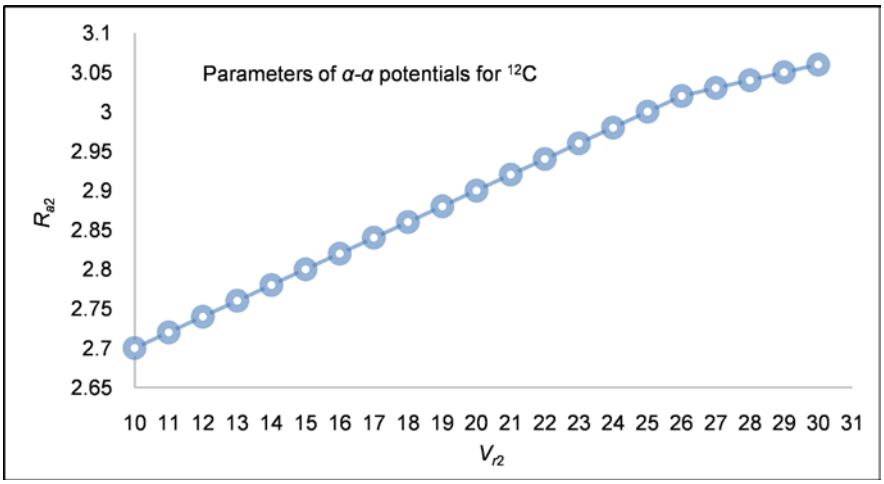

Figure 2. The dependence of the attractive range parameter (R_{a2}) on the repulsive depth parameter (V_{r2}) for the second potential.

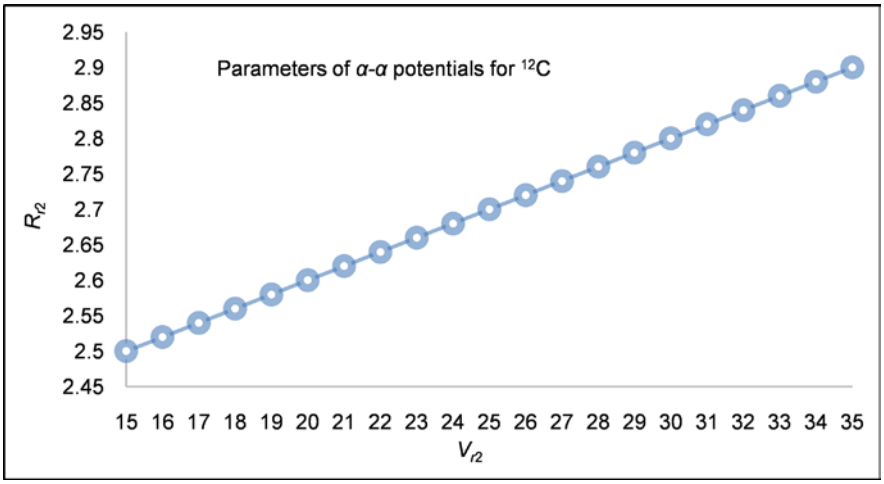

Figure 3. The dependence of the repulsive range parameter (R_{r2}) on the repulsive depth parameter (V_{r2}) for the second potential.

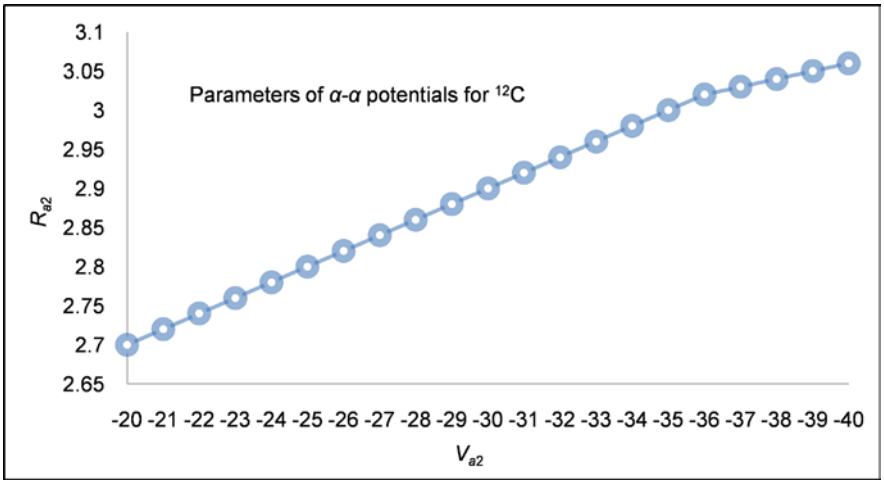

Figure 4. The dependence of the attractive range parameter (R_{a2}) on the attractive depth parameter (V_{a2}) for the second potential.

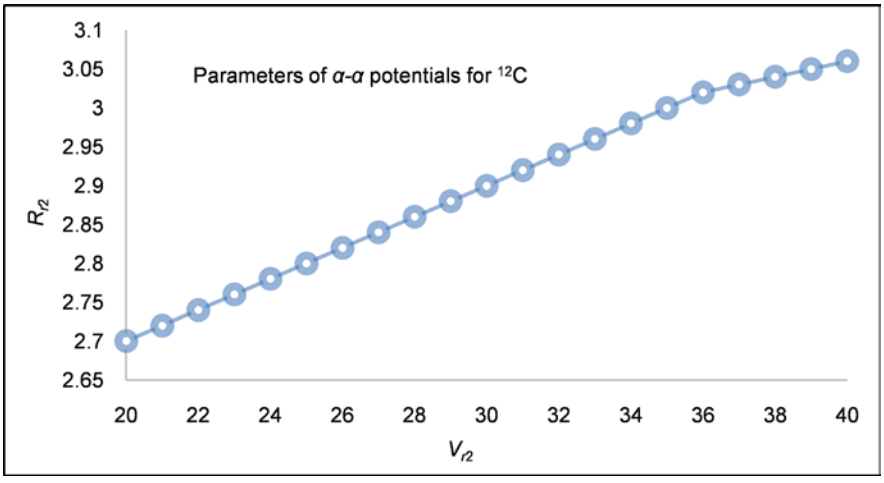

Figure 5. The dependence of the repulsive range parameter (R_{r2}) on the repulsive depth parameter (V_{r2}) for the second potential.

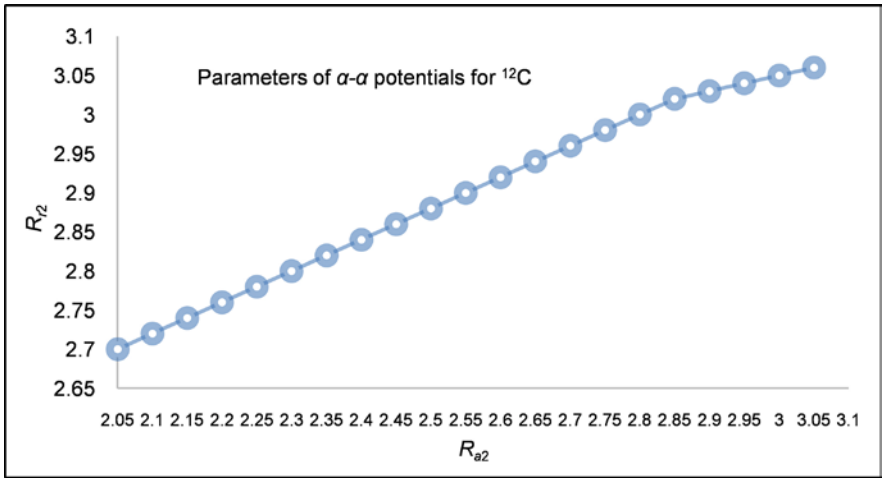

Figure 6. The dependence of the repulsive range parameter (R_{r2}) on the attractive range parameter (R_{a2}) for the second potential.

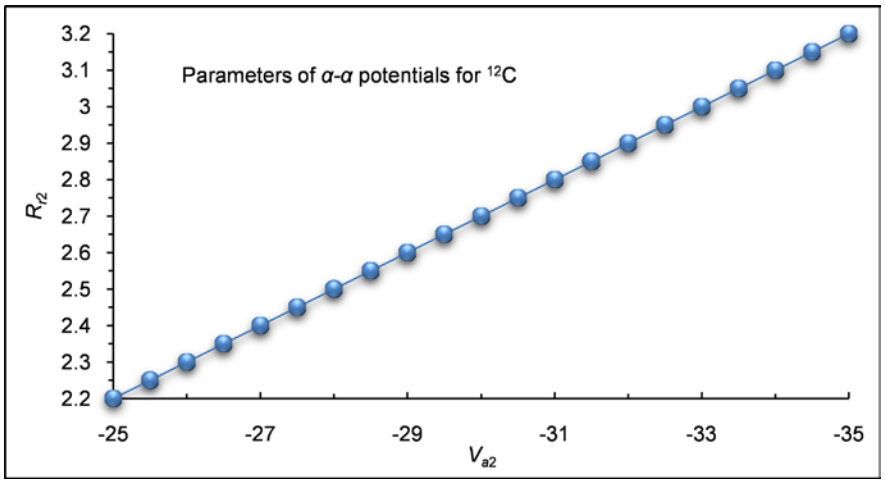

Figure 7. The dependence of the repulsive range parameter (R_{r2}) on the attractive depth parameter (V_{a2}) for the second potential.

Indeed, each cluster-cluster potential is a collection of nucleon-nucleon Gaussian potentials arranged in some way in order to average the effects of their mutual interactions in the whole nucleus.

Concerning the calculated ground-state nuclear wave function of the nucleus ^{12}C, it results as a superposition of three S-states with symmetry type [444]. Other symmetry types are occurred in the total ground state nuclear wave function, such as: [4431], and [422].

6. Conclusion

The unitary scheme model, which is formulated by a precise application of the group theoretical methods, achieved great success in describing the ground and excited states of light nuclei by using nucleon-nucleon interactions and three-body interactions. It is then of interest to investigate the behavior of the ground state of nuclei by using four-body interactions in the form of $\alpha-\alpha$ in-

teractions. A nucleus consisting of multiples of α particles is then very interesting to be investigated in the framework of the unitary scheme model by using $\alpha-\alpha$ interactions. Accordingly, our goal in the present paper is to investigate the behavior of the ground state of the carbon nucleus in the framework of the unitary scheme model by using $\alpha-\alpha$ interactions. Two types of these suggested interactions are obtained. The first of which is an attractive Gaussian interaction and the second is a superposition of repulsive and attractive Gaussian forces. The two potentials produced good results for the binding energy of the ^{12}C nucleus.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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