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Using TODIM Approach with TOPSIS and Pythagorean Fuzzy Sets for MCDM Problems in the Bullwhip Effect

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Abstract

The aim of this paper was to explore decision-making using Pythagorean fuzzy sets with an acronym in Portuguese for Interactive multi-criteria decision making (TODIM) approach. We propose a novel decision-making model that TODIM integrates the technique for order of preference by similarity to the ideal solution (TOPSIS) and fuzzy concepts to solve problems related to the core alternatives facing innovative companies under the bullwhip effect (BWE). Our findings indicate that demand forecast updating (A_1) represents a compromise solution. The proposed model offers a novel approach to select the best alternative to manage the bullwhip effect by optimizing decision-making. We proposed a new concept of PF-TODIM approach, Q_i based on preferences, the optimal similarity-based measure, S_i and the optimal distance-based measure, d_i to complete the optimal research method. This research is the first to apply TODIM method for multi-criteria decision making (MCDM) problems in ambiguous environment of the BWE, thereby providing decision-makers with directions in designing management models and setting business strategies to determine optimal target markets. From the perspective of innovative enterprises, precise handling of uncertainly factors is the principal strategy for success.

Keywords

The Bullwhip Effect, TODIM Approach, TOPSIS, MCDM

1. Introduction

Decision-making in an uncertain environment involves selecting from alternatives with unclear boundaries and conditions (Bellman & Zadeh, 1970). In today's business environment full of uncertainties and opportunities, strategy formulation and management are important tasks faced by decision-makers. Drawing on a wealth of literature on the bullwhip effect (BWE), this paper proposes applying the TODIM approach to explore the revenue of innovative enterprises. To achieve this, we introduce the Pythagorean fuzzy set (PFS), defined by Yager et al. (Yager & Abbasov, 2013), which has satisfied the condition of the square sum of membership degree and nonmembership degree is less than or equal to 1, which can deal with IFS for which the sum of membership degree and nonmembership degree is less than or equal to 1, cannot manage problems.

Zadeh (Zadeh, 1965) reported that the properties of Pythagorean fuzzy sets (PFS) depend on the degree of membership. We also review research on the intuitionistic fuzzy set (IFS) and the hesitant fuzzy set (HFS), with a focus on determining the ways that fuzzy sets affect decision-making. This concept is structured around three core areas: definitions, axioms, and applications. We further discuss integrating PF-TODIM approach for MCDM problems of the BWE in PF-environment.

1.1. Trajectory of Development for Fuzzy Set Theory

Fuzzy logic has grown from the first proposal by Zadeh (Zadeh, 1965) of a fuzzy set to discuss concepts that are vague and not easily defined or measured. Subsequently, Atanassov (Atanassov, 1986) proposed an intuitionistic fuzzy set (IFS) to solve the problem of ambiguity. Yager (Yager & Abbasov, 2013) then proposed a Pythagorean fuzzy set (PFS) to further expand the applicability of the IFS (**Figure 1**).

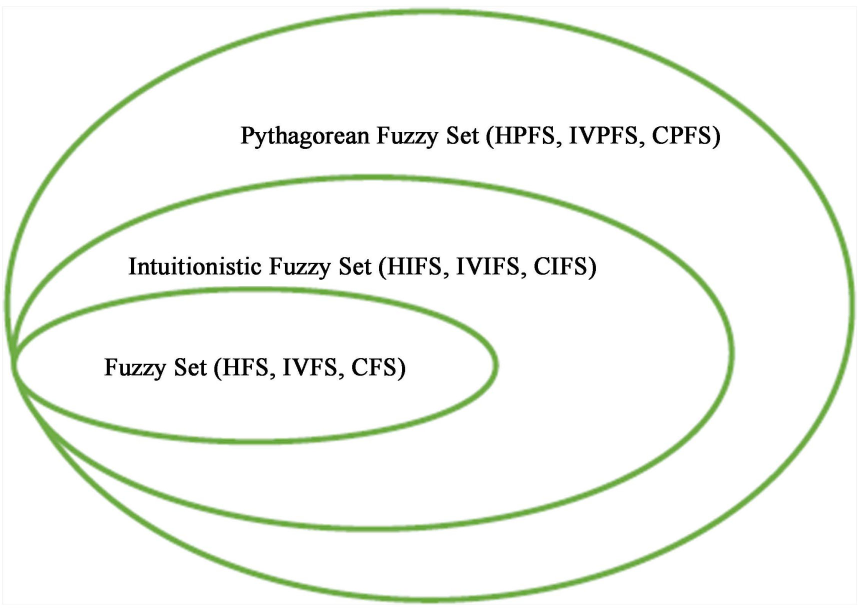

Figure 1. Relationships among types of fuzzy sets (Constructed by Shu-Mei Lin, 2020).

Over five decades of development, the field of fuzzy logic has moved through nine important stages. In 1965, Zedeh (Zadeh, 1965) proposed the concept of fuzzy logic and defined fuzzy sets, which have only one geometric interpretation. In 1986, intuitionistic fuzzy sets (IFSs) were proposed by Atanassov (Atanassov, 1986), for which each element $x \in E$ has degrees of membership and non-membership <μ, ν>, which are equal to <1, 0> if IFS *A* is a subset of IFS *B*, equal to <0, 1>, IFS *B* is a subset of IFS *A*, and *B* contains real numbers in the interval [0, 1] for which $\mu + \nu \leq 1$. For arbitrary two IFSs *A* and *B*, $A \cup B = B$ if $A \cap B = A$. In 1999, Atanassov (Atanassov, 1999) developed interval-valued intuitionistic fuzzy sets (IVIFSs). IVIFS *A* over *E* is defined as an object of the form = $\left\{\left\langle x, M_A(x), N_A(x)\right\rangle \mid x \in E\right\}$, where $M_A(x) \subset [0,1]$ and $N_A(x) \subset [0,1]$ are intervals, and for all $x \in E$, $\sup M_A(x) + \sup N_A(x) \leq 1$. M_A and N_A are interpreted as membership functions. In 2002, crisp sets were extended to fuzzy sets, referred to as complex fuzzy sets (CFSs), by Ramot et al. (Ramot, Milo, Friedman, & Kandel, 2002). For CFSs, the membership function is defined by two variables, the range of which is not limited to [0, 1], but extended to the unit circle in the complex plane. In 2010, hesitant fuzzy sets (HFSs) were proposed by Torra (Torra, 2010); an HFS is an IFS with a non-empty closed interval in 2012, Alkouri et al. (Alkouri & Salleh, 2012) proposed complex-intuitionistic fuzzy sets (CIFSs), which are IFSs with complex-valued membership and nonmembership functions. In 2013, Pythagorean fuzzy sets (PFSs) were proposed by Yager (Yager & Abbasov, 2013). PFSs involve the idea of Pythagorean membership grades: a PFS comprises pairs (μ, ν) that satisfy the condition that $\mu^2 + \nu^2 \leq 1$. These sets are capable of handling a wider range of ambiguity than IFSs. In 2015, interval-valued Pythagorean fuzzy sets (IVPFSs) were proposed by Liang et al. (Liang, Zhang, & Liu, 2015). These sets build a weighted arithmetic averaging operator to establish an optimization model for determining the weights of criteria for each expert. This constructs a minimizing consistency optimal model to derive the weights of criteria for the decision-making group. In 2020, Ullah et al. (Ullah, Mahmood, Ali, & Jan, 2020) proposed complex Pythagorean fuzzy sets (CPFSs). CFSs cannot handle "yes" and "no" data types and CIFSs are only applicable to a limited range of values. CPFSs overcome these limitations. The above development trajectory is depicted in **Figure 2**.

1.2. Brief to the Bullwhip Effect Concept

Forrester (Forrester, 1961) pointed out that consumer demand is always unstable, but companies need to optimize the allocation of inventory and other resources by predicting consumer demand. In this endeavor, Lee et al. (Lee, Padmanabhan, & Whang, 1997b) warned against distortion in demand information; that is, manufacturers who only observe their immediate order data will be misled by the amplified demand pattern, which has a serious impact on costs. This phenomenon was termed "the bullwhip effect". Manufacturers and retailers

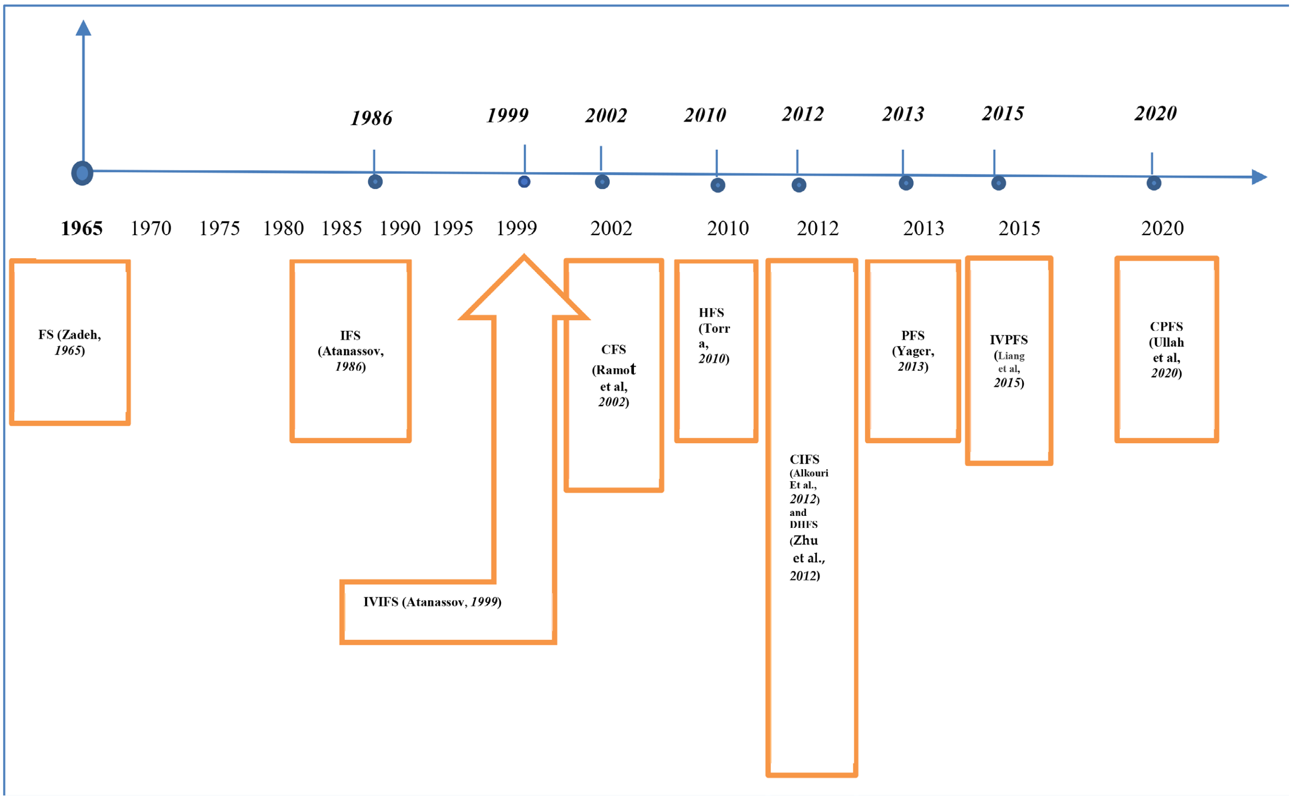

Figure 2. Chronological development of fuzzy logic (Constructed by Shu-Mei Lin, 2020).

tend to carry stock to protect against variability in the supply chain. The bullwhip effect causes upstream suppliers to experience more inventory uncertainty than that experienced by supply chain members who are closer to the customer. In essence, the end customer holds the whip handle, resulting in order fluctuation upstream (Ouyang, 2007). van Engelenburg et al. (van Engelenburg, Janssen, & Klievink, 2018) proposed applying a blockchain architecture to reduce the bullwhip effect. Zhao et al. (Zhao, Mashruwala, Pandit, & Balakrishnan, 2019) introduced the concept of supply chain relational capital to the discussion. Adnan and Ozelkan (Adnan & Ozelkan, 2020) proposed the revenue-sharing contract while Zhu et al. (Zhu, Balakrishnan, & da Silveira, 2020) considered the problem within the context of the oil and gas industries. Lee et al. (Lee, Padmanabhan, & Whang, 1997a, 1997b) highlighted the factors of demand forecast updating, order batching, price fluctuation, and rationing and shortage gaming as the standard drivers of the BWE. These are described in Table 1.

1.3. Brief to the TODIM Approach

Gomes and Lima (Gomes & Lima, 1992) proposed TODIM model to handle MCDM problems. Later, Krohling et al. (Krohling, Pacheco, & Siviero, 2013) modified the TODIM model to encompass cases involving uncertainty, resulting in the F-TODIM, while Ren et al. (Ren, Xu, & Gou, 2016) expanded the original model to include fuzzy environments to create the PF-TODIM. According to

Table 1. The standard drivers of the bullwhip effect.

Alternatives	Criteria
Demand forecast updating (DFU) A_1	Historical record of the relationship between orders and consumers (C_1)
Order batching (OB) (A_2)	Procuring times (C_2)
Price fluctuation (PF) (A_3)	Based on price fluctuations as a reference for inventory determination (C_3)
Rationing and Shortage gaming (R and SG) (A_4)	In the expected supply and demand of the market, the relationship of the number of orders received by upstream manufacturers and real market demand (C_4)

multi-criteria decision making (MCDM) method is always looking for a solution corresponding to the maximum value of certain global value measures of the DM, the TODIM approach is based on the prospect theory (Kahneman & Tversky, 1979) to evaluate the DM's behavior preferences and find the optimal alternative for the DM.

1.4. Brief to the TOPSIS Method

Hwang and Yoon (Goodwin & Franklin, 1994) developed the technique for order of preference by similarity to the ideal solution (TOPSIS). It selects the optimal decision by finding the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS) and addition closeness coefficient to identified close with PIS index or NIS index to ensure rigor for measurements. In the recently, İç (İç, 2012) and Zhang and Xu (Zhang & Xu, 2014) expanded this technique to apply to fuzzy environments through the development of the PF-TOPSIS.

1.5. Brief to the MCDM Problems Concept

Opricovic and Tzeng (Opricovic & Tzeng, 2004) mentioned that the MCDM analysis model proposed in 1980 is deals with decision-making involving the consideration of multiple or conflicting criteria, attributes, goals and determined the optimal alternative. The processing stages of MCDM are as follows:

1) Establish survey criteria for linking system functions and goals.

2) Develop an alternative system to achieve the goal (i.e., generate alternatives).

3) Evaluate the alternatives according to the criteria (i.e., find the value of the criteria function).

4) Apply standardized multi-criteria analysis to rank alternatives.

5) Choose the best option.

6) If the final result is not suitable, collect new data and iterate the procedure.

2. Preliminary

In this section we review important concepts related to our research aims.

2.1. Pythagorean Fuzzy Sets

Pythagorean fuzzy sets (PFSs) were proposed by Yager and Abbasov (Yager & Abbasov, 2013). A PFS comprises pairs (μ, ν) that satisfy the condition that $\mu^2 + \nu^2 \le 1$. PFSs are more useful than IFSs, as PFSs can represent a larger range of uncertainty. For example, suppose a decision-maker generates an alternative with the value $\frac{\sqrt{3}}{2}$, with non-membership represented by $\frac{1}{2}$. The sum of these two values is greater than 1, which makes this case unsuited to an IFS. However, since $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \le 1$, a PFS may be applied.

Definition 1. Yager and Abbasov (Yager & Abbasov, 2013)

Let *Z* be a universe of discourse. A PFS *p* includes *Z*.

$$p = \left\{ \left\langle z, \mu_p(z), \nu_p(z) \right\rangle \mid z \in Z \right\} \tag{1}$$

where $\mu_p : Z \to [0,1]$ represents membership and $\nu_p : Z \to [0,1]$ represents non-membership of element $x \in Z$ for set *P*. For each $z \in Z$, $0 \le (\mu_p(z))^2 + (\nu_p(z))^2 \le 1$. The degree of hesitation for each fuzzy member of the set is determined using the following function:

$\pi_p(z) = \sqrt{1 - (\mu_p(z))^2 + (\nu_p(z))^2}$. For the sake of convenience, Zhang and Xu [30] recommend using $p = (\mu_p, \nu_p)$ to represent a Pythagorean fuzzy number.

Definition 2. Zhang and Xu (Zhang & Xu, 2014)

For any two Pythagorean fuzzy numbers $\varrho_1 = p(\mu_{\varrho_1}, \nu_{\varrho_1})$ and $\varrho_2 = p(\mu_{\varrho_2}, \nu_{\varrho_2})$, let $s(\varrho_1)$ and $s(\varrho_2)$ represent the score function of ϱ_1 and ϱ_2, respectively.

1) If $s(\varrho_1) < s(\varrho_2)$, *then* $\varrho_1 \prec \varrho_2$.

2) If $s(\varrho_1) > s(\varrho_2)$, *then* $\varrho_1 \succ \varrho_2$.

3) If $s(\varrho_1) = s(\varrho_2)$, *then* $\varrho_1 \sim \varrho_2$.

Definition 3. Zhang (Zhang, 2016)

To solve comparison of PFSs $\beta = (\mu_\beta, \nu_\beta)$, proposed closeness index $c(\beta)$.

If $\beta = (\mu_\beta, \nu_\beta)$ is a Pythagorean fuzzy number, defined $c(\beta)$ is the closeness index, as follows:

$$c(\beta) = \frac{1 - (\nu_\beta)^2}{2 - (\mu_\beta)^2 - (\nu_\beta)^2} \tag{2}$$

According to closeness index $c(\beta)$, the comparison method for ranking PFN is indicated:

1) If $c(\beta_1) < c(\beta_2)$, then $\beta_1 \prec \beta_2$.

2) If $c(\beta_1) > c(\beta_2)$, then $\beta_1 \succ \beta_2$.

3) If $c(\beta_1) = c(\beta_2)$, then $\beta_1 \sim \beta_2$.

Definition 4. Yager and Abbasov (Yager & Abbasov, 2013)

If $\beta = (\mu_\beta, \nu_\beta) = (\gamma_\beta, \gamma_\beta)$ represents a Pythagorean fuzzy number, then *V* is defined as follows:

$$V(\beta)=\frac{1}{2}+\gamma_\beta\left(d_\beta-\frac{1}{2}\right)=\frac{1}{2}+\gamma_\beta\left(\frac{1}{2}-\frac{2\theta_\beta}{\pi}\right) \tag{3}$$

$V(\beta)$ can be used to compare two PFSs, as follows:

1) If $V(\beta_1)>V(\beta_2)$, then $\beta_1 \succ \beta_2$;

2) If $V(\beta_1)=V(\beta_2)$, then $\beta_1 \sim \beta_2$.

Definition 5. Peng et al. (Peng, Yuan, & Yang, 2017)

If *A*, *B* represent two Pythagorean fuzzy sets, then the operations can be defined as follows:

1) $A_c=\left\{(\varrho,\nu_A(\varrho),\mu_A(\varrho))\mid \varrho\in T\right\}$;

2) $A\subseteq B$ iff $\forall\varrho\in T$, $\mu_A(\varrho)\le\nu_B(\varrho)$ and $\nu_A(\varrho)\ge\nu_B(\varrho)$;

3) $A=B$ iff $\forall\varrho\in T$, $\mu_{A'}(\varrho)=\mu_B(\varrho)$ and $\nu_A(\varrho)=\nu_B(\varrho)$;

4) $\varnothing_A=\left\{(\varrho,1,0)\mid\varrho\in T\right\}$;

5) $\varnothing_A=\left\{(\varrho,0,1)\mid\varrho\in T\right\}$;

6) $A\cap B=\left\{(\varrho,\mu_A(\varrho)\wedge\mu_B(\varrho),\nu_A(\varrho)\vee\nu_B(\varrho))\mid\varrho\in T\right\}$;

7) $A\cup B=\left\{(\varrho,\mu_A(\varrho)\vee\mu_B(\varrho),\nu_A(\varrho)\wedge\nu_B(\varrho))\mid\varrho\in T\right\}$;

8) $A\oplus B=\left\{\varrho,\sqrt{\mu_A^2(\varrho)+\mu_B^2(\varrho)-\mu_A^2(\varrho)}\right\}$;

9) $A\otimes B=\left\{\left(\varrho,\mu_A(\varrho)\mu_B(\varrho),\sqrt{\nu^2(\varrho)+\nu_B^2(\varrho)-\nu_A^2(\varrho)\nu_B^2(\varrho)}\right)\mid\varrho\in T\right\}$;

Definition 6. Peng et al. (Peng, Yuan, & Yang, 2017)

If α,β and O be three PFSs on *X*, a distance measure $d(\alpha,\beta)$ is a mapping $d:\mathrm{PFS}(x)\times\mathrm{PFS}(x)\to[0,1]$, have the properties as follows:

(D1) $0\le d(\alpha,\beta)\le 1$;

(D2) $d(\alpha,\beta)=d(\beta,\alpha)$;

(D3) $d(\alpha,\beta)=0$ iff $\alpha=\beta$.

(D4) $d(\alpha,\alpha^c)=1$ iff α is a crisp set (omitted).

(D5) If $\alpha\subseteq\beta\subseteq 0$, then $d(\alpha,\beta)\le d(\alpha,\mathrm{O})$ and $d(\beta,\mathrm{O})\le d(\alpha,\mathrm{O})$.

Definition 7. Peng et al. (Peng, Yuan, & Yang, 2017)

If α,β and O be three PFSs on *X*, a similarity measure $s(\alpha,\beta)$ is a mapping $s:\mathrm{PFS}(x)\times\mathrm{PFS}(x)\to[0,1]$, have the properties as follows:

(S1) $0\le s(\alpha,\beta)\le 1$;

(S2) $s(\alpha,\beta)=s(\beta,\alpha)$;

(S3) $s(\alpha,\beta)=1$ if $\alpha=\beta$.

(S4) $s(\alpha,\beta)=0$ if α is a crisp set (omitted).

(S5) If $\alpha\subseteq\beta\subseteq 0$, then $s(\alpha,\mathrm{O})\le s(\alpha,\beta)$ and $s(\alpha,\mathrm{O})\le s(\beta,\mathrm{O})$.

Definition 8. Peng and Yang (Peng & Yang, 2015)

If $\varrho=(\mu_\varrho-\nu_\varrho)$ be a Pythagorean fuzzy number and defined $a(\varrho)$ be the accuracy function, as follows:

$$a(\varrho)=(\mu_\varrho)^2-(\nu_\varrho)^2 \tag{4}$$

where $a(x)\in[0,1]$.

For any arbitrary PFS ϱ_1,ϱ_2,

1) If $s(\varrho_1) > s(\varrho_2)$, then $\varrho_1 \succ \varrho_2$;

2) If $s(\varrho_1) = s(\varrho_2)$, then.

a) If $a(\varrho_1) > a(\varrho_2)$, then $\varrho_1 \succ \varrho_2$;

b) If $a(\varrho_1) = a(\varrho_2)$, then $\varrho_1 \sim \varrho_2$;

c) If $a(\varrho_1) < a(\varrho_2)$, then $\varrho_1 \prec \varrho_2$;

3) If $s(\varrho_1) < s(\varrho_2)$, then $\varrho_1 \prec \varrho_2$.

Pythagorean fuzzy membership allows for a larger ambiguity than intuitionistic fuzzy membership, because a Pythagorean fuzzy number can be modeled in both directions. The relationship between an intuitionistic fuzzy number and a Pythagorean fuzzy number is depicted in Figure 3.

2.2. TODIM Approach

In 1990, Gomes and Lima (Gomes & Lima, 1992) developed the TODIM approach based on prospect theory (Kahneman & Tversky, 1979) to effectively deal with MCDM problems. The mathematical form of the TODIM approach is presented below (Gomes) (Gomes, 2009):

1) Compute the dominance of alternative A_i based on every alternative A_t on criterion C_j using the comparison matrix $\varnothing_j = \left[\varnothing_j(A_i, A_t)\right]_{m\times m}$ where

$$\varnothing_j(A_i, A_t) = \begin{cases} \sqrt{\dfrac{w_{jr} d(A_{ij}, A_{tj})}{\sum_{j=1}^{n} w_{jr}}}, & \text{if } f_{ij} - f_{tj} > 0 \\ 0, & \text{if } f_{ij} - f_{tj} = 0 \\ \dfrac{1}{-\theta}\sqrt{\left(\sum_{j=1}^{n} w_{jr}\right)\dfrac{d(A_{ij}, A_{tj})}{w_{jr}}}, & \text{if } f_{ij} - f_{tj} < 0 \end{cases} \quad (5)$$

In the above, $d(A_{ij}, A_{tj})$ is the distance measure between two alternatives where θ represents the attenuation factor of the losses,

2) $\left[\delta(A_i, A_t)\right]_{m\times m}$ is the dominance matrix of alternative A_i for every alternative A_t where

$$\delta(A_i, A_t) = \sum_{j=1}^{n} \varnothing_j(A_i, A_t), (i, t = 1, 2, \cdots, m) \quad (6)$$

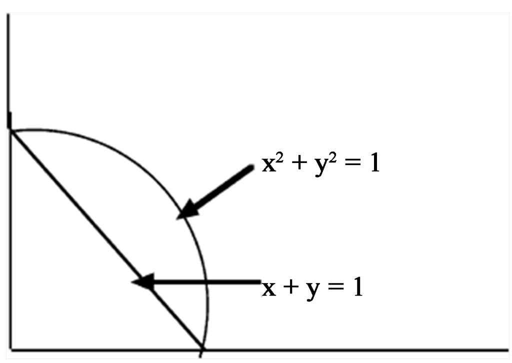

Figure 3. Graphical depictions of Pythagorean and intuitionistic fuzzy numbers (Cited from Yager, 2013).

3) $A_i\left(i=1,2,\cdots,m\right)$ is an alternative total value, which is obtained using the following:

$$\varnothing_j\left(A_i\right)=\frac{\sum_{t=1}^{m}\delta\left(A_i,A_t\right)-\min_{i=1}^{m}\left[\sum_{t=1}^{m}\delta\left(A_i,A_t\right)\right]}{\max_{i=1}^{m}\left[\sum_{t=1}^{m}\delta\left(A_i,A_t\right)\right]-\min_{i=1}^{m}\left[\sum_{t=1}^{m}\delta\left(A_i,A_t\right)\right]} \quad (7)$$

From the above three steps, we can confirm that the partial of advantage degrees for every alternative is better than the others. Ultimately, we can rank the alternatives.

2.3. TOPSIS

Hwang and Yoon (Hwang & Yoon, 1981) developed the TOPSIS method in 1981. It defines a relative index based on closeness or similarity as follows:

Step 1. Normalize ratings:

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}x_{ij}^2}},i=1,\cdots,m;\ j=1,\cdots,n. \quad (8)$$

Step 2. Weight normalized ratings:

$$\upsilon_{ij}=w_j r_{ij},i=1,\cdots,m;\ j=1,\cdots,n, \quad (9)$$

where w_j is the weight of the jth attribute.

Step 3. Determine PIS and NIS:

$$A^+=\left\{\upsilon_1^+,\cdots,\upsilon_n^+\right\} \quad (10)$$

$$A^-=\left\{\upsilon_1^-,\cdots,\upsilon_n^-\right\} \quad (11)$$

where A^+ represents the PIS and A^- represents the NIS. If the *j*th criteria is a benefit criteria, then $\upsilon_j^+=\max\left\{v_{ij},i=1,\cdots,m\right\}$ and $\upsilon_j^-=\min\left\{v_{ij},i=1,\cdots,m\right\}$. If the *i*th criteria is a cost criteria, then $\upsilon_j^+=\min\left\{v_{ij},i=1,\cdots,m\right\}$ and $\upsilon_j^-=\max\left\{v_{ij},i=1,\cdots,m\right\}$.

Step 4. Compute the distances from each alternative to PIS and NIS:

A^+ is a positive result, calculated according to the following formula:

$$D_i^+=\sqrt{\sum_{j=1}^{n}\left(v_{ij}-v_j^+\right)^2},i=1,\cdots,m. \quad (12)$$

A^- is a negative result, calculated according to the following formula:

$$D_i^-=\sqrt{\sum_{j=1}^{n}\left(v_{ij}-v_j^-\right)^2},i=1,\cdots,m. \quad (13)$$

Step 5. Compute distance and similarity by finding the relative closeness coefficient to the ideal solution:

$$C_{d_i}=\frac{D_i^-}{D_i^++D_i^-},i=1,\cdots,m, \quad (14)$$

$$C_{s_i}=\frac{s_i^-}{s_i^++s_i^-},i=1,\cdots,m, \quad (15)$$

$$C_i = \frac{C_{d_i}}{C_{s_i}}. \tag{16}$$

Step 6. Ranking the alternatives:

Rank alternatives based on C_i in descending order.

2.4. Bullwhip Effect

The first was proposed by Forrester in Industrial Dynamics. In general business activities, customer demand is always unstable, and enterprises always need to optimize the allocation of resources such as inventory by predicting customer demand. Forecasts are based on statistics and cannot be completely accurate, so companies tend to reserve some extra inventory as safety stock in their operations. In the supply chain, from downstream to upstream, from the end customer to the original supplier, each component requires more and more safety stock. During periods of rising demand, downstream firms increase the number of upstream orders, and during periods of declining demand, downstream firms reduce or stop ordering. This change in demand will be amplified as supply chains recover. The magnification of this information distortion is much like a whip on a graphic display, so it is figuratively called the whip effect. The most downstream client is the root of the whip, and the most upstream supplier is the tip of the whip. If there is a slight jiggle at one end of the root, a large wave is transmitted to the tip. Another upstream of this impact is the supply chain, where the greater the change and the further away from the end customer, the greater the impact (Forrester, 1961).

This leads to a kind of information distortion which is vividly described by the term 'the bullwhip effect'. The end-customer is represented by the handle while upstream suppliers are represented by the whip end. A small shift in the handle end results in a large fluctuation at the tip of the whip. Lee et al. (Lee, Padmanabhan, & Whang, 1997a, 1997b) proposed four crucial factors underlying the bullwhip effect: demand forecast updating, order batching, price fluctuation, and rationing and shortage gaming. Ouyang (Ouyang, 2007) analyzed these using linear programming and time-invariant inventory management. Ouyang and Daganzo (Ouyang & Daganzo, 2008) suggested that Markovian jump linear systems could reduce the bullwhip effect, while van Engelenburg et al. (van Engelenburg, Janssen, & Klievink, 2018) applied a blockchain architecture. Zhao et al. (Zhao, Mashruwala, Pandit, & Balakrishnan, 2019) used multivariate regression, and Adnan and Ozelkan (Adnan & Ozelkan, 2020) used a stochastic model.

2.5. MCDM Problems

MCDM was proposed by Yoon & Hwang in 1981. It is a powerful technique for making-decisions under complex conditions to help decision-makers in a limited number of feasible plans, based on the particularity of each at-tribute of each plan, from the feasible plans, make the ranking of pros and cons for each measure and select a solution to satisfying the decision maker's ideals (Hwang & Yoon, 1981).

MCDM helps in the selection of an optimal solution from among a range of alternatives based on criteria weighting (Zaeri, Sadeghi, Naderi, Fasihy, Shorshani, & Poyan, 2011) MCDM criteria are divided into two types: benefits and costs. Let us consider a typical MCDM problems with alternatives ($A_1, A_2, \cdots, A_m$) and criteria ($C_1, C_2, \cdots, C_n$). This scenario can be depicted as follows:

$$X = \left[x_{ij}\right]_{m\times n},\ W = \left[w_j\right]_n$$

where X is a decision matrix expressing the performance of the ith alternative performance according to the jth criterion and W is a weight vector expressing the weight of each criterion. In practice, a decision-making problem such as this likely involves uncertainty. In these cases, the application of fuzzy numbers is useful (Zhu, Xu, & Xia, 2012). Also, there was other researcher proposed using others research methods to be solving MCDM problems (Mary & Suganya, 2016).

3. Applying TODIM Approach Based on Pythagorean Fuzzy Sets to Describe Bullwhip Effect

Lee et al. (Lee, Padmanabhan, & Whang, 1997a, 1997b) proposed that the bullwhip effect is driven by four important factors: demand forecast updating, order batching, price fluctuation, and rationing and shortage gaming. Suppose an innovation enterprise is faced with a set of four alternatives: A_1 (demand forecast updating), A_2 (order batching), A_3 (price fluctuation), and A_4 (rationing and shortage gaming). The criteria are C_1 (historical records of the relationship between orders and consumers), C_2 (procurement times), C_3 (using price fluctuations as a reference for inventory determination), and C_4 (relationship between the number of orders received by upstream manufacturers and real market demand). The manager is tasked with selecting from among these alternatives. Raj and Kumar (Raj & Kumar, 1999) proposed a ranking concept for fuzzy numbers and linguistic variables; it involves applying fuzzy weights ($\bar{w}_i$) to alternatives (A_i) using fuzzy arithmetic. It further utilizes the concepts of max and min to determine total utility or ordering value for each of the alternatives. In this paper, we propose a novel approach using the PF-TODIM approach combined with the optimal distance-based measure of PF-TOPSIS to solve PF-MCDM problems related to the bullwhip effect.

3.1. Research Framework

To achieve our aims, we applied the research framework depicted in **Figure 4**.

1) Build the PF-values matrix; 2) Input PF-values to PF-TODIM algorithm; 3) Transform PF-TODIM values to PF-TOPSIS technique process; 4) Compute the

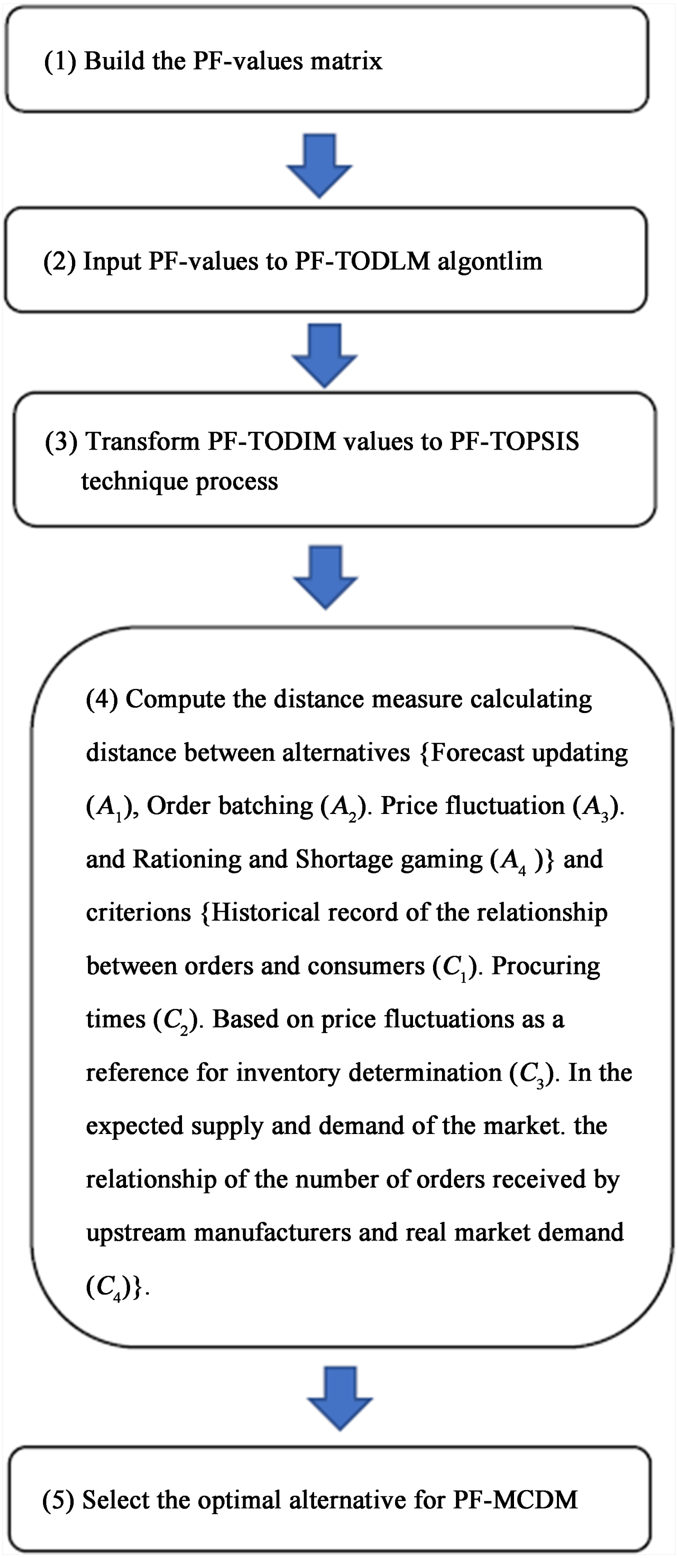

Figure 4. Research framework (Constructed by Shu-Mei Lin, 2020).

distance measure; 5) Select the optimal alternative for PF-MCDM.

A compromise solution is determined as shown in Figure 5.

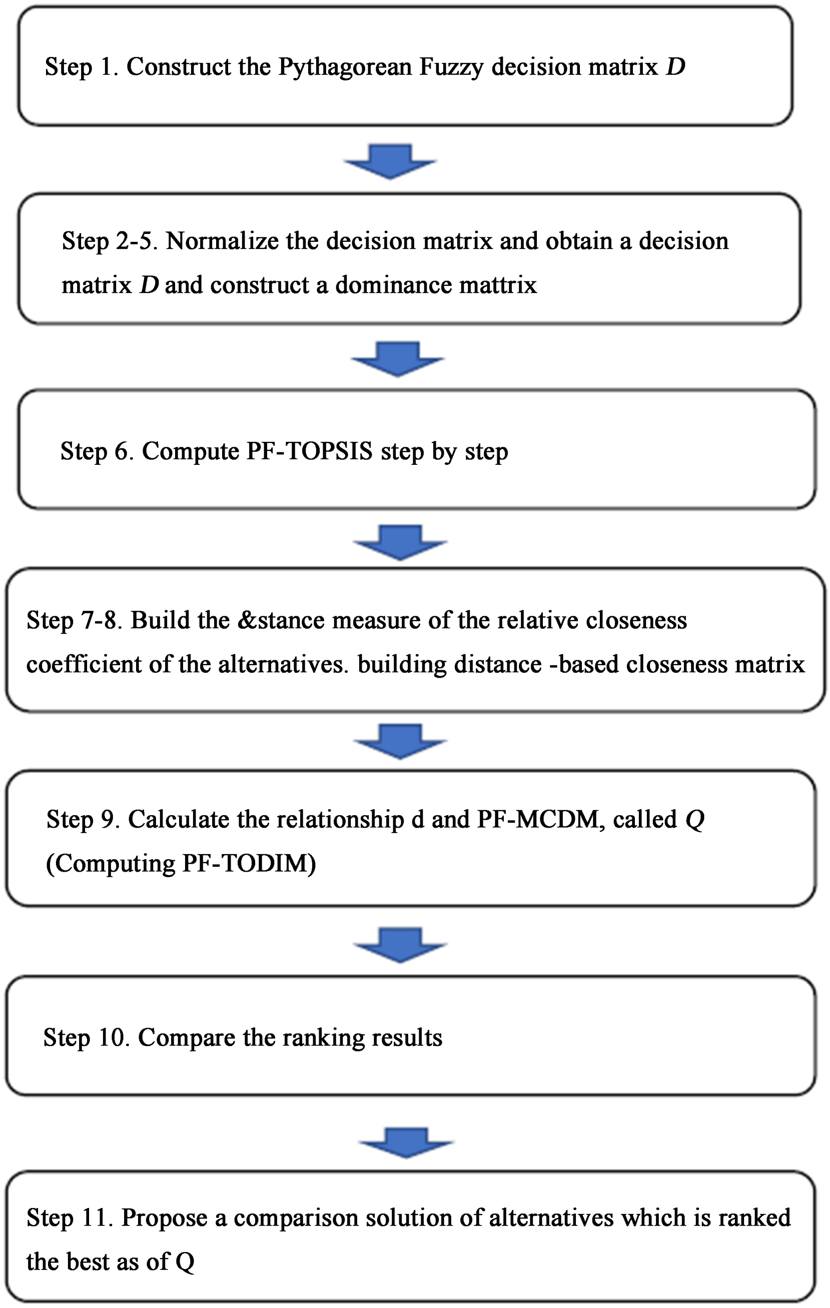

Figure 5. Procedure of proposed approach (Constructed by Shu-Mei Lin, 2020).

3.2. Numerical Experiment

In the above example with alternatives $A = \{A_1, A_2, A_3, A_4\}$ and criteria $C = \{C_1, C_2, C_3, C_4\}$, the decision-maker is tasked with selecting a strategy to minimize the negative impact of the bullwhip effect. In this paper, the evaluation of alternatives is based on the given Pythagorean fuzzy numbers; these values are used to construct the Pythagorean fuzzy decision matrix shown in Table 2.

Applying PF-TOPSIS method to PF-MCDM:

Step 1 Given Table 2, PF decision matrix D as follows:

Table 2. PF decision matrix.

D	C_1	C_2	C_3	C_4
A_1	(0.8, 0.1)	(0.7, 0.1)	(0.3, 0.1)	(0.6, 0.7)
A_2	(0.3, 0.2)	(0.8, 0.2)	(0.5, 0.4)	(0.7, 0.5)
A_3	(0.7, 0.6)	(0.4, 0.7)	(0.2, 0.8)	(0.2, 0.3)
A_4	(0.2, 0.5)	(0.5, 0.3)	(0.4, 0.2)	(0.6, 0.5)

$$D = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & 0.1 & 0.1 & 0.1 & 0.6 \\ A_2 & 0.2 & 0.2 & 0.4 & 0.5 \\ A_3 & 0.6 & 0.4 & 0.2 & 0.2 \\ A_4 & 0.2 & 0.3 & 0.2 & 0.5 \end{pmatrix}.$$

Step 2. Obtain benefit-criterion normalized decision matrix D':

$$D = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & 0.1 & 0.1 & 0.1 & 0.6 \\ A_2 & 0.2 & 0.2 & 0.4 & 0.5 \\ A_3 & 0.6 & 0.4 & 0.2 & 0.2 \\ A_4 & 0.2 & 0.3 & 0.2 & 0.5 \end{pmatrix}$$

$$\Rightarrow$$

$$D' = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & \frac{1}{0.1} & \frac{1}{0.1} & \frac{1}{0.1} & \frac{1}{0.6} \\ A_2 & \frac{1}{0.2} & \frac{1}{0.2} & \frac{1}{0.4} & \frac{1}{0.5} \\ A_3 & \frac{1}{0.6} & \frac{1}{0.4} & \frac{1}{0.2} & \frac{1}{0.2} \\ A_4 & \frac{1}{0.2} & \frac{1}{0.3} & \frac{1}{0.2} & \frac{1}{0.5} \end{pmatrix}$$

$$\Rightarrow$$

$$D' = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & 10 & 10 & 10 & 1.66 \\ A_2 & 5 & 5 & 2.5 & 2 \\ A_3 & 1.66 & 2.5 & 5 & 5 \\ A_4 & 5 & 3.33 & 5 & 2 \end{pmatrix}.$$

Step 3. Calculate the weight of the criteria: $w = (0.227, 0.261, 0.182, 0.18)$.

$$D' = \begin{pmatrix} & 0.227 & 0.261 & 0.182 & 0.18 \\ & C_1 & C_2 & C_3 & C_4 \\ A_1 & 10 & 10 & 10 & 1.66 \\ A_2 & 5 & 5 & 2.5 & 2 \\ A_3 & 1.66 & 2.5 & 5 & 5 \\ A_4 & 5 & 3.33 & 5 & 2 \end{pmatrix}$$

Table 3. Candidate priority ranking.

	Preference				Rank
	C_1	C_2	C_3	C_4	
A_1	2.27	2.61	1.82		1
A_2					
A_3				0.9	2
A_4					

$$\Rightarrow$$

$$D' = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & 2.27^* & 2.61^* & 1.82^* & 0.3^- \\ A_2 & 1.135 & 1.307 & 0.45 & 0.36 \\ A_3 & 0.37^- & 0.65^- & 0.91 & 0.9^* \\ A_4 & 1.13 & 0.87 & 0.91 & 0.9 \end{pmatrix}$$

$$\Rightarrow$$

Candidate priority ranking: $A_1 \succ A_3$, A_2 and A_4 are excluded (**Table 3**).

Step 4. Calculate the weight ratio as follows:

$$w_r = \max\{w_j \mid j = 1, 2, \cdots, n\} = \max\{0.227, 0.261, 0.182, 0.09\}$$

$$w_{jr} = w_j / w_r$$

$$w_{1r} = 0.869,\quad w_{2r} = 1,\quad w_{3r} = 0.697,\quad w_{4r} = 0.344$$

$$w_{jr} = (0.869, 1, 0.697, 0.344)$$

Step 5. Obtain the dominance matrix as follows:

$$f_{ij} = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & 2.27^+ & 2.614^+ & 1.826^+ & 0.301^- \\ A_2 & 1.135 & 1.3 & 0.456 & 0.361 \\ A_3 & 0.378^- & 0.653^- & 0.913 & 0.904^+ \\ A_4 & 1.135 & 0.871 & 0.913 & 0.361 \end{pmatrix}$$

$$f_j^+ = (2.27, 2.614, 1.826, 0.904)$$

$$f_j^- = (0.378, 0.653, 0.456, 0.301)$$

$$w_j = (0.227, 0.261, 0.182, 0.18)$$

Step 6. Perform TOPSIS as follows:

6.1) Normalize ratings:

To ensure that different types of quantities can be compared, they must be normalized. Normalization is performed as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, (i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m). \tag{17}$$

6.2) Weight normalized ratings:

$$\upsilon_{ij} = w_j r_{ij}, i = 1,\cdots,m; j = 1,\cdots,n. \tag{18}$$

The normalized decision matrix is expressed using

$$x = \left(r_{ij}\right)_{n\times m}$$

where w_j is the weight of the *j*th attribute.

6.3) Identify PIS and NIS:

$$A^+ = \left\{\upsilon_1^+,\cdots,\upsilon_n^+\right\}, \tag{19}$$

$$A^- = \left\{\upsilon_1^-,\cdots,\upsilon_n^-\right\} \tag{20}$$

where A^+ represents the PIS and A^- represents the NIS. If the *j*th criterion is a benefit criterion, then $\upsilon_j^+ = \max\left\{v_{ij}, i = 1,\cdots,m\right\}$ and $\upsilon_j^- = \min\left\{v_{ij}, i = 1,\cdots,m\right\}$. In contrast, if the *i*th criterion is a cost criterion, then $\upsilon_j^+ = \min\left\{v_{ij}, i = 1,\cdots,m\right\}$ and $\upsilon_j^- = \max\left\{v_{ij}, i = 1,\cdots,m\right\}$.

6.4) Calculate the distances from each alternative to PIS and NIS:

The distance from the PIS to the *i*th alternative is calculated as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^+\right)^2}, i = 1,\cdots,m. \tag{21}$$

The distance from the NIS to the *i*th alternative is calculated as follows:

$$D_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2}, i = 1,\cdots,m. \tag{22}$$

6.5) Calculate the distance and relative similarity (*i.e.*, closeness) to the ideal solution:

$$C_{d_i} = \frac{D_i^-}{D_i^+ + D_i^-}, i = 1,\cdots,m, \tag{23}$$

$$C_{s_i} = \frac{s_i^-}{s_i^+ + s_i^-}, i = 1,\cdots,m, \tag{24}$$

$$C_i = \frac{C_{d_i}}{C_{s_i}}. \tag{25}$$

6.6) Rank alternatives:

Select the alternative with the largest C_i or rank alternatives based on C_i in descending order.

To calculate the dominance of the alternative, perform the following steps:

Step (i) Calculate alternative trend A_i and preference criteria for each one ($i = 1,2,\cdots,5$; $t = 1,2,\cdots,5$). Set the value of θ at 2.5. Based on decision matrix *D*, construct five dominances $\varnothing_1,\cdots,\varnothing_5$, as follows:

$$w_{jr} = \frac{w_j}{w_r} = \left(0.869, 1, 0.697, 0.689\right)$$

$$\sum_{j=1}^{4} w_{jr} = 0.869 + 1 + 0.697 + 0.689 = 3.255$$

Table 4. PF decision matrix D.

D	C_1	C_2	C_3	C_4
A_1	(0.8, 0.1)	(0.7, 0.1)	(0.3, 0.1)	(0.6, 0.7)
A_2	(0.3, 0.2)	(0.8, 0.2)	(0.5, 0.1)	(0.7, 0.5)
A_3	(0.7, 0.6)	(0.4, 0.7)	(0.2, 0.8)	(0.2, 0.3)
A_4	(0.2, 0.5)	(0.5, 0.3)	(0.4, 0.2)	(0.6, 0.5)

PF decision matrix D (Table 4)

$$D=\begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & A_{11}=p(0.8,0.1) & A_{12}=p(0.7,0.1) & A_{13}=p(0.3,0.1) & A_{14}=p(0.6,0.7) \\ A_2 & A_{21}=p(0.3,0.2) & A_{22}=p(0.8,0.2) & A_{23}=p(0.5,0.1) & A_{24}=p(0.7,0.5) \\ A_3 & A_{31}=p(0.7,0.6) & A_{32}=p(0.4,0.7) & A_{33}=p(0.2,0.8) & A_{34}=p(0.2,0.3) \\ A_4 & A_{41}=p(0.2,0.5) & A_{42}=p(0.5,0.3) & A_{43}=p(0.4,0.2) & A_{44}=p(0.6,0.5) \end{pmatrix}$$

$$d(\beta_1,\beta_2)=\frac{1}{2\left(\left|\left(\mu_{\beta_1}\right)^2-\left(\mu_{\beta_2}\right)^2\right|+\left|\left(\upsilon_{\beta_1}\right)^2-\left(\upsilon_{\beta_2}\right)^2\right|+\left|\left(\pi_{\beta_1}\right)^2-\left(\pi_{\beta_2}\right)^2\right|\right)}$$

$$\beta_1=p\left(\mu_{\beta_1},\nu_{\beta_1}\right)\ \text{ and }\ \beta_2=p\left(\mu_{\beta_2},\nu_{\beta_2}\right),$$

where $\mu_\beta,\upsilon_\beta\in[0,1],\pi_\beta=\sqrt{1-\left(\mu_\beta\right)^2-\left(\upsilon_\beta\right)^2}$. (PFSs)

Example:

$$d\left(A_{ij},A_{ij}\right)=\frac{1}{2}\left(\left|(0.8)^2-(0.7)^2\right|+\left|(0.1)^2-(0.1)^2\right|+\left|\left(\pi_{A_{ij}}\right)^2-\left(\pi_{A_{ij}}\right)^2\right|\right)$$

$$d\left(A_{ij},A_{ij}\right)=\frac{1}{2}\left(|0.64-0.49|+|0|+\left|\sqrt{1-(0.15-0)^2-(1-0-0)^2}\right|\right)$$

$$=\frac{1}{2}\left(0.15+0+\left|\sqrt{1-0.0225-1}\right|\right)=\frac{1}{2}(0.15+0+0.15)=0.15$$

Steps (i) - (ii) Calculate the closeness coefficients for each alternative.

$$CC_i=\frac{d_i^-}{d_i^-+d_i^+},(i=1,2,\cdots,n).$$

Step (iii) Alternatives are ranked based on the value of CC_i; a bigger CC_i means a better a_i.

Step 7

7.1) Construct distance-based closeness matrix (Table 5):

$$d_i=\{6.892,4.954,2.969,3.999\}$$

Ranking: $\{6.892\succ 4.964\succ 3.999\succ 2.969\}$

7.2) Construct similarity-based closeness matrix (Table 6):

Ranking: $\{5.844\succ 2.273\succ 2.2\succ 2.045\}$

Table 5. Distance-based closeness matrix.

	C_1	C_2	C_3	C_4	R_i	Rank
d_{1i}	0.751	1.289	1.289	1.781	6.892	1
d_{2i}	1.23	0.751	1.23	1.23	4.964	2
d_{3i}	0.765	0.484	0.751	0.484	2.969	4
d_{4i}	0.812	0.812	0.812	0.751	3.999	3

Table 6. Similarity-based closeness matrix.

	C_1	C_2	C_3	C_4	S_i	Rank
S_{1i}	0.255	0.202	0.202	0.692	2.045	4
S_{2i}	0.419	0.255	0.419	0.419	2.273	2
S_{3i}	1.45	0.164	0.255	0.164	2.2	3
S_{4i}	1.466	1.466	1.466	1.445	5.844	1

7.3) Compute TODIM approach

$$\varnothing_j\left(A_i, A_t\right)=\begin{cases}\sqrt{\dfrac{w_{jr} d\left(A_{ij}, A_{tj}\right)}{\sum_{j=1}^{n} w_{jr}}}, & \text{if } f_{ij}-f_{tj}>0 \\ 0, & \text{if } f_{ij}-f_{tj}=0 \\ \dfrac{1}{-\theta} \sqrt{\left(\sum_{j=1}^{n} w_{jr}\right) \dfrac{d\left(A_{ij}, A_{tj}\right)}{w_{jr}}}, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$(\varnothing_1)$

$$\varnothing_1\left(A_1, A_1\right)=\begin{cases}\sqrt{\dfrac{\dfrac{0.227}{0.868(0.454)}}{1.307}}, & \text{if } f_{ij}-f_{tj}>0 \\ 0, & \text{if } f_{ij}-f_{tj}=0 \\ \dfrac{1}{-2.5} \sqrt{1.307\left(\dfrac{0.454}{0.227}\right)}, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$$\Rightarrow$$

$$\varnothing_1\left(A_1, A_1\right)=\begin{cases}0.301, & \text{if } f_{ij}-f_{tj}>0 \\ 0, & \text{if } f_{ij}-f_{tj}=0 \\ -0.646, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$(\varnothing_2)$

$$\varnothing_2\left(A_2, A_2\right)=\begin{cases}\sqrt{\dfrac{0.261(1.063)}{1.307}}, & \text{if } f_{ij}-f_{tj}>0 \\ 0, & \text{if } f_{ij}-f_{tj}=0 \\ \dfrac{1}{-2.5} \sqrt{1.307\left(\dfrac{1.063}{0.261}\right)}, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$$\Rightarrow$$

$$\varnothing_2\left(A_2,A_2\right)=\begin{cases}0.461, & \text{if } f_{ij}-f_{tj}>0\\ 0, & \text{if } f_{ij}-f_{tj}=0\\ -0.922, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$(\varnothing_3)$

$$\varnothing_3\left(A_3,A_3\right)=\begin{cases}\sqrt{\dfrac{0.261(1.32)}{1.307}}, & \text{if } f_{ij}-f_{tj}>0\\ 0, & \text{if } f_{ij}-f_{tj}=0\\ \dfrac{1}{(-2.5)(2.569)}, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$$\Rightarrow$$

$$\varnothing_3\left(A_3,A_3\right)=\begin{cases}0.513, & \text{if } f_{ij}-f_{tj}>0\\ 0, & \text{if } f_{ij}-f_{tj}=0\\ -1.027, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$(\varnothing_4)$

$$\varnothing_4\left(A_4,A_4\right)=\begin{cases}\sqrt{\dfrac{0.261(-0.068)}{1.307}}, & \text{if } f_{ij}-f_{tj}>0\\ 0, & \text{if } f_{ij}-f_{tj}=0\\ \dfrac{1}{-2.5}\sqrt{1.307\left(-\dfrac{0.068}{0.261}\right)}, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

$$\Rightarrow$$

$$\varnothing_4\left(A_4,A_4\right)=\begin{cases}-0.117, & \text{if } f_{ij}-f_{tj}>0\\ 0, & \text{if } f_{ij}-f_{tj}=0\\ 0.234, & \text{if } f_{ij}-f_{tj}<0\end{cases}$$

7.4) Calculate the dominance matrix:

$$D'=\begin{pmatrix} & C_1 & C_2 & C_3 & C_4\\ A_1 & 2.27 & 2.614 & 1.826 & 0.301\\ A_2 & 1.135 & 1.307 & 0.456 & 0.361\\ A_3 & 0.378 & 0.653 & 0.913 & 0.904\\ A_4 & 1.135 & 0.871 & 0.913 & 0.361\end{pmatrix}$$

$$\times$$

$$D''=\begin{pmatrix} & A_1 & A_2 & A_3 & A_4\\ C_1 & 2.27 & 1.135 & 0.378 & 1.135\\ C_2 & 2.614 & 1.307 & 0.653 & 0.871\\ C_3 & 1.826 & 0.456 & 0.913 & 0.913\\ C_4 & 0.301 & 0.361 & 0.904 & 0.361\end{pmatrix}.$$

7.5) Perform TODIM approach:

($\varnothing_1$)

$$\varnothing_1 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0.000 & 0.301 & 0.301 & -0.646 \\ A_2 & -0.922 & 0.000 & -0.922 & -0.922 \\ A_3 & -1.027 & 0.513 & 0.000 & 0.513 \\ A_4 & -0.117 & -0.117 & -0.117 & 0.000 \end{pmatrix}$$

($\varnothing_2$)

$$\varnothing_2 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0.000 & 0.301 & 0.301 & -0.646 \\ A_2 & -0.922 & 0.000 & -0.922 & -0.922 \\ A_3 & -1.027 & 0.513 & 0.000 & 0.513 \\ A_4 & 0.234 & 0.234 & 0.234 & 0.000 \end{pmatrix}$$

($\varnothing_3$)

$$\varnothing_3 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0.000 & 0.301 & 0.301 & -0.646 \\ A_2 & -0.922 & 0.000 & -0.922 & -0.922 \\ A_3 & -0.027 & 0.513 & 0.000 & 0.513 \\ A_4 & -0.117 & -0.117 & -0.117 & 0.000 \end{pmatrix}$$

($\varnothing_4$)

$$\varnothing_4 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0.000 & 0.301 & 0.301 & -0.646 \\ A_2 & -0.922 & 0.000 & -0.922 & -0.922 \\ A_3 & -1.027 & 0.513 & 0.000 & 0.513 \\ A_4 & 0.274 & -0.56 & -0.56 & 0.000 \end{pmatrix}$$

This gives us the following dominance matrix:

$$\bar{D} = \begin{pmatrix} & C_1 & C_2 & C_3 & C_4 \\ A_1 & 2.27^{+} & 2.614^{+} & 1.826^{+} & 0.301^{-} \\ A_2 & 1.135 & 1.307 & 0.456 & 0.361 \\ A_3 & 0.378^{-} & 0.653^{-} & 0.913 & 0.904^{+} \\ A_4 & 1.135 & 0.871 & 0.913 & 0.361 \end{pmatrix}.$$

7.6) Calculate PIS $\bar{D}^{+}$ and NIS $\bar{D}^{-}$:

$$\bar{D}^{+} = (2.27, 2.614, 1.826, 0.9045)$$

$$\bar{D}^{-} = (0.378, 0.653, 0.365, 0.301)$$

7.7) Set υ at 0.4, and compute minimum similarity S_i and maximum distance D_i:

($\varnothing_1$)

$$\varnothing_1(A_1, A_1) = \begin{cases} 0.301, & \text{if } f_{ij} - f_{tj} > 0 \\ 0, & \text{if } f_{ij} - f_{tj} = 0 \\ -0.646, & \text{if } f_{ij} - f_{tj} < 0 \end{cases}$$

$(\varnothing_2)$

$$\varnothing_2(A_2, A_2) = \begin{cases} 0.461, & \text{if } f_{ij} - f_{tj} > 0 \\ 0, & \text{if } f_{ij} - f_{tj} = 0 \\ -0.922, & \text{if } f_{ij} - f_{tj} < 0 \end{cases}$$

$(\varnothing_3)$

$$\varnothing_3(A_3, A_3) = \begin{cases} 0.513, & \text{if } f_{ij} - f_{tj} > 0 \\ 0, & \text{if } f_{ij} - f_{tj} = 0 \\ -1.027, & \text{if } f_{ij} - f_{tj} < 0 \end{cases}$$

$(\varnothing_4)$

$$\varnothing_4(A_4, A_4) = \begin{cases} -0.117, & \text{if } f_{ij} - f_{tj} > 0 \\ 0, & \text{if } f_{ij} - f_{tj} = 0 \\ 0.234, & \text{if } f_{ij} - f_{tj} < 0 \end{cases}$$

$$\max_{i=1}^{m} \sum_{t=1}^{m} \varnothing_j(A_i, A_t) = (0.301 + 0.461 + 0.513 + (-0.117)) = 1.158$$

$$\sum_{t=1}^{m} \varnothing_j(A_i, A_t) = 0$$

$$\min_{i=1}^{m} \sum_{t=1}^{m} \varnothing_j(A_i, A_t) = ((-0.646) + (-0.922) + (-1.027) + 0.234) = 1.779$$

$$d(\bar{D}_j^+, \bar{D}_{ij}) = \max_{i=1}^{m} \sum_{t=1}^{m} \varnothing_j(A_i, A_t) - \sum_{t=1}^{m} \varnothing_j(A_i, A_t) = 1.158 - 0 = 1.158$$

$$d(\bar{D}_j^+, \bar{D}_j^-) = \max_{i=1}^{m} \sum_{t=1}^{m} \varnothing_j(A_i, A_t) - \min_{i=1}^{m} \sum_{t=1}^{m} \varnothing_j(A_i, A_t)$$
$$= 1.158 - (-1.779) = -0.621$$

$$S_i = \min_{i=1}^{m} \sum_{j=1}^{n} w_j \frac{d(\bar{D}_j^+, \bar{D}_{ij})}{d(\bar{D}_j^+, \bar{D}_j^-)}$$

$$S_{11} = 0.679,\ S_{12} = 0.333,\ S_{13} = -1.864,\ S_{14} = -0.539$$

$$S_{1i} = -1.391$$

$$S_{21} = -0.903,\ S_{22} = 0.514,\ S_{23} = 1.004,\ S_{24} = 0.612$$

$$S_{2i} = 1.227$$

$$S_{31} = -0.838,\ S_{32} = 2.064,\ S_{33} = 0.451,\ S_{34} = 0.9$$

$$S_{3i} = 2.577$$

$$S_{41} = -1.619,\ S_{42} = 0.178,\ S_{43} = 0.787,\ S_{44} = -0.927$$

$$S_{4i} = 0.273$$

$$S_i = \{-1.391, 1.227, 2.577, 0.273\}$$

Priority ranking: $2.577 \succ 1.227 \succ 0.2732 \succ -1.391$

Ranking of candidates: $A_3 \succ A_2 \succ A_4 \succ A_1$

$$d_i = \max_{j=1}^{n} \sum_{j=1}^{n} w_j \frac{d\left(\bar{D}_j^+, \bar{D}_{ij}\right)}{d\left(\bar{D}_j^+, \bar{D}_j^-\right)}$$

$$d_{11} = -0.591,\ d_{12} = 0.064,\ d_{13} = -0.626,\ d_{14} = -0.5$$

$$d_{1i} = -0.471$$

$$d_{21} = -0.156,\ d_{22} = -0.527,\ d_{23} = -0.137,\ d_{24} = -0.389$$

$$d_{2i} = -0.897$$

$$d_{31} = -0.108,\ d_{32} = -0.732,\ d_{33} = 0.506,\ d_{34} = -0.963$$

$$d_{3i} = -1.297$$

$$d_{41} = -0.481,\ d_{42} = -0.303,\ d_{43} = 0.282,\ d_{44} = 0.664$$

$$d_{4i} = 0.162$$

$$d_i = \{-0.471, -0.897, -1.297, 0.162\}$$

Priority ranking: $0.162 \succ -0.471 \succ -0.897 \succ -1.297$

Ranking of candidates: $A_4 \succ A_1 \succ A_2 \succ A_3$.

Step 8. Compute the relationship between s_i and d_i:

$$Q_i = \frac{(1-\upsilon)\left(s_i - S^-\right)}{S^+ - S^-} + \frac{\upsilon\left(d_i - d^-\right)}{d^+ - d^-}$$

where $S^- = \max_{i=1}^{m} s_i, S^+ = \min_{i=1}^{m} s_i, d^- = \min_{i=1}^{m} d_i, d^+ = \max_{i=1}^{m} d_i$.

υ is the weight of the max similarity measure. Despite the loss of generality, let us suppose that the value of υ is set to 0.5. Then we have

$$Q_1 = 1,\ Q_2 = 0.366,\ Q_3 = -0.459,\ Q_4 = 0.131$$

$$Q_i = \{1, 0.366, -0.459, 0.131\}$$

Ranking of Q_i: $Q_1 \succ Q_2 \succ Q_4 \succ Q_3$.

Step 9. Rank alternatives in descending order of S_i, d_i, Q_i ($i = 1, 2, \cdots, m$) to output three types of rankings (Table 7).

Ranking of d_i: $A_1 \succ A_2 \succ A_4 \succ A_3$

Ranking of Q_i: $A_1 \succ A_2 \succ A_4 \succ A_3$

Table 7. Ranking of S_i, d_i, and Q_i.

Alternative A_i	S_i	Rank	d_i	Rank	Q_i	Rank
A_1	2.045+	1	6.892+	1	1	1
A_2	2.273	3	4.954	2	0.366	2
A_3	2.2	2	2.969−	4	−0.459	4
A_4	5.844−	4	3.999	3	0.131	3

Result A_1 is the optimal candidate.

Step 10. Posit alternative $A'^{(1)}$ as a compromise solution:

The max of Q is not satisfied; m is the number of alternatives:

$$Q\left(A'^{(2)}\right)-Q\left(A'^{(1)}\right)\geq\frac{1}{m-1}$$

$$Q\left(A'^{(2)}\right)-Q\left(A'^{(1)}\right)=0.366-1=-0.634$$

$$\frac{1}{m-1}=\frac{1}{4-1}=0.33$$

where $-0.634 < 0.33$ as $Q\left(A'^{(2)}\right)<Q\left(A'^{(1)}\right)$

Therefore, A_1 is the optimal solution.

3.3. Results

The above calculations demonstrate that A_1 represents a compromise solution. That means demand forecast updating is the best strategy for management of the bullwhip effect. Lee et al. (Lee, Padmanabhan, & Whang, 1997b) proposed that the information transmitted in the form of orders is often distorted and may mislead upstream inventory and affect production decisions. This alternative argues that order information available in the orders received from downstream should be used with great caution; information related to order statuses downstream is key factor to improving and reducing the bullwhip effect. Because alternative A_1 is associated with mapping criterion C_1, "Historical record of the relationship between orders and consumers" is the most important criterion of mapping with the bullwhip effect phenomenon four alternatives, as depicted in **Figure 6**:

According to **Figure 6**, the bullwhip effect amplifies order fluctuations based on the dynamic flow of alternatives. In order to compare alternatives, we consider the farthest distance from the starting point under the alternative.

Ranking of d_i: $A_1 \succ A_2 \succ A_4 \succ A_3$

Ranking of Q_i: $A_1 \succ A_2 \succ A_4 \succ A_3$

Therefore, A_1 is the optimal candidate.

This article is the first to discuss selection of optimal strategies in the face of the bullwhip effect. Our results serve as valuable reference for the decision-makers of innovative enterprises.

3.4. Managerial Implications

This article integrates TODIM approach, TOPSIS, and PFS to manage the bullwhip effect for innovative companies. The proposed approach is versatile and applicable to complex problems, providing reliable, accurate, and strict theoretical results. Based on the work of Lee et al. (Lee, Padmanabhan, & Whang, 1997a, 1997b), we considered four strategies for managing the bullwhip effect: demand forecast updating, order batch processing, price fluctuation, and rationing and shortage. We proposed a new concept of PF-TODIM approach,

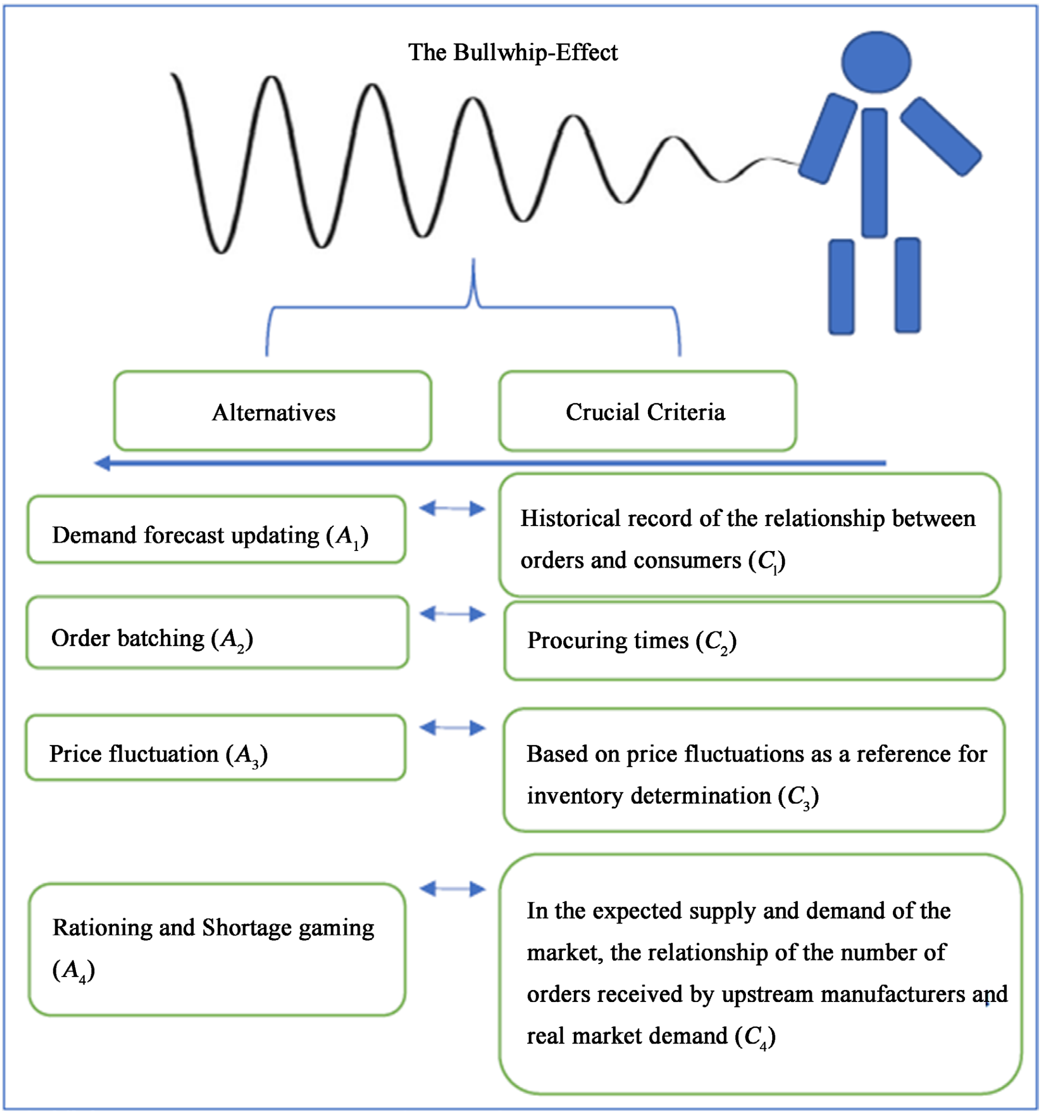

Figure 6. Relationship of bullwhip effect to four underlying criteria (Constructed by Shu-Mei Lin, 2020).

Q_i based on preferences the optimal similarity-based measure, s_i and the optimal distance-based measure, d_i to complete the optimal research method, which is the first study proposed using TODIM approach combined with TOPSIS for MCDM problems in the bullwhip effect.

4. Conclusion

4.1. Proposed a New Decision Framework

This paper proposed a new decision framework (**Figure 6**) for selecting optimal strategies to reduce the negative impact of the bullwhip effect. Depended on the work of Ren et al. (Liang, Zhang, & Liu, 2015) was proposed a new research framework (**Figure 4**) to describe an innovative research framework for combining PFS with the PF-TODIM approach and dealing with PF-MCDM problems.

4.2. Generate three Matrix Formulas

The model was the first to merge the PF-TODIM approach and PF-TOPSIS methods. We are applying the proposed model to the case of the bullwhip effect for the supply chain management. It comprises three important matrix formulas:

1) $s_i = \min_{i=1}^{m} \sum_{j=1}^{n} w_j \frac{d\left(\bar{D}_j^+, \bar{D}_{ij}\right)}{d\left(\bar{D}_j^+, \bar{D}_j^-\right)}$ (similarity-based measure),

2) $d_i = \max_{j=1}^{n} \sum_{j=1}^{n} w_j \frac{d\left(\bar{D}_j^+, \bar{D}_{ij}\right)}{d\left(\bar{D}_j^+, \bar{D}_j^-\right)}$ (distance-based measure),

3) $Q_i = (1-\upsilon)\frac{s_i - S^-}{S^+ - S^-} + \upsilon \frac{d_i - d^-}{d^+ - d^-}$ (PF-TODIM approach).

4.3. Contribution

In this study, we sought the optimal solution for the PF-MCDM problem in an uncertain environment. We offer accurate handling of the trend of uncertain factors from the perspective of enterprises. We further prove that a scientific approach to decision-making in the case of the bullwhip effect is the best way of maximizing competitive advantage.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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