Development of Sparse Reconstruction Algorithm of Cone-Beam CT

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Abstract

CT technology has been widely used in various fields such as medical treatment, industry, and materials. Recently years, cone-beam CT (CBCT) used in the medical field is replacing the traditional spiral CT slowly due to its unique advantages. Improving the performance of CBCT and the image reconstruction algorithms could obtain higher quality of images, meanwhile, these also reduce the exposure time of X-ray irradiation. And image reconstruction techniques based on sparse angles have benefits for both. This article briefly introduces the advantages of CBCT and the shortcomings of spiral CT, then the traditional filtered projection algorithm Feldkamp is explained. The development of the CBCT reconstruction algorithms based on incomplete data is analyzed from three aspects: TV model, dictionary learning and compressed sensing sampling. The advantages and disadvantages of these algorithms are analyzed for the development of new algorithms.

Subject Areas

Image Processing, Nuclear Engineering, Radiology & Medical Imaging

Keywords

Cone-Beam CT, Sparse Angle, Reconstruction Algorithm, Review

1. Introduction

With the discovery of Roentgen rays in 1895, CT technology began to develop. Subsequently, the Austrian mathematician Radon confirmed the capacity that the internal information could be reconstructed from its projection data through the derivation formula of mathematical theory, which is the theoretical foundation for CT imaging [1]. In clinical, the commonly used CT scanning methods mainly include spiral CT and cone-beam CT (CBCT). Spiral CT scans the hu-

man body in a continuous rotation through the X-ray tube, while the scanning bed automatically enters at an even speed. Therefore, the scan line is spiral on the body surface of patient. CBCT means that the X-ray emitter performs a circular scan around the object to be inspected. The scan line trajectory in each projection angle is a cone-beam shape, which has the advantages of high spatial resolution, short data acquisition time and high utilization efficiency of X-ray. Compared with a two-dimensional parallel beam and fan-beam CT, a three-dimensional CBCT system could achieve higher resolution and better use of photons [2], so it is widely used in radiotherapy positioning, stomatology, and small animal pre-clinical aspects.

At present, three-dimensional (3D) reconstruction algorithms of CBCT are divided into analytical algorithms and iterative algorithms. The analytical algorithms could be divided into two types: precise reconstruction algorithms and approximate reconstruction algorithms [3]. According to the central slice theorem, the Fourier transform of each projection is equivalent to the segment that passes through the origin in the direction parallel to the detector in the two-dimensional (2D) Fourier transform of the original image. Therefore, the covering of the entire Fourier space of the reconstructed image is essential to achieve high-precision reconstruction, it requires that the projection space contains all the projection data within the 180-angle range, and all the data in the projection space is also called a complete projection data set [4].

According to the different conditions of data, incomplete data problems could be divided into the following three categories: interior problem, exterior problem, incomplete view problem [5]. Among them, the incomplete angle problem could be divided into sparse-view problem and the limited-angle problem [6]. The sparse angle problem refers to the problem of a certain interval between adjacent scanning angles. Normally, the scanning angles are equally spaced, meanwhile the total range of scanning angles covers the 180-degree.

The application of sparse angle projection for image reconstruction could significantly reduce the times of X-ray projections sampled by the imaging system, which is an effective way to reduce radiation dose. It could also effectively shorten the exposure time of each X-ray scanning at the same time. So the motion artifacts caused by the shaking, heartbeat and breathing motion of patients are reduced. Finally, the imaging quality is improved.

2. Cone-Beam CT

2.1. Limitations of Spiral CT

Multi-slice spiral CT (MSCT) is a common method of image examination because it could provide sufficient diagnostic information for general anatomical images. However, the imaging performance of MSCT still has several limitations [7]:

1) The continuous projection data of any cross-section of the scanned object cannot be obtained during MSCT scanning, so the longitudinal interpolation

method is required to approximate reconstruction of the cross-sectional image, which lead to longitudinal artifacts in the reconstructed image.

2) Although multi-slice spiral CT ccould obtain a 3D image by overlapping a series of 2D axial images, the spatial resolution of the three axes is not isotropic, the Z-axis resolution is worse than the axial resolution usually.

3) The patient receives overlapping X-rays during adjacent rotations, high-dose radiation is also a problem with MSCT.

2.2. Advantages of CBCT

The emergence of cone-beam CT (CBCT) fundamentally makes up for the short-comings of MSCT, especially when using flat panel detector (FPD) [8]. The FPD could provide CBCT imaging with advantage of high spatial resolution, high dynamic range, good linearity, and geometric distortion is avoided. The FPD bring great development for CBCT in many medical applications. Due to the large detection area of FPD, FPD-CBCT brings a breakthrough in large-scale 3D CT imaging and isotropic resolution. One rotation is performed during CBCT scanning, there is no overlapping radiation to the patient, so the box radiation dose is reduced compared with MSCT. The above-mentioned characteristics of FDP-CBCT make it the most promising imaging method in many medical fields such as small animal imaging, breast imaging and bone imaging. In recent years, the application of CBCT in medicine has mainly focused on radiotherapy and diagnosis [9].

With the rapid development of CBCT, the 3D reconstruction algorithm of CBCT has gradually become the most popular frontier topic in the medical field. On account of CBCT is widely used in modern medicine and industry, the requirement for reconstruction speed is more stringent.

3. The Feldkamp Reconstruction Algorithm

At present, the mainstream of CT reconstruction algorithms is still the filtered back-projection reconstruction algorithm. The more famous one is the Feldkamp algorithm. The cone-beam image reconstruction algorithm of Feldkamp is specially designed for the circular focus orbit of the cone beam. It is developed by Feldkamp, Davis, and Kress. The advantages of the Feldkamp algorithm are practical and stable. When the cone opening angle of the cone beam is relatively small (for example, less than 10°), the Feldkamp algorithm can reconstruct a better image. The Feldkamp algorithm can be divided into the following steps:

1) Preprocessing the projection data, that is, multiplying by a cosine function $\cos\alpha$;

2) One-dimensional ramp filtering is performed on the processed data line by line;

3) Make the filtered data into the weighted back-projection of the conical beam, and the weight function in the back projection depends on the distance from the reconstruction point to the focus.

The slope filtering in the Feldkamp algorithm is performed line by line, without filtering in the direction of the axis of rotation. Assuming that the direction of the rotation axis is the z-direction, the Feldkamp algorithm can be expressed as:

$$f(\gamma,\varphi,z)=\frac{1}{2}\int_0^{2\pi}\frac{D}{D-S}\int_{-\infty}^{+\infty}\frac{D}{\sqrt{D^2+\ell^2+\hat{z}^2}}g(\ell,\hat{z},\beta)h(\ell'-\ell)\mathrm{d}\ell\mathrm{d}\beta \qquad (1)$$

where $h(\ell)$ is the convolution kernel of the one-dimensional ramp filter, and D is the focal length, $g(\ell,\hat{z},\beta)$ is the projected data of the sector beam, ℓ is the linear coordinate on the detector. $S=\gamma\sin(\varphi-\beta)$, $\ell'=\dfrac{D\gamma\cos(\varphi-\beta)}{D-\gamma\sin(\varphi-\beta)}$, $\sqrt{D^2+\ell^2+\hat{z}^2}$ are the cosine functions of the angle of incidence of the cone-beam ray.

4. Sparse Reconstruction Algorithm

4.1. TV Minimization Reconstruction Model

Candes *et al.* [10] first proposed the TV regularization model and achieved high MRI image reconstruction accuracy under very small sampling conditions in 2006. Based on the research results of Candes, Sidky *et al.* [11] designed a CT image reconstruction model based on constrained TV minimization and achieved good reconstruction result under sparse angle sampling. According to the different calculation methods of the gradient operator, it could be divided into isotropic TV and anisotropic TV. The research of Chen *et al.* [12] in 2011 and Li *et al.* [13] in 2014 showed that anisotropic TV adapts to the difference of images in different directions, so the model will further improve the solution performance. In 2012, Liu *et al.* [14] proposed an adaptive weighted TV minimization algorithm, which considers the anisotropic edge characteristics between adjacent image voxels, it could be adaptively adjusted by local image gradients to preserve edge details. In 2013, Bian *et al.* [15] proposed an adaptive steepest descent weighted convex set projection algorithm. In the same year, Zhang *et al.* [16] proposed an alternate direction TV minimization algorithm, which reformulated the TV problem as a problem in which the objective function of linear equality constraints is separable, then through ADM, the augmented Lagrangian function is minimized and divided into sub-problems to solve.

The TV model is mainly based on the sparsity mining of the first-level domain of the image. It is difficult to accurately restore the features of the sliced continuous image that is sparse in the high-level domain, and it produces the block artifacts and the texture blur. To better protect image edges and texture details, some sparse models based on high-gradient domains have been studied in recent years. In 2014, Hu *et al.* [17] proposed a generalized HDTV (Higher Degree TV) regularization, model. The sparsity model of the high-gradient domain could better fit the polynomial function and avoid the staircase effect, however, it is easy to cause the sharpness of the edge to be reduced. Therefore, some mixed-order models that integrate a degree of gradient and a high degree of the gra-

dient are studied. Among them, the classic mixed-order models are the LOT model and the TV-Stokes model. In 2014, Liu *et al.* [18] applied the TV-Stokes model to sparse angle reconstruction and achieved good results in overcoming the ladder effect of TV. In 2010, Birdies *et al.* [19] proposed a new first-order and high-gradient fusion model the total generalized variation (TGV) model. In 2016, Zhang *et al.* [20] proposed a Generalized model of TGpV (Total Generalized P-variation) to optimize the sparsity of TGV, a good effect was achieved. However, the second-order TGV model can better maintain the balance between the first-order degree and the second-order degree by introducing intermediate variables and is currently considered to be the most excellent model among many mixed-order models [21]. Yan *et al.* [22] focused on the study of the uniqueness of the TV minimization model and proposed a method of sampling quantification for precise reconstruction of a fixed phantom and fixed geometric parameters with a limited viewpoint. In sparse optimization models such as the TV minimization reconstruction model, the reconstruction model has and only one solution means that theoretically accurate reconstruction results can be obtained in the reconstruction model.

In generally, the current mainstream CT image reconstruction model is the TV regularization model and its improved form. The research of this model has attracted more attention and has achieved good application results. However, different models have certain limitations and could not be applied to all problems. In the field of sparse angle reconstruction, designing a suitable reconstruction model and reducing the number of projection samples while ensuring the quality of the reconstructed image is the focus of research.

4.2. Dictionary Learning

With the development and deepening of research, a new branch has emerged on the way of searching for sparse transformation and sparse expression of images, namely dictionary learning. It is different from the classic sparse transformation in the above algorithm, the dictionary learning method solves a set of over-complete bases (a dictionary) through a sparse optimization algorithm. The idea of a complete dictionary was proposed by Mallat *et al.* [23] in 1993. The idea pointed out that the number of atoms in the dictionary is much larger than the dimension of the signal feature, and a matching pursuit algorithm was proposed to solve the sparse optimization problem of a complete dictionary. The earliest dictionary learning method is the optimal directions algorithm (MOD), which was proposed by Engan *et al.* [24]. Dictionary learning has always played an important role in computer vision and other aspects. Mairal *et al.* [25] deeply studied the application of dictionary learning in image processing and proved that dictionary learning could solve image processing problems such as image restoration, image desiccation and image classification. It is different from the TV reconstruction model, the reconstruction algorithm based on dictionary learning focuses more on capturing the local structural information of the image to be

reconstructed. Literature [26] applied the dictionary learning method to low-dose CT reconstruction and achieved good reconstruction results. In 2012, the dual dictionary reconstruction algorithm was used in incomplete angle reconstruction [27]. In 2013, Zhao K. *et al.* [28] proposed a reconstruction algorithm based on K-singular value decomposition (KSVD) and orthogonal pursuit (OMP). With the widespread application of deep learning in the field of image classification, in 2016, Snigdha *et al.* [29] proposed the greedy deep dictionary learning (DDL) method, which aims to obtain effective information of more complex images. The reconstruction algorithm based on dictionary learning not only opens up a new direction in the sparse representation of the model, but its model solving algorithm is also different from the first-order type sparse optimization method mentioned above but uses the mainstream algorithm for solving the l_0-norm optimization problem.

4.3. Compressed Sensing Sampling Reconstruction

Reconstruction algorithms based on sparse optimization have achieved good application results in incomplete angle reconstruction. However, there is a lack of a perfect theoretical basis for the sampling conditions used in the actual imaging process. In 2009, Pan *et al.* [5] tried to explain the meaning of compressed sensing (CS) for image reconstruction, they wanted to establish the connection between CS and CT reconstruction, and a preliminary RIP analysis of the discretized system matrix is conducted. In 2011, Jørgensen *et al.* [30] proposed the concept of hadamard full sampling and proposed the simulation experiment estimation method for the amount of data required in CT reconstruction by using CS theory, which provides a certain basis for the estimation of the range of data required by the sparse reconstruction algorithm. In 2012, Jørgensen *et al.* [31] further discussed the sufficient sampling conditions (SSCs) of the amount of data required by iterative reconstruction and CS theory, which provides a further reference for the estimation of the amount of data required in actual reconstruction. In 2014, Wang *et al.* [32] provided the necessary conditions for sampling angle estimation, which, combined with sufficient sampling estimation, made some progress in the research on the quantitative relationship between image sparseness and the minimum number of projection angles. However, due to the limitation of system matrix size and computational complexity, this method cannot be directly applied to the system matrix of 3d imaging. In 2016, Kulkarni *et al.* [33] proposed an invisible net based on Convolutional Neural Network (CNN) to realize the non-iterative compressed perception image reconstruction. Compared with the traditional algorithm, the biggest advantage of the method is that it reduces the computational complexity. In 2017, Yao *et al.* [34] modified ReconNet architecture by adding residual connections to improve reconstruction performance. Dai *et al.* [35] enforced the previously captured long-term spatially dependent images, and the improved method can obtain better image quality reconstruction.

5. Discussion

CBCT applied in the medical field is replacing the original spiral CT due to its unique advantages. To obtain high-quality images and reduce exposure time of X-ray, in addition to improving the performance of CBCT, it could also start with image reconstruction algorithms. Image reconstruction technology based on sparse angles has great benefits for both. Both analytical algorithms and traditional iterative algorithms are difficult to obtain high-quality reconstruction results for the sparse angle reconstruction. The reconstruction technology based on sparse optimization mines the inherent sparse characteristics of the image, then a sparse optimization model is established. The corresponding reconstruction algorithm is designed to reduce the amount of sampled data. The algorithm has been successfully applied in sparse angle reconstruction. In this reconstruction algorithm, research on to relax the number of system samples is the main research direction. To reduce the system samples, the main research point is studying the relationship between imaging systems and image sparse characteristics to mine sparse prior characteristics furtherly. However, the core of this problem is to design better reconstruction models and develop more efficient solving algorithms.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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