Open Access Library Journal
2020, Volume 7, e6075
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Stability and Adaptive Control with Sychronization of 3-D Dynamical System

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How to cite this paper: Aziz, M.M. and Merie, D.M. (2020) Stability and Adaptive Control with Sychronization of 3-D Dynamical System. *Open Access Library Journal*, **7**: e6075.
https://doi.org/10.4236/oalib.1106075

Received: January 14, 2020
Accepted: February 25, 2020
Published: February 28, 2020

Abstract

A three-dimensional system is presented with unknown parameters that employs two nonlinearities terms. The basic characteristics of the system are studied. The stability is measured by Characteristic equation roots, Routh stability criteria, Hurwitz stability criteria and Lapiynov function, all show that the system unstable. Then, Chaoticity is measured by maximum Lapiynov exponent of ($L_{\max} = 2.509426$) and "Kaplan-Yorke" dimension ($D_L = 2.22349544$). The system is controlled effectively and synchronized by designed adaptive controllers. Furthermore, the theoretical and graphic results of the system before and after control are compared.

Subject Areas

Dynamical System

Keywords

Stabilization, Dissipative System, Adaptive Control, Lapiynov Exponent

1. Introduction

Researches in recent years on chaotic phenomena have increased a lot, because of the increasing frontiers of applications of chaos in engineering and non-engineering systems. "Chaos is a phenomenon which results from the exhibits sensitivity to perturbation in the structural parameters and initial conditions of some classes of dynamical systems" [1]. "Chaotic signals have a random-like nature and broadband spectrum and are non-periodic" [2]. "For a system to be chaotic, the following conditions must be satisfied. Firstly, it must be sensitive to perturbation in its initial conditions which should lead to unpredictability of its future trajectories, secondly, it is not topologically transitive and thirdly, the chaotic orbits are dense in the phase space" [3]. "Among some evolved chaotic attractors in the literature are the Chen's" [4], "3-D, 4-wing attractor" [5], "Sundarapandian-Pehlivan" [6], "Morphous one parameter attractor" [7], "Rabinovich system" [8]. "When chaotic attractors possess one positive Lapiynov exponent, then the system is chaotic. However, the system which has two or more positive Lapiynov exponents it is a highly chaotic system and becomes hypersensitive to small perturbations in its system dynamics" [9] [10]. "Chaos Control subject has received widespread attention of research because controllability and synchronizability of chaotic attractors are index of utility in different designs such as in secure communications and robotics" [11] [12] [13].

"In the context of stability and stabilization, the principle of Lapiynov stability continued to enjoy large applications; it can effectively stabilize the dissipative systems" [14] [15].

This paper is organized as: Section 2, present a description of 3-d system. Section 3, basic analysis such as stability, dissipativity, Lapiynov dimension "Kaplan-Yorke dimension". Section 4, we designed adaptive control law of the chaotic system. Section 5, a comparison of the analysis results before and after control. Section 6, we derive results for the adaptive synchronization of identical highly chaotic system. Finally Section 7, summarization of the main results.

2. System Description

A three-dimensional dynamical system [16] consist of three ordinary differential equations with state variables x_i, (i = 1, 2, 3) and four unknown parameters (ρ, a, δ and φ), employs six terms include two quadratic cross-product nonlinear terms. Given by:

$$\begin{aligned} \dot{x}_1 &= \rho\left(x_2 - x_1\right) \\ \dot{x}_2 &= a x_1 - \delta x_1 x_3 \\ \dot{x}_3 &= \varphi x_1 x_2 - x_3 \end{aligned} \tag{1}$$

The parameters values are taken as

$$\rho = 10, \delta = 40, a = 296.5, \varphi = 10 \tag{2}$$

3. System Analysis

In this section essential, the system (1) is invested and has the following characteristics.

3.1. Equilibrium Points

The first step to analyze a system is to find its equilibrium points, so we need to solve the nonlinear equations as follows

$$-10x_1 + 10x_2 = 0$$

$$296.5x_1 - 40x_1x_3 = 0$$

$$10x_1x_2 - x_3 = 0$$

We get the following equilibrium points:

$$E_0 = (0,0,0), \quad E_1 = \left(\frac{\sqrt{\frac{593}{2}}}{20}, \frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80} \right), \quad E_2 = \left(-\frac{\sqrt{\frac{593}{2}}}{20}, -\frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80} \right)$$

3.2. Stability Analysis

3.2.1. Characteristic Equation Roots

"A necessary and sufficient condition for the system to be stable is that the real parts of the characteristic equation have negative real parts".

When the parameters values are taken as in (2), the Jacobian matrix of system (1) at $E_0 = (0,0,0)$ is:

$$J = \begin{bmatrix} -10 & 10 & 0 \\ 296.5 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\det(J - \lambda I) = 0$$

$$\Rightarrow \lambda^3 + 11\lambda^2 - 2955\lambda - 2965 = 0 \quad (3)$$

By using Horner's Ruffini method [17] we get:

$$\lambda_1 = -1, \quad \lambda_2 = 49.68089, \quad \lambda_3 = -59.68089$$

Similarly, we find Jacobian matrix at E_1 and E_2, then we obtain the eigenvalues, as shown in **Table 1**.

So the system (1) is unstable.

3.2.2. Routh Stability Criteria

"The system is considered stable by the Routh stability states (all poles in OLHP (Open Loop Half plane)) if and only if all elements of the first column in the Routh array are positive. In addition, number of poles not in the OLHP is equal to the number of sign changes in the first column" [18].

$$a_0 = -2965$$

$$a_1 = -2955$$

$$a_2 = 11$$

$$a_3 = 1$$

$$b_1 = a_1 - \frac{a_3 a_0}{a_2} = -2685.4545$$

Since, there are two negative elements in the first column. Therefore, the system (1) is unstable.

3.2.3. Hurwitz Stability Criteria

"This criterion is applied using determinants formed from coefficients of the characteristic equation. If the small minors of the square matrix J of the system (1) are all positive then the system (1) is stable, otherwise it's unstable" [18].

If $n = 3$ (n denote the degree of the square matrix)

From Equation (3) we find:

$$\Delta_1 = a_2 = 11 > 0$$

$$\Delta_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = a_2 a_1 - a_3 a_0 = -29.540 < 0$$

$$\Delta_3 = \begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{vmatrix} = a_2 a_1 a_0 - a_0^2 a_3 = 87586100 > 0$$

Since the values of one minors is less than zero, so the system (1) is unstable.

3.2.4. Lapiynov Function

We can use quadratic function for system (1).

We assume that

$$V(x_1, x_2, x_3) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$$

$$\dot{V}(x_1, x_2, x_3) = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 \tag{4}$$

If $\dot{V}(x_1, x_2, x_3) < 0$ then the system is stable.

By substituting (1) in Equation (4) we get:

$$\dot{V}(x_1, x_2, x_3) = -10x_1^2 + 306.5x_1x_2 - 30x_1x_2x_3 - x_3^2$$

Since $\dot{V}(x_1, x_2, x_3) > 0$ therefore the system (1) is unstable.

3.3. Dissipativity

Let $f_1 = \frac{\mathrm{d}x_1}{\mathrm{d}t}$, $f_2 = \frac{\mathrm{d}x_2}{\mathrm{d}t}$ and $f_3 = \frac{\mathrm{d}x_3}{\mathrm{d}t}$ in the system (1).

Then we get for the vector field

$$(\dot{x}_1, \dot{x}_2, \dot{x}_3)^{\mathrm{T}} = (f_1, f_2, f_3)^{\mathrm{T}}$$

thus the divergence of the vector field $\boldsymbol{V}$ on R^3 yields to:

$$\nabla \cdot (\dot{x}_1, \dot{x}_2, \dot{x}_3)^{\mathrm{T}} = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -(\rho + 1) = f$$

Note that $f = -(\rho + 1) = -11$, so the system (1) is dissipative for all positive values of ρ, and an exponential rate is:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = fV \Rightarrow V(t) = V_0 \mathrm{e}^{ft} = V_0 \mathrm{e}^{-11t}$$

From above equation, the volume element V_0 is contracted by the flow into a volume element $V_0\mathrm{e}^{-11t}$ at the time t.

3.4. Numerical and Graphical Results

For the numerical solution, we use Runge-Kutta method of order 5th to solve system (1). With initial states $x|_{x_1(0), x_2(0), x_3(0)} = [-2, 7, 12]$

3.4.1. Wave Form of the System (1)

The wave-form $x_1(t), x_2(t), x_3(t)$ for the system (1) is characteristic with non-periodic shape, shown in Figures 1(a)-(c) which is one of the basic characteristic behaviors of chaotic dynamical system.

3.4.2. Phase Portrait of the System (1)

Figures 2(a)-(d) and Figures 3(a)-(c) are shows chaotic attractor for system (1) in (x_1, x_2, x_3), (x_1, x_3, x_2), (x_2, x_1, x_3), (x_3, x_1, x_2) space, and 2-D attractor of system (1) in (x_1, x_3), (x_1, x_2), (x_2, x_3) plane.

The orbit is dense in each graph which means the system exhibit two-scroll hyper chaotic attractor.

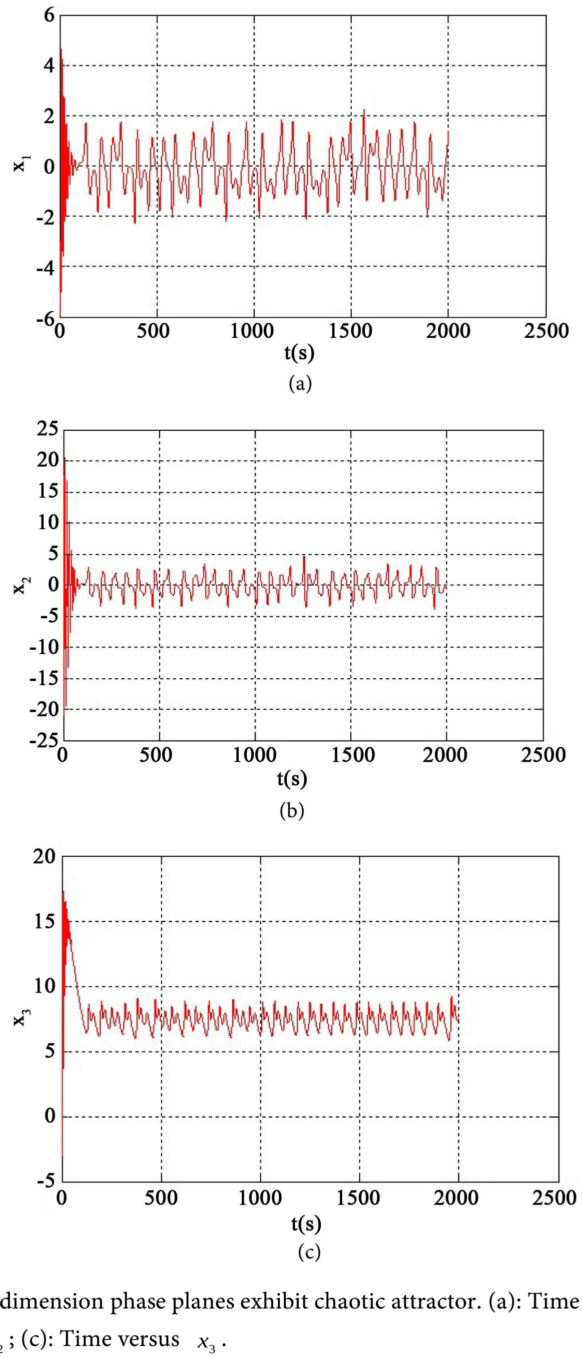
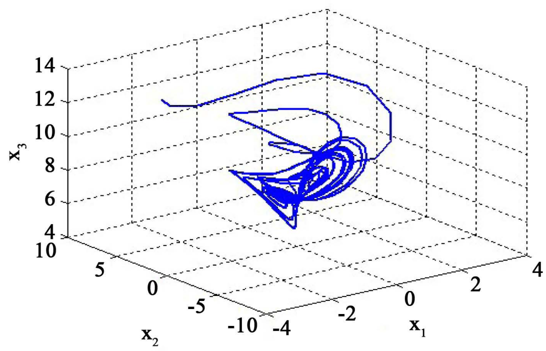
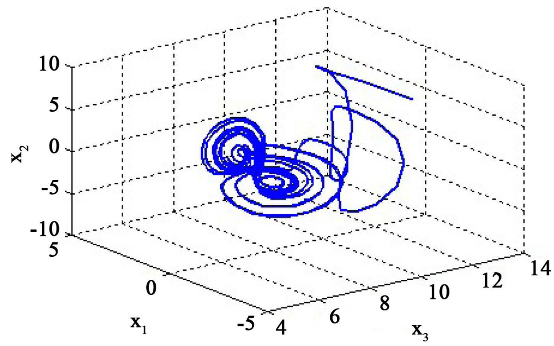


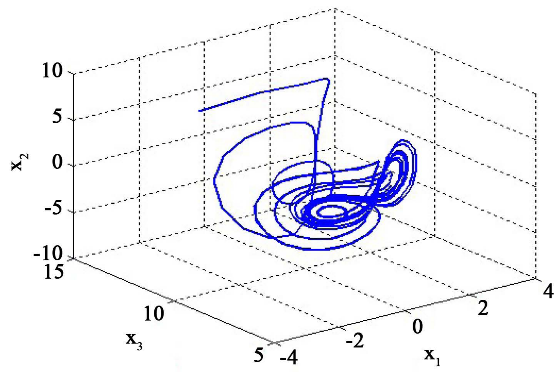
Figure 1. Two-dimension phase planes exhibit chaotic attractor. (a): Time versus x_1; (b): Time versus x_2; (c): Time versus x_3.



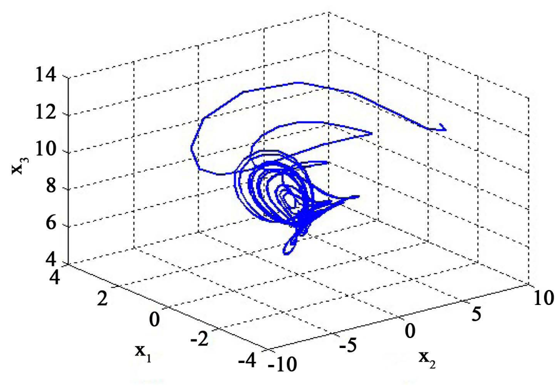
(a)



(b)



(c)



(d)

Figure 2. 3-D Attractor of the system (1).

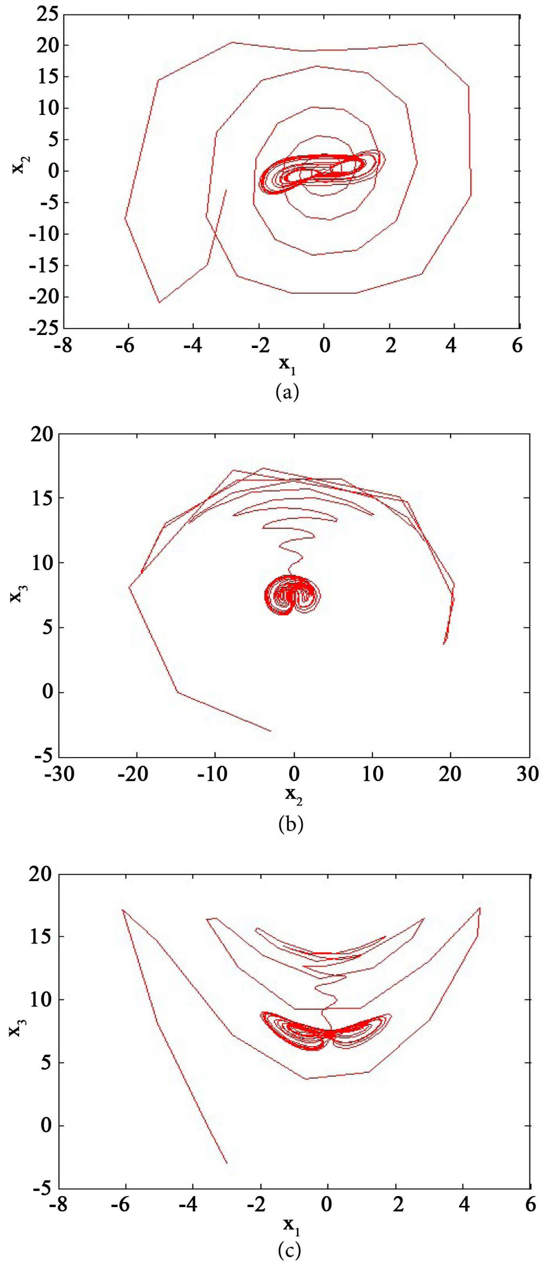


Figure 3. 2-D Attractor of the system (1).

3.5. Lapiynov Exponent and Lapiynov Dimension

"As a rule the Lapiynov exponents refer to the average exponential rates of divergence or convergence of nearby trajectories in the phase space. The system is chaotic if there is at least one Lapiynov exponent greater than zero". The values of lapiynov exponents are: ($L_1 = 2.509426$, $L_2 = 0.132019$ and $L_3 = -11.818787$). Therefore, the Lapiynov dimension "Kaplan-Yorke dimension" is:

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.22349544$$

So the system (1) is Highly Chaotic System, as shown in **Figure 4**.

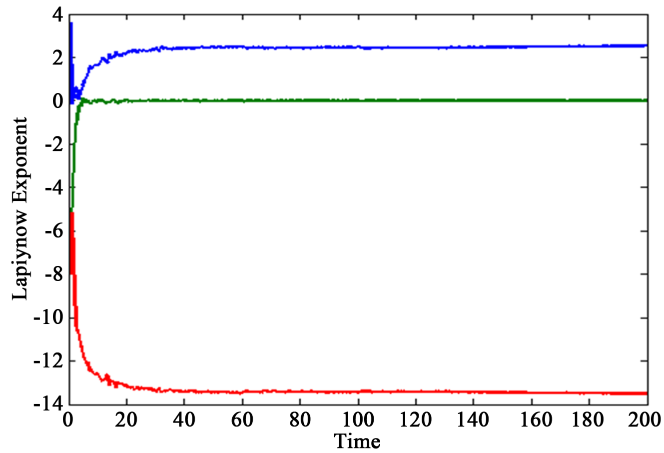


Figure 4. Lapiynov exponent of system (1).

4. Adaptive Control Strategy

4.1. Theoretical Results

To stabilize highly chaotic system (1), an adaptive control law is designed with unknown parameter a.

As follows:

$$\begin{aligned} \dot{x}_1 &= 10(x_2 - x_1) + \alpha_1 \\ \dot{x}_2 &= a x_1 - 40 x_1 x_3 + \alpha_2 \\ \dot{x}_3 &= 10 x_1 x_2 - x_3 + \alpha_3 \end{aligned} \tag{5}$$

when $\alpha_1, \alpha_2, \alpha_3$ are the feedback controllers.

The adaptive control functions are:

$$\begin{aligned} \alpha_1 &= -10(x_2 - x_1) - \mu_1 x_1 \\ \alpha_2 &= -\hat{a} x_1 + 40 x_1 x_3 - \mu_2 x_2 \\ \alpha_3 &= -10 x_1 x_2 + x_3 - \mu_3 x_3 \end{aligned} \tag{6}$$

where the constants μ_i, (i = 1, 2,3) are positive , $\hat{a}$ is the parameter estimate of a.

Substituting (6) into (5), we get

$$\begin{aligned} \dot{x}_1 &= -\mu_1 x_1 \\ \dot{x}_2 &= (a - \hat{a}) x_1 - \mu_2 x_2 \\ \dot{x}_3 &= -\mu_3 x_3 \end{aligned} \tag{7}$$

Let the parameter estimation error

$$e_a = a - \hat{a} \tag{8}$$

Using (8), the dynamics (7) can be written compactly as

$$\begin{aligned} \dot{x}_1 &= -\mu_1 x_1 \\ \dot{x}_2 &= e_a x_1 - \mu_2 x_2 \\ \dot{x}_3 &= -\mu_3 x_3 \end{aligned} \tag{9}$$

The Lapiynov approach is used for derivation of update law for adjusting the parameter estimate $\hat{a}$.

Consider the lapiynov function

$$V(x_1,x_2,x_3)=\frac{1}{2}\left(x_1^2+x_2^2+x_3^2+e_a^2\right) \tag{10}$$

Notice V is positive-definite on R^4.

Also

$$\dot{e}_a=-\dot{\hat{a}} \tag{11}$$

Differentiating V with substituting (9) and (11), we get:

$$\dot{V}=-\mu_1x_1^2-\mu_2x_2^2-\mu_3x_3^2+e_a\left[x_1x_2-\dot{\hat{a}}\right] \tag{12}$$

In Equation (12), we update estimated parameter by:

$$\dot{\hat{a}}=x_1x_2+\mu_4e_a \tag{13}$$

where the constant μ_4 is a positive.

Now, we substitute (13) into (12), we obtain

$$\dot{V}=-\mu_1x_1^2-\mu_2x_2^2-\mu_3x_3^2-\mu_4e_a^2 \tag{14}$$

Notice $\dot{V}$ is negative definite on R^4.

Thus, by lapiynov stability, Routh-array criteria, Eigenvalues and Hurwitz stability criteria we get the below result.

Proposition 1. The chaotic system (5) with unknown parameter is stabilized for every initial value by adaptive control (6), where the estimated parameter is obtained by (13) and μ_1,μ_2,μ_3,μ_4 are greater than zero.

4.2. Numerical Results

To simulate the controlled highly chaotic system (7) we take the initial values $x|_{x_1(0),x_2(0),x_3(0)}=[12,9,17]$ and $[\mu_1,\mu_2,\mu_3]=[30,50,30]$.

Figure 5 shows Controlled trajectories of system (1).

5. System Comparison Tables before & after Control

See Tables 1-6.

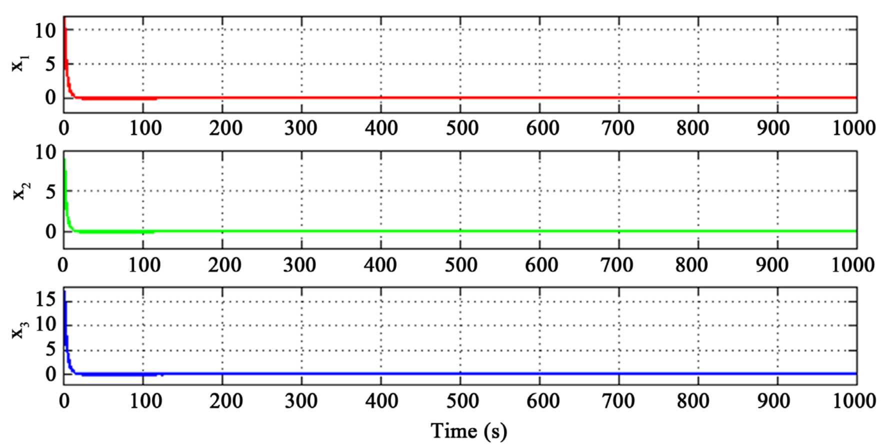


Figure 5. Controlled state trajectories of system (1).

Table 1. Eigenvalues of the system (1) before and after control.

Equilibrium point	Before control	After control
$(0,0,0)$	$\lambda_1 = -1$ $\lambda_2 = 49.68089$ $\lambda_3 = -59.68089$	$\lambda_1 = -30$ $\lambda_2 = -50$ $\lambda_3 = -30$
$\left(\frac{\sqrt{\frac{593}{2}}}{20}, \frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$	$\lambda_1 = 1.39097$ $\lambda_2 = 42.622$ $\lambda_3 = -55.013$	$\lambda_1 = -30$ $\lambda_2 = -50$ $\lambda_3 = -30$
$\left(-\frac{\sqrt{\frac{593}{2}}}{20}, -\frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$	$\lambda_1 = 1.39097$ $\lambda_2 = 42.622$ $\lambda_3 = -55.013$	$\lambda_1 = -30$ $\lambda_2 = -50$ $\lambda_3 = -30$

Table 2. Hurwitz criteria of the system (1) before and after control.

Equilibrium point	Before control	After control
$(0,0,0)$	$\Delta_1 = 11$ $\Delta_2 = -29.540$ $\Delta_3 = 87586100$	$\Delta_1 = 110$ $\Delta_2 = 384000$ $\Delta_3 = 1728 \times 10^{10}$
$\left(\frac{\sqrt{\frac{593}{2}}}{20}, \frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$	$\Delta_1 = 11$ $\Delta_2 = -29243.5$ $\Delta_3 = -95377675.25$	$\Delta_1 = 110$ $\Delta_2 = 384000$ $\Delta_3 = 1728 \times 10^{10}$
$\left(-\frac{\sqrt{\frac{593}{2}}}{20}, -\frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$	$\Delta_1 = 11$ $\Delta_2 = -29243.5$ $\Delta_3 = -95377675.25$	$\Delta_1 = 110$ $\Delta_2 = 384000$ $\Delta_3 = 1728 \times 10^{10}$

Table 3. Routh array criteria of the system (1) before and after control.

Equilibrium point	λ	Before Control		After Control	
$(0,0,0)$	λ^3	1	−2955	1	3900
	λ^2	11	−2965	110	45,000
	λ^1	−2685.5	0	3490.9	0
	λ^0	−2965	0	45,000	0
$\left(\frac{\sqrt{\frac{593}{2}}}{20}, \frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$	λ^3	1	−2362	1	3900
	λ^2	11	3261.5	110	45,000
	λ^1	−5634.5	0	3490.9	0
	λ^0	3261.5	0	45,000	0
$\left(-\frac{\sqrt{\frac{593}{2}}}{20}, -\frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$	λ^3	1	−2362	1	3900
	λ^2	11	3261.5	110	45,000
	λ^1	−5634.5	0	3490.9	0
	λ^0	3261.5	0	45,000	0

Table 4. Lapiynov function of system (1) before and after control.

(x_1, x_2, x_3)	$\dot{V}(x_1, x_2, x_3)$	
	Before control	**After control**
$(0.6815, 1.3625, 6.7879)$	44.79278685	-1486.699813
$(-2, 7, 12)$	565	-6925

Table 5. Lapiynov exponent of system (1) before and after control.

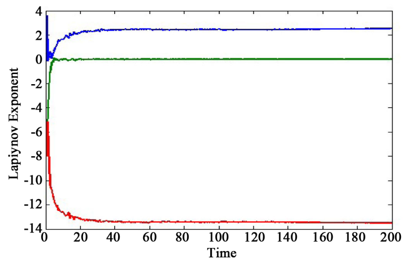
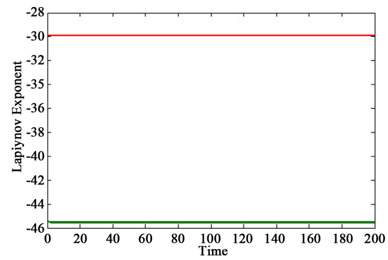
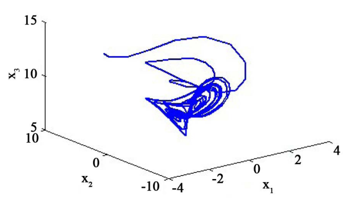
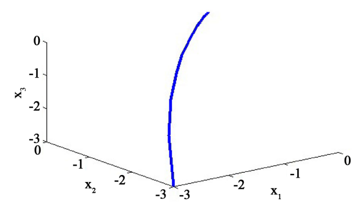
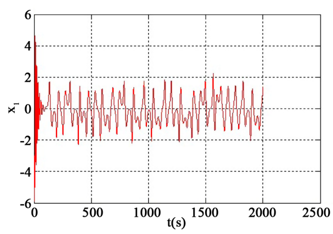
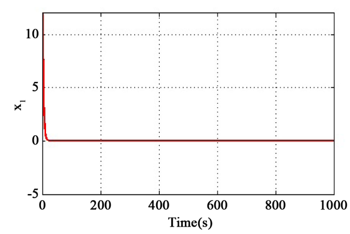
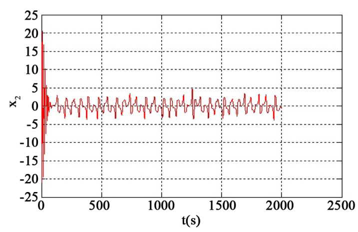
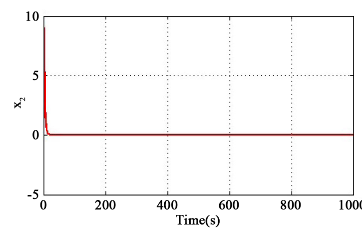
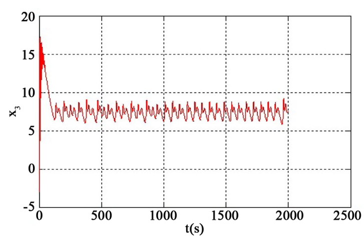
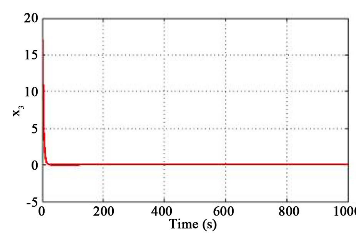
Before control	After control
	

Table 6. Of phase portrait of the system (1) before and after control.

Before control	**After control**
	
	
	
	

6. Adaptive Synchronization Technique

6.1. Theoretical Results

We apply adaptive synchronization technique of highly chaotic system with unknown parameter a.

The drive system is

$$\begin{aligned}\dot{x}_1 &= 10(x_2 - x_1)\\ \dot{x}_2 &= ax_1 - 40x_1x_3\\ \dot{x}_3 &= 10x_1x_2 - x_3\end{aligned} \tag{15}$$

where x_i, ($i = 1, 2, 3$) are state variables.

As the response system, the controlled highly chaotic dynamics given by

$$\begin{aligned}\dot{y}_1 &= 10(y_2 - y_1) + \alpha_1\\ \dot{y}_2 &= ay_1 - 40y_1y_3 + \alpha_2\\ \dot{y}_3 &= 10y_1y_2 - y_3 + \alpha_3\end{aligned} \tag{16}$$

where $\alpha_1, \alpha_2, \alpha_3$ are nonlinear controllers to be designed, and the state variables are y_i, ($i = 1, 2, 3$).

The synchronization error is defined by

$$e_i = y_i - x_i, (i = 1, 2, 3) \tag{17}$$

then the error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= 10(e_2 - e_1) + \alpha_1\\ \dot{e}_2 &= ae_1 - 40(e_1e_3 + x_3e_1 + x_1e_3) + \alpha_2\\ \dot{e}_3 &= 10(e_1e_2 + x_2e_1 + x_1e_2) - e_3 + \alpha_3\end{aligned} \tag{18}$$

The adaptive control functions $\alpha_1(t), \alpha_2(t), \alpha_3(t)$ define as

$$\begin{aligned}\alpha_1 &= -10(e_2 - e_1) - \mu_1e_1\\ \alpha_2 &= -\hat{a}e_1 + 40(e_1e_3 + x_3e_1 + x_1e_3) - \mu_2e_2\\ \alpha_3 &= -10(e_1e_2 + x_2e_1 + x_1e_2) + e_3 - \mu_3e_3\end{aligned} \tag{19}$$

where the constants μ_1, μ_2, μ_3 greater than zero, and $\hat{a}$ is the estimated value of the parameter a.

Substitute (19) into (18), to obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= -\mu_1e_1\\ \dot{e}_2 &= (a - \hat{a})e_1 - \mu_2e_2\\ \dot{e}_3 &= -\mu_3e_3\end{aligned} \tag{20}$$

Now, the parameter estimation error is

$$e_a = a - \hat{a} \tag{21}$$

By substituting (21) into (20), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -\mu_1e_1\\ \dot{e}_2 &= e_ae_1 - \mu_2e_2\\ \dot{e}_3 &= -\mu_3e_3\end{aligned} \tag{22}$$

From Lapiynov approach we derive the updated law to adjust the estimation of the parameter.

The quadratic lapiynov function is

$$V = e_1^2 + e_2^2 + e_3^2 + e_a^2 \tag{23}$$

which be a positive definite on R^4.

Note that

$$\dot{e}_a = -\dot{\hat{a}} \tag{24}$$

Differentiating V and substituting (22) & (24) in it, we get:

$$\dot{V} = -\mu_1 e_1^2 - \mu_2 e_2^2 - \mu_3 e_3^2 - e_a \left[e_1 e_2 - \dot{\hat{a}} \right] \tag{25}$$

update the estimated parameter in Equation (25) by the following

$$\dot{\hat{a}} = e_1 e_2 + \mu_4 e_a \tag{26}$$

where the constant μ_4 is greater than zero.

From (25) and (26), we obtain:

$$\dot{V} = -\mu_1 e_1^2 - \mu_2 e_2^2 - \mu_3 e_3^2 - \mu_4 e_a^2 \tag{27}$$

We note that (27) is negative definite on R^4.

Hence, by Lapiynov stability [14], "it is immediate that the parameter error and synchronization error decay exponentially to zero with time for all initial values".

Thus, we proved the results below.

Proposition 2. The drive and response identical chaotic systems (15) and (16) with unknown parameter a are synchronized for all initial values by adaptive control law (19), where the estimated parameter given by (26) and $\mu_1, \mu_2, \mu_3, \mu_4$ are constants greater than zero.

6.2. Numerical Results

To get the results numerically, we used the 4th-order Runge-Kutta method to solve systems (15) & (16), and solve system (18) with adaptive control law (19).

We take $x\big|_{x_1(0),x_2(0),x_3(0)} = [2,15,10]$ and $y\big|_{y_1(0),y_2(0),y_3(0)} = [18,6,4]$ as initial states of the drive system (15) and the response system (16) respectively. Also take a = 296.5 and $\mu_i = 8$ for i = 1, 2, 3, 4.

Figure 6 shows adaptive synchronization of the highly chaotic system.

Figure 7 shows the convergent for system (18) with controller (19).

7. Conclusion

A three-dimensional dynamical system is dealt in this paper, it has quadratic cross-product nonlinear terms. The basic characteristics are analyzed by equilibrium points, stability analysis (such as characteristic equation roots, Routh criterion, Hurwitz criterion and Lapiynov function) all methods of stability shows that the system is unstable. Then, dissipativity analysis indicates system (1) is dissipative for positive values of the parameter ρ. Lapiynov exponent, lapiynov dimension and wave-form analysis present the hyper chaos behavior when the

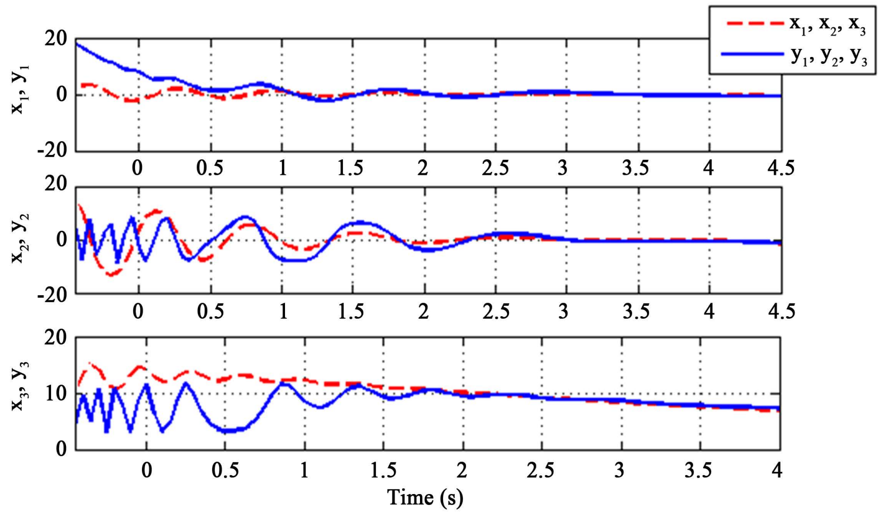


Figure 6. The synchronization trajectories for system (15) and (16).

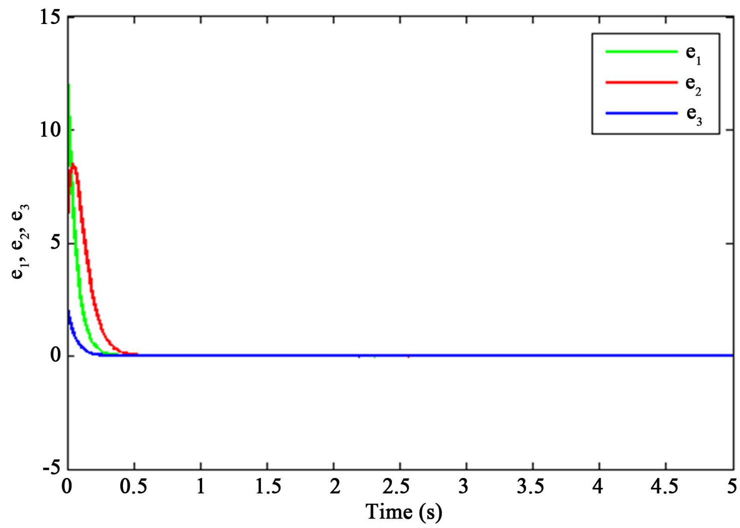


Figure 7. The convergent for system (18) with adaptive control (19).

parameters taken as $\rho = 10, a = 296.5, \delta = 40, \varphi = 10$, and the maximum values of lyapenov exponents are: $L_1 = 2.509426$, $L_2 = 0.132019$ and $L_3 = -11.818787$, lapiynov dimension "Kaplan-Yorke dimension" of the system is $D_L = 2.22349544$, which means that the system is highly chaotic system. Moreover, to stabilize the highly chaotic system, we produced an adaptive control strategy. Finally, we proposed adaptive synchronization scheme for identical highly chaotic system with upadate law for the estimiation of system parameter. Synchronization schemes are established by Lapiynov stability. Furthermore, we compared theoretical and graphical results of the system before and after control.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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