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Suzuki-Type Fixed Point Theorem in b_2-Metric Spaces

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Abstract

In this paper, we establish a fixed point theorem for two mappings under a contraction condition in b_2-metric space, and this theorem is related to a Suzuki-type of contraction.

Subject Areas

Mathematical Analysis

Keywords

Common Fixed Point, b_2-Metric Space, Generalized Suzuki-Type Contraction

1. Introduction

Banach [1] proved a principle, and this famous Banach contraction principle has many generalizations, see [2]-[7], and in 2008, Suzuki [8] established one of those generalizations, and this generalization is called Suzuki principle.

The aim of this paper is to prove a fixed point result generalized from the above mentioned principle in b_2-metric space [9].

2. Preliminaries

Before giving our results, these definitions and results as follows will be needed to present.

Definition 2.1 [9] Let X be a nonempty set, $s \geq 1$ be a real number and let d: $X \times X \times X \to R$ be a map satisfying the following conditions:

1) For every pair of distinct points $x, y \in X$, there exists a point $z \in X$ such that $d(x, y, z) \neq 0$.

2) If at least two of three points x, y, z are the same, then $d(x, y, z) = 0$,

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3) The symmetry:

$d(x,y,z)=d(x,z,y)=d(y,x,z)=d(y,z,x)=d(z,x,y)=d(z,x,y)$ for all $x,y,z\in X$.

1) The rectangle inequality:

$d(x,y,z)\le s\left[d(x,y,a)+d(y,z,a)+d(z,x,a)\right]$, for all $x,y,z,a\in X$.

Then d is called a b_2 metric on X and (X,d) is called a b_2 metric space with parameter s. Obviously, for $s=1$, b_2 metric reduces to 2-metric.

Definition 2.2 [9] Let $\{x_n\}$ be a sequence in a b_2 metric space (X,d).

1) A sequence $\{x_n\}$ is said to be b_2-convergent to $x\in X$, written as $\lim_{n\to\infty}x_n=x$, if all $a\in X$ $\lim_{n\to\infty}d(x_n,x,a)=0$.

2) $\{x_n\}$ is Cauchy sequence if and only if $d(x_n,x_m,a)\to 0$, when $n,m\to\infty$. for all $a\in X$.

3) (X,d) is said to be complete if every b_2-Cauchy sequence is a b_2-convergent sequence.

Definition 2.3 [9] Let (X,d) and (X',d') be two b_2-metric spaces and let $f:X\to X'$ be a mapping. Then f is said to be b_2-continuous, at a point $z\in X$ if for a given $\varepsilon>0$, there exists $\delta>0$ such that $x\in X$ and $d(z,x,a)<\delta$ for all $a\in X$ imply that $d'(fz,fx,a)<\varepsilon$. The mapping f is b_2-continuous on X if it is b_2-continuous at all $z\in X$.

Definition 2.4 [9] Let (X,d) and (X',d') be two b_2-metric spaces. Then a mapping $f:X\to X'$ is b_2-continuous at a point $x\in X'$ if and only if it is b_2-sequentially continuous at x; that is, whenever $\{x_n\}$ is b_2-convergent to x, $\{fx_n\}$ is b_2-convergent to $f(x)$.

Lemma 2.5 [9] Let (X,d) be a b_2-metric space and suppose that $\{x_n\}$ and $\{y_n\}$ are b_2-convergent to x and y, respectively. Then we have

$\frac{1}{s^2}d(x,y,a)\le\liminf_{n\to\infty}d(x_n,y_n,a)\le\limsup_{n\to\infty}d(x_n,y_n,a)\le s^2d(x,y,a)$, for all a in X. In particular, if $y_n=y$ is a constant, then

$\frac{1}{s}d(x,y,a)\le\liminf_{n\to\infty}d(x_n,y,a)\le\limsup_{n\to\infty}d(x_n,y,a)\le sd(x,y,a)$, for all a in X.

Lemma 2.6 [10] Let (X,d) be a b_2 metric space with $s\ge 1$ and let $\{x_n\}_{n=0}^{\infty}$ be a sequence in X such that

$$d(x_n,x_{n+1},a)\le\lambda d(x_{n-1},x_n,a), \tag{2.1}$$

for all $n\in N$ and all $a\in X$, where $\lambda\in\left[0,\frac{1}{s}\right)$. Then $\{x_n\}$ is a b_2-Cauchy sequence in (X,d).

3. Main Results

Theorem 3.1. Let (X,d) be a complete b_2-metric space. Let $f,g:X\to X$ be two self-maps and $\phi:[0,1)\to\left(\frac{1}{2},1\right]$ be defined as follows

$$\phi(\rho)=\begin{cases}1, 0\le\rho\le\dfrac{\sqrt{5}-1}{2}\\ \dfrac{1-\rho}{\rho^2}, \dfrac{\sqrt{5}-1}{2}\le\rho\le\dfrac{1}{\sqrt{2}}\\ \dfrac{1}{1+\rho}, \dfrac{1}{\sqrt{2}}\le\rho<1\end{cases} \tag{3.1}$$

Assume there exists $\rho\in[0,1)$ such that for every $x,y\in X$, the following condition is satisfied

$$\begin{aligned}&\frac{1}{s}\phi(\rho)\min\{d(x,fx,a),d(fx,fy,a)\}\le d(x,y,a)\\ &\Rightarrow \max\{d(gx,gy,a),d(gx,fy,a),d(fx,fy,a),d(gy,fx,a)\}\le\frac{\rho}{s^2}d(x,y,a).\end{aligned} \tag{3.2}$$

Then f,g have a unique common fixed point $z\in X$.

Proof in (3.2), we take $y=fx$

$$\begin{aligned}&\frac{1}{s}\phi(\rho)\min\{d(x,fx,a),d(x,gx,a)\}\le d(x,gx,a)\\ &\Rightarrow \max\{d(gx,g^2x,a),d(gx,fgx,a),d(fx,fgx,a),d(g^2x,fx,a)\} \text{ for } x\in X. \\ &\le\frac{\rho}{s^2}d(x,gx,a),\end{aligned} \tag{3.3}$$

therefore,

$$d(gx,fgx,a)\le\frac{\rho}{s^2}d(x,gx,a). \tag{3.4}$$

Now we take $y=fx$ in (3.2)

$$\begin{aligned}&\frac{1}{s}\phi(\rho)\min\{d(x,fx,a),d(x,gy,a)\}\le d(x,fy,a)\\ &\Rightarrow \max\{d(gx,gfy,a),d(gx,f^2y,a),d(fx,f^2x,a),d(gfx,fx,a)\} \text{ for all } x\in X. \\ &\le\frac{\rho}{s^2}d(x,fx,a),\end{aligned} \tag{3.5}$$

therefore,

$$d(fx,f^2x,a)\le\frac{\rho}{s^2}d(x,fx,a), \tag{3.6}$$

and

$$d(gfx,fx,a)\le\frac{\rho}{s^2}d(x,fx,a). \tag{3.7}$$

Given an arbitrary point x_0 in X then by $x_{2n+1}=gx_{2n}$ and $x_{2n+1}=fx_{2n+1}$ we construct a sequence $\{x_n\}$, for $n\in N$.

From (3.4), we get

$$d(x_{2n+1},x_{2n+2},a)=d(gx_{2n},fgx_{2n},a)\le\frac{\rho}{s^2}d(x_{2n},gx_{2n},a)=\frac{\rho}{s^2}d(x_{2n},x_{2n+1},a). \tag{3.8}$$

From (3.7) and (3.8) we get

$$d\left(x_{2n+1},x_{2n},a\right)=d\left(gfx_{2n-1},fx_{2n-1},a\right)\le\frac{\rho}{s^2}d\left(x_{2n},fx_{2n-1},a\right)=\frac{\rho}{s^2}d\left(x_{2n-1},x_{2n},a\right),$$

that is,

$d\left(x_{n+1},x_n,a\right)\le\frac{\rho}{s^2}d\left(x_n,x_{n-1},a\right)$, since $\frac{\rho}{s^2}\in[0,1)$, by Lemma 2.6, we get $\{x_n\}$ is a Cauchy sequence.

Since X is complete, there exists z in X, such that $\lim_{n\to\infty}x_n=z$, that is $\lim_{n\to\infty}gx_{2n}=\lim_{n\to\infty}x_{2n+1}=z$, and $\lim_{n\to\infty}fx_{2n+1}=\lim_{n\to\infty}x_{2n+2}=z$.

Now let us give that

$d\left(fx,z,a\right)\le\rho d\left(x,z,a\right)$, for every $x\ne z$. For $\left\{d\left(x_{2n},gx_{2n},a\right)\right\}$ is convergent to 0, and by Lemma 2.5, we get

$\frac{1}{s}d\left(x,z,a\right)\le\limsup_{n\to\infty}d\left(x_{2n},x,a\right)$, thus we have $\limsup_{n\to\infty}d\left(x_{2n},x,a\right)>0$, thus from the above relation, there exists a point x_{2n_k} in X such that

$$\frac{1}{s}\phi(\rho)\min\left\{d\left(x_{2n_k},gx_{2n_k},a\right),d\left(x_{2n_k},fx_{2n_k},a\right)\right\}\le d\left(x_{2n_k},x,a\right).$$

For such x_{2n_k}, (3.2) implies that

$$\begin{aligned}&d\left(gx_{2n_k},fx,a\right)\\&\le\max\left\{d\left(gx_{2n_k},gx,a\right),d\left(gx_{2n_k},fx,a\right),d\left(fx_{2n_k},fx,a\right),d\left(gx,fx_{2n_k},a\right)\right\}\\&\le\frac{\rho}{s_2}d\left(x_{2n_k},x,a\right),\end{aligned}$$

therefore by Lemma 3.5,

$$\frac{1}{s}d\left(fx,z,a\right)\le\limsup_{n\to\infty}d\left(gx_{2n_k},fx,a\right)\le\frac{\rho}{s^2}\limsup_{n\to\infty}d\left(x_{2n_k},x,a\right)\le\frac{\rho}{s}d\left(x,z,a\right),$$

therefore we get

$$d\left(fx,z,a\right)\le\rho d\left(x,z,a\right),\text{ for each } x\ne z. \tag{3.9}$$

Now we show that for each $n\in N$,

$$d\left(f^nz,z,a\right)\le d\left(fz,z,a\right), \tag{3.10}$$

It is obvious that the above inequality is true for $n=1$, assume that the relation holds for some $m\in N$. We get (3.10) is true when we have $f^mz=fz$ if $f^mz=z$, then if $f^mz\ne z$, we get the following relation from (3.9) and induction hypothesis, and that is

$$\begin{aligned}d\left(z,f^{m+1}z,a\right)&\le\rho d\left(z,f^mz,a\right)\le\rho^2d\left(z,f^{m-1}z,a\right)\le\cdots\le\rho^{m+1}d\left(z,fz,a\right)\\&\le\rho d\left(fz,z,a\right)\le d\left(fz,z,a\right),\end{aligned}$$

then (3.10) is proved.

Now we consider the following two possible cases in order to prove that f has a fixed point z in X, and that is $fz=z$.

Case 1 $0\le\rho<\frac{1}{\sqrt{2}}$, therefore, $\phi(\rho)\le\frac{1-\rho}{\rho^2}$. First, we prove the following relation

$$d\left(f^n z, fz, a\right) \le \frac{\rho}{s} d\left(fz, z, a\right), \text{ for } n \in N. \quad (3.11)$$

When $n=1$ it is obvious, and it follows from (3.6) when $n=2$, from (3.10) and take $a = fz$ we have

$d\left(f^n z, z, fz\right) \le d\left(fz, z, fz\right) = 0$, then we get $d\left(f^n z, fz, z\right) = 0$.

Now suppose that (3.11) holds for some $n>2$,

$$\begin{aligned} d\left(fz, z, a\right) &\le s\left(d\left(z, f^n z, a\right) + d\left(f^n z, fz, a\right) + d\left(f^n z, fz, z\right)\right) \\ &\le sd\left(z, f^n z, a\right) + sd\left(z, fz, a\right), \end{aligned}$$

Therefore, we get

$$(1-\rho) d\left(z, fz, a\right) \le sd\left(z, f^n z, a\right), \text{ that is } d\left(z, fz, a\right) \le \frac{s}{1-\rho} sd\left(z, f^n z, a\right), \quad (3.11.1)$$

then by taking $x = f^{n-1} z$ in (3.6)

$$d\left(f^n z, f^{n+1} z, a\right) \le \frac{\rho}{s^2} d\left(f^{n-1} z, f^n z, a\right) \le \cdots \le \frac{\rho^n}{s^{2n}} d\left(z, fz, a\right), \quad (3.11.2)$$

using the above two relations, (3.11.1) and (3.11.2) we have

$$\begin{aligned} &\frac{1}{s}\phi(\rho)\min\left\{d\left(gf^n z, f^n z, a\right), d\left(f^n z, f^{n+1} z, a\right)\right\} \\ &\le \frac{1-\rho}{s\rho^2} d\left(f^n z, f^{n+1} z, a\right) \le \frac{1-\rho}{s\rho^n} d\left(f^n z, f^{n+1} z, a\right) \\ &\le \frac{1-\rho}{s\rho^n} \cdot \frac{\rho^n}{s^{2n}} d\left(z, fz, a\right) = \frac{1-\rho}{s^{2n+1}} d\left(z, fz, a\right) \\ &\le \frac{1-\rho}{s^{2n+1}} \cdot \frac{s}{1-\rho} d\left(z, f^n z, a\right) \le \frac{1}{s^{2n}} d\left(z, f^n z, a\right) \le d\left(z, f^n z, a\right). \end{aligned}$$

From (3.2) and (3.10) with $x = f^n z$ and $y = z$, we have

$$\begin{aligned} &\max\left\{d\left(gf^n z, gz, a\right), d\left(gf^n z, fz, a\right), d\left(f^{n+1} z, fz, a\right), d\left(gz, f^{n+1} z, a\right)\right\} \\ &\le \frac{\rho}{s^2} d\left(z, f^n z, a\right) \le \frac{\rho}{s^2} d\left(z, fz, a\right) \le \frac{\rho}{s} d\left(z, fz, a\right). \end{aligned}$$

Therefore,

$$d\left(f^{n+1} z, fz, a\right) \le \frac{\rho}{s} d\left(fz, z, a\right). \quad (3.12)$$

So by induction we prove the relation of (3.11).

Now (3.11) and $fz \ne z$ show that for every $n \in N$ $f^n z \ne z$, thus, (3.9) shows that

$$d\left(z, f^{n+1} z, a\right) \le \rho d\left(z, f^n z, a\right) \le \rho^2 d\left(z, f^{n-1} z, a\right) \le \cdots \le \rho^n d\left(z, fz, a\right).$$

Therefore $\lim_{n\to\infty} d\left(z, f^{n+1} z, a\right) = 0$. Furthermore by using Lemma 2.5, we get

$$\frac{1}{s} d\left(z, \liminf_{n\to\infty} f^{n+1} z, a\right) \le \liminf_{n\to\infty} d\left(z, f^{n+1} z, a\right) = 0,$$

so

$$d\left(z, \liminf_{n\to\infty} f^{n+1} z, a\right) = 0.$$

In the same way,

$d\left(z, \limsup_{n\to\infty} f^{n+1}z, a\right) = 0$, thus we have $d\left(z, \lim_{n\to\infty} f^{n+1}z, a\right) = 0$, that is $f^{n+1}z \to z$, and by using Lemma 2.5 in (3.12), we get

$\frac{1}{s} d(z, fz, a) \le \limsup_{n\to\infty} d\left(f^{n+1}z, fz, a\right) \le \frac{\rho}{s} d(z, fz, a)$, which claims that $d(z, fz, a) = 0$, and that is a contraction.

Case 2. $\frac{1}{\sqrt{2}} \le \rho < 1$, and that is when $\phi(\rho) = \frac{1}{1+\rho}$. We now prove that we can find a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that

$$\frac{1}{s(1+\rho)} \min\left\{d\left(x_{n_k}, gx_{n_k}, a\right), d\left(x_{n_k}, fx_{n_k}, a\right)\right\} \le d\left(x_{n_k}, z, a\right), \text{ for } k \in N. \quad (3.13)$$

The contraries of the above relation are as follows

$$\frac{1}{s(1+\rho)} d(x_n, fx_n, a) \ge \frac{1}{s(1+\rho)} \min\{d(x_n, gx_n, a), d(x_n, fx_n, a)\} > d(x_n, z, a),$$

and

$$\frac{1}{s(1+\rho)} d(x_n, fx_n, a) \ge \frac{1}{s(1+\rho)} \min\{d(x_n, gx_n, a), d(x_n, fx_n, a)\} > d(x_n, z, a),$$

for $n \in N$. If n is even we have

$$\begin{aligned}
&\frac{1}{s(1+\rho)} d(x_{2n}, gx_{2n}, a) \\
&\ge \frac{1}{s(1+\rho)} \min\{d(x_{2n}, gx_{2n}, a), d(x_{2n}, fx_{2n}, a)\} \\
&> d(x_{2n}, z, a),
\end{aligned}$$

if n is odd then we get

$$\begin{aligned}
&\frac{1}{s(1+\rho)} d(x_{2n+1}, fx_{2n+1}, a) \\
&\ge \frac{1}{s(1+\rho)} \min\{d(x_{2n+1}, gx_{2n+1}, a), d(x_{2n+1}, fx_{2n+1}, a)\} \\
&> d(x_{2n+1}, z, a),
\end{aligned}$$

for $n \in N$. By (3.8) we have

$$\begin{aligned}
&d(x_{2n}, x_{2n+1}, a) \\
&\le s\left(d(x_{2n}, z, a) + d(x_{2n+1}, z, a) + d(x_{2n}, x_{2n+1}, z)\right) \\
&< \frac{s}{s(1+\rho)} d(x_{2n}, gx_{2n}, a) + \frac{s}{s(1+\rho)} d(x_{2n+1}, fx_{2n+1}, a) \\
&\quad + \frac{s}{s(1+\rho)} d(x_{2n}, gx_{2n}, x_{2n+1}) \\
&= \frac{1}{1+\rho}\left(d(x_{2n}, x_{2n+1}, a) + d(x_{2n+1}, x_{2n+2}, a) + d(x_{2n}, x_{2n+1}, x_{2n+1})\right)
\end{aligned}$$

$$\begin{aligned}&\le \frac{1}{1+\rho}d\left(x_{2n},x_{2n+1},a\right)+\frac{\rho}{s^2\left(1+\rho\right)}d\left(x_{2n+1},x_{2n},a\right)\\&\le \frac{1}{1+\rho}d\left(x_{2n},x_{2n+1},a\right)+\frac{\rho}{1+\rho}d\left(x_{2n+1},x_{2n},a\right)\\&= d\left(x_{2n},x_{2n+1},a\right),\end{aligned}$$

this is impossible. Therefore, one of the following relations is true for every $n \in N$,

$$\frac{1}{s}\phi(\rho)\min\left\{d\left(x_{2n},gx_{2n},a\right),d\left(x_{2n},fx_{2n},a\right)\right\}\le d\left(x_{2n},z,a\right),$$

or

$$\frac{1}{s}\phi(\rho)\min\left\{d\left(x_{2n+1},gx_{2n+1},a\right),d\left(x_{2n+1},fx_{2n+1},a\right)\right\}\le d\left(x_{2n+1},z,a\right).$$

That means there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that (3.13) is true for every $k \in N$. Thus (3.2) shows that

$$\begin{aligned}&d\left(gx_{2n},fz,a\right)\\&\le \max\left\{d\left(fx_{2n},gz,a\right),d\left(fz,gx_{2n},a\right),d\left(fx_{2n},fz,a\right),d\left(gz,fx_{2n},a\right)\right\}\\&\le \frac{\rho}{s^2}d\left(x_{2n},z,a\right).\end{aligned}$$

or

$$\begin{aligned}&d\left(fx_{2n+1},fz,a\right)\\&\le \max\left\{d\left(gx_{2n+1},gz,a\right),d\left(fz,gx_{2n+1},a\right),d\left(fx_{2n+1},fz,a\right),d\left(gz,fx_{2n+1},a\right)\right\}\\&\le \frac{\rho}{s^2}d\left(x_{2n+!},z,a\right).\end{aligned}$$

From Lemma 2.5, we have

$$\frac{1}{s}d\left(z,fz,a\right)\le \limsup_{n\to\infty} d\left(gx_{2n},fz,a\right)\le \frac{\rho}{s^2}\limsup_{n\to\infty} d\left(x_{2n},z,a\right)\le \frac{\rho}{s}d\left(z,z,a\right)=0,$$

or

$$\frac{1}{s}d\left(z,fz,a\right)\le \limsup_{n\to\infty} d\left(fx_{2n+1},fz,a\right)\le \frac{\rho}{s^2}\limsup_{n\to\infty} d\left(x_{2n+1},z,a\right)\le \frac{\rho}{s}d\left(z,z,a\right)=0,$$

Therefore $d\left(z,fz,a\right)\le 0$, which is impossible unless $fz = z$. hence *z* in *X* is a fixed point of *f*. From the process of the above proof, we know $fz = z$, then by

$$0=\frac{1}{s}\phi(\rho)\min\left\{d\left(z,fz,a\right),d\left(z,gz,a\right)\right\}\le d\left(z,fz,a\right),$$

it implies

$$\begin{aligned}d\left(gz,z,a\right)&\le \max\left\{d\left(gz,gfz,a\right),d\left(gz,f^2z,a\right),d\left(fz,f^2z,a\right),d\left(gfz,fz,a\right)\right\}\\&\le \frac{\rho}{s^2}d\left(fz,z,a\right)=0,\end{aligned}$$

this proves that $gz = z$. By (3.2) we can prove the uniqueness of the common fixed point *z*,

$\frac{1}{s}\phi(\rho)\min\{d(z,fz,a),d(z,gz,a)\}\le d(z,z',a)$, so (3.2) shows that

$$
\begin{aligned}
d(z,z',a) &= \max\{d(gz,gz',a),d(fz,fz',a),d(gz,fz',a),d(gz',fz,a)\} \\
&\le \frac{\rho}{s^2}d(z,z',a),
\end{aligned}
$$

which is impossible unless $z=z'$. □

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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