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An Aggregate Production Plan for a Biscuit Manufacturing Plant Using Integer Linear Programming

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Abstract

Effective planning, scheduling, and synchronization of all production activities are the key responsibilities of the management of a manufacturing plant. Therefore, it is necessary for the management of the plant to design the production process so that the total production cost is minimized, subject to the available resources that cannot be compromised. In this study, a biscuit manufacturing plant is selected and an integer linear programming (ILP) model is formulated to determine aggregate number of batches that the plant should produce from each product per month so that monthly demand is satisfied with available resources. The objective is to minimize the monthly production cost of the plant. The required data were collected from the production plant for a period of one month, and then, the objective function and constraints were formulated. The management has given a paramount importance in satisfying the demand so that there will not be any unsatisfied customer. According to the managerial requirement, any feasible solution obtained by the model must satisfy the demand. Therefore, demand constraint is considered as a hard constraint. The management is forced to adjust the labour and machine requirements more frequently according to the monthly demand. Thus, labour and machine hour constraints are considered as soft constraints. Formulated ILP model was implemented as a spreadsheet model in Excel and solved using Excel Solver which uses the simplex algorithm and incorporates the integer requirement of the model when finding the optimal solution. Total available labour and machine hours can be changed within a particular range until a feasible solution is found. The solved model determines the number of batches to be produced from each product and the corresponding minimum cost per month. By implementing this production plan, manufacturing excess of biscuits can be avoided and hence utilizes the physical and human resources to the optimum manner. Additionally, the machine and labour idle times and the needed overtime hours can be identified using the solution while the additional overtime cost will be added to the monthly production cost.

Subject Areas

Corporate Governance

Keywords

Integer Linear Programming, Soft Constraints, Hard Constraints, Spreadsheet Model, Excel Solver, Simplex Algorithm

1. Introduction

Proper functioning of a manufacturing plant requires efficient planning, scheduling, and synchronization of all production activities. Thus, a production plan (PP) is an important part of a business plan that the manufacturing or production department is responsible for developing. It is necessary for the management of the plant to determine the total amount of output that the manufacturing plant should produce from each period in the planning horizon in order to satisfy the customer demand with the limited available resources in the plant. The output is generally expressed in terms of units of measurement such as tons, liters, kilograms and batches. A quantitative solution for the PP problem can be found in different ways. One method is to use a mathematical model to solve a PP problem. The applications of Operations Research (OR) are widely used in industrial sector due to its ability to optimize a given scenario resulting in maximum possible gain. In practical situations, linear programming is a part of a very essential area of mathematics termed "optimization techniques". A mathematical model can be formulated using linear programming (LP) [1] with the objective of minimizing the cost or maximizing the profit without violating the constraints of limited resources and satisfying the demand constraint in order to determine the number of units to be produced from each item. Tingley [2] revealed that LP is a conventionally used technique. However, in order to apply LP to a production planning problem, the data input should be uniquely determined.

In this study, a quantitative approach is used by means of operations research to solve an aggregate production planning (APP) problem for a biscuit manufacturing plant. By implementing this production plan, manufacturing excess of biscuits can be avoided and hence utilizes the physical and human resources to the optimum manner. This can be achieved by building up a mathematical model using LP. LP is the procedure of taking different linear inequalities considering some situations and finding the best feasible solution subject to those requirements. Using LP techniques to solve PP problems accounts for vast amount of studies found in the literature [3]-[10]. In these studies, where LP techniques were used, many authors attempted to solve the APP problem. In APP, the aim is to obtain overall (aggregate) production quantities for each product. Other approaches such as goal programming have been used to solve the APP problem in the literature [11] [12] [13]. Since defining goals is a managerial decision, the problem is reduced to a linear programming problem. In real-world APP problems, the input data or parameters, such as demand, resources, cost and objective function are often uncertain (fuzzy) because some information is incomplete or unavailable. Therefore, Fuzziness is included in most models in previous studies to find a solution to APP [7] [8] [14]. Moreover, Wang and Fang [15] incorporated the fuzzy nature of parameters and presented a fuzzy linear programming method for solving the APP. However, the data such as cost and resources for this study were obtained from the manufacturing plant. The complexity of the problem reduces as the monthly demand is decided by the top management and informed the production managers at the beginning of each month. Thus, as mentioned earlier, an LP model can be developed to solve this PP problem considering only one month as the data input is uniquely determined. The aim of this study is to determine the number of batches to be produced from each product per month in order to satisfy the monthly demand with the available resources while minimizing the production cost. Owing to the fact that the plant does not produce partial batches, all the variables are restricted to be integers. Thus, in this study, an integer linear programming (ILP) model [16] is used to solve the PP problem. In our mathematical model all the decision variables are restricted to be integers. However, when only a set of variables are integers, the problem can be solved using mixed integer linear programming (MILP) problem [17] [18].

If the plant has the capability of producing biscuits according to the monthly demand, based on their past experience, without using further quantitative analytical methods, then most of the biscuits will remain in the finished-goods stores for a longer period, before they are sent to the warehouses or other distributing areas. As a result, more often, by the time customer consumes the product, biscuits are reaching their expiration date, so that the moisture level of biscuits will be increased, the appearance and the colour may have changed, and the crispiness will be reduced. Once such low-quality products are released to the market, the goodwill of the customer regarding the product would be lost. An improved PP for the production plant is extremely important since a better production plan will reduce the time that the finished goods remain in the stores, thus, prevent the additional inventory costs and the quality of the product will be fresh by the time consumer consumes it. In addition, the product can be sent to the market according to the demand without unnecessary delays.

In the production process, the wet dough starts to process at the mixing and passes through different types of machineries in forming section such as lamination, gauge rolls, relaxing web, and moulding, and subsequently through oven, cooling web and stacking web. This is a continuous process and the plant has only one production line and therefore, parallel production is not possible (see

Figure 1 for the production process). The plant is producing various types of products, which are mainly categorized as soft dough and hard dough biscuits. In addition, there are certain products, which are unique and need further manufacturing process such as oiling, creaming and flavouring. These deviations are taken into account when the mathematical model is formulated. The range of products that are produced in the plant is presented in Table 1. The table categorises the products as soft dough and hard dough and specifies the biscuits which use the additional processing; creaming and flavouring. Furthermore, the coefficients for both objective function and constraints are calculated to develop the mathematical model by considering the process of the biscuit plant.

2. Materials and Methods

A mathematical model to determine the number of batches to be produced from each item (see Table 1 for all the available products in this plant) was formulated using ILP with the objective of minimizing the cost without violating the

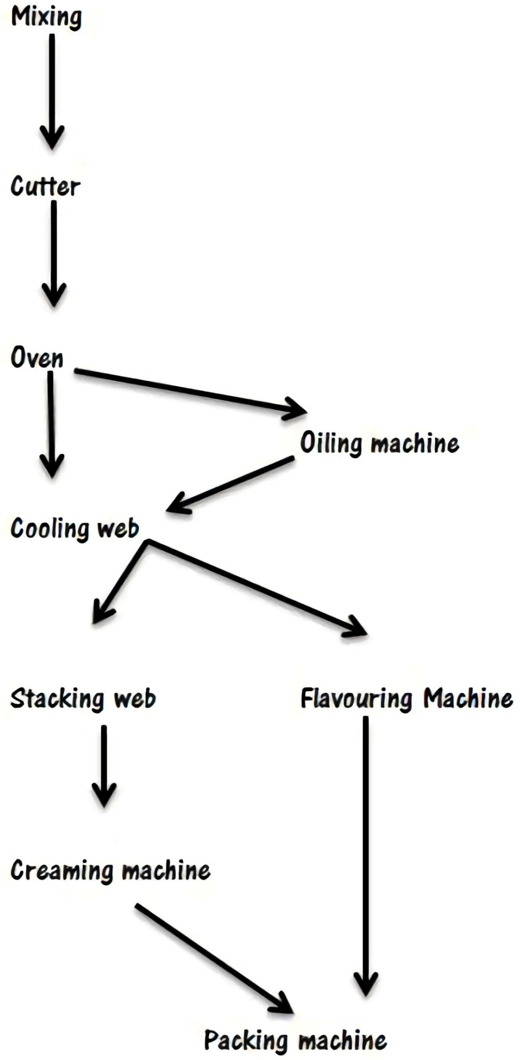

Figure 1. Production process.

Table 1. Range of products.

Type	Product	Creaming	Flavouring
Hard dough	Cheese and Onions (P_8)		Yes
	Cheese Cuts (P_7)		Yes
	Cream Cracker (P_1)		
	Hot Chilly Byte (P_9)		Yes
	Lemon Puff (P_{10})	Yes	
	Marie (P_5)		
	Onion Byte (P_6)		Yes
Soft dough	Chocolate Cream (P_{11})	Yes	
	Nice (P_2)		
	Sorties (P_3)		
	Teasty (P_4)		

constraints of limited resources and satisfying the demand constraint. All the necessary data were collected and the coefficients corresponding to the objective and constraints were estimated as demonstrated in the following sections.

2.1. Formulating the Mathematical Model

Let x_i be the number of batches to be produced from the i^{th} product (P_i) per month, where $i = 1, 2, \cdots, 11$.

2.1.1. Objective

The objective is to minimize the monthly manufacturing cost of unpacked biscuits (baked biscuits, which are not packed but stacked into the bins). The production cost per kilogram of biscuits for unpacked biscuit was collected as raw data. It involves raw material cost, labour cost, fuel cost for the oven, electricity etc. After incorporating these costs, the objective function was formulated as follows:

Let C_i be the production cost per finished batch of the i^{th} product.

$$\min Z = \sum_{i=1}^{11} C_i x_i + OC;$$

where C_i is the cost per batch of the i^{th} product and OC is the overtime cost.

The wastage produced in the process of biscuit manufacturing and weight losses at each section are illustrated in Table 2. Afterward, all the wastages and weight losses mentioned in Table 2 were reduced from the weight of the wet dough mixture. The model is developed and solved assuming that these are the only wastages that will occur in the production process. All the necessary data are exhibited in Table 3. It enumerates the weight of a baked biscuit batch (B_i). Given that the cost per kilogram (k_i) was collected as raw data, the cost per batch of the i^{th} product (C_i) can be calculated using the following formulas. The computations are done in Table 4.

Table 2. The wastage produced in the process of biscuit manufacturing and weight losses at each section.

Product (P_i)	Dough Weight (Kg)	Baking loss %	Waste at mixing (kg)	Waste at cutter (kg)	Waste in cooling & stacking % (from oven output)
P_1	550	26	1	1.5	0.5
P_2	650	13	0.75	2	0.5
P_3	615	13	0.75	2	0.5
P_4	580	13	0.75	2	0.5
P_5	619	23	1	1.5	0.5
P_6	525	27	1	1.5	0.5
P_7	600	27	1	1.5	0.5
P_8	600	27	1	1.5	0.5
P_9	580	27	1	1.5	0.5
P_{10}	540	22	1	1.5	0.5
P_{11}	630	12	0.75	2	0.5

Table 3. Weights of baked biscuit batches.

Product (P_i)	Dough weight (Kg)	Waste in Mixing (Kg)	Oven input (Kg)	Baking loss (kg)	Oven output (kg)	Waste in cooling & stacking (kg) (from oven output)	Final weight of the baked biscuit batch (Kg)—B_i
P_1	550	2.5	547.5	142.35	405.15	2.03	403.12
P_2	650	2.75	647.25	84.14	563.11	2.82	560.29
P_3	615	2.75	612.25	79.59	532.66	2.66	529.99
P_4	580	2.75	577.25	75.04	502.21	2.51	499.7
P_5	619	2.5	616.5	141.8	474.71	2.37	472.33
P_6	525	2.5	522.5	141.08	381.43	1.91	379.52
P_7	600	2.5	597.5	161.33	436.18	2.18	433.99
P_8	600	2.5	597.5	161.33	436.18	2.18	433.99
P_9	580	2.5	577.5	155.93	421.58	2.11	419.47
P_{10}	540	2.5	537.5	118.25	419.25	2.1	417.15
P_{11}	630	2.75	627.25	75.27	551.98	2.76	549.22

Table 4. Cost per batch for each product.

Product (P_i)	Production cost per kilogram (Rs)—K_i	Final weight of the baked biscuit batch (Kg)—B_i	Cost per batch (Rs)—C_i
P_1	150.0015876	403.12	60,468.64
P_2	133.000464	560.29	74,518.83
P_3	117.0009245	529.99	62,009.32
P_4	115.9991795	499.7	57,964.79

Continued

P_5	124.0003811	472.33	58,569.1
P_6	149.9991568	379.52	56,927.68
P_7	260.0024655	433.99	112,838.47
P_8	220.0020968	433.99	95,478.71
P_9	129.9991179	419.47	54,530.73
P_{10}	170.0015342	417.15	70,916.14
P_{11}	180.0000364	549.22	98,859.62

Table 5. Total demand for each product.

Product (P_i)	SKU (g)	No of packets in a box	Demand (in boxes)	Total demand for different SKUs (kg)	Total demand for each product type (Kg)
	125	50	1475	9218.75	
P_1	190	24	1875	8550	19,769
	500	4	1000	2000	
P_2	100	50	2500	12,500	15,500
	500	4	1500	3000	
P_3	75	50	47,500	178,125	19,200
	320	4	15,000	19,200	
P_4	60	50	45,000	135,000	21,000
	300	4	17,500	21,000	
P_5	80	50	4250	17,000	18,050
	300	4	875	1050	
P_6	30	20	8750	5250	10,320
	130	20	1950	5070	
P_7	170	24	2250	9180	12,015
	210	10	1350	2835	
P_8	25	20	22,500	11,250	11,250
P_9	30	20	20,000	12,000	12,000
P_{10}	100	48	2150	10,320	17,520
	200	24	1500	7200	
P_{11}	100	50	2625	13,125	14,725
	400	4	1000	1600	

$$B_i = \text{Dough Weight} - \text{Waste in Mixing} - \text{Baking Loss} - \text{Waste in Cooling \& Stacking}$$

$$C_i = \text{Production cost per kilogram of biscuits}\left(K_i\right) \times \text{Weight of a baked biscuit batch}\left(B_i\right).$$

Hence, the objective function for the concerned problem can be written as:

$$\min Z = 60468.64x_1 + 74518.83x_2 + 62009.32x_3 + 57964.79x_4 + 58569.10x_5 + 56927.68x_6 + 112838.47x_7 + 95478.71x_8 + 54530.73x_9 + 70916.14x_{10} + 98859.62x_{11}.$$

2.1.2. Constraints

Satisfying all constraints is essential for the feasibility. However, many models contain two categories of constraints: hard constraints that must be satisfied by any feasible solution and the soft constraints of different relative importance may or may not be satisfied. Hvolby and Steger-Jensen [19] discussed using soft constraints and hard constraints in production planning. The hard constraints stipulate the set of feasible solutions, and the soft constraints stipulate a function to be optimized in deciding between the feasible solutions. If both kinds of constraints exist in a model, soft constraints can be adjusted, until a feasible solution is found.

1) Hard Constraint

- **Demand constraint**

The management has given utmost importance in satisfying the demand so that there will not be any unsatisfied customer. Any feasible solution obtained by the model must satisfy the demand. Therefore, for this study, demand constraint was considered as a hard constraint.

Since x_i is the number of batches to be produced from the i^{th} product, $B_i x_i$ gives the total number of kilograms that have to be produced from the i^{th} product per month. This amount should be equal to the demand of each product (in kilograms) to satisfy the demand as the plant does not want to produce any excess amount of biscuits. Thus, the demand constraint, in general, can be written as follows:

$$B_i x_i \geq D_i \quad \text{for} \quad i = 1, 2, \cdots, 11;$$

where D_i is the monthly demand in kilograms for the i^{th} product.

The monthly demand is decided by the top management which is passed away to the biscuit plant at the beginning of every month. The demand is expressed as the number of boxes of biscuits needed for different stock keeping units (SKU) from each product. These data were collected from the manufacturing plant and monthly demand in kilograms for each product was calculated utilizing these data. Table 5 enumerates all the necessary calculations to find the total demand in kilograms per month from each product.

As illustrated in Table 1, the product types, Lemon Puff (P_{10}) and Chocolate cream (P_{11}) have a cream layer. Thus, in Table 6, the cream weight was reduced

Table 6. Biscuit weight without cream.

Product (P_i)	Cream percentage %	Total Weight (kg)	Cream weight (kg)	Weight without cream (kg)
P_{10}	30	17,520	5256	12,264
P_{11}	25	14,725	3681.25	11,043.75

from the total demand which was calculated in Table 5 for P_{10} and P_{11} in order to calculate the total amount of baked biscuits that should be produced.

After calculating all the coefficient values and right-hand side values which are the requirements, the demand constraints for each product can be written as follows:

$$403.12425x_1 \geq 19769$$

$$560.2919625x_2 \geq 15500$$

$$529.9942125x_3 \geq 19200$$

$$499.6964625x_4 \geq 21000$$

$$472.331475x_5 \geq 18050$$

$$379.517875x_6 \geq 10320$$

$$433.994125x_7 \geq 12015$$

$$433.994125x_8 \geq 11250$$

$$419.467125x_9 \geq 12000$$

$$417.15375x_{10} \geq 12264$$

$$549.2201x_{11} \geq 11043.75 .$$

2) Soft Constraints

The manufacturing company has limited amount of resources. Therefore, it is essential to consider available resources in the plant when minimizing the manufacturing cost. When the biscuits are manufactured, manufacturing process cannot exceed consuming the available amount of resources such as the number of labourers and the machine capacity. The management is repeatedly adjusting the labour and machine requirements according to the monthly demand. Thus, in this study, labour and machine hour constraints were considered as soft constraints.

- **Machine Hours**

A batch of each product takes a certain processing time. The required total processing time for the monthly production should be less than or equal to total available machine hours per month. If the demand cannot be satisfied with the available total machine hours, number of working hours per day and working days per month can be increased up to some level in the model solving step. Therefore, machine hour constraint was considered as a soft constraint. Then, the machine hour constraint, in general, is as follows:

$$\sum_{i=1}^{11} t_i x_i \leq T \quad \text{for} \quad i = 1, 2, \cdots, 11 ,$$

where t_i is the total processing time (in minutes) per batch of the i^{th} product and T is the available machine time (in minutes) per month.

Available machine hours per month are calculated in Table 7 while processing time for a batch of each product is illustrated in Table 9. The machine hour constraint can be written as follows:

Table 7. Available machine hours per month.

Number of working hours per day	Number of working days per month	Total available machine hours (T)	T (min)
8	25	200	12000

$$\begin{aligned}&30x_1+25x_2+20x_3+26x_4+40x_5+50x_6+45x_7\\&+50x_8+50x_9+30x_{10}+30x_{11}\le 12000\end{aligned}.$$

- **Labor hour constraint**

The required labour hours for the monthly production should be less than the available labour hours per month. Thus, the labour hour constraint for each section; mixing, cutter, baking and cooling, and stacking can be formulated, in general, as follows:

$$\sum_{i=1}^{11} t_i l_I x_i \le L,$$

for each of the four sections mixing, cutter, baking and cooling, and stacking, where l_i is the number of labourers needed for each production section, t_i is the processing time (in minutes) per batch of the i^{th} product through each section and L is the total available labour hours.

A fixed number of labourers are allocated to each section of the production plant (see Table 8). Among these labourers, different numbers of labourers are assigned into each section during the production of different products (see Table 9). Moreover, the calculations of the required labour time for each section during the production are demonstrated in Table 9 and the available labour time in minutes is shown in Table 8.

Since the number of working days or hours can be adjusted until a feasible solution is found, this constraint is also identified as a soft constraint. Therefore, labour constraint for each section for this PP problem can be presented as follows:

Mixing:

$$\begin{aligned}&300x_1+300x_2+240x_3+312x_4+400x_5+500x_6+450x_7\\&+500x_8+500x_9+300x_{10}+360x_{11}\le 144000\end{aligned}.$$

Cutter:

$$\begin{aligned}&180x_1+125x_2+100x_3+130x_4+240x_5+300x_6+270x_7\\&+300x_8+300x_9+180x_{10}+150x_{11}\le 72000\end{aligned}.$$

Baking and Cooling:

$$\begin{aligned}&150x_1+125x_2+100x_3+130x_4+200x_5+250x_6+225x_7\\&+250x_8+250x_9+150x_{10}+150x_{11}\le 72000\end{aligned}.$$

Stacking:

$$\begin{aligned}&420x_1+450x_2+360x_3+468x_4+800x_5+300x_6+270x_7\\&+300x_8+300x_9+180x_{10}+540x_{11}\le 240000\end{aligned}.$$

In addition, all the x_i values should be positive integers as the plant does not produce partial batches. That is, $x_i \ge 0$ and integer for $i=1,2,\cdots,11$.

Table 8. Number of labours assigned for each section and available labour hours per month.

Section	Available number of labours	Number of days	Number of hours	L (h)	L (min)
Mixing	12	25	8	2400	144,000
Cutter	6	25	8	1200	72,000
Baking and Cooling	5	25	8	1200	72,000
Stacking	20	25	8	4000	240,000

Table 9. Required labour time.

Product (P_i)	Processing time per batch (min)—t_i	Number of labourers needed				Required labour time (min)			
		Mixing	Cutter	Baking & Cooling	Stacking	Mixing	Cutter	Baking & Cooling	Stacking
P_1	30	10	6	5	14	300	180	150	420
P_2	25	12	5	5	18	300	125	125	450
P_3	20	12	5	5	18	240	100	100	360
P_4	26	12	5	5	18	312	130	130	468
P_5	40	10	6	5	20	400	240	200	800
P_6	50	10	6	5	6	500	300	250	300
P_7	45	10	6	5	6	450	270	225	270
P_8	50	10	6	5	6	500	300	250	300
P_9	50	10	6	5	6	500	300	250	300
P_{10}	30	10	6	5	6	300	180	150	180
P_{11}	30	12	5	5	18	360	150	150	540

2.2. Solving the Formulated Mathematical Model

The most prominent algorithm to solve LP problems is the Simplex Algorithm developed by Dantzig in 1947. However, the formulated model has integer decision variables and therefore it is referred to as Integer Linear Programming (ILP). Thus, the problem cannot be solved using the simplex method alone. One class of exact algorithms that can be used to solve ILP is cutting plane methods [20]. This method first uses LP relaxation and afterward adds linear constraints so that it will lead the solution towards being integer without excluding any integer feasible points. Another class of exact algorithms is variants of the branch and bound method [21]. Many problems are intractable since ILP is NP-hard (see [22]), thus heuristic methods are used instead. Hill climbing (see [23]), simulated annealing (see [24]), and ant colony optimization (see [25]) are some of the heuristic methods that can be applied to solve ILPs. However, in this study spreadsheet paradigm was used to solve the formulated ILP problem.

Model in Microsoft Excel

The above integer linear programming model was implemented in Microsoft

Excel (see **Figure 2**). Major spreadsheet packages come with a built-in optimization tool called Solver. MacDonald [26] explained how to use Microsoft Excel Solver to solve LP problems. Many authors in the literature have used the Excel Solver to solve different linear and integer linear programming problems [27] [28]. Once a model was implemented in a spreadsheet, the optimal values of the decision variables and the optimal objective function value can be found using Excel Solver. In the Solver dialog box, Simplex LP was selected to solve the ILP implemented in Excel as a spreadsheet model, where Solver uses a Branch and Bound to solve the model. The demand constraint should be satisfied by any solution of this problem as it was identified as a hard constraint. The number of working hours, working days were changed within a particular range acceptable to the management, and a feasible solution was found when the number of working hours was increased up to 8.5 hours, while number of labours assigned for each section considered constant. Since the production process is continuous, labourers and machines in all sections should work for additional 0.5 hours. As a result, the right-hand side of the machine hour and labour hour constraint were changed as displayed in the spreadsheet model in **Figure 2**. The overtime cost can be calculated separately, thus, it is not included in the spreadsheet model.

3. Results and Discussion

3.1. Results

According to the obtained optimum solution, the number of batches to be

	Products	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11			
	Number of batches	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11			
	(per month)	50	28	37	43	39	28	28	26	29	30	21			
Objective	Cost coefficient	60468.6	74518.8	62009.3	57964.8	58569.1	56927.7	112838.5	95478.7	54530.7	70916.1	98859.6		Min Z	25201810.9
													Sum Product		
Constraint 1(Demand)	Cream Cracker(P1)	403.124	0	0	0	0	0	0	0	0	0	0	20156.21	≥	19,769
	Nice(P2)	0	560.292	0	0	0	0	0	0	0	0	0	15688.17	≥	15500
	Shorties(P3)	0	0	529.994	0	0	0	0	0	0	0	0	19609.79	≥	19200
	Teasty(P4)	0	0	0	499.696	0	0	0	0	0	0	0	21486.95	≥	21000
	Marie(P5)	0	0	0	0	472.331	0	0	0	0	0	0	18420.93	≥	18050
	Onion Byte(P6)	0	0	0	0	0	379.518	0	0	0	0	0	10626.5	≥	10320
	Cheese Cuts(P7)	0	0	0	0	0	0	433.9941	0	0	0	0	12151.84	≥	12015
	Cheese & Onion(P8)	0	0	0	0	0	0	0	433.994	0	0	0	11283.85	≥	11250
	Hot Chilly Byte(P9)	0	0	0	0	0	0	0	0	419.467	0	0	12164.55	≥	12000
	Lemon Puff(P10)	0	0	0	0	0	0	0	0	0	417.154	0	12514.61	≥	12264
	Chocolate Cream(P11)	0	0	0	0	0	0	0	0	0	0	549.22	11533.62	≥	11043.75
Constraint 2(Machine Hours)	For all products	30	25	20	26	40	50	45	50	50	30	30	12558	≤	12750
Constraint 3(Labor Hours)	Mixing	300	300	240	312	400	500	450	500	500	300	360	131956	≤	153000
	Cutter	180	125	100	130	240	300	270	300	300	180	150	72160	≤	76500
	Baking and Cooling	150	125	100	130	200	250	225	250	250	150	150	62790	≤	63750
	Stacking	420	450	360	468	800	300	270	300	300	180	540	147444	≤	255000

Figure 2. The spreadsheet model and solution as exhibited in Excel.

produced from each product per month is displayed in **Figure 2**, and according to **Figure 2**, monthly demand can be satisfied using the available resources with a minimum cost of approximately Rs.25.2 million. In order to achieve that, the plant should produce following number of batches from each product:

x_1 (Cream Cracker) = 50, x_2 (Nice) = 28, x_3 (Sorties) = 37, x_4 (Teasty) = 43, x_5 (Marie) = 39, x_6 (Onion byte) = 28, x_7 (Cheese cuts) = 28, x_8 (Cheese and Onion) = 26, x_9 (Hot chilly Byte) = 29, x_{10} (Lemon Puff) = 30, x_{11} (Chocolate Cream) = 21.

Moreover, in order to satisfy the customer demand, the manufacturing plant should function 25 days and 8.5 hours continuously each day for this particular month.

3.2. Discussion

The Excel does not provide sensitivity report, as the variables are integers. Therefore, sensitivity analysis cannot be performed for the optimal solution. The total monthly cost can be reduced by Rs. 474,705 if the integer constraint is removed and this additional monthly cost is due to the excess amount of biscuits that will be produced due to the integer constraint. Consequently, if the company can produce partial batches, then the monthly additional cost of producing additional biscuits could be reduced and the excess amount of biscuits produced could be avoided.

In addition, the solved model (using sum product column) identifies the number of working hours and working days of the production plant per month and the total kilograms of biscuits from each product should be produced by the end of month. However, the minimum number of hours that the plant should function to get a feasible solution is 8.372 hours. However, it was round up to 8.5 considering the convenience of paying overtime. If we assume that all the 43 labourers are working 8.5 hours for all the 25 days even though for some products some labourers are kept idle, the overtime cost will be $(8.5 - 8) \times 25 \times 43 \times \text{OTR}$, where OTR is the overtime rate. Moreover, idle times of labourers and machines can be obtained by observing the spreadsheet model.

Even though this study was carried out based on the data of a particular month, this model can be generalized to determine the number of batches to be produced for any month. As D_i's are changing from month to month, applying new D_i's to the formulated spreadsheet model would give the number of products to be produced in each month. In addition, number of working hours and working days can be appropriately changed in the model until it reaches feasibility.

For further analysis of the business, the management can analyze the future demand after identifying a probability distribution for the monthly demand and modelling a simulation to take an expected monthly demand. In addition, at the moment the management has not defined any goals but if they do, then a goal programming model can be used for this problem.

4. Conclusions

Based on the results obtained above, the solved integer linear programming model gives a feasible solution and it minimizes the monthly manufacturing cost. It could be concluded that by implementing the production plan suggested by the solution, the monthly demand could be satisfied using the available resources while minimizing the monthly production cost.

Moreover, the management can schedule the production for each day, as the aggregate number of batches to be produced for each product and the numbers of labour hours needed for the month are predetermined. If the plant overproduced the biscuits, the biscuits would be remained in the finished goods stores for an additional period, which results in excessive amount of inventory cost. This will also lead to low quality products entering into the market, causing bad reputation to the production plant. Therefore, by implementing this production plan, manufacturing excess of biscuits can be avoided and hence utilizes the physical and human resources to the optimum manner. This will enable manufacturing plant to reduce the production cost. Moreover, using the proposed mathematical model and with the help of Excel spreadsheet, the company can plan the production for the coming years.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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