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Wind Wave Growth

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ABSTRACT

A recent formula for the lift force on a low speed wing of circular arc cross-section [1] is adapted to the upward pressure force on the crests of a surface gravity wave propagating in the wind. In both cases, the main feature is the utilization of the air's compressibility. At and near a wave crest, it is predicted that the air density is increased over the ambient value and that the air density decreases inversely as the square of the upward distance from the radius of curvature of the crest. As a consequence, the air pressure also decreases upward inversely as the square of the same distance. Therefore, an upward pressure force on each crest occurs which presumably will make the crests grow. Growth rates are largest for small wavelengths and large mean slopes of the wave surface. Contrary winds should produce wave growth (not damping) as well as no wind at all.

1. INTRODUCTION

Given an initial surface gravity wave of amplitude *a*, and a steady wind blowing in the same direction as the propagating wave, but with greater speed, how does the wave amplitude increase as time increases? A theory of the lift force on a circular arc wing, moving at low speed [1], is here adapted to this problem. What is new to the general subject of wind wave generation is that compressibility of the air is taken into account, although turbulence, included in some theories, is left out. It is proposed that on each wave crest there is a thin boundary layer in which the air density is higher than that in the environment. This leads to a lift force on each crest in accordance with the dynamical development reported previously for the wing.

Inside a wave crest's attached air boundary layer both the pressure and the density theoretically decrease upward at the same rate resulting in an upward pressure force. Presumably that force will cause wave growth because a fluid is such a material that it will move in the presence of the least amount of force. Curiously the wind speed does not come out of the derivation of the pressure force to be included as a factor although it is present in most theories of wave generation and growth by wind. Therefore immediately comes the prediction that a contrary wind will not damp down the waves, but rather continue to make them grow!

Even if there is no wind at all, there will still be an upward pressure force on each crest during propagation because of the propagation of the wave through the air. An amazing fact not completely explained is that surface gravity waves have been traced from a storm in the Indian Ocean to the shores of California: half way around the world [2]! The proposed upward pressure force along the way has the tendency to enhance the propagation distance. Also helpful in this regard is the recent suggestion that the orbital motion of the fluid particles within the depth of wave influence has no associated friction according to the Navier-Stokes equations in polar coordinates [3].

Quite a few different theories of the wave growth by wind are available since the 1950s, and then reviews of these theories came into existence shortly afterwards. A relatively recent review [4] outlines these earlier ones as well as describing the individual theories. However, the present approach lies completely outside the range of work done before on this topic.

2. METHOD

At the start of the dynamical discussion of low speed flow past a wing of circular arc cross-section [1] are three equations in the three unknowns: pressure (p), density (ρ) and velocity (u). Bernoulli's law in its usual form is the first equation. Second is the equation of state of air as a perfect gas. Third is the force balance for fluid particles traveling along curved streamlines above the wing's top surface: upward centrifugal force equals downward pressure gradient. One differential equation in the pressure is easily obtained (it is derived in [1]) and solved to give

$$p = \frac{const}{r^2} \tag{1}$$

where r is the distance from the center of the circle of which the arc surface of the wing is a part, and the constant is

$$const = \rho_0 h r_0 S \tag{2}$$

where ρ_0 is the ambient air density, h is the maximum thickness of the wing, r_0 is the radius of the wing's top surface, and $S = RT$, where R is the gas constant for air and T is the air temperature taken constant.

From Equation (1) it is easily seen, by taking the derivative with respect to r, that there is an upward pressure force at the top of the wing. To apply this force to the top of a sinusoidal wave crest the radius of curvature R is needed. It is

$$R = \frac{1}{ak^2} \tag{3}$$

where k is the wave number. Then the upward pressure force at the crest top is

$$\frac{dp}{dr} = \frac{4\pi\rho_0 (ak)^3 S}{l} \tag{4}$$

where l is the wave length, displayed separately in spite of its inverse relation to the wave number, and (ak) is proportional to the mean wave slope.

3. DISCUSSION

Certain characteristics of the upward pressure force on a wave crest in Equation (4) stand out. For example, there is a fairly strong dependence on the mean wave slope: the greater the mean slope the faster the growth rate. This feature has not shown up in any earlier theories as far as I am aware. Also smaller wavelengths are predicted to grow faster than longer ones, if everything else is kept constant, which qualitatively agrees with an abundance of observations.

In contrast to the present analysis of the amplitude of a single wave component the vast majority of theoretical and experimental work on wind generated surface gravity waves is dominated by the total wave energy and a complete spectrum of frequencies. Because of the difference in language comparisons are difficult.

Observations of significance for evaluating the proposed mechanism of wave growth by wind are air densities over wave crests as a function of height. It is presumed that no such data are available at this time. Experimental work in the laboratory might be easier to carry out than in the open ocean, which is a project for the future.

4. CONCLUSION

The main conclusion is a theory of wave growth by wind waiting to be verified by future observations. When air moves past a crest of a propagating surface gravity wave, it is predicted that the compression of air at and near the crest will increase the density there and cause the pressure to decrease with increasing upward distance. As a consequence wave crests will grow in response to the upward pressure force. Wave growth should also take place on the wave crests by this method by an opposing wind and even if there is no wind at all. The growth rate is proportional to the cube of the mean slope of the wave surface.

CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

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