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The Asymmetry of Shanghai Composite Index Volatility

—Stochastic Volatility Models Based on GHST Distribution

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Abstract

In this paper, we analyzed how the asymmetric stochastic volatility models with GHST distribution capture the asymmetry of stock index volatility in China. Under the setting of fat-tail distribution, we introduced the correlation parameter ρ of two error terms to refine the classification of ASV model from two aspects of Contemporaneous correlation and Subsequent correlation. So we could compare the effect of ASV model in demonstrating the asymmetry of stock index volatility under the above different settings. Using the daily returns of Shanghai stock composite index, we concluded that the ASV model with GHST distribution and Subsequent correlation between error terms can better describe the asymmetry of the stock index in China. The DIC value and Kupiec test verified the adequacy and the effectiveness of risk measurement of the above model respectively.

Keywords

Asymmetric Stochastic Volatility Model, Asymmetry, Gibbs Sampling, Shanghai Composite Index, VaR Risk Measurement

1. Introduction

With the rapid development of economic globalization and financial integration, the transaction scale of global financial market continues to expand, and the operation efficiency is significantly improved. With the unprecedented development of financial market, the volatility of asset prices is increasing as well. The volatility has always been an important topic in the field of financial econometrics and time series analysis. At first, volatility is considered to be constants. Later, a series of researches have proved that, affected by time, dividend and related information, volatility is more constant than time variant and regular.

As a systematic method to research volatility, generalized autoregressive conditional heteroscedasticity (GARCH) models and stochastic volatility (SV) models have been developed fully and widely in recent years. On the basis of time-varying, scholars use the above two models to study the typical characteristics of volatility, such as persistence, clustering and asymmetry. GARCH models assume that the return is a stochastic process, and the variance equation isn't affected by the error term. The typical characteristics of volatility can be described by transforming the variance equation. The SV models assume that the volatility is random that both the yield and volatility equations consider the random error term, which increases the flexibility of volatility measurement as well as the complexity of the measurement to a certain extent.

In the underlying researches, SV models are applied to the study of option pricing in the form of continuous time series (Taylor, 1982; Hull & White, 1987). The discrete SV models are more convenient for empirical research, and have been widely used in the study of the return on financial assets such as stock market and foreign exchange market (Taylor, 1986; Jacquier et al., 1994; Kim et al., 1998). The discrete SV model is as follows,

$$y_t = \exp(0.5h_t) w_t. \tag{1}$$

$$h_t = \omega + \phi(h_{t-1} - \omega) + \sigma_\nu \nu_t. \tag{2}$$

The volatility $\sigma_t = \exp(0.5h_t)$ acts as a constant scale factor in return equation, and h_t is the unobserved latent volatility in logarithmic volatility equation. In order to ensure the strict stationarity and ergodicity of the stochastic process, the persistence parameter $|\phi| < 1$ is assumed as well as $h_0 = \omega$, $\nu_0 \sim N\left(0, \sigma^2 / (1-\phi^2)\right)$. w_t and ν_t are the random error terms. Theoretically, when ν_t follows standard normal distribution, $h_t \sim N\left(\omega, \sigma^2 / (1-\phi^2)\right)$, h_t is a stationary process of AR (1).

The basic SV models assume that error terms w_t and ν_t of returns and volatility respectively following independent normal distribution. However, the return of most financial assets doesn't conform to the characteristics of normal distribution, but presents the phenomenon of leptokurtosis and fat-tail. That is, compared with normal distribution, the value of skewness and kurtosis of asset return are larger, even slightly extreme. The generalized hyperbolic distribution (Barndorff-Nielsen, 1977) is a general term for a wide range of parametric distributions, such as hyperbolic distribution, normal inverse Gaussian (NIG) distribution, skew-t distribution, and so on. In the case of affine transformation, conditionalization and marginalization, these distributions are still closed (Nakajima & Omori, 2012), which can describe the fat-tailed characteristics flexibly. The generalized hyperbolic skew-t distribution (GHST), as a special case of GH distribution, could describe the skewness and fat tails of volatility because of its properties that two tails are polynomial and exponential respectively. Aas & Haff (2006) compared the fitting effects of normal inverse Gaussian distribution, skew-t distribution and GHST distribution on four financial time series of stocks, bonds, foreign exchange and interest rates, and proved that GHST distribution is the distribution most in line with the characteristics of skewness and fat tail, and is simpler in form setting, avoiding over parameterization and reducing the difficulty of model estimation (Nakajima & Omori, 2012), while fat-tailed NIG is suitable for biased but not severe heavy tailed sequences, biased t distribution could well fit fat tailed data, but couldn't deal with extreme skewness. In addition to the characteristics of leptokurtosis, there is obvious asymmetric effect in the financial market, especially in the volatile stock market. The asymmetry of volatility is defined as the negative correlation between future conditional volatility and current yield. In contrast to good news, bad news may bring about more intensive volatility. The basic accept of asymmetric SV models is as follows: there is a negative impact on the price level when ρ is negative, which will lead to lager volatility, in contrast to the negative impact of the same scale, the positive price shock will lead to the decline of volatility when ρ is positive, and there is no correlation between the error terms when ρ is 0, therefore the model is symmetric. Different settings of error term should be considered in the process of measure the asymmetry of volatility, such as normal distribution, t distribution, GHST distribution and skew-t distribution. In the past empirical studies, Harvey & Shephard (1996) used CRSP and S&P500 data to estimate the asymmetric SV model with normal inverse Gaussian distribution, and Jacquier et al. (2004) concluded that volatility has significant asymmetry using CRSP index and two sets of exchange rate data of UK against US dollar, Deutsche Mark and Canadian relative to the US dollar. It is demonstrated that the SV model with negative correlation coefficient ρ and following t distribution is better than the asymmetric normal distribution, symmetric t distribution and normal distribution. Nakajima & Omori (2009) adapted S&P500 and Topix index data to fit a variety of SV models with different settings. Among them, the fat-tailed ASV model following t distribution and introducing gamma scale could capture the asymmetric characteristics of fluctuations efficiently.

Using different SV models, the fat tail characteristics and leverage effect of Chinese stock index returns were also verified. Yang and Su (2013) concluded that the SV model under t distribution is easier to describe the fat tail characteristics of Chinese stock index by comparing the SV model with normal distribution and t distribution; Yang and Wu (2016) analyzed the fat tail and leverage effect of Chinese GEM stock index using ASV-t model, and demonstrated that the fitting effect was better than that of the asymmetric SV-N model.

Because of its relatively uncomplicated form, t distribution could describe the sufficiency of volatility significantly, but its fitting efficiency is lower than that of GHST distribution. As the only distribution with exponential tail and polynomial tail in GH distribution, GHST distribution has more advantages in analyzing the asymmetry of volatility, fitting return on assets data and VaR risk measurement. The innovation of this paper is that, considering the asymmetry of volatility of SV-GHST model under the contemporary and subsequent correlation. We also test the effectiveness of VaR risk measurement, verify the advantage of GHST distribution from the perspective of risk measurement, and fill in the gap of domestic research on risk measurement under this complex distribution.

The paper is organized as follows. Section 2 outlines the asymmetric SV models with GHST distribution and student t distributions as well as the Gibbs sampling scheme in detail. In Section 3, the application of ASV models using Shanghai composite index is developed, which includes the parameter estimation and comparison of models in Section 2 as well as the VaR measure by formula method. Finally Section 4 concludes.

2. SV Models and Estimation

2.1. The Setting of SV Models

In SV model, the yield rate of financial assets is affected by two random shocks w_t and ν_t. In order to ensure the finiteness of return, the error term ν_t is generally admitted as the standard normal distribution, while the error term w_t is divided into different classification adhering to different distributions. In this paper, we use the SV model of generalized hyperbolic skew *t* distribution setting by Nakajima & Omori (2012), and combine the correlation of two error terms w_t and ν_t to describe the asymmetry of volatility.

Firstly, the error term w_t of basic SV model in Equation (1) is expressed as a random variable following GHST distribution,

$$w_t = \mu_w + \beta Z_t + \sqrt{Z_t} u_t . \tag{3}$$

$$u_t \sim N(0,1), Z_t \sim IG(0.5\nu, 0.5\nu). \tag{4}$$

The error term w_t expressed by the above formula is a mixture of normal variance -mean, and the mixed distribution Z_t is the inverse gamma distribution with degree of freedom of $\nu/2$. In order to guarantee the finite variance, the value of ν must be greater than 4 and $\mu_w = -\beta\mu_z$. At the same time, in order to ensure $E(w_t) = 0$, there is $\mu_z \equiv E(Z_t) = \nu/(\nu-2)$. Substituting the expression of w_t back to Equation (1), there is the following SV model with generalized hyperbolic skew *t* distribution:

$$y_t = \mu + \exp(0.5h_t)\left\{\beta(Z_t - E(Z_t)) + \sqrt{Z_t} u_t\right\}. \tag{5}$$

$$h_{t+1} = \omega + \phi(h_t - \omega) + \sigma_\nu \nu_{t+1}. \tag{6}$$

$$Z_t \sim IG(0.5\nu, 0.5\nu). \tag{7}$$

where, (β, ν) is the common variable that determines the skewness and thick tail characteristics of models. In particular, when $\beta = 0$ the above Equation (5) is simplified as SV model in which the error term follows the student distribution.

Secondly, the classification of SV models can be refined according to the correlation between two error terms u_t and ν_s ($s = t$ or $s = t+1$), so as to further investigate whether the efficiency of different models to capture the asymmetry of volatility is different.

When $\text{cov}(u_t, \nu_s) = 0$, the relationship between the error terms u_t and ν_s

follows the basic zero correlation setting. Most of the two error terms in the basic SV model follow the null correlation hypothesis (Taylor, 1982, 1986; Hull & white, 1987), which could describe the clustering of volatility. However, when the asymmetry and long memory of volatility are considered, the explanatory ability of this model will decline (Nakajima & Omori, 2009, 2012; Jacquier et al., 2004; Berg & Yu, 2004). When describing the asymmetry of volatility, it is necessary to consider the correlation coefficient $\rho(\rho \neq 0)$ between the error term u_t and ν_s ($s = t$ or $s = t + 1$) based on the basic SV model.

Furthermore, the research on the correlation between the error terms (u_t and ν_s) of ASV model can be basically divided into two types: subsequential correlation (SC) represented by Harvey & Shephard (1996) and contemporary correlation (CC) represented by Jacquier et al. (2004). When the correlation between the error terms is assumed as synchronous, the error term of volatility equation could be written as $\nu_t = \rho u_t + \sqrt{1-\rho^2}\eta_t$, and η_t follows the standard normal distribution. In order to ensure ν_t still follows the standard normal distribution and $\operatorname{cov}(u_t, \nu_t) = \rho$, assuming u_t and η_t is uncorrelated, i.e. $\operatorname{cov}(u_t, \eta_t) = 0$. When it is assumed that there is a lag correlation between the error terms, the term could be written as $\nu_{t+1} = \rho u_t + \sqrt{1-\rho^2}\eta_{t+1}$, $\eta_{t+1} \sim N(0,1)$. Similarly, supposing $\operatorname{cov}(u_t, \eta_{t+1}) = 0$ to ensure $\operatorname{cov}(u_t, \nu_{t+1}) = \rho$ and $\nu_{t+1} \sim N(0,1)$. In particular, if $\rho = 0$, there is no correlation between the above two terms.

When the error terms are correlated, the impact path of price shocks on volatility is as follows. Taking the lag correlation as an example, that is, assuming that the correlation coefficient ρ between the error terms is less than 0, ν_{t+1} will produce a larger volatility than the positive impact of the same size through the related transition between ν_{t+1} and u_t when the yield y_t is negatively impacted (i.e. $u_t < 0$), so the uncertainty of the yield increases in the next period as a result. Similarly, the same conclusion can also be drawn when the two error terms are contemporaneous correlation, that is, negative shocks will be associated with higher contemporaneous and subsequent volatility, while positive shocks will reduce volatility (Jacquier et al., 2004). However, it is worth noting that the current negative impact in lag correlation is not equal to the negative impact in the next period. Therefore, the current negative impact on the yield can only ensure that the probability of the raise of the returns in the next period will increase, but it couldn't guarantee that the sign of the value must be negative (Kong, 2017). In theory, the error term of contemporaneous correlation couldn't explain the leverage effect $E\left(\ln\left(\sigma_t^2 \mid X_t, \sigma_t\right)\right)$ as well as conform to the hypothesis of efficient market because it is not martingale difference sequence, meanwhile the empirical data of S&P and CRSP yields also verified that the ASV model with lag correlation can capture the asymmetry of volatility better (Yu, 2005). Therefore, lag correlation is easier to explain the negative correlation between price and volatility than the ASV model with contemporaneous correlation. However, for the sake of the integrity of the model type, this paper still considers the situation of the same period correlation to verify the conclusion from the empirical perspective of Chinese stock index return.

To sum up, the expression of SV model can be written as follows,

$$y_t = \mu + \exp(0.5h_t)\left\{\beta\left(Z_t - E(Z_t)\right) + \sqrt{Z_t}\rho^{-1}\left(\upsilon_s - \sqrt{1-\rho^2}\eta_t\right)\right\}. \quad (8)$$

$$h_s = \omega + \phi(h_{s-1} - \omega) + \sigma_v \nu_s \quad (s = t, t+1). \quad (9)$$

when $s = t+1$, it is an asymmetric SV model following GHST distribution with subsequential correlation between error terms (ASV-GHSTSC); when $s = t$, it is an asymmetric SV model following GHST distribution with contemporaneous correlation (ASV-GHSTSC). In particular, when $\rho = 0$, the above models are simplified to be symmetric, that is, there is non-correlation (SV-GHST); when $\beta \equiv 1, s = t+1$, asymmetric SV model following student t distribution with subsequent correlation, and When $\beta \equiv 1, s = t$, asymmetric SV model following T distribution with contemporaneous correlation.

2.2. Gibbs Sample

BUGS (Bayesian Inference Using Gibbs Sampling) is a software package that runs SV model based on Bayesian Markov Chain Monte Carlo (MCMC) method. It can easily set the prior distribution and error distribution by modifying the code, and then improve the efficiency of Gibbs sampling. In this paper, the single-move Gibbs algorithm in OpenBUGS software is used to estimate the ASV models whose error term follows the generalized hyperbolic skew-t distribution by setting the normal variance-mean mixed distribution, and the mixed distribution is inverse gamma distribution (Nakajima & Omori, 2012). OpenBUGS could give the results of convergence, parameter estimation, autocorrelation posterior density map and DIC value. Generally speaking, the steps of Gibbs sampling are as follows:

1) initialize θ, h, z
2) sample $\phi \mid \sigma, \rho, \mu, \beta, \nu, h, z, y$
3) sample $(\sigma, \rho) \mid \phi, \mu, \beta, \nu, h, z, y$
4) sample $\mu \mid \phi, \sigma, \rho, \beta, \nu, h, z, y$
5) sample $\beta \mid \phi, \sigma, \rho, \mu, \nu, h, z, y$
6) sample $\nu \mid \phi, \sigma, \rho, \mu, \beta, h, z, y$
7) sample $z \mid \theta, h, y$
8) sample $h \mid \theta, z, y$
9) go to 2

In this paper, we use the prior distributions of parameters for reference which are setting by Nakajima & Omori (2012), and Kong (2017) has made some improvement based on the consideration of the correlation, persistence and model fitting effect of volatility equation. The prior distribution of parameters includes[1]: $\mu \sim N(0,100)$, $\nu \sim G(16,0.8)(\nu > 4)$, $\omega \sim N(0,10)$, $\rho \sim U(-1,1)$, $\sigma_v \sim IG(2.5,0.025)$, $0.5(\phi+1) \sim Beta(20,1.5)$. In this paper, the single chain Gibbs algorithm is used to improve the accuracy of the estimation results by ite-

[1]Where N is the normal distribution, Beta is the beta distribution, G is the gamma distribution, IG is the inverse gamma distribution, and U is the uniform distribution.

rating 15,000 times and discarding the first 5000 times as burn-in period.

3. Application

3.1. Data

We consider daily closing price of Shanghai Composite Index from January 2001 to July 2020 as the research object, and calculate the logarithmic rate of returns, $y_t = 100 * \ln(p_t / p_{t-1})$, where p_t represents the daily closing price of Shanghai Composite Index on trading day *t*. The sample size is 4746 for the returns of Shanghai composite index, and it includes all the stocks in Shanghai Stock Exchange with A-shares and B-shares. Table 1 shows the descriptive statistics of y_t. The value of mean is 0.0095, close to zero, which means that the number of positive and negative logarithmic returns is equal. This also reflects the high volatility and instability of the stock index return. At the same time, the skewness of the log return is −0.3987 and the kurtosis is 7.8630. We can see that the stock index return presents left deviation and excess kurtosis. Because of the large JB statistic, we could also judge that the return rate of Shanghai stock index is non-normal distribution.

3.2. Parameter Estimates

3.2.1. Convergence

In the estimation results in Table 2, MC error is used to measure the uncertainty of posterior mean estimation under sampling. The smaller the value is, the higher the estimation accuracy of posterior mean is, and the estimation accuracy of posterior mean may be ±2 times of MC_error. It can be seen from Table 2 that the posterior mean values of each parameter have very high accuracy, indicating that they all converge to the prior distribution.

Table 1. Summary statistic for log returns.

mean	Stdev.	skewness	kurtosis	median	max	min	JB stat
0.0095	1.5678	−0.3987	7.8630	0.0573	9.4008	−9.2561	4802

Table 2. Estimation results of Gibbs sampling of SV models.

	μ	ω	β	ϕ	ρ	σ_v	ν
	0.0046*	0.1455*	−0.1643	0.9889	−0.2553	0.1363	7.059
SV-GHSTSC	0.0166	0.1941	0.0503	0.0029	0.0682	0.0136	0.8031
	6.573E−4	0.0040	0.0028	1.787E−4	0.0054	0.0012	0.0648
	0.0193*	0.1759*	−0.0853	0.9894	−0.2483	0.1335	6.858
ASV-GHSTCC	0.0160	0.1876	0.0453	0.0029	0.0600	0.0134	0.8182
	4.516E−4	0.0046	0.0023	1.853E−4	0.0044	0.0012	0.0677
	0.0170**	0.1494*	−0.1127	0.991		0.1118	6.898
SV-GHST	0.0164	0.2136	0.0410	0.0028		0.0136	0.6118
	4.952E−4	0.0037	0.0018	1.968E−4		0.0014	0.0430

Continued

	0.0322	0.1458*	0.9886	−0.2919	0.1184	7.141
ASV-TCC	0.0151	0.1779	0.0027	0.0529	0.0095	0.7196
	4.028E−4	0.0057	1.429E−4	0.0038	8.252E−4	0.0546
	0.0296	0.0965*	0.9892	−0.1968	0.1096	6.631
ASV-TSC	0.0152	0.1936	0.0029	0.0637	0.0120	0.722
	4.518E−4	0.0052	1.936E−4	0.0049	0.0011	0.0599

Note: In the above table, the first, second and third row are the posterior mean value and standard deviation and MC error of each parameter respectively. The parameters with "**" are significantly 0 at 1% significance level, and the parameters with "*" are significantly 0 at 5% significance level, and other parameters are significantly not 0 at 1% significance level.

In the meantime, the sample autocorrelation graph, sample path and posterior distribution map of parameter iteration also show that the sample autocorrelation of each parameter decays rapidly, the sample path is stable, and the model has high convergence. Each parameter has only one peak value, and the curve is smooth, which indicates that the sampling iteration of each parameter converges to the target prior, so the Gibbs sampling method is fast and effective.

3.2.2. The Sufficiency of Model Fitting

Deviation information criterion (DIC) was proposed by Spiegelhalter et al. (2002) to measure the fitting effect of several Bayesian models. The so-called deviation refers to the difference between the logarithmic likelihood of the fitting model and that of the perfect replica model. DIC is composed of $\bar{D}$ and pD, which is especially suitable for comparing Bayesian models with posterior distribution obtained by MCMC simulation. Among them, $\bar{D}$ is the posterior mean value of logarithmic likelihood, which is used to measure the fitting degree of the model. The better the fitting effect is, the smaller the $\bar{D}$ value is. pD is the complexity degree of the model, which is expressed as the deviation of logarithm similar to the posterior mean value minus the deviation of the posterior mean value of parameters. The greater the pD value, the greater the penalty for the complexity of the model. Yu (2005) compared eight SV models including real models by using the bias information criterion to verify that DIC can identify the real model of the generated data, and then proves the applicability of DIC in the selection process of financial time series model, so as to introduce and apply it to SV model. The calculation of DIC value could be directly implemented in OpenBUGS software, which is easier to get than the marginal likelihood value of Bayesian model.

DIC value reflects the Bayesian measurement of model goodness of fit and model complexity, so it can better explain the adequacy of the model, rather than simply measure the goodness of fit of the model (Kong, 2017). DIC is used to compare the ability of different models in predicting the same kind of new data, but it is unable to evaluate whether one model is good or bad value (Spiegelhalter et al., 2002). When comparing different models with the same data, the smaller DIC value indicates that the model has better prediction ability. It is worth noting that DIC and marginal likelihood criteria have different concerns. DIC indicates the degree of prediction of future data by posterior data, while marginal likelihood criterion focuses on the prediction degree of prior data for observation data, so the comparison results of DIC and marginal likelihood criterion may be slightly different (Berg & Yu, 2004).

It can be seen from the results in **Table 3** that in terms of the degree of model fitting, $\bar{D}$ value of the ASV-TCC model is the smallest as well as the fitting effect is best, and ASV-GHSTSC and ASV-GHST are in the second and third place respectively. However, as for the complexity of the model, the pD value of ASV-TCC model is the largest, which indicates that the model has the largest reduction of uncertainty due to estimation, and the pD value of symmetric SV-GHST model is the smallest among the five models, indicating that the complexity of the model is the lowest, but also because of its relatively simple, its fitting effect is the worst. Generally speaking, the DIC value of ASV-GHSTSC model is the smallest, and the model fitting is the best, while the ASV-GHST model is the worst.

3.2.3. The Result of Model Estimation

Firstly, for the yield equation of models, it can be seen from the estimation results that the constants μ of the two ASV models with the error term following the GHST distribution are significantly 0 at the level of 5%, and the constants term μ of the two ASV models following the *t* distribution aren't significantly 0 at the level of 1%. In SV-GHST model, the posterior mean value of parameter β is −0.1029 (Stdev is 0.0436), the posterior mean value of β in ASV-GHSTSC model is −0.1127 (0.0410), and the posterior mean value of β in ASV-GHSTSC model is −0.1643 (0.0503), and they are not 0 under the significance level of 1%, which indicates that the three models could capture the negative skewness of return. Parameter estimation results in Kong (2017) demonstrate that the skewness parameter β of ASV-GHSTSC model and ASV-GHSTSC model is significantly 0 at the level of 1%. In the conclusion of this paper, the skewness parameter of all ASV models following GHST distribution is significantly not 0. This is because the data size used in this paper is larger, and the volatility of stock index return is more severe under random impact, therefore this kind of model is of great significance to capture the negative skewness parameter.

The estimated values of the degree of freedom of the SV model discussed are far less than that of Nakajima & Omori (2012) using S&P500 index (the posterior mean value of ν is 12.513, the standard deviation is 1.4522) and the TOPIX index (the posterior mean value of ν is 29.791, and the standard deviation is 4.4430), indicating that the return rate of Shanghai Composite Index has a larger fat tail characteristics.

Secondly, for the parameter estimation of volatility equation, as shown in **Table 2**, the value of persistence parameters ϕ of Shanghai composite index volatility is approximately equal to 0.99, and the volatility has high persistence when impacted, which indicates that the SV model discussed above could capture the volatility clustering well. At the same time, the half-life of the asymmetric SV model is considered to be about 63 days. The half-life of the symmetric SV-GHST model is 76 days. It shows that the time of duration of the volatility affected by price is much longer. Compared with symmetric SV models, the adjusting speed of asymmetric SV models is faster. The time is shorter the adjustment of the ASV model is faster than that of the symmetric model in comparison.

Finally, the ability of ASV models to capture volatility asymmetry is investigated. The posterior mean value of the correlation coefficient ρ of ASV-GHSTSC model is −0.2553 (0.0683), which is significantly not 0 at the level of 1%. The 95% confidence interval is [−0.382, −0.119], indicating that there is a significant negative correlation between the error terms u_t and ν_t. The posterior mean value of the correlation coefficient ρ of ASV-GHSTCC model is −0.2483 (0.0600), and the 95% confidence interval is [−0.3631, −0.1306]. Compared with ASV-GHSTSC model, the absolute value of ρ is smaller and the confidence interval is closer to 0. Therefore, the negative correlation between error items is not significant by contrast. Similarly, the posterior mean value of parameter ρ of ASV-TSC model is −0.1968 (0.0637), which is significantly not 0 at the level of 1%. The 95% confidence interval is [−0.316, −0.1981], the posterior mean value of parameter ρ of ASV-TCC model is −0.2919 (0.0529), and the 95% confidence interval is [−0.3969, −0.1922]. ASV-TCC model could better describe the asymmetry of volatility.

3.3. Risk Measurement and Posterior Test

VaR (value at risk), as a common index of risk measurement, measures the maximum possible loss of a financial asset or portfolio under a certain confidence level, as shown in Equation (11),

$$P_r\left[L_t(l) \le VaR_{1-p}\right] \ge 1-p\,. \tag{11}$$

This paper uses formula method to calculate the VaR value of 244 trading days from July 31, 2019 to July 31, 2020, and verifies it by Kupiec test (1995). There is the intra sample prediction formula of VaR by Equtaion (12), where μ is the average yield of 0.0094, σ the standard deviation of each model at different times, namely $\sigma_t = \sqrt{e^{h_t}}$, and the alpha number after each distribution was simulated for 100,000 times as quantile $Z_{1-\alpha}$.

$$VaR_t = -\mu + \sigma_{t-1} Z_{1-\alpha}\,. \tag{12}$$

Kupiec test is also called failure frequency test. When the actual loss of a trading day is less than the estimated value of its VaR, it is recorded as the success of the experiment. Supposing the actual number of days is T and the number of days of failure is N, then the actual failure rate $p = N/T$, the expected failure rate $p^* = 1-\alpha$, and α is the confidence level. At this time, the predicted VaR value could be tested by verifying whether the expected failure rate is equal to the actual failure rate, that is $H_0: p = p^*$.

$$LR = 2\left\{\ln\left(\left(1-p\right)^{T-N} p^N\right) - \ln\left(\left(1-p^*\right)^{T-N} p^{*N}\right)\right\}. \quad (13)$$

when H_0 holds, the likelihood ratio $LR \sim \chi^2(1)$. The critical value of $\chi^2(1)$ is 3.84 at 95% confidence level. As shown in Table 3, except for ASV-TCC model, the LR statistics of other models are less than 3.84, so we couldn't reject the null hypothesis, that is, the other four models could better measure the market risk of Shanghai composite index return rate. The failure days under different settings are similar, which indicates that the risk measurement of the four models is more accurate, and does not overestimate or underestimate the market risk of Shanghai stock index return.

Among them, the LR statistics of SV-GHST model is the smallest, which can effectively measure the market risk of return rate, and the data fitting degree is the best, while the LR statistics of ASV-GHSTSC model is the largest, in comparison, the accuracy of risk measurement and data fitting effect are worse. At 99% confidence level, the failure rate and LR statistics of the five models are almost the same, and far less than the critical value, so it is no longer explained.

To sum up, although ASV-TCC model could capture the asymmetry of volatility significantly, the adequacy of the model fitting effect is not as good as that of ASV-GHST model because the error term follows the t distribution, so it couldn't effectively measure the risk status of return. Compared with other models, ASV-GHSTSC model can not only describe the asymmetry of volatility significantly, but also measure the market risk of Shanghai composite index return rate more accurately.

4. Conclusion

This paper focuses on the ability of SV model to depict the asymmetry of the volatility of Shanghai Composite Index, and verifies the effectiveness of the model in VaR risk measurement. It fits a more sufficient and efficient model for the volatility stock market. In the setting of ASV model, GHST fat-tailed distribution (t distribution as a special case) is used for the error term. Meanwhile, the asymmetry of volatility is described by measuring the correlation coefficient ρ between return and volatility error term. The correlation between error items is subdivided into contemporaneous correlation, subsequent correlation and

Table 3. DIC values and LR stats.

	$\bar{D}$	pD	DIC	rank	Failed days	Failure rate	LR stats
ASV-TCC	14,460	1108	15,570	3	20	8.23%	4.5061
ASV-TSC	14,670	922	15,600	4	18	7.79%	2.5480
SV-GHST	14,890	750	15,640	5	17	5.74%	1.7805
ASV-GHSTSC	14,490	1034	15,530	1	19	7.79%	3.4356
ASV-GHSTCC	14,560	992.5	15,550	2	18	7.41%	2.5990

non-correlation. In the empirical study, the Gibbs sampling method in Open-BUGS software is used to estimate the model, and the formula method is used to estimate the VaR value and carry out a posterior test.

It can be concluded that the parameters of the models discussed have high convergence, and the parameters of persistence, asymmetry and degree of freedom in SV models are significantly not 0. Therefore, ASV model can capture the asymmetry, clustering and fat tail characteristics of volatility in Chinese stock market. As far as the sufficiency of model fitting is concerned, the DIC value of ASV-GHSTSC model is the smallest, and the fitting is the most sufficient. The DIC value of symmetric SV-GHST model is the largest, and the fitting effect of ASV models with GHST distribution is generally better than that of ASV model with asymmetry. As for the VaR risk measurement, SV-GHST model can measure the market risk of return rate most effectively, and the fitting effect is the best. Although the VaR value of ASV-GHSTSC model is large, it also passes Kupiec test, which could effectively measure the market risk of Shanghai composite index return rate. Therefore, our research could provide an ASV-GHSTSC model with sufficient fitting and effective measurement of stock market risk for Shanghai stock market, which provides experience for other financial markets to some extent.

Our research has limitations as well. Firstly, we concentrated on the stock yields only using the most commonly using Shanghai composite index, ignoring other typical financial assets data such as GEM, exchange rate and so on. Then, only the GHST and *T* distributions are adopted in our paper, and the results may be limited because of the lack of adequate comparison distributions. In the future, we can expand the research object to other financial assets such as foreign exchange, futures, etc. to investigate the asymmetry of volatility under the biased GED distribution of SV model. We use the long memory SV model to study the long memory of the yield series, and further study the effectiveness of VaR under the SV complex distribution model.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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