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Einstein's Concept of Clock Synchronization Conflicts with the Second Relativity Postulate

Steven D. Deines

Donatech Corporation, Inc., Fairfield, Iowa, USA
Email: physicspublishing@gmail.com

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Abstract

Einstein defined clock synchronization whenever photon pulses with time tags traverse a fixed distance between two clocks with equal time spans in either direction. Using the second relativity postulate, he found clocks mounted on a rod uniformly moving parallel with the rod's length cannot be synchronized, but clocks attached to a stationary rod can. He dismissed this discrepancy by claiming simultaneity and clock synchronization were not common between inertial frames, but this paper proves with both Galilean and Lorentz transformations that simultaneity and clock synchronization are preserved between inertial frames. His derivation means moving clocks can never be synchronized in a "resting" inertial frame. Ultraprecise atomic clocks in timekeeping labs daily contradict his results. No algebraic error occurred in Einstein's derivations. The two cases of clocks attached to a rod reveal three major conflicts with the current second postulate. The net velocity between a photon source and detector plus the "universal" velocity $\boldsymbol{c}$ is mathematically equivalent to Einstein's clock synchronization method. As the ultraprecise timekeeping community daily synchronizes atomic clocks on the moving Earth with ultraprecise time uncertainty well below Einstein's lowest limit of synchronization, the theoretical resolution of the apparent conflict is accomplished by expanding the second relativity postulate to incorporate the net velocity between the photon source and detector with the emitted velocity $\boldsymbol{c}$ as components of the total velocity $\boldsymbol{c}$. This means the magnitude of the total photon velocity can exceed the speed limit (299792458 m/s) set by the standard velocity $\boldsymbol{c}$.

Keywords

Special Relativity, Simultaneity, Clock Synchronization, Photon Speed, Lorentz Transformation, Galilean Transformation

1. Preface

The author chose the accurate translation of Einstein's 1905 paper in the appendix of Arthur I. Miller's book [1] with § denoting sections followed by line numbers. Miller was a physics professor at Harvard and Lowell Universities, who transitioned as a science historian. He translated numerous correspondences involving Einstein and discussed the many issues debated by Einstein's many supporting and opposing contemporaries. The translation in the appendix differs from previous (and, in places, unacceptable) English translations. Typographical errors in the original Annalen version are flagged, which went into the Teubner edition, and additional errors appear in the Dover reprinted volume The Principle of Relativity. Footnotes from Einstein and Sommerfeld are annotated in that book.

In this review, photon velocity combines the photon speed in m/s with vector direction. The author defines simultaneous events as occurring when two or more phenomena separate or intersect at one coordinate point at one instantaneous coordinate time. This preserves simultaneity between inertial frames because points and time instants of simultaneous events remain unchanged as points and time instants in other inertial frames. Synchronous events occur at separate coordinate locations at one instantaneous coordinate time.

The author requires multiple observers concurrently recording an experiment must have the same results, especially when those results are transformed into one common reference frame. Else, experimental physics is a waste of time if multiple observers collecting such concurrent data do not agree on the results. Theoretical physics is then useless since there is no agreed lab result for theory to explain the outcome.

2. Einstein's Thought Experiment: Reflecting Light between Rod Ends

Einstein's 1905 relativity paper gives a logical discourse without any reference to other papers or experiments. The first section covered the procedure to establish coordinate time by synchronizing clocks throughout a reference frame. "Let us consider a coordinate system in which the equations of Newtonian mechanics hold. For precision of demonstration and to distinguish this coordinate system verbally from others which will be introduced later, we call it the 'resting system'." ([1], §1, lines 1-5)

Einstein considered simultaneous events to occur with a time associated with a single location. His example was a train that arrived at 7 o'clock, which he meant the small hand of his watch pointed at 7 as a train arrived. ([1], §1, lines 13-17)

Einstein considered a clock A at position A and an identical clock B at location B, but no connection existed between "A time" or "B time" for locations A and B ([1], §1, lines 33-41). He stated, "The latter time can now be defined by requiring that *by definition* (italicized in Einstein's German publication) the 'time' necessary for light to travel from A to B be identical to the 'time' necessary to travel from B to A. Let a ray of light start at the 'A time' t_A from A toward B, let it at the

'B time' t_B be reflected at B in the direction of A, and arrive again at A at the 'A time' t'_A." ([1], §1, lines 41-45) He concluded the two clocks run in synchronization if ([1], §1, Equation (1.1))

$$t_B - t_A = t'_A - t_B$$

This definition requires two one-way transmissions. He further claimed: (1) if a clock at B synchronizes with the clock at A, the clock at A synchronizes with the clock at B, and (2) if the clock at A synchronizes with the clock at B and also the clock at C, then clocks at B and C synchronize with each other ([1], §1, lines 48-60). He added the requirement ([1], §1, Equation (1.2)) for length AB as $2AB/\left(t'_A - t_A\right) = c$ where c is the postulated universal constant for photon speed. This is Einstein's synchronization method based on his definition. Equation (1.1) stipulates that opposite one-way time traverses over a fixed distance are equal. The roundtrip time span, $t'_A - t_A$, is divided by 2, which matches the one-way transmission span, to advance the broadcast time tag from the master clock to set the remote clock. This setting between the remote and master clocks is his synchronization procedure. In practice, multiple transmissions are needed to ensure the remote clock's time interval matches the master clock's interval (*i.e.*, seconds). Many physicists considered this as coordinate time throughout the reference frame. The time t is the time recorded by stationary synchronized clocks in the resting frame. Einstein did not explicitly state it, but all pairs of clocks maintain fixed distances between them without rotation in an inertial frame. That is why he envisioned a rod with two clocks at the rod's endpoints for his mental tests and subsequent derivations.

In § 2, Einstein defined his two postulates of special relativity ([1], §2, lines 1-12). The translation lists

1) "The laws of which the states of physical systems undergo changes are independent of whether these changes of state are referred to one or the other of two coordinate systems moving relatively to each other in uniform translational motion."

2) "Any ray of light moves in the 'resting' coordinate system with the definite velocity c, which is independent of whether the ray was emitted by a resting or by a moving body."

He added, "Consequently, *velocity* = (*light path*)/(*time interval*) where time interval is to be understood in the sense of the definition in §1". Newtonian forces are the derivatives of momentum, in which a constant velocity v of an unchanging mass results in zero force for any inertial frame (those frames that translate linearly by a constant velocity). The first postulate preserves the system's state, because a zero force will not change the equations of motion defining the physical state.

Einstein considered that a rigid rod at rest of length L is measured by a measuring rod or ruler at rest. He added two clocks to the two ends, A and B, of the rod, that were synchronized with the clocks of the resting system. He further added two moving observers, each fixed to each moving clock. He stated, "Let a ray of light depart from A at the time t_A, let it be reflected at B at the time t_B, and reach A again at the time t'_A. Taking into consideration the principle of the constancy of the velocity of light we find that

$$t_B - t_A = \frac{r_{AB}}{c-v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c+v}$$

where r_{AB} denotes the length of the moving rod—measured in the resting system. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the resting system would declare the clocks to be synchronous." ([1], §2, lines 47-54)

Einstein wrote, "Thus, we see that we can attribute no *absolute* meaning to the concept of simultaneity, but that two events which, examined from a coordinate system, are simultaneous, can no longer be interpreted as simultaneous events when examined from a system which is in motion relatively to that system." ([1], §2, lines 55-58) Einstein never proved this unsupported claim. Simultaneous events are preserved between moving inertial reference frames, which the proof is given in the next paragraph. To be practical, a small acceptable neighborhood must encompass the point location and a time uncertainty with the time instant of any simultaneous events. Einstein's example was a train arrived at the station where the observer stood on a platform inside the station with the observer's watch indicating 7 o'clock with an implied uncertainty in position and time.

Any transformation between reference frames must be a one-to-one, onto function that assigns points from one frame to another. This guarantees that the function associates unique pairs of points, so that an inverse function or inverse transformation assigns the same unique pairs of points. The author's definition of simultaneous events is when the events separate or intersect at a point in space at one instant of time. So, a point at time t in the K frame is still a unique point and unique time instant in the k frame, preserving simultaneity between inertial frames. Imagine an observer in the K frame sees two events A and B that occur at the same x-coordinate $x_A = x_B$ and same time, $t_A = t_B$. From the Lorentz transformation,

$$\begin{aligned} \tau_A &= \gamma\left[t_A - \left(v/c^2\right)x_A\right] = \gamma\left[t_B - \left(v/c^2\right)x_B\right] = \tau_B \\ \zeta_A &= \gamma\left(x_A - vt_A\right) = \gamma\left(x_B - vt_B\right) = \zeta_B \end{aligned} \tag{1}$$

Two simultaneous events in the K frame are also simultaneous for the second observer in the k frame, which is described in a textbook. [2] For the Galilean transformation, $\gamma = 1$ so that $\zeta_A = \zeta_B$ as $\tau = t$ everywhere. This proof contradicts Einstein's claim of simultaneity.

Einstein's derivation for transmitted time spans was the first discrepancy, which should yield equal time spans. Einstein gave no derivation for the two above unnumbered equations, but the derivation is shown in the next section. His formulas are a contradiction between time spans that his definition required for synchronization by Equation (1.1). The first postulate requires system states to be unaffected by a constant linear velocity between reference frames. The resting frame with the stationary rod and synchronized clocks will be in the same state of clock synchronization for the moving inertial frame attached to the moving rod and its clocks.

Einstein considered a resting frame K and a moving frame k with coordinate axes mutually parallel between frames where k was constrained to move its origin and ζ-axis along the x-axis of K at a constant speed v ([1], §3, lines 4-20). The Galilean transformation between K and k inertial frames is simply $\zeta = x - vt$ and $\tau = t$. If two synchronized clocks maintain $t_A = t_B$ in the K frame, then those clocks have $\tau_A = \tau_B$ in the k frame to preserve synchronization. Instead, he derived the Lorentz transformation equations between time t and coordinate x in the K frame with time τ and coordinate ζ in the k frame. One Lorentz transformation is $\zeta = \gamma(x - vt)$ [§3.27] in his paper. Apply subtraction between two Lorentz transformations for two locations and times at A and B to get the difference equation of

$$\zeta_B - \zeta_A = \gamma\left(\left[x_B - x_A\right] - v\left[t_B - t_A\right]\right). \tag{2}$$

For clock synchronization in the K frame, $t_A = t_B$ at x_A and x_B, and the distance between clocks A and B is $L = |x_B - x_A|$ in (2). Substitute the absolute L into (2) and divide by speed c where $\gamma|L|/c = |\zeta_B - \zeta_A|/c$.

$$\begin{aligned} |\tau_B - \tau_A| &= \frac{|\zeta_B - \zeta_A|}{c} = \frac{\gamma|x_B - x_A|}{c} = \cdots \\ &= \frac{\gamma|x_A - x_B|}{c} = \frac{|\zeta_A - \zeta_B|}{c} = |\tau_A - \tau_B| \end{aligned} \tag{3}$$

According to Einstein's Equation (1.1), the stationary synchronized clocks A and B in the resting frame K are still synchronized by his definition in the moving inertial k frame under the Lorentz transformation.

$$\Delta\tau_{BA} = |\tau_B - \tau_A| = |\tau_A - \tau_B| = \Delta\tau'_{AB} \tag{4}$$

If the second postulate of relativity is exact, then Einstein derived that the uniformly moving rod with its attached clocks had unequal time spans of transmission between its endpoints of $r/(c+v)$ and $r/(c-v)$, which would mean those moving clocks can never be synchronized according to his synchronization method by Equation (1.1). That derivation had no algebraic error. He dismissed this discrepancy by claiming without proof that synchronized clocks stationary in one inertial (*i.e.*, "resting") frame are not synchronized in the moving inertial frame.

The above proof shows two synchronized clocks that keep a fixed distance apart will maintain $t_A = t_B$ in one inertial frame and $\tau_A = \tau_B$ in a different moving inertial frame by the Lorentz transformation. Einstein's argument of synchronization versus nonsynchronization between different inertial frames is incorrect.

In mathematical proofs, an initial claim with established postulates is used to derive logical steps and results that lead to a conclusion. If the conclusion is contradictory or false, then that claim is proven to be false. Einstein's discrepancy means his wording of the second relativity postulate is inexact as revealed in the next section.

3. Photon Paths between Ends of Stationary and Uniformly Moving Rods

The equivalent formula after Einstein's second postulate is *time interval* = (*light* *path*)/(*velocity*).

In quantum electrodynamics (QED), photons emitted from endpoint A combine with the reflective atoms at endpoint B to create excited electrons in higher orbits. New photons are emitted from deexcited electrons in B and travel to endpoint A, so that virtually all photons from B to A reinforce various paths with different amplitudes. The square of the amplitudes is the probability where photons traveled along merged paths. After combining all routes, the most probable traverse is often the path of least action (e.g., Snell's law in this case). In the roundtrip setup, one beam traverses the distance $A \rightarrow B$ and the other beam $B \rightarrow A$ in the macroscopic scale. Any reflection involves two beams with QED. Einstein incorrectly interpreted photons were reflected by a perfect mirror at the rod's end to maintain one continuous beam with no change in its magnitude (i.e., speed), because quantum mechanics was unknown in 1905.

Orient a uniformly moving rod with its length parallel to its velocity, $\boldsymbol{v}$, relative to the resting frame. When the rod is stationary, the photons traverse a distance of L, the length of the rod. Each one-way time span is $\Delta t = L/c$, and each roundtrip distance over the resting rod is $2L$.

However, photons traveling with or opposite to the moving rod experience different transmission distances. Algebra can directly solve the distances that photons traverse to intercept the opposite end of the moving rod. In general, the photon's constant velocity from endpoint A to endpoint B is the constant $\boldsymbol{c}_{AB}$, and the photon's velocity from endpoint B to endpoint A is the constant $\boldsymbol{c}_{BA}$, which these magnitudes of velocity may be equal or different to the magnitude of $\boldsymbol{c}$. In the resting frame, the rod moves with velocity $\boldsymbol{v}$ so that the endpoint B is at B' where the parallel photons from A overtake the receding endpoint at B' in the time span of Δt_{AB} as shown in **Figure 1**. The newly emitted antiparallel photons from endpoint B intercept the approaching endpoint A at the position A' in the time span of Δt_{BA} as shown in **Figure 2**.

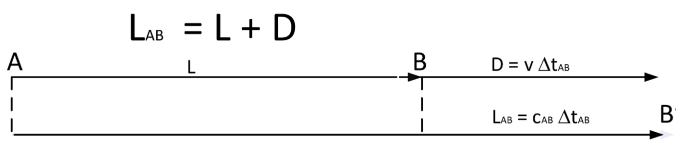

Figure 1. Photons overtaking receding endpoint.

Solve for D in $D/v = (L + D)/c_{AB}$ and replace D in $L_{AB} = L + D$.

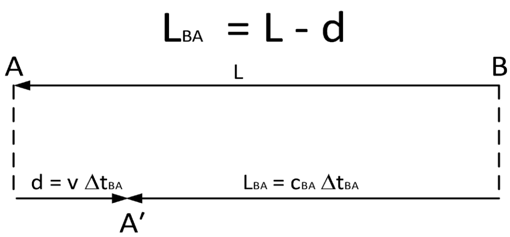

Figure 2. Photons intercepting approaching endpoint.

Solve for d in $d/v = (L-d)/c_{BA}$ and replace d in $L_{BA} = L - d$. The resulting distances in the resting frame are:

$$L_{AB} = \frac{Lc_{AB}}{c_{AB} - v}, \text{ and} \tag{5}$$

$$L_{BA} = \frac{Lc_{BA}}{c_{BA} + v}. \tag{6}$$

It is immediately apparent that $L_{AB} > L$ if $c_{AB} > v$, and $L_{BA} < L$ if $v > 0$. If both c_{AB} and c_{BA} are equal to the standard speed c, the roundtrip distance is greater than $2L$. Simply add Equations (4) and (5) with the standard speed for $\boldsymbol{c}$ for the roundtrip distance:

$$\frac{Lc}{c-v} + \frac{Lc}{c+v} = \frac{2Lc^2}{c^2 - v^2} = \frac{2L}{1 - \frac{v^2}{c^2}} = 2L\gamma^2 > 2L. \tag{7}$$

With a universal speed c, the photons have a longer roundtrip distance traversing the uniformly moving rod in the resting frame than if that rod was stationary. Length contraction from special relativity undercompensates, as length contraction is $L' = L/\gamma$ for the moving rod relative to the stationary observer, leaving an extra γ in the roundtrip distance with the moving rod. For the stationary observer, photons have a greater roundtrip traverse for the moving rod than for the stationary rod.

If photons always move with a universal constant c, then Equation (7) shows that velocity of the inertial frame attached to the moving rod can be independently determined within the resting frame. "Measurements made entirely within a given system must be incapable of distinguishing that system from all others moving uniformly with respect to it. This *postulate of equivalence* requires that physical laws must be phrased in an identical manner for all uniformly moving systems." [3] Using the second relativity postulate, Equation (7) contradicts the equivalence postulate (*i.e.*, first relativity postulate) by calculating v in γ, which is the second contradiction with the second relativity postulate.

If one divides the one-way distances L_{AB} and L_{BA} by their respective constant photon speeds, unequal time spans to intercept the opposite ends of the moving rod occur for the stationary observer while an observer fixed with the moving rod concurrently records equal time spans. The results are:

$$\Delta t_{AB} = \frac{L}{c_{AB} - v}, \text{ and} \tag{8}$$

$$\Delta t_{BA} = \frac{L}{c_{BA} + v}. \tag{9}$$

Equations (8) and (9) are identical in form to Einstein's unnumbered equations for his unequal time intervals that the photons traverse the moving rod, except he postulated the speed c in place of c_{AB} and c_{BA}, which are not necessarily equal.

The derivation above shows Einstein made no algebraic error. It was proven in Equations (3) through (4) with the Lorentz transformation that the synchronization between clocks keeping a fixed distance apart is preserved between moving inertial frames.

Einstein correctly required equal time spans, $\Delta t_{AB} = \Delta t_{BA}$, in his Equation (1.1) for synchronizing remote clocks. Equations (8) and (9) with universal *c* from the second postulate contradict (1.1), because a universal *c* allows the clocks to determine the relative velocity *v* of the moving rod and its attached inertial frame with the resting frame, which violates the first postulate (*i.e.*, equivalence postulate). If one divided (7) by the universal speed *c* and let $L/c = \Delta t$ in the roundtrip time span of the stationary rod, then the roundtrip time span for the uniformly moving rod is $2\Delta t = 2\gamma^2\Delta t$. Time dilation predicts $\Delta t = \gamma\Delta t$, which leaves an unaccounted *γ* factor for the roundtrip time span. It was proven with the Lorentz transformation that synchronized clocks in one inertial frame maintain a fixed distance between them in another moving inertial frame and have the same one-way time span for photons to traverse that distance in either direction. By Einstein's equation (1.1), the synchronized clocks in the first frame will remain synchronized in the second frame. This is what the first relativity postulate requires for the state of a system to be identical between inertial reference frames (*i.e.*, equivalence principle). However, Einstein derived the result without algebraic errors that the moving clocks were not synchronized because the time spans to traverse the distance between the clocks were unequal when he assumed a universal photon velocity for all reference frames.

Equation (4) showed that the moving clocks still maintained synchronization between them. Simultaneity is proven by (1) that simultaneous events in one reference frame are simultaneous and unique in another frame. Using Einstein's own notation, t_A is the broadcast time of a photon emitted from moving endpoint *A*. The time of absorption of the photon at the moving endpoint *B* is t_B. The time of emission of a new photon from endpoint *B* is virtually identical and labeled as t_B. The later time of absorption of the photon from *B* into the moving endpoint *A* is t'_A. Only the four broadcast messages of times from *A* and *B* are received by either a stationary observer or a moving observer with a uniform velocity between them. Then, $t_B - t_A = t'_A - t_B$. Another equivalent way uses (4) that proved $\Delta\tau_A = \Delta\tau_B$, and the time difference equation with the inverse Lorentz transformation for time will obtain $\Delta t_A = \Delta t_B$. This contradicts Einstein's result of unequal time spans to traverse the moving rod in opposite directions in the resting frame, which was rigorously derived from his version of the second relativity postulate. This is the third contradiction with the second postulate's universal ***c***. Therefore, the second relativity postulate is not exact and is incongruous with the first relativity postulate that requires equal time spans to traverse a stationary or uniformly moving rod.

Einstein stated that only when the clocks on the rod are stationary in the resting frame can $\Delta t_{AB} = \Delta t_{BA}$, but he claimed the clocks attached to the uniformly moving rod are not synchronized in the resting frame. As shown by (8) and (9), $\Delta t_{AB} \neq \Delta t_{BA}$ where the difference between time spans is

$L/(c-v) - L/(c+v) = 2Lv/(c^2 - v^2) \approx 2Lv/c^2$ as $c \gg v$. Depending on the values of L and v, the universal photon speed prevents clock synchronization for the moving clocks because the transmission time spans are unequal by a scale factor of v/c. For example, if two clocks are held 5 m apart and Earth's orbital velocity is about 30,000 m/s parallel to that distance, then the time difference is 3.33E−12 s for parallel versus antiparallel transmission that stops true synchronization between moving clocks with uncertainties less than 3.33E−12 s by the strict wording of the second postulate.

The best pendulum clocks circa 1905 were accurate to 1E−08 s (half a second lost in a year). The first atomic clock was built by the National Bureau of Standards (now the National Institute of Standards and Technology [NIST]) in 1948 using a stream of stimulated ammonia atoms. The first accurate atomic clock was built in June 1955 by Essen and Parry at the National Physical Laboratory (NPL) using cesium-133 atoms. Its cesium clock was used to calibrate the current definition of the second in a collaborative study from 1955 through 1958 with the US Naval Observatory (USNO). Improvements in the cesium atomic clock have resulted in NIST developing its NIST-F1 (5E−16 in δf/f frequency inaccuracy [4]) and NIST-F2 (1E−16 [5]) as the US primary time standards for civil usage. NIST has the ultraprecise ability to synchronize its clocks below Einstein's limits that claim it is impossible to synchronize moving clocks. Earth's surface rotates with a tangential velocity and Earth has an orbital velocity, but ultraprecise laboratories daily synchronize their atomic clocks in both directions.

Optical clocks have reached absolute accuracies approaching 1E−18, and optical oscillators such as cavity-stabilized lasers have provided timing stability less than 1 fs over many seconds of operation [6]. NIST has demonstrated full unambiguous synchronization of two-way links between two optical clocks through the atmosphere of up to 4 km with a time deviation below 1 fs operating from 0.1 to 6500 s while the effective path length changed up to 10 cm due to atmospheric density changes and building sway. The physical distance between the optical clocks remained fixed in the building. Over 2 days, the time wander was 40 fs peak to peak, and the time frequency transfer was below 225 attoseconds (as) [6]. NIST's demonstrations disprove Einstein's theoretical result that moving clocks cannot be synchronized.

Based on NIST's testing, $\Delta t_{AB} = \Delta t_{BA}$ for photon traverses between ends A and B of the moving rod. Equate Δt_{AB} in (8) to $\Delta t = L/c$ time intervals and, next, Δt_{BA} in (9) to the same Δt traverse time span.

$$\frac{L_{AB}}{c_{AB}} = \frac{L}{c_{AB} - v} = \frac{L}{c} \Rightarrow c_{AB} = c + v \text{, and} \tag{10}$$

$$\frac{L_{BA}}{c_{BA}} = \frac{L}{c_{BA} + v} = \frac{L}{c} \Rightarrow c_{BA} = c - v. \tag{11}$$

To satisfy Einstein's requirement of equal time transmission spans to synchronize clocks by Equation (1.1), Equations (10) and (11) show that parallel and antiparallel photon velocities differ from the standard speed c due to moving photon sources relative to the photon detectors. The emitted photons from endpoint A obtain the additional speed, $+v$, in the parallel velocity to overtake endpoint B. The second beam of ejected photons from endpoint B after absorption gets the opposite speed, $-v$, in the antiparallel direction of the uniformly moving rod relative to the photon detector. These results comply with the addition law of photon velocities, due to the mutual velocity between the source and detector. When the rod is stationary in the resting frame, $v = 0$ and the one-way time span is L/c for either direction by Equations (8) and (9). The addition law of velocities makes $c_{AB} = c + v$ and $c_{BA} = c - v$, so that both Equations (8) and (9) predict equal one-way time spans of L/c. As long as the distance between two clocks is fixed, synchronization is possible whether the clocks are stationary or uniformly moving relative to an inertial frame.

The roundtrip photon speed experiments of Fizeau (1849), Foucault (1850 and 1862), and Michelson (1877-1879 and 1926) indicated an apparent constant photon speed. Roundtrip transmissions with a single reflection have an average speed equal to the inherent magnitude of c as $\left(\left[c+v\right]+\left[c-v\right]\right)/2=c$. By induction, all roundtrip traverses have an average speed over multiple reflections identical to the standard speed or magnitude of c [7]. A universal speed c prevents synchronization with a sufficiently small time uncertainty between uniformly moving clocks that maintain a fixed distance apart.

About a hundred atomic timekeeping laboratories are scattered around the world, and many can maintain an uncertainty of 1E−15 s with pulses or frequencies transmitted in both directions. The second relativity postulate should be modified to include vector velocity addition due to the mutual velocity between the photon source and detector. This modification will permit equal time transmissions between clocks maintaining a fixed distance apart.

Special relativity, rigorously derived from the second postulate, has its own contradictions. Special relativity predicts any slightly accelerated mass measured parallel to its velocity will be γ times more massive than measured perpendicular ([1], §10, 1-81). No standards institute has reported a diurnal variation in mass measurements (e.g., 1000 kg mass measured parallel to Earth's orbital velocity could be 5 μg more than measured perpendicular six hours later).

One can test directly if Einstein's second postulate is totally correct in one direction. According to special relativity, length contraction is $\Delta L_{\text{resting}} = \gamma\Delta L_{\text{moving}}$ and time dilation is $\gamma\Delta\tau_{\text{resting}} = \Delta\tau_{\text{moving}}$. Let the integer $k = 299792458$, $\Delta L_{\text{resting}} = 1$ meter, $\Delta\tau_{\text{resting}} = 1$ second. As the speed of light is currently defined,

$$c = k\cdot\frac{\text{meters}}{\text{second}} = k\frac{\gamma\Delta L_{\text{moving}}}{\Delta\tau_{\text{moving}}/\gamma} = k\gamma^2\frac{\Delta L_{\text{moving}}}{\Delta\tau_{\text{moving}}} = c' \tag{12}$$

This means that the speed of photons in a constant moving inertial frame has shorter meters and longer second intervals than units in a resting frame such that one-way photon speeds do not have the same universal constant as $k \neq k\gamma^2$ (e.g., the orbital speed of Earth at apogee and perigee would produce a difference of about 0.199 m/s for $k\gamma^2$). Equation (12) reveals the contradiction that special relativity cannot maintain the universal numerical photon speed between moving inertial frames, regardless of the direction. It would take at least 10 significant figures to test this.

Special relativity does not preserve the same photon speed between inertial frames in the formula $c = f\lambda$. Let frequency f denote cycles/second and λ as wavelength in meters with n and N being the real numbers of units for frequency and wavelengths, and denote *second* and *meter* as the units in the first inertial frame. Let *Second* and *Meter* be the units in the moving inertial frame with a constant velocity $\boldsymbol{v}$ relative to the first inertial frame. Then,

$$c = n\frac{\text{cycles}}{\text{second}}N \cdot \text{meters} = n\frac{\text{cycles}}{\text{Second}/\gamma}N \cdot \gamma \cdot \text{Meters} = \gamma^2 n\frac{\text{cycles}}{\text{Second}}N \cdot \text{Meters} = \gamma^2 c' \tag{13}$$

Special relativity predicts length contraction and time dilation, but it has no effect on *cycles* or absolute numbers (n and N). Equation (13) reveals special relativity cannot preserve a universal photon speed regardless of vector direction. It has the same effect as Equation (12).

Some prior tests claim that the standard velocity c was measured for a moving particle's photon emission. One example is γ rays from the decay of π^0 mesons with more than 6 GeV were measured absolutely by timing over a known distance [8]. The test was designed to measure $c + kv$. For moving mesons ($\gamma > 45$), $k = (-3 \pm 13) \times 10^{-5}$. Two unidentified detectors were 31.450 m apart to measure the time interval the γ rays traveled between detectors along the line of γ ray propagation, resulting in the measured standard c. According to QED, the first detector intercepted the initial γ ray photons by absorption, creating excited orbital electrons in the atoms of the detector that triggers timing pulses, which deexcite and emit new photons with the standard intrinsic velocity c relative to the first detector. The second photon detector measured the time after new photons were emitted with the standard c from the first detector. Both photon detectors were stationary in the laboratory, producing $k \approx 0$, which is equivalent to $v = 0$ in Equations (10) and (11) to measure the standard c speed. Photons that penetrate through the first detector without interception could not be detected as photons have no charge, and electrons with a charge can be easily detected by magnetic fields. If these penetrating photons were recorded by the second detector, that information would be discarded as noise as there was no corresponding data by the first detector to calculate a time span for $31.450/(c + kv)$. This and similar tests must be scrutinized to ensure the photon velocities were measured correctly without prior interception or interference by the detectors. This test failed to measure the actual incident γ ray velocities.

4. Summary and Conclusion

Before Einstein delved into his special relativity theory, he devoted the first section of his 1905 relativity paper to synchronizing pairs of clocks in an inertial frame, which would be required for recording timed events before conducting any experiments to compare between laboratories. He assumed perfect clocks were stationary throughout the "resting" frame. Synchronizing remote clocks to a master clock, he prescribed that the time span for a beam of photons transmitted from point A to B (a distance of L) and reflected back from B to A be recorded by the clock at A. The roundtrip distance is $2L$, and the roundtrip time span is $2L/c$ with a universal photon velocity $\boldsymbol{c}$. In his synchronization method, he divided the roundtrip time span by 2 and required that the future transmission from the clock at A send its time tag to the remote clock at B with the time span of transmission or L/c. The operator at B would add this time span to the time tag to set the clock at B for synchronization. Einstein defined that clocks can only be synchronized in the reference frame (his Equation 1.1) if the transmission time span using photon beams was the same in either direction. In his mental experiment, he had a rod with clocks A and B at the endpoints of the rod to maintain a fixed distance for clock synchronization. When the rod had a constant velocity v parallel to its length, he found the time spans between the clocks were unequal in the resting frame, because photons had to travel a longer distance than L to overtake the receding end of the rod and a shorter distance than L to intercept the approaching end. His results obtained unequal time spans of $t_{AB} = L/(c-v)$ from A to B and $t_{BA} = L/(c+v)$ from B to A. If the time uncertainty was less than $|t_{AB} - L/c|$ or $|t_{BA} - L/c|$, he concluded that a pair of moving clocks in the resting reference frame could not be synchronized. From these results, Einstein made the unproven claims that events of simultaneity and clock synchronization at locations in the resting frame are nonsimultaneous and nonsynchronous in other moving inertial frames.

The atomic timekeeping community would dispute Einstein's results. Ultraprecise atomic clock synchronization is maintained daily by scores of timekeeping laboratories that are often their national time standards. Their uncertainties in synchronization are well below and often orders smaller than Einstein's results. The predicted offset of $\left[L/(c+v) - L/c\right]$ or $\left[L/(c-v) - L/c\right]$ is about 1.67E−12 s using the speed due to Earth's orbit with a spacing of L = 5m between clocks.

For example, NIST has demonstrated full unambiguous synchronization of two-way links between two optical clocks. Over 2 days, the time wander was 40 fs peak to peak, and the time frequency transfer was below 225 attoseconds (as) [6]. For all practical purposes, the transmission time spans of t_{AB} and t_{BA} are identical between the two NIST optical clocks. This paper shows that Einstein's derivation has no algebraic error. It is also proven that both the Galilean and Lorentz transformations between inertial reference frames do preserve simultaneity and clock synchronization, which shows Einstein's claims are incorrect. Simultaneous events in one reference frame are still simultaneous in another moving inertial frame. Clocks broadcasting time in synchronization in one inertial frame synchronically broadcast time in another inertial frame (although an observer must account for different delays of reception if each clock is a different distance from the observer).

Assuming t_{AB} and t_{BA} are identical transmission spans between A and B of a moving rod as demonstrated by NIST, Equations (10) and (11) show what conditions allow that. QED theory identifies two photon beams are transmitted: c_{AB} from A to B and c_{BA} from B to A—not Einstein's continuous beam with a reflection at B. Equations (10) and (11) reveal that $c_{AB} = c + v$ and $c_{BA} = c - v$, which demonstrates the net magnitude of velocity between the photon source and detector is part of the magnitude of total velocity of a photon in the reference frame. This net velocity was denied in Einstein's original second relativity postulate. Even if one chose a nonrotating, freely falling reference frame with the origin at Earth's center, which Einstein stated would be equivalent to an inertial frame, all timing laboratories are at different latitudes with different tangential rotational velocities. (e.g., NIST at 40.01˚ latitude would have about 356 m/s tangential velocity to such an Earth-centered, nonrotating frame, and Einstein's formulae would predict the lowest limit of 1.85E−14 s with a 5 m distance between clocks for synchronization.) Yet, all atomic clocks are synchronized with smaller time uncertainties far below what Einstein's derivation would predict. Equations (10) and (11) reveal that the one-way photon velocity is not universal, although emitted photons do have a common intrinsic velocity of c. In general, $c_{AB} \neq c \neq c_{BA}$ for a uniformly moving rod.

The roundtrip photon speed experiments of Fizeau (1849), Foucault (1850 and 1862), and Michelson (1877-1879 and 1926) indicated an apparent constant photon speed. Note that these tests obtained the average speeds from the two legs of the roundtrip course with reflection, but a velocity measurement would be zero as the total displacement is zero. The average roundtrip photon speed is $\left[(c-v)+(c+v)\right]/2 = c$. It is the author's opinion that these tests influenced Maxwell to develop his electromagnetic theory (1861-1862) with a constant c and Einstein to state photon velocity was a universal constant velocity $\boldsymbol{c}$ in his special relativity paper (1905).

Einstein's original second relativity postulate is an excellent approximation due to the high speed of photons compared to typical velocities of photon sources relative to the detectors. The two contradictions of special relativity show <u>one-way</u> photon speeds are not numerically identical in a vacuum between moving inertial frames as required by the second relativity postulate (e.g., $k \neq k\gamma^2$) as proven in equations (12) and (13). Special relativity also predicts a slowly accelerated body would have two simultaneously different masses when measured perpendicular or parallel to the mass's velocity ([1], §10, 1-81), but no mass variation has been reported in metrology. If simultaneously different masses could be measured parallel versus perpendicular, which the difference is a multiplicative factor of γ in special relativity, then absolute velocity of the laboratory within the universe could be obtained from γ for speed and direction is obtained where the largest mass is measured (*i.e.*, largest mass is measured parallel to the object's velocity), which immediately contradicts the first relativity postulate (*i.e.* equivalence principle). Since special relativity was rigorously derived from Einstein's second postulate, these special relativity contradictions point to some inaccuracy in the current wording of the second postulate.

To the author's best knowledge, this is the first definitive, analytical study of relativity combined with ultraprecise time measurements that confirm the total photon velocity includes the net velocity between the photon source and detector with the intrinsic photon velocity $\boldsymbol{c}$ (e.g., magnitude of 299792458 m/s). Einstein's derivation of moving clocks predicted transmission time spans of $L/(c \pm v)$ instead of the ideal L/c for stationary clocks when assuming a universal constant speed c in his resting frame. This difference often requires 15 significant digits of timed measurements to verify precise clock synchronization between moving clocks in an arbitrary inertial frame. Older atomic clocks often operated with a time uncertainty of 1E−13 s, which is not enough to validate synchronization with moving clocks below Einstein's predicted lowest limit. The recent development of optical clocks has the needed ultraprecision (e.g., below 1E−15 s) for synchronizing moving clocks.

The author's logic concludes, both theoretically and physically, that photon speed may exceed the "maximum" intrinsic photon velocity upon emission. The logical process is straightforward. Follow Einstein's derivation except generalize photon velocity with $\boldsymbol{c}_{AB}$ and $\boldsymbol{c}_{BA}$ instead of $\boldsymbol{c}$. (If needed, one can later equate them to $\boldsymbol{c}$.) This obtains Equations (8) and (9). NIST has demonstrated clock synchronization with time span errors in transmission between clocks to be less than 1E−15 s. The nearest nonrotational, freely falling frame as an equivalent inertial frame would be at Earth's center, and the velocity of NIST relative to that frame would be due to Earth's rotation for a tangential velocity of 356 m/s. Einstein's derivation predicts the smallest time span transmission uncertainty of $\left[L/(c+v)-L/c\right]$ or $\left[L/(c-v)-L/c\right]$, which is 1.85E−14 s in this case. However, NIST obtained synchronization with a transmission time error of 0.225E−15 s. By Einstein's definition of synchronized clocks, the transmission times are equal ($\Delta t_{AB} = \Delta t_{BA}$) within experimental error, which is less than Einstein's lowest predicted error.

Satisfying his synchronization requirement of equal time intervals for the traversing photons over a moving rod, Equations (10) and (11) prove that speeds $c_{AB} = c + v$ and $c_{BA} = c - v$, which is the addition law due to the photon source moving relative to the detector. This is the requirement for photon velocity to maintain clock synchronization whether the pair of clocks are stationary or uniformly moving within a given inertial frame. For the rod with uniform velocity in the resting frame, photons moving $A \rightarrow B$ have a longer distance to traverse ($L_{AB} > L$) and a faster one-way photon speed ($c_{AB} > c$) such that $L_{AB}/c_{AB} = \Delta t = L/c$. Photons moving $B \rightarrow A$ have a shorter distance to traverse ($L_{BA} < L$) and a slower one-way photon speed ($c_{BA} < c$) such that $L_{BA}/c_{BA} = \Delta t = L/c$. This theoretically satisfies Einstein's synchronization requirement of equal time spans for photon pulses to traverse the stationary or moving rod in either direction, which satisfies the first relativity postulate (*i.e.*, equivalence principle).

For theory to allow daily ultraprecise clock synchronization, the second relativity postulate should combine the net velocity between the source and detector with the intrinsic velocity $\boldsymbol{c}$ as part of the total photon velocity. This study verifies that total photon speed can vary as $c \pm v$, depending on how the net velocity of the photon source and detector move relative to the photon emission and direction within the reference frame. This brings into question whether Minkowski 4-D space-time is accurate if the time axis is no longer rigid with a variable total $\boldsymbol{c}$. Some invariant quantities containing c or constants defined with c may not be static. With ultraprecise measurements containing high enough significant digits of total wave transmissions, classical electromagnetism (EM) will need to be reexamined as two of Maxwell's equations contain c as the static quantity c, but the total velocity $\boldsymbol{c}$ is a variable. All quantum entanglement tests that observe interactions can expect faster communications between remote test particles with the total $\boldsymbol{c}$ than with the standard speed c. With a variable photon speed, the definition of the meter is not rigorous nor exact. (See [9] for other relativity issues, ultraprecise tests, and ramifications)

Gravity does affect photon velocity (e.g., Shapiro time delay where the gravitational field changed the photon's direction to traverse a curved path), which Einstein did not deliberate in 1905. That topic is beyond special relativity but will be considered in a future paper. Any discovery of gravitational effects with photon velocity may additionally alter the second relativity postulate. Even though the second relativity postulate is an excellent approximation, ultraprecise measurements by the current technology now reveal inaccuracies in that approximation, and the second relativity postulate should be expanded to incorporate the net velocity between the photon sources and detectors with the intrinsic velocity of photons as part of the total photon velocity.

Richard Feynman, Nobel recipient for co-developing QED, summed up the scientific pursuit of knowledge as follows [10].

> "In general, we look for a new law by the following process. First, we guess it. Then, we compute the consequences of the guess to see what would be implied if this law that we guessed is right. Then, we compare the result of the computation to nature, with experiment or experience, compare it directly with observations, to see if it works.
>
> If it disagrees with experiment, it is wrong. In that simple statement is the key to science. It does not make any difference how beautiful your guess is. It does not make any difference how smart you are, who made the guess, or what his name is—if it disagrees with experiment, it is wrong."

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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