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A Novel Classical Model of the Free Electron

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Abstract

Previous models of the free electron using classical physics equations have predicted attributes that are inconsistent with the experimentally observed attributes. For example, the magnetic moment has been calculated for the observed spinning electric charge. For the calculated moment to equal the observed moment, the electron would either have to spin at two hundred times the speed of light or have a charge radius two hundred times greater than the classical radius. A similar inconsistency results when the mass derived from the spin angular momentum is compared with the observed mass. A classical model is herein proposed which eliminates the magnetic moment inconsistency and also predicts the radius of the electron. The novel feature of the model is the replacement of a single charge with two opposite charges, one on the outer surface of the electron and the other at the center.

Keywords

Classical Electron Model, Free Electron, Electron Structure, Electron Charge, Electron Radius, Electron Spin, Electron Shape, Electron Compressibility

1. Introduction

Some attributes of the electron that have been measured are charge, mass, angular momentum, and magnet moment. Angular momentum has been assumed to result from the spinning of the mass. Magnetic moment has been assumed to result from the spinning of the charge. Previous classical models of the electron have attempted to relate these attributes using classical physics equations. The result has been inconsistent on the order of two orders of magnitude. Attempts to resolve the inconsistencies have predicted very large radii or rotation speeds greatly exceeding the speed of light. As a consequence, many have concluded that the classical laws of physics do not apply in the quantum domain of the electron.

The following article proposes a novel model of the free electron using classical physics equations. The model has the following features:

- replaces the single charge in previous models with two opposite charges, one on the outer surface of the core and one at the center;
- eliminates the inconsistency between the observed charge radius and the charge radius deduced from the spin magnetic moment;
- assumes a spin rotation speed that is close to, but does not exceed, the speed of light;
- predicts a radius that is close to the calculated classical and experimentally measured radii;
- suggests that the core shape might be a ring rather than a sphere;
- does not rely on tensile strength of the electron material to hold it together;
- there is no compression or tensile force on the electron core material at the equator;
- predicts a force holding the electron together greater than the nuclear Strong Force.

Except where otherwise noted, all constants, such as those in **Table 1**, and equations in this article are expressed in cgs units.

2. Magnetic Moment and Spinning Charge

2.1. Background

Consider a model of the electron wherein the charge q is distributed across the surface of a sphere of radius R. Assume the sphere rotates very near the speed of light c.

The magnetic dipole moment M of a spinning charged spherical shell is:

$$M = \frac{q}{3}\omega r^2 \quad \text{[MKS] [2]} \quad M = \frac{q}{3c}\omega r^2 \quad \text{[cgs]}$$

where q = uniformly distributed charge;

r = radius;

$\omega = \frac{2\pi}{T}$, where T = period of rotation.

Table 1. Electron constants.

constant	*symbol*	*value* [*cgs*]
charge	q	-4.8032×10^{-10}
mass	m	9.1094×10^{-28}
classical radius	R	2.82×10^{-13}
spin angular momentum	S	9.1329×10^{-28}
magnetic dipole moment	M	$-9.284764 \times 10^{-21}$
Planck's constant	h	6.6261×10^{-27}
speed of light	c	$2.99792458 \times 10^{10}$

These constants were measured or derived from experimental observations [1]. Spin angular momentum S was derived from the equation $S = \sqrt{3}\frac{h}{4\pi}$.

Assuming the electron model has a spherically charged shell of radius R, its spin magnetic moment M can be expressed as:

$$M = \frac{2\pi q}{3Tc} R^2$$

$$\text{rotation speed at the equator} = \frac{2\pi R}{T} = \frac{3Mc}{qR} = 205.6c$$

To generate the observed magnetic moment by spinning the observed charge, the electron equator would have to spin at more than 200 times the speed of light. Since mass cannot spin faster than the speed of light, an alternative explanation for the large observed spin magnet moment might be a radius larger than R. Now assume that the rotation speed is less than but very close to the speed of light c. The required radius r can be calculated as follows:

$$M = \frac{2\pi q}{3Tc} r^2,$$

where

$$T = \frac{2\pi}{c} r$$

$$r = \frac{3M}{q} = 5.799 \times 10^{-11} = 205.6R$$

The value calculated for r is close to that calculated in [3], which is 3.86×10^{-11}. The difference could be attributed to [3] assuming the charge is concentrated in a ring, rather than distributed across a sphere.

The classical model of an electron with a spherical charge shell predicts a spin rotation speed of more than 200 times the speed of light. Or, if the spin speed is limited to the speed of light, the radius of the shell would be more than 200 times the classical electron radius. If the charge is assumed to be uniformly distributed throughout the interior of the sphere, the speed or charge shell radius would be even greater. Such large inconsistencies have caused many to believe that classical mechanics and electrodynamics cannot be used to model the electron.

2.2. Proposed Charge Model

A model is proposed wherein the electron is comprised of two opposite charges. The spinning outer charge q^+ creates the observed magnetic moment. The inner charge q^- located at the center has a very small radius, such that it does not significantly contribute to the magnetic moment. The electric fields from the inner and outer charges combine such that the net electric field of the electron appears to be created by a negative charge q of the observed value.

$$q = q^+ + q^-$$

The cgs unit for charge is $\text{cm}^{3/2}\ \text{g}^{1/2} \cdot \text{s}^{-1}$. Length, mass, and time all change on a speeding platform relative to a stationary platform according to Einstein's Special Relativity equations. However, when combined within the unit for charge, the three changes all cancel each other out. Therefore, charge is invariant under speed. A value of charge is the same whether observed on a stationary platform or a platform moving near the speed of light [4].

Magnetic dipole moment M of a spinning charged spherical shell:

$$M = \frac{q^+}{3}\frac{\omega}{c}r^2,$$

where q^+ = positive charge;

r = radius;

$\omega = \frac{2\pi}{T}$, where T = period of rotation $= \frac{2\pi}{c}r$.

For sign consistency, the value of M for the electron is considered to be negative, corresponding to a negative charge. Since M is actually being generated in the dual-charge model by a spinning positive charge, instead of a negative charge in the single-charge model, the spin direction in the dual-charge model must be reversed from that in the single-charge model. M after reversing the spin rotation:

$$M = -\frac{q^+}{3}r$$

For M = electron magnetic dipole moment:

$$q^+ = -\frac{3M}{r}$$

$$q^- = q - q^+ = q + \frac{3M}{r}$$

3. Mass and Spin Angular Momentum

The angular momentum of a rotating ring is

$$S = mrv,$$

where m = mass of the ring;

v = speed of rotation;

r = radius.

The speed of rotation of the ring is:

$$v = \frac{S}{mr}$$

Let the ring have an angular momentum equal to the spin angular momentum S of the electron, a mass m equal to that of the electron and a radius r equal to that of its classical radius R:

$$v = 3.555\times10^{12} = 118.6c$$

Special Relativity tells us that the speed of a mass must be less than that of light. Limiting the ring speed v to slightly less than the speed of light c, the ring mass m' would have to be slightly greater than:

$$m' = \frac{S}{rc}$$

Consider a non-rotating sphere having a uniformly distributed mass throughout its volume. Slice the sphere into many concentric cylinders, each having a mass m_n. Now spin the sphere about the cylinder axes at a speed v. The relativistic mass m'_n of each cylinder spinning at a speed v_n is given by the Special Relativity equation:

$$m'_n = \frac{m_n}{\sqrt{1-\left(\frac{v_n}{c}\right)^2}}$$

The total mass m' of the rotating sphere is $m' = \sum_n m'_n$.

An approximation using ten cylinders and a rotation speed of the outer cylinder very near the speed of light shows that almost all of the mass m' is concentrated in the outer cylinder, or ring, to within about 1%. Therefore, for the following calculations, the relativistic mass m' of a rotating sphere will be considered to be uniformly concentrated along a ring of radius r at the equator. For the purposes of modeling the electron, the equation $m' = \frac{S}{rc}$ will be assumed to be a very close approximation for m'.

4. Radius

An electron can be modeled as originating from a spherical shell of charge q^+ having a very large radius. At the center of the shell is a charge q^-. The electron radius to be calculated is R'. The two charges are:

$$q^+ = -\frac{3M}{R'}$$

$$q^- = q + \frac{3M}{R'}$$

The charge increments on the shell tend to repel each other. Each increment at distance r from the center sees the remaining increments as a point charge at the center.

Coulomb's law for force f between two charges separated by a distance r: $f = \frac{q^1 q^2}{r^2}$.

The total repulsive force for all electron charge increments on the spherical shell is:

$$\left(\frac{q^+}{r}\right)^2 = \left(\frac{3M}{R'}\right)^2 \frac{1}{r^2}$$

The attractive force between q^+ and q^- is:

$$\frac{q^+ q^-}{r^2} = -\left[\frac{3Mq}{R'} + \left(\frac{3M}{R'}\right)^2\right]\frac{1}{r^2}$$

The sum of the repulsive and attractive forces is a net inward force of:

$$-F = -\frac{3Mq}{R'}\frac{1}{r^2}$$

The outer charge shell will collapse under the inward force *F*. The electrostatic potential energy lost when the shell's radius contracts from infinity to *r* is:

$$E = -\int_{\infty}^{r}\frac{3Mq}{R'}\frac{1}{x^2}\mathrm{d}x = \frac{3Mq}{R'}\frac{1}{r}$$

The energy *E* lost is transferred to the spinning electron energy E':

$$E' = m'c^2 = \frac{Sc}{R'}$$

The electron radius R' is the solution *r* to the equation $E' = E$.

$$R' = \frac{3Mq}{Sc} = 4.8864\times10^{-13} = 1.73R$$

The radius for the proposed dual-charge electron model with a spherical core is 73% greater than the classical radius *R*.

5. Internal Forces

In this section, the internal forces of the electron model are calculated as a function of radius *r*, given that charge *q*, magnetic moment *M*, and spin angular momentum *S* are constants.

The outward forces tending to push the electron apart are:

- centrifugal force on the spin mass ring associated with the spin angular moment *S*;
- mutual repulsion of the charge increments on the outside charge shell;
- outward repulsion of the charge moving through its own magnetic field;
- compression force of the electron core material.

The inward force is the attraction of the spherical charge shell to the opposite charge at the center of the electron.

The model assumes that the spin speed *v* at the electron equator is slightly less than the speed of light *c*:

$$\text{centrifugal force} = \frac{m'v^2}{r} \cong \frac{m'c^2}{r} = \frac{Sc}{r^2}$$

$$\text{mutual repulsion of the outer charge shell} = \frac{(3M)^2}{r^4}$$

magnetic repulsion of outer charge shell—

A charge increment on the outer shell spins through the magnet field created by all of the other spinning charge increments. To simplify calculations, the spinning sphere was approximated by a spinning ring at the equator of the electron. To calculate the magnetic force on the ring, the ring was split into two rings very close to each other. Each ring had one half the total charge. Each ring spins in the magnetic field of the other. The two rings attract each other. The force on each ring was calculated. The net force on the ring pair was then calculated. By comparison, the magnetic outward force was found to be about ten thousand times weaker that the electric outward force. It is therefore not significant in the following force calculations.

$$\text{inward force} = \frac{q^+ q^-}{r^2} = -\frac{3Mq}{r^3} - \frac{(3M)^2}{r^4}$$

Note that positive forces are repulsive (outward) and negative forces are attractive (inward).

The total internal force F is the sum of the centrifugal, repulsive, and inward forces:

$$-F = \frac{Sc}{r^2} - \frac{3Mq}{r^3}$$

For $r = R' = \frac{3Mq}{Sc}$, the sum of all the internal forces F is zero.

For $r = R'$, the internal forces are balanced. Unfortunately, the balance is unstable. A small change in radius will cause the force to increase such that it causes a greater change in radius, and so forth. The forces within the electron must always be balanced for it to have stable attributes, such as radius.

A stable internal force balance can be achieved by introducing an incompressible or compressible core. An inward force—F will be counteracted by an outward force F' from the compressed core. The force balance will be stable when

$$\frac{\mathrm{d}F'}{\mathrm{d}r} \geq \frac{\mathrm{d}F}{\mathrm{d}r}.$$

For $\frac{\mathrm{d}F'}{\mathrm{d}r} > \frac{\mathrm{d}F}{\mathrm{d}r}$, a small decrease in radius will cause the outward force F' from the core to increase more than the inward force F, resisting the change in radius. A small increase in radius will cause the outward force F' to decrease more than the inward force F. The net force change will be inward, resisting the radius change.

For $\frac{\mathrm{d}F'}{\mathrm{d}r} = \frac{\mathrm{d}F}{\mathrm{d}r}$, a small change in radius will cause the outward and inward forces to change by the same amount, so the net change in force will be zero.

For $\frac{\mathrm{d}F'}{\mathrm{d}r} < \frac{\mathrm{d}F}{\mathrm{d}r}$, a small decrease in radius will cause the net inward force F to increase more than the resisting compression force F', resulting in a net force imbalance.

Up to this point, the model of the electron core has been spherical and consisting of an incompressible material. The force components at the surface of the sphere are not uniform in magnitude. The electrical forces are, but the centrifugal force is not. The centrifugal force is greatest at the equator and decreases very rapidly away from the equator. It is zero along the spin axis. The rapid decrease is mainly due to the concentration of mass m' at the equator.

For an incompressible core, it is obvious that $\frac{\mathrm{d}F'}{\mathrm{d}r} > \frac{\mathrm{d}F}{\mathrm{d}r}$. The total force balance is stable.

The stability of a compressible core is considered in the following:

The compressibility constant K for a sphere is defined by:

$$K=-\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}P}\text{, where } V=\text{volume}=\frac{4\pi}{3}r^3$$

$$P=\text{pressure}=\frac{F'}{A}\text{, where}$$

$$F' = \text{total force on the area } A$$

$$A=4\pi r^2$$

$$\mathrm{d}V=\left(4\pi r^2\right)\mathrm{d}r$$

$$\mathrm{d}P=-\frac{1}{K}\frac{1}{V}\mathrm{d}V=\frac{\mathrm{d}F'}{A}$$

$$\frac{\mathrm{d}F'}{\mathrm{d}r}=-\frac{12\pi}{K}r$$

$$\frac{\mathrm{d}F}{\mathrm{d}r}=\frac{2Sc}{r^3}-\frac{9Mq}{r^4} \quad \text{near the equator}$$

$$\frac{\mathrm{d}F}{\mathrm{d}r}=-\frac{9Mq}{r^4} \quad \text{away from the equator}$$

For a stable force balance, $\frac{\mathrm{d}F'}{\mathrm{d}r}\geq\frac{\mathrm{d}F}{\mathrm{d}r}$:

$$\frac{12\pi}{K}R'\geq\frac{9Mq}{R'^4}-\frac{2Sc}{R'^3} \quad \text{near the equator}$$

$$\frac{12\pi}{K}R'\geq\frac{9Mq}{R'^4} \quad \text{away from the equator}$$

6. Electron Shape, Size, and Charges

The modeled shape of the electron is a function of the compressibility of the electron material. For incompressible material, $K = 0$ and the shape can be spherical. The upper limits to the values of K for electron radius R' are approximately:

$$K=\frac{12\pi R'^5}{9Mq-2ScR'}=7.85\times10^{-32} \quad \text{near the equator}$$

$$K=\frac{4\pi R'^5}{3Mq}=2.62\times10^{-32} \quad \text{away from the equator}$$

For $K>0$, the core is compressible and not a perfect sphere. The shape will tend toward that of a ring. It will bulge outward at the equator. For $K>\frac{4\pi R'^5}{3Mq}=2.62\times10^{-32}$, the core will be collapsed along its spin axis. As K increases further, the core will become a ring spinning around the axis.

All of the above equations containing the constant M assume M was calculated for a sphere.

The magnetic moment for a spinning ring is

$$M = \frac{q}{2}\omega R^2 \quad \text{[MKS] [5]} \quad M = \frac{q}{2c}\omega R^2 \quad \text{[cgs]}$$

Therefore, for a spinning ring, M in the equations assuming a spherical core must be replaced by $\frac{2}{3}M$ and the equations recalculated. The equations for the radii of the spherical core and the ring core are:

$$R'(\text{sphere}) = \frac{3Mq}{Sc} = 4.8864\times10^{-13} = 1.73R$$

$$R'(\text{ring}) = \frac{2Mq}{Sc} = 3.2576\times10^{-13} = 1.16R$$

The radius of the ring core is very close to the calculated value R [6] and the approximate value measured by X-ray diffraction [7]. This correlation provides some evidence that the electron core is better modeled as a ring rather than a sphere, as suggested in [7].

The upper limit of K for a ring core of radius R' **is**:

$$K = \frac{6\pi R'^5}{3Mq - ScR'} = 1.5505\times10^{-32}$$

For K greater than the upper limit, the ring core will collapse.

The positive and negative internal charges are the same for both the spherical and ring core shapes, and are:

$$q^+(\text{sphere}) = -\frac{3M}{R'(\text{sphere})} = q^+(\text{ring}) = -\frac{2M}{R'(\text{ring})}$$
$$= 5.7004\times10^{-8} = 118.7q$$

$$q^-(\text{sphere}) = q + \frac{3M}{R'(\text{sphere})} = q^-(\text{ring}) = q + \frac{2M}{R'(\text{ring})}$$
$$= -5.7484\times10^{-8} = 119.7q$$

For the dual-charge model of the electron, the tensile force on the core material is zero. By comparison, the tensile force on the core for the single-charge model is:

$$\frac{Sc}{R^2} + \left(\frac{q}{R}\right)^2 = 3.5\times10^8$$

The internal binding force for the dual-charge model with a ring core is quite large:

$$\frac{2Mq}{(R')^3} + \frac{(2M)^2}{(R')^4} = 3.1\times10^{10}$$

The nuclear Strong Force, which binds the nucleus of atoms together, is: 2.5 × 10^9 [8], so the binding force in the dual-charge electron is about ten times stronger than the Strong Force.

7. Summary

A model of the electron has been proposed which has two opposite electrical charges. The positive charge q^+ resides on the outer surface of the electron. The negative charge q^- resides at the center of the electron. It has a radius small enough so that the spinning negative charge does not significantly contribute to the net magnetic moment. The shape of the electron can be spherical, a ring, or a shape in between the two. The charges are the same for shapes within this range:

$$q^+ = 5.7004\times10^{-8} = 118.7q\,,\quad q^- = -5.7484\times10^{-8} = 119.7q$$

The radius depends on the shape of the electron core:

$$R'(\text{sphere}) = 4.8864\times10^{-13} = 1.73R\,,\quad R'(\text{ring}) = 3.2576\times10^{-13} = 1.16R$$

The radius of the ring core is very close to the calculated and experimental values, suggesting that the electron is better modeled as a ring rather than a sphere.

The single-charge model has a large inconsistency between the observed spin magnet moment and the calculated moment due to the spinning classical charge. Eliminating the inconsistency would require a radius of more than 200 times the classical radius or a spin rotation speed of more than 200 times the speed of light. The proposed dual-charge model eliminates the inconsistency with a radius close to the classical radius and a spin rotation speed slightly less than the speed of light c.

Most of the relativistic spinning mass is concentrated in a ring around the electron equator, even for the spherical core shape.

The classical single-charge electron model implicitly depends on great tensile strength of the electron material to hold the electron together. The dual-charge model does not require any tensile strength. The model depends on an incompressible or compressible core to provide a stable internal force balance. For a spherical core, there will be compressive pressure on the core, except at the equator. For a ring core, there is no compressive force. The maximum compressibility constant for stable internal force balance is $K = 1.5508\times10^{-32}$.

The internal binding force that holds the electron ring core together is one order of magnitude greater than the nuclear Strong Force.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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