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Remarkable Findings in Fundamental Theory of Quantum Mechanics

—Matter Wave and Discrete Time in Physics

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Abstract

The relation of matter wave, which is well-known as a hypothesis proposed by de Broglie in 1923, gave basis for establishing the quantum mechanics. After that, experimental results revealed that a micro particle has a wave nature. However, the theoretical validity of the relation itself has never been revealed since his proposal. Theoretical basis that a micro particle has a wave nature has been thus disregarded in the unsolved state. The diffusion equation having been accepted as Fick's second law was derived from the theory of Markov process in mathematics. It was then revealed that the diffusivity D depends on an angular momentum of a micro particle in a local space. The fact being unable to discriminate between micro particles in a local space resulted in having to accept the existence of minimum time $t_0(>0)$ in the quantum mechanics. Based on t_0 and D obtained here, the theoretical validity of relation of matter wave was confirmed. Denying the density theorem in mathematics for time in physics indicates that the probabilistic interpretation is essentially indispensable for understanding the quantum mechanics. The logical necessity of quantum theory itself is thus understandable through introducing t_0 into the Newton mechanics. It is remarkable that the value of t_0 between $1.14\times10^{-17}\ \text{s} \le t_0 \le 1.76\times10^{-14}\ \text{s}$ obtained here is extremely larger than that of the well-known Planck time $t_P = 5.39\times10^{-44}\ \text{s}$.

Keywords

Quantum, Diffusion Particle, Matter Wave, Planck Time, Minimum Time

1. Introduction

There are sometimes reproducible phenomena expressible by a relation under the given conditions in physics. When we cannot theoretically reveal the validity of its relation, it has been accepted as a law or a principle. Further, such hypotheses as de Broglie's hypothesis relevant to the matter wave, Planck's hypothesis relevant to the photon energy, Bohr's hypothesis relevant to the atomic model and so on, have been also often accepted in the history of physics. Physics has developed in the theoretical frame based on such laws, principles or hypotheses. For example, Newton's laws are valid in the following preconditions.

Precondition [A]: the absolute time of $t' = t$ is accepted between the different coordinate systems of (t, x, y, z) and (t', x', y', z').

Precondition [B]: the mathematical density theorem is valid in arbitrary variables of coordinate system, that is, $\lim_{t_1 \to t_2} (t_1 - t_2) = 0$, $\lim_{x_1 \to x_2} (x_1 - x_2) = 0$, and so on.

Here, when we found a new fact contradictory to the existing laws, principles or hypotheses, themselves or their preconditions should be examined again. For example, Einstein's relativity, which is one of the modern physics, was established by denying the above precondition [A], accepting the constant principle of light speed in contradiction to Newton's law. On the other hand, the quantum theory of another modern physics was established by accepting the hypothesis of de Broglie [1], which had never been understandable in the Newton mechanics until recently [2].

In 1926, Schrödinger [3] derived the wave equation of a micro particle from the hypothesis proposed by de Broglie in 1923. The so-called Schrödinger equation has been in conformity with each behavior of micro particles. Judging from the theoretical frame of physics, however, the quantum theory has been still essentially incomplete without revealing the causality for the Newton mechanics, even if it is justifiable. In fact, we have the unsolved "proposition" having to verify the theoretical basis for wave nature of a micro particle.

To solve the proposition in those days, it seems that Einstein, Bohm, and others tried to transform the diffusion equation of micro particles into the wave equation of Schrödinger. However, their projects ended in failure. In actuality, the above proposition has been disregarded and the quantum theory has developed as an afterthought in the matter of fundamental problems. Incidentally, the diffusion equation has been accepted as a low proposed by Fick in 1855. As far as we thus accept it as a law, the diffusivity is only a mathematical operator in the partial differential equation and we cannot grasp its physical meaning then. Here, Okino [4] thought that their failures are caused by accepting the diffusion equation as a law. To grasp the essential meaning of diffusivity in physics, therefore, deriving the diffusion equation from the theory of Markov process in mathematics was first considered then. As a result, it was revealed that the diffusivity D depends on an angular moment of a micro particle in a closed local space and $D = \hbar/2m$ is valid then, where $\hbar$ and m are $\hbar = h/2\pi$ for the Planck constant h and a mass of micro particle.

The photon energy indicates that the discrimination between two micro particles in a local space is essentially impossible. Here, we accept the matter as an impossible principle of discrimination between micro particles. In that case, the impossible principle of discrimination between micro particles results in the fact that there is a minimum time t_0 as a real time in physics in contradiction to the density theorem of real time in mathematics [5].

As a result, the wave equation of Schrödinger is reasonably derived from the diffusion equation for micro particles by using the impossible principle of discrimination between micro particles and the diffusivity $D = \hbar/2m$. Here, the wave nature of a micro particle was theoretically revealed. Further, the validity of the relation itself of matter wave was reasonably revealed [6]. In addition, such theoretical basis that the probabilistic interpretation is indispensable for the quantum theory is also reasonably revealed.

Judging from the theoretical frame of physics, it is essentially important to understand the logical necessity reaching from the Newton mechanics to the quantum mechanics. Nevertheless, the elucidation of logical necessity has been disregarded for a long time in the unsolved state. Thus, the elucidation is a main purpose in the present work.

As a result, such theoretical bases that a micro particle has wave nature and that the probabilistic interpretation is indispensable for the quantum theory were reasonably revealed in introducing the conception of t_0 into the Newton mechanics. In other words, we will notice that the quantum theory is established by denying $\lim_{t_1 \to t_2}(t_1 - t_2) = 0$ in the precondition [B] mentioned above.

2. Verification of Matter Wave

For a micro particle of mass m moving with a speed v in space-time (t, x, y, z), the partial differential equation of wave function $\Psi = \Psi(t, x, y, z)$ yielding

$$i\hbar \frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\langle \tilde{\nabla} | \nabla \rangle \Psi \tag{1}$$

was derived by Schrödinger [3] from the hypothesis of de Broglie [1] of

$$\lambda = h/p, \tag{2}$$

where $i, \hbar, \lambda$ and p are a unit imaginary number, $\hbar = h/2\pi$ for the Planck constant h, a wave length of matter wave and a momentum $p = mv$. In addition, the nabla vector ∇ is expressed by the Dirac bracket and the notation $\langle \tilde{\nabla} | = -|\nabla\rangle^\dagger$ is then defined because of the Hermite conjugate.

For the concentration $C = C(t, x, y, z)$ of diffusion particles, the nonlinear diffusion equation of moving coordinate system given by

$$\frac{\partial C}{\partial t} = D\langle \tilde{\nabla} | \nabla \rangle C \quad \text{for} \quad D = \frac{(\Delta r)^2}{2\Delta t}, \quad r = \sqrt{x^2 + y^2 + z^2} \tag{3}$$

was derived from the theory of Markov process in mathematics [7]. For a diffusion particle in the closed local space, Equation (3) shows that the diffusivity D is rewritten as $D = \Delta rp/2m$ relevant to an angular momentum of the diffusion particle, because of $\Delta rp = \sqrt{\langle \Delta r \times \tilde{p} | \Delta r \times p \rangle}$ in the present case. This means that the diffusion particle in a local space makes a circuit around the center point of local space.

On the other hand, the quantum condition $r_n p = n\hbar\ (n = 1, 2, \cdots)$ in the atomic model of Bohr is also able to rewrite as $\Delta r p = \hbar$ for an orbital electron because of $\Delta r = r_n - r_{n-1}\ (r_0 = 0)$. After confirming that the relation $\Delta r p = \hbar$ is even valid in an arbitrary motion of electron because of $\Delta r p = \langle \Delta r | p \rangle$ in the present case, applying the equipartition law to a free electron in such material as metal revealed that the relation of

$$\Delta r p = \hbar \tag{4}$$

is also valid for an arbitrary micro particle [6]. Therefore, substituting Equation (4) into $D = \Delta r p / 2m$ yields

$$D = \frac{\hbar}{2m} \tag{5}$$

for a micro particle in local space [5].

Accepting the impossibility of discriminating between micro particles in a local space, the investigation of an elastic collision process between two micro particles of the same kind revealed that there is a minimum time t_0 as a real time in physics [8]. As can be seen from **Figure 1**, although we cannot understand behavior of the particle 1 between $-\Delta r < r < \Delta r$, it seems then that the particle

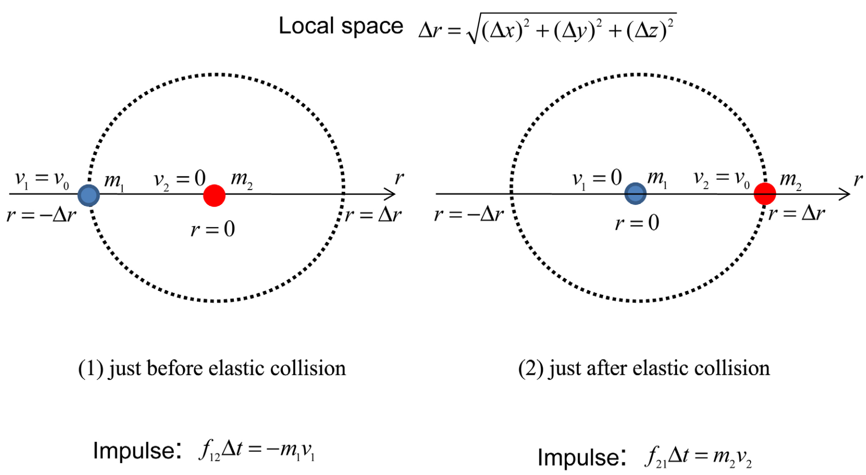

Figure 1. Elastic collision between two micro particles of the same kind. The figure shows an elastic collision between a particle 1 having a mass $m_1 = m$ and a velocity $v_1 = v_0$ at $r = -\Delta r$ and a particle 2 of the same kind having the mass $m_2 = m$ and the velosity $v_2 = 0$ at $r = 0$ in the initial state. In the Newton mechanics, those impulses are rewritten as $f_{12} = -m_1 \Delta r / (\Delta t)^2$ and $f_{21} = m_2 \Delta r / (\Delta t)^2$. If we cannot discriminate them, however, the relations $\bar{f}_{12} = m_1 \Delta r / (\Delta t)^2$ and $\bar{f}_{21} = -m_2 \Delta r / (\Delta t)^2$ obtained by replacement of each suffix 1 and suffix 2 should be then equivalent to the original expressions f_{12} and f_{21}, respectively. Therefore, the relations $\bar{f}_{12} \to f_{12}$ and $\bar{f}_{21} \to f_{21}$ resulting from the impossibility of discrimination between those particles correspond to rewriting $\Delta t \to \pm i \Delta t$ in each equation of $\bar{f}_{12} = m_1 \Delta r / (\Delta t)^2$ and $\bar{f}_{21} = -m_2 \Delta r / (\Delta t)^2$. The density theorem in mathematics is thus not valid for the time in physics, but it is still valid for the space. In that meaning, the conception of time is different from that of space.

1 moved from $r=-\Delta r$ to $r=\Delta r$ without incident through the impossibility of discriminating between the particle 1 and the particle 2. In other words, consequently we seem as if the particle 2 was nonexistent from the beginning. As mentioned in the caption of **Figure 1**, we must accept the imaginary time $\pm i\Delta t$ in physics for $0\le\Delta t<t_0$ in mathematics then, denying the mathematical density theorem. It was thus revealed that the minimum time t_0 is existent in the quantum theory and an arbitrary time t_j is expressed as a discrete time yielding $t_j=jt_0$ for $j=-\infty,\cdots,-2,-1,0,1,2,\cdots,\infty$.

In accordance with the limit theory, the existence of the minimum time t_0 reveals that the differential operators $\partial/\partial t$ and $\partial/\partial x$ become

$$\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\left(\frac{\partial}{\partial t}\right)x\to\left(\mp i\frac{\partial}{\partial t}\right)x=\mp i\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\lim_{\Delta t\to 0}1\bigg/\left(\frac{\Delta x}{\pm i\Delta t}\right)$$

and

$$\lim_{\Delta x\to 0}\frac{\Delta t}{\Delta x}=\left(\frac{\partial}{\partial x}\right)t\to\left(\pm i\frac{\partial}{\partial x}\right)t=\pm i\lim_{\Delta x\to 0}\frac{\Delta t}{\Delta x}=\lim_{\Delta t\to 0}1\bigg/\left(\frac{\Delta x}{\pm i\Delta t}\right)$$

in the differential equation for a micro particle. Judging from eigenvalues of these operators, therefore, the differential operators $\partial/\partial t$ and $|\nabla\rangle$ in the Newton mechanics should be rewritten as

$$\partial/\partial t\to i\,\partial/\partial t,\quad |\nabla\rangle\to -i|\nabla\rangle \tag{6}$$

in the quantum mechanics [6].

Here, substituting Equations (5) and (6) into Equation (3) and rewriting $\hbar C\to\Psi$ give the Schrödinger Equation (1). At this point, the wave nature of an arbitrary micro particle was theoretically verified in accordance with the causality for the Newton mechanics because of the reasonable transformation from the equation of micro particle into the wave equation. At the same time, the wave length λ of matter wave for an arbitrary micro particle is expressed as

$$\lambda=2\pi\Delta r \tag{7}$$

from the wave characteristic. Here, the above proposition having been disregarded for a long time was thus theoretically solved. Further, Equation (2) was theoretically derived for the first time in the history of quantum theory by eliminating Δr from Equations (4) and (7). Thus, the relation of matter wave is now not a hypothesis but a basic equation in physics judging from the theoretical frame of physics.

In addition, another relation of matter wave was also obtained as

$$\lambda=h\Big/\sqrt{\alpha_n m\left(k_{\mathrm{B}}T+\varepsilon\right)} \tag{8}$$

in the analytical process, where $k_{\mathrm{B}},T,\varepsilon$ and α_n are the Boltzmann constant, an absolute temperature in material, a correction term at $T=0$ in relation to the uncertain principle and a degree of freedom of micro particle composed of n atoms [6]. Eliminating m from Equations (2) and (8), a period T_{P} of matter wave is obtained as

$$T_{\mathrm{P}}=\frac{\lambda}{v}=\frac{h}{\alpha_n\left(k_{\mathrm{B}}T+\varepsilon\right)}. \tag{9}$$

When a micro particle passes through a local space of the size $l=2\Delta r$, Equations (7) and (9) show that the taken time t_{α_n} is expressed as

$$t_{\alpha_n}=\frac{l}{v}=\frac{h}{\pi\alpha_n\left(k_{\mathrm{B}}T+\varepsilon\right)}. \tag{10}$$

It is thus remarkable that the time T_{P} and t_{α_n} depend only on α_n and *T*.

The mathematical solution $\Psi=\Psi(t,x,y,z)$, which is obtained in accordance with the theorem of unique solution for the differential Equation (1), corresponds to either $\Psi=\Psi(t_j,x,y,z)$ or $\Psi=\Psi(t_{j+1},x,y,z)$ between $t_j\le t\le t_{j+1}$ with a certain probability for every *j* value, because of the fluctuation caused by the existence of discrete time $t_j=jt_0$ in the quantum theory. This means that we cannot principally apply the theorem of unique solution for a differential equation in mathematics to analyzing differential equations in the quantum mechanics. At the same time, this indicates that we must accept the probabilistic interpretation as a basic conception in the quantum theory. However, the behavior of a micro particle corresponds to the mathematical solution $\Psi=\Psi(t,x,y,z)$, as far as we do not determine the functional value.

Using a probability factor $A_j\left(0\le A_j\le 1\right)$ for the solution $\Psi(t_j,x,y,z)$, the physical solution $\Psi_{\mathrm{P}}(t,x,y,z)$ corresponding to $\Psi(t,x,y,z)$ is expressed as

$$\Psi_{\mathrm{P}}(t,x,y,z)=A_j\Psi(t_j,x,y,z)+A_{j+1}\Psi(t_{j+1},x,y,z)\quad\text{for}\quad A_j+A_{j+1}=1, \tag{11}$$

using a superposition of wave functions for every *j* value. In that case, it seems as if $\Psi_{\mathrm{P}}(t,x,y,z)$ interferes with itself because of the interference between $\Psi(t_j,x,y,z)$ and $\Psi(t_{j+1},x,y,z)$, resulting from accepting the discrete time $t_j=jt_0$ in the present theory. Here, we can now understand the theoretical evidence that a wave function interferes with itself in the quantum theory.

In addition, it seems that Einstein did not accept the probabilistic interpretation in the quantum theory in relation to the theorem of unique solution for a differential equation in mathematics. However, we now suppose that he would accept it in those days if he noticed the correlation between $\Psi(t,x,y,z)$ and $\Psi_{\mathrm{P}}(t,x,y,z)$ mentioned above. It is, therefore, essentially important that the minimum time t_0 is existent in the quantum theory.

3. Revision of Diffusion Theory

In general, we have no such a conception that the space itself moves in physics. However, it is considered that the space within a diffusion region moves relatively with respect to the surface of diffusion region because of the following reason. The expansion or shrinkage of diffusion region is caused by a thermal influence. In other words, an observer on the surface of diffusion region seems that the space within the diffusion region moves then. This means that the coordinate system setting a coordinate origin at a point of space within the closed diffusion region is a moving coordinate system with respect to the outside of diffusion region.

It was confirmed that the nonlinear diffusion Equation (3) is reasonably transformed into the usual expression of the fixed coordinate system given by

$$\frac{\partial C}{\partial t} = \left\langle \tilde{\nabla} \middle| D\nabla \right\rangle C, \tag{12}$$

which has been accepted as a law of Fick for a long time [7]. The diffusion Equation (3) has not been recognized as a nonlinear partial differential equation of a moving coordinate system, in spite of the indispensable one for understanding the diffusion theory. In addition, the universal diffusivity expression of

$$D = \frac{\hbar}{2m} \exp\left[\frac{U - Q}{k_B T + \varepsilon}\right] \tag{13}$$

applicable to an arbitrary micro particle in a material with an activation energy Q was also reasonably obtained, where U is a potential energy between a micro particle in local space and micro particles around the local space.

Judging from the theoretical frame of physics, the diffusion Equation (12) is now not a law but a basic equation in physics. Thus, the finding obtained here gives us a lesson that we should sometimes try to reexamine the relation having been accepted without the demonstration, even if it has been accepted as a law for a long time in physics. It is essentially indispensable for analyzing diffusion problems to discuss the coordinate systems used for the diffusion equation. This means that the existing fundamental theory of diffusion should be revised in accordance with the discussion between the coordinate systems used for the diffusion equation [4]. For example, although the conception of intrinsic diffusion has been widely accepted for a long time, we will notice that such a conception, which was empirically assumed in relation to the Kirkendall effect in those days, is nonexistent from the beginning as if it has been an illusion [7].

Even the general solutions of the concentration $C = C(t, x)$ and the diffusivity $D = D(t, x)$ in case of one dimension space for Equation (12) were not obtained. Then, Boltzmann [9] in 1894 transformed Equation (12) in case of the coordinate system (t, x) into the nonlinear ordinary differential equation of

$$-\frac{\xi}{2}\frac{\mathrm{d}C}{\mathrm{d}\xi} = \frac{\mathrm{d}}{\mathrm{d}\xi}\left\{D\frac{\mathrm{d}C}{\mathrm{d}\xi}\right\} \tag{14}$$

in the parabolic space $\xi = x/\sqrt{t}$. Nevertheless, the general solutions of $C = C(\xi)$ and $D = D(\xi)$ of Equation (14) had not been also obtained for a long time. In that situation, recently the general solutions of Equation (14) were first obtained [4]. Using them for the diffusion problems of many elements system, the reasonable analytical method has been thus established [7].

4. Minimum Time in Physics

Oriental people have been used to the word "setsuna" [刹那] defined as a minimum time in the world, resulting from the Sanskrit word in ancient India. The existence of minimum time was also theoretically clarified in physics. The impossibility of discriminating between micro particles in a local space revealed the existence of minimum time t_0 in the quantum theory then. In comparison with Einstein's relativity established by denying the above precondition [A], the quantum theory developed here is established by denying the precondition [B], *i.e.*, by accepting $\lim_{t_1 \to t_2}(t_1 - t_2) = \pm t_0$ in contradiction to the Newton mechanics [6].

The finding obtained here reveals the theoretical evidence that the chronon (quantum-time) proposed by Levi [10] in 1927 as a hypothesis is existent in the quantum mechanics. After that, Caldirola [11] in 1980 reported the time $\theta_0 = e^2/6\pi\varepsilon_0 mc^3 \left(= 6.27\times10^{-24}\ \text{s}\right)$ as a value of chronon in the electron theory, where e, ε_0, c and m are the elementary charge, the dielectric constant, the light speed and the mass of electron. Further, the Planck time $t_\text{p} = \sqrt{\hbar G/c^5} \left(= 5.39\times10^{-44}\ \text{s}\right)$ expressed by $\hbar, c$ and the gravitational constant G is also well-known as a minimum time in physics.

In relation to Equation (10), the minimum time t_0 is estimated in the following. In general, the temperature effect is not considered in analyzing Equation (1). Using the room temperature $T_\text{R} \left(\cong 290\ \text{K}\right)$ for Equation (10), therefore, Equation (10) is rewritten as

$$t_{\alpha_n} = \frac{2\hbar}{\alpha_n k_\text{B} T_\text{R}}, \tag{15}$$

where $\varepsilon \cong 0$ is acceptable in the present case. If it is possible that Equation (15) corresponds to the minimum time t_0, the relation $t_0 \le t_{\alpha_1} \left(= 1.76\times10^{-14}\ \text{s}\right)$ is valid then because of using $\alpha_n = 3$ for a monatomic molecule ($n = 1$).

The reasonable transformation from the diffusion Equation (3) or (12) into the wave Equation (1) of Schrödinger indicates that the random movement of micro particles is closely relevant to each wave nature of them and further that the parabolic law shown in the concentration profile corresponds to the matter wave. The correlation between the diffusion theory and the quantum theory is thus close with each other. In other words, when the self-diffusion phenomena are observed in a material, a micro particle constituting the material has the wave nature then.

When a vacant local space is generated by a thermal fluctuation in the gas state, a molecule in a neighboring local space jumps to the vacant local space in accordance with the elementary process of diffusion. The random movement of molecules occurs through such iteration. The behavior of gas molecules can be investigated by using not only the diffusion equation but also the state equation for ideal gas.

In the following, the present minimum time t_0 is roughly estimated by using the state equation for ideal gas. In the gas state, when a molecule in local space jumps to the neighboring vacant one in relation to the diffusion phenomena, we think for the present that the jumping time corresponds to a minimum time t_0 resulting from a collision process mentioned above.

Here, Avogadro's law shows that gas molecules of $N_\text{A} \left(= 6.02\times10^{23}\right)$ numbers coexist in the volume $V_0 \left(= 2.24\times10^{-2}\ \text{m}^3\right)$ at the temperature $T_0 \left(= 273\ \text{K}\right)$

and the pressure 1013 hPa. For a size of local space occupied by a molecule at the same pressure, it is roughly considered as $l_0 = \left(V_0 T_R / N_A T_0\right)^{1/3} \left(= 3.41\times10^{-9}\ \text{m}\right)$ at the room temperature $T = T_R$. Therefore, the minimum time is roughly obtained as $t_0 \ge 1.14\times10^{-17}\ \text{s}$ because of $t_0 > l_0/c$.

In the present study, it was found that minimum time t_0 depends on a physical system of a micro particle concerned. As a result, the relation of

$$1.14\times10^{-17}\ \text{s} \le t_0 \le 1.76\times10^{-14}\ \text{s} \tag{16}$$

is thus obtained. In addition, we can roughly discuss a size of micro particle to take account of the quantum effect from Equations (15) and (16), as discussed in the following.

Substituting $t_{\alpha_n} = t_0 \left(= 1.14\times10^{-17}\ \text{s}\right)$ into Equation (15), the value of $\alpha_n = 4.62\times10^3$ is obtained as a degree of freedom of a micro particle at the room temperature $T = T_R$. If we can then determine atom numbers n corresponding to the degree of freedom α_n, it is considered that a micro particle composed of atoms fewer than n atoms has a wave nature. On the other hand, it is also considered that the size of micro particle should be smaller than $l_0 \left(= 3.41\times10^{-9}\ \text{m}\right)$ in relation to the size of local space. In addition, the various material structures are possible for a micro particle, for example, a giant molecule, a nanoparticle of metal, and so on. In that situation, since the degree of freedom α_n depends on the complicated structure of each micro particle concerned, a matter for the correlation between α_n and n will be accepted as a subject in the future, but $n = 1.54\times10^3$ is possible if $\alpha_n = 3n$ is simply acceptable.

For the difference between t_p and t_0, the author thinks that the time t_p is not actual judging from the theoretical frame of physics, because we cannot suppose matters like the physical quantities c and $\hbar$ resulting from denying preconditions [A] and [B] in the Newton mechanics are simultaneously used with the gravitational constant G. The expression of θ_0 corresponds only to the charged particle, but t_0 is valid for an arbitrary micro particle.

Here, it is remarkable that a minimum time of order 10^{-17} s satisfying Equation (16) has been reported in relation to the uncertainty principle [12]. In that situation, Equation (10) indicates that the time t_{α_n} does not depend on each mass m of micro particles but α_n and T. On the other hand, the diffusivity having the close correlation with the quantum theory depends on a mass of micro particle. Therefore, the detailed estimation of the discrete time t_0 pointed out here should be widely investigated in the future, judging from the importance of fundamental theory in the quantum mechanics. In any case, if the time t_0 obtained here is just valid in the quantum mechanics, some fundamental theories in physics may be unexpectedly influenced by a fluctuation resulting from the minimum time t_0.

5. Discussion and Conclusions

Judging from the theoretical frame of physics, the essential equation in physics for micro particles is just considered to be Equation (3) itself, which is theoretically derived from the theory of Markov process in mathematics. The reason is as follows. Equation (3) is transformable into Equation (12) using Equation (13) applicable to an arbitrary diffusion field under the condition of $t_0 = 0$. On the other hand, Equation (3) corresponds to Equation (1) through substituting Equations (5) and (6) into Equation (3) and rewriting $\hbar C \to \Psi$ under the condition of $t_0 > 0$. Here, the determination of either $t_0 > 0$ or $t_0 = 0$ depends only on whether we investigate behavior of a single micro particle or that of its collective motion.

Since the establishment of quantum theory, some basic problems have been disregarded in the unsolved state. We have been thus unable to understand the theoretical basis that a micro particle has a wave nature. In other words, the theoretical evidence that the hypothesis of de Broglie is valid has never been revealed. In addition, the theoretical bases that the probabilistic interpretation is indispensable for the quantum theory and the matter wave interferes with itself have not been also essentially understood even if it has been plausibly explained in textbooks using a slit.

In that situation, recently those bases were theoretically proved by obtaining the essential diffusivity expression relevant to an angular momentum of micro particle from the theoretical derivation of diffusion equation having been accepted as a law for a long time, and at the same time by revealing the existence of discrete time t_j in the world of a micro particle. Thus, we could first theoretically solve the problems having been disregarded for a long time in the basic theory of quantum mechanics.

Including the fact that Equation (2) is now not a hypothesis but a basic equation, the new fundamental theory of quantum mechanics resulting from the causality for the Newton mechanics should be discussed in elementary textbooks of physics from a viewpoint of the education for younger people. Further, misunderstanding problems shown in the existing textbooks for the fundamental theory of diffusion should be suitably revised as soon as possible. The diffusion theory should be thus developed in such a way as to start not from accepting the diffusion equation as Fick's law but from deriving itself from the mathematical theory.

Judging from the discussion developed in the present work, there is no doubt that the conception of minimum time is indispensable for understanding behavior of a micro particle. It will be thus no exaggeration to say that the quantum theory is established by incorporating the conception of discrete time t_j into the Newton mechanics. In conclusion, the author hopes that the values of a minimum time t_0 are highly discussed from various viewpoints in physics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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