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Flavour and Colour of Quarks in Spin Topological Space

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Abstract

An assumption that *all* the six flavour quarks are attributed to be the components of *a same*, *a common* isospin multiplets space named **STS** is proposed. Base on **Pauli Exclusion Principle**, every quark is assigned to different flavour marks in STS. Every flavour quark possesses *its own colour spectral line array* specially appointed. The collection of colour spectral line arrays of the six flavour quarks constructs together the **CSDF**, Colour Spectrum Diagram of Flavour, further baryons and mesons could be constructed from **CSDF**. STS, Spin Topological Space is a math frame with infinite dimensional matrix representation for spin angular momentum. Flavours is an isospin angular momentum coupling phenomena of the three-colour-quarks.

Keywords

Pauli Exclusion Principle, **STS**, Spin Topological Space, **STC**, Spin Topological Coordinate, Colour Spectral Line Array, **CSDF**, Colour Spectrum Diagram of Flavour

1. Introduction

In isotopic spin space of **Standard Model**, **SM**, Gell-Mann M [1] and Zweig G [2], the isospin quantum number I and the third component I_3, for flavour quarks u, d are 1/2, +1/2 and 1/2, −1/2 respectively, for flavour quarks s,c,b,t are 0, 0. u ($I_3=+1/2$) and d ($I_3=-1/2$) quarks are assigned to an isodoublet with $I=1/2$, the dimension of matrix representation of the isodoublet is equal to $2\times 1/2+1=2$. This matrix representation is an analogy with ordinary angular momentum $\vec{\sigma}/2$, $\vec{\sigma}$ is Pauli matrix. And the remaining four flavour quarks $s(I_3=0)$, $c(I_3=0)$, $b(I_3=0)$, $t(I_3=0)$ are assigned to four isosinglets with $I=0$, respectively, the dimension of matrix representation of each isosinglet is equal to $2\times 0+1=1$. Further, there is an isodoublet and there are four isosinglets in isospin scheme for flavour quarks [3].

It is a curious question, what will happen? if the above six flavour quarks are all put into *a common* multiplet, that is, if these flavours are treated equally in one isotopic spin space. According to **Pauli Exclusion Principle**, **PEP**, each of those values of $I_3(q_i)$ of six flavour quarks should not be the same each other. Following the unified math symmetry picture, all eigenvalues of $I_3(q_i)$ of flavour q_i quark are proposed to be half-integers, to be +5/2, +3/2, +1/2, −1/2, −3/2, −5/2. $q_i = q_t, q_c, q_u, q_d, q_s, q_b, i = t, c, u, d, s, b$ respectively are shown in **Table 1**.

Table 1. Flavours quarks from **SM** to **STS**.

Flavour	I	$I_3(q_i)$	matrix	**PEP**	I	$I_3(q_i)$	matrix
Quark	**SM**			$\Rightarrow$	**STS**		infinite dimension
t	0	$I_3(t)=0$	1 dimension		1/2	$I_3(t)=+5/2$ ♦	infinite dimension
c	0	$I_3(c)=0$	1 dimension		1/2	$I_3(c)=+3/2$ ♦	infinite dimension
u	1/2	$I_3(u)=+1/2$	2 dimension		1/2	$I_3(u)=+1/2$	infinite dimension
d	1/2	$I_3(d)=-1/2$	2 dimension		1/2	$I_3(d)=-1/2$	infinite dimension
s	0	$I_3(s)=0$	1 dimension		1/2	$I_3(s)=-3/2$ ♦	infinite dimension
b	0	$I_3(b)=0$	1 dimension		1/2	$I_3(b)=-5/2$ ♦	infinite dimension

In **Table 1**, the right side is more graceful and elegant than the left side, but how can we obtain *those* third component eigenvalues $I_3(q_i)$ of *isospin* 1/2 *particles*, that labelled by mark ♦, *which are greater than* +1/2 or *less than* −1/2 in **Table 1**? Next, we resort to **Spin Topological Space** ([4] [5] [6] [7]), abbreviation **STS**, that can help us to construct what we want to get the right side in **Table 1**.

2. STS, Spin Topological Space

Spin angular momentum $\vec{\pi}$ of a spin particle in STS math frame, is labelled by two subscripts j,k (if in real region):

$$\vec{\pi}_{j,k} = \left(\pi_{1;j,k}, \pi_{2;j,k}, \pi_{3;j,k}\right) \tag{1}$$

$\vec{\pi}_{j,k}$ satisfy angular momentum commutation rule (2)

$$\vec{\pi}_{j,k} \times \vec{\pi}_{j,k} = i\vec{\pi}_{j,k} \tag{2}$$

$$\pi_{1;j,k} = \frac{1}{2}\left(\pi_j^+ + \pi_k^-\right) \tag{3.1}$$

$$\pi_{2;j,k} = \frac{1}{2i}\left(\pi_j^+ - \pi_k^-\right) \tag{3.2}$$

$$\pi_{3;j,k} = \frac{1}{2}\left(\pi_j^+\pi_k^- - \pi_k^-\pi_j^+\right) \tag{3.3}$$

$j,k \subset \mathrm{STS}$. $\pi_{1;j,k}$ and $\pi_{2;j,k}$ are two infinite dimensional Non-Hermitian Matrices. $\pi_{3;j,k}$ is an infinite dimensional Hermitian Matrix [4] [7]. Using the three components of $\vec{\pi}_{j,k}$, we get the expressions for the eigenvalue of Casimir Operator $\pi_{j,k}^2$ and the eigenvalue of the third component $\pi_{3;j,k}$ of $\vec{\pi}_{j,k}$ below

$$\pi_{j,k}^2 = \pi_{1;j,k}^2 + \pi_{2;j,k}^2 + \pi_{3;j,k}^2 = \frac{1}{4}\left\{(j-k)^2 - 1\right\} I_0 \tag{4}$$

$$\pi_{3;j,k} = \pi_0(0) + \frac{1}{2}(j+k+1) I_0 \tag{5}$$

$$\pi_0(0) = \mathrm{diag}\{\cdots,5,4,3,2,1,\underline{0},-1,-2,-3,-4,-5,\cdots\} \tag{6}$$

$$I_0 = \mathrm{diag}\{\cdots,1,1,1,1,1,\underline{1},1,1,1,1,\cdots\} \tag{7}$$

formulas (4), (5) show $\pi_{j,k}^2$ and $\pi_{3;j,k}$ are diagonal infinite dimensional matrices. Here $\pi_0(0)$ is the vacuum background spin angular momentum of $\pi_{3;j,k}$. If in case of no confusion, it is convenient to instead of (5) to use (9) to deal with I_3, then obtain following expressions

$$\pi_{j,k}^2 = \frac{1}{4}\left(S_{j,k}^2 - 1\right) \tag{8}$$

$$\pi_{3;j,k} = \frac{1}{2}\left(A_{j,k} + 1\right) \tag{9}$$

$$S_{j,k} = j - k, \quad A_{j,k} = j + k \tag{10}$$

$$(j,k) = \left(\frac{1}{2}\left(A_{j,k} + S_{j,k}\right), \frac{1}{2}\left(A_{j,k} - S_{j,k}\right)\right) \tag{11}$$

Call (j,k), **STC**, Spin Topological Coordinate of spin particle in STS.

Addition of $\vec{\pi}_{j,k}$ and $\vec{\pi}_{r,s}$, $\vec{\Pi}_{j,k;r,s}$ is given below

$$\vec{\pi}_{j,k} \times \vec{\pi}_{j,k} = i\vec{\pi}_{j,k}, \quad \vec{\pi}_{r,s} \times \vec{\pi}_{r,s} = i\vec{\pi}_{r,s} \tag{12}$$

$$\vec{\Pi}_{j,k;r,s} \times \vec{\Pi}_{j,k;r,s} = i\vec{\Pi}_{j,k;r,s} \tag{13}$$

$$\vec{\Pi}_{j,k;r,s} = \frac{1}{2}\left(\vec{\pi}_{j,k} + \vec{\pi}_{r,s}\right) \tag{14}$$

$$\Pi_{j,k;r,s}^2 = \frac{1}{16}\left(\left(S_{j,k} + S_{r,s}\right)^2 - 4\right) = \frac{1}{4}\left(\left(S_{j,k}/2 + S_{r,s}/2\right)^2 - 1\right) \tag{15}$$

$$\Pi_{3;j,k;r,s} = \frac{1}{2}\left(\pi_{3;j,k} + \pi_{3;r,s}\right) \tag{16}$$

$\Pi_{j,k;r,s}^2$ and $\Pi_{3;j,k;r,s}$ are Casimir operator and the third component of spin particle $\vec{\Pi}_{j,k;r,s}$ in STS.

3. Flavour Quarks in STS

Now we continue **Table 1** quark model in STS, in *flavour isotopic space*, $\vec{\pi}$ is replaced by $\vec{I}(q_i)$, then obtain Casimir Operator $I^2(q_i)$ (18) and the third component eigenvalues $I_3(q_i)$ (19) of flavour q_i quarks (*fermion* $I(q_i) = \frac{1}{2}$ (17)). Details are shown in **Table 2**.

Table 2. Flavour quantum number of quarks in **STS** (isospin $I = \hbar/2$).

$I_{3;j,k}(q_i)$	$=\frac{1}{2}(A_{j,k}(q_i)+1)$	$A_{j,k}(q_i)$	$S_{j,k}(q_i)$	$(j,k)_{q_i}$
$I_{3;+3,+1}(t) = +5/2^{\blacklozenge}$	$\frac{1}{2}(+4+1)$	$A_{+3,+1}(t) = +4$	$S_{+3,+1}(t) = +2$	$(+3,+1)_t$
$I_{3;+2,0}(c) = +3/2^{\blacklozenge}$	$\frac{1}{2}(+2+1)$	$A_{+2,0}(c) = +2$	$S_{+2,0}(c) = +2$	$(+2,0)_c$
$I_{3;+1,-1}(u) = +1/2$	$\frac{1}{2}(0+1)$	$A_{+1,-1}(u) = 0$	$S_{+1,-1}(u) = +2$	$(+1,-1)_u$
$I_{3;0,-2}(d) = -1/2$	$\frac{1}{2}(-2+1)$	$A_{0,-2}(d) = -2$	$S_{0,-2}(d) = +2$	$(0,-2)_d$
$I_{3;-1,-3}(s) = -3/2^{\blacklozenge}$	$\frac{1}{2}(-4+1)$	$A_{-1,-3}(s) = -4$	$S_{-1,-3}(s) = +2$	$(-1,-3)_s$
$I_{3;-2,-4}(b) = -5/2^{\blacklozenge}$	$\frac{1}{2}(-6+1)$	$A_{-2,-4}(b) = -6$	$S_{-2,-4}(b) = +2$	$(-2,-4)_b$

Note: $A_{j,k}(q_i)$ is named as *flavour quantum number* of quarks, which are even numbers.

$$I(q_i) = \frac{1}{2} \tag{17}$$

$$I_{j,k}^2(q_i) = \text{diag}\left\{\cdots, \frac{+3}{4}, \frac{+3}{4}, \frac{+3}{4}, \frac{+3}{\underline{4}}, \frac{+3}{4}, \frac{+3}{4}, \frac{+3}{4}, \cdots\right\} \tag{18}$$

$$I_{3;j,k}(q_i) = \text{diag}\left\{\cdots, \frac{+7}{2}\blacklozenge, \frac{+5}{2}\blacklozenge, \frac{+3}{2}\blacklozenge, \frac{+1}{\underline{2}}, \frac{-1}{2}, \frac{-3}{2}\blacklozenge, \frac{-5}{2}\blacklozenge, \cdots\right\}, i = t, c, u, d, s, b \tag{19}$$

4. Colour Quarks in STS

Now **Suppose** that except *flavour quantum number* marked $A_{j,k}(q_i)$ in STS (**Table 2**), quark could even possess *colour quantum number array* that called as **Colour Spectral Line Array** labelled by q_{RGB} (20), which is an array comprised of *three colour quantum numbers*, marked q_{R}, q_{G} and q_{B}, they are third-fractions.

$$q_{\text{RGB}} \equiv (q_{\text{R}}, q_{\text{G}}, q_{\text{B}}) \equiv (A(q_{\text{R}}), A(q_{\text{G}}), A(q_{\text{B}})) \tag{20}$$

Different flavour quark possesses its own colour spectral line array, for example, array $u_{\text{RGB}} \equiv (u_{\text{R}}, u_{\text{G}}, u_{\text{B}}) = \left(\frac{+2}{3}, \frac{+5}{3}, \frac{+11}{3}\right)$ is the colour spectral line array of flavour u quark, and array $d_{\text{RGB}} \equiv (d_{\text{R}}, d_{\text{G}}, d_{\text{B}}) = \left(\frac{-16}{3}, \frac{-13}{3}, \frac{-7}{3}\right)$ is colour spectral line array of flavour d quark. Flavour $A_{j,k}(q_i)$ and colour q_{RGB} are the identities of quark particles.

Definition CSDF, Colour Spectrum Diagram of Flavour is composed of six (or more) colour spectral line arrays of flavour quarks, somewhat similar to the Gene diagram of chromosomes. Explicit scheme of **CSDF** is given below.

CSDF, Colour Spectrum Diagram of Flavour

	$t,\frac{+5}{2}$	$c,\frac{+3}{2}$	$u,\frac{+1}{2}$	$d,\frac{-1}{2}$	$s,\frac{-3}{2}$	$b,\frac{-5}{2}$
Quark $I_3(q_{\text{RGB}})$	$I_3(t_{\text{R}},t_{\text{G}},t_{\text{B}})$	$I_3(c_{\text{R}},c_{\text{G}},c_{\text{B}})$	$I_3(u_{\text{R}},u_{\text{G}},u_{\text{B}})$	$I_3(d_{\text{R}},d_{\text{G}},d_{\text{B}})$	$I_3(s_{\text{R}},s_{\text{G}},s_{\text{B}})$	$I_3(b_{\text{R}},b_{\text{G}},b_{\text{B}})$
	$\left(\frac{+41}{6},\frac{+44}{6},\frac{+50}{6}\right)$	$\left(\frac{+23}{6},\frac{+26}{6},\frac{+32}{6}\right)$	$\left(\frac{+5}{6},\frac{+8}{6},\frac{+14}{6}\right)$	$\left(\frac{-13}{6},\frac{-10}{6},\frac{-4}{6}\right)$	$\left(\frac{-31}{6},\frac{-28}{6},\frac{-22}{6}\right)$	$\left(\frac{-49}{6},\frac{-46}{6},\frac{-40}{6}\right)$
	$\bar{t},\frac{-5}{2}$	$\bar{c},\frac{-3}{2}$	$\bar{u},\frac{-1}{2}$	$\bar{d},\frac{+1}{2}$	$\bar{s},\frac{+3}{2}$	$\bar{b},\frac{+5}{2}$
Anti-Quark $I_3(\bar{q}_{\bar{\text{R}}\bar{\text{G}}\bar{\text{B}}})$	$I_3(\bar{t}_{\bar{\text{R}}},\bar{t}_{\bar{\text{G}}},\bar{t}_{\bar{\text{B}}})$	$I_3(\bar{c}_{\bar{\text{R}}},\bar{c}_{\bar{\text{G}}},\bar{c}_{\bar{\text{B}}})$	$I_3(\bar{u}_{\bar{\text{R}}},\bar{u}_{\bar{\text{G}}},\bar{u}_{\bar{\text{B}}})$	$I_3(\bar{d}_{\bar{\text{R}}},\bar{d}_{\bar{\text{G}}},\bar{d}_{\bar{\text{B}}})$	$I_3(\bar{s}_{\bar{\text{R}}},\bar{s}_{\bar{\text{G}}},\bar{s}_{\bar{\text{B}}})$	$I_3(\bar{b}_{\bar{\text{R}}},\bar{b}_{\bar{\text{G}}},\bar{b}_{\bar{\text{B}}})$
	$\left(\frac{-41}{6},\frac{-44}{6},\frac{-50}{6}\right)$	$\left(\frac{-23}{6},\frac{-26}{6},\frac{-32}{6}\right)$	$\left(\frac{-5}{6},\frac{-8}{6},\frac{-14}{6}\right)$	$\left(\frac{+13}{6},\frac{+10}{6},\frac{+4}{6}\right)$	$\left(\frac{+31}{6},\frac{+28}{6},\frac{+22}{6}\right)$	$\left(\frac{+49}{6},\frac{+46}{6},\frac{+40}{6}\right)$
	↖ ↑ ↗	↖ ↑ ↗	↖ ↑ ↗	↖ ↑ ↗	↖ ↑ ↗	↖ ↑ ↗
	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘
	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘	❘ ❘ ❘
	↙ ↓ ↘	↙ ↓ ↘	↙ ↓ ↘	↙ ↓ ↘	↙ ↓ ↘	↙ ↓ ↘
	$t,+4$	$c,+2$	$u,0$	$d,-2$	$s,-4$	$b,-6$
Quark q_{RGB}	$(t_{\text{R}},t_{\text{G}},t_{\text{B}})$	$(c_{\text{R}},c_{\text{G}},c_{\text{B}})$	$(u_{\text{R}},u_{\text{G}},u_{\text{B}})$	$(d_{\text{R}},d_{\text{G}},d_{\text{B}})$	$(s_{\text{R}},s_{\text{G}},s_{\text{B}})$	$(b_{\text{R}},b_{\text{G}},b_{\text{B}})$
	$\left(\frac{+38}{3},\frac{+41}{3},\frac{+47}{3}\right)$	$\left(\frac{+20}{3},\frac{+23}{3},\frac{+29}{3}\right)$	$\left(\frac{+2}{3},\frac{+5}{3},\frac{+11}{3}\right)$	$\left(\frac{-16}{3},\frac{-13}{3},\frac{-7}{3}\right)$	$\left(\frac{-34}{3},\frac{-31}{3},\frac{-25}{3}\right)$	$\left(\frac{-52}{3},\frac{-49}{3},\frac{-43}{3}\right)$
	$\bar{t},-6$	$\bar{c},-4$	$\bar{u},-2$	$\bar{d},0$	$\bar{s},+2$	$\bar{b},+4$
Anti-Quark $\bar{q}_{\bar{\text{R}}\bar{\text{G}}\bar{\text{B}}}$	$(\bar{t}_{\text{R}},\bar{t}_{\text{G}},\bar{t}_{\text{B}})$	$(\bar{c}_{\bar{\text{R}}},\bar{c}_{\bar{\text{G}}},\bar{c}_{\bar{\text{B}}})$	$(\bar{u}_{\bar{\text{R}}},\bar{u}_{\bar{\text{G}}},\bar{u}_{\bar{\text{B}}})$	$(\bar{d}_{\bar{\text{R}}},\bar{d}_{\bar{\text{G}}},\bar{d}_{\bar{\text{B}}})$	$(\bar{s}_{\bar{\text{R}}},\bar{s}_{\bar{\text{G}}},\bar{s}_{\bar{\text{B}}})$	$(\bar{b}_{\bar{\text{R}}},\bar{b}_{\bar{\text{G}}},\bar{b}_{\bar{\text{B}}})$
	$\left(\frac{-44}{3},\frac{-47}{6},\frac{-53}{6}\right)$	$\left(\frac{-26}{3},\frac{-29}{3},\frac{-35}{3}\right)$	$\left(\frac{-8}{3},\frac{-11}{3},\frac{-17}{3}\right)$	$\left(\frac{+10}{3},\frac{+7}{3},\frac{+1}{3}\right)$	$\left(\frac{+28}{3},\frac{+25}{3},\frac{+19}{3}\right)$	$\left(\frac{+46}{3},\frac{+43}{3},\frac{+37}{3}\right)$

5. Hypothesis

Flavours are coupling phenomena of isospin angular momenta of three-colour-quarks.

To track the idea, we make use of **CSDF**, and of angular momentum formulae (21) and (22) of three spin particles below. Further obtain **STC array** of flavour quarks in Table 3. In other words, obtain the relationships **STC array**, $(j,k)_{q_i}=\left((j,k)_{q_{\text{R}}},(j,k)_{q_{\text{G}}},(j,k)_{q_{\text{B}}}\right)$ between flavour quantum number, $A_{j,k}(q_i)$ and Colour Spectral Line Array $q_{\text{RGB}}=(q_{\text{R}},q_{\text{G}},q_{\text{B}})$.

$$I^2(3q)=\frac{1}{36}\left(\left(S(q_{\text{R}})+S(q_{\text{R}})+S(q_{\text{B}})\right)^2-9\right)=\frac{3}{4} \tag{21}$$

$$I_3(3q)=\frac{1}{3}\left(I_3(q_{\text{R}})+I_3(q_{\text{G}})+I_3(q_{\text{B}})\right)=\frac{1}{2}\left(\frac{A(3q)}{3}+1\right) \tag{22.1}$$

$$I_3(q)=\overline{I_3(3q)}=\frac{1}{3}I_3(3q) \tag{22.2}$$

Table 3. STC array of colour quantum numbers of flavour quarks.

$(j,k)_{q_i}$	$(j,k)_t$	$(j,k)_c$	$(j,k)_u$	$(j,k)_d$	$(j,k)_s$	$(j,k)_b$
	$(j,k)_{t_R}$	$(j,k)_{c_R}$	$(j,k)_{u_R}$	$(j,k)_{d_R}$	$(j,k)_{s_R}$	$(j,k)_{b_R}$
$(j,k)_{q_R}$	$\left(\frac{+44}{6},\frac{+32}{6}\right)_{t_R}$	$\left(\frac{+26}{6},\frac{+14}{6}\right)_{c_R}$	$\left(\frac{+8}{6},\frac{-4}{6}\right)_{u_R}$	$\left(\frac{-10}{6},\frac{-22}{6}\right)_{d_R}$	$\left(\frac{-28}{6},\frac{-40}{6}\right)_{s_R}$	$\left(\frac{-46}{6},\frac{-58}{6}\right)_{b_R}$
	$(j,k)_{t_G}$	$(j,k)_{u_G}$	$(j,k)_{u_G}$	$(j,k)_{u_G}$	$(j,k)_{s_G}$	$(j,k)_{u_G}$
$(j,k)_{q_G}$	$\left(\frac{+47}{6},\frac{+35}{6}\right)_{t_G}$	$\left(\frac{+29}{6},\frac{+17}{6}\right)_{c_G}$	$\left(\frac{+11}{6},\frac{-1}{6}\right)_{u_G}$	$\left(\frac{-7}{6},\frac{-19}{6}\right)_{d_G}$	$\left(\frac{-25}{6},\frac{-37}{6}\right)_{s_G}$	$\left(\frac{-43}{6},\frac{-55}{6}\right)_{b_G}$
	$(j,k)_{u_B}$	$(j,k)_{u_B}$	$(j,k)_{u_B}$	$(j,k)_{u_B}$	$(j,k)_{s_B}$	$(j,k)_{u_B}$
$(j,k)_{q_B}$	$\left(\frac{+53}{6},\frac{+41}{6}\right)_{t_B}$	$\left(\frac{+35}{6},\frac{+23}{6}\right)_{c_B}$	$\left(\frac{+17}{6},\frac{+5}{6}\right)_{u_B}$	$\left(\frac{-1}{6},\frac{-13}{6}\right)_{d_B}$	$\left(\frac{-19}{6},\frac{-31}{6}\right)_{s_B}$	$\left(\frac{-37}{6},\frac{-49}{6}\right)_{b_B}$

6. Baryons and Mesons in STS

Due to baryons all are "white colour" particles, which are made of colourful quarks. To help with **CSDF**, picking up three colour quantum numbers: $q_R^1 \subseteq q_{RGB}^1$, $q_G^2 \subseteq q_{RGB}^2$ and $q_B^3 \subseteq q_{RGB}^3$, respectively from any three quarks q^1, q^2 and q^3 (q^1, q^2 and q^3 can be any flavour), then various visible baryons could be produced. In this way, for example, baryon decuplet is constituted as shown in **Table 4**.

According to SM, the "colourless phenomena" of all mesons could be satisfied by blending with a quark with colour and antiquark with anti-colour, that is to say, $q_R \bar{q}_{\bar{R}}$, or $q_G \bar{q}_{\bar{G}}$, or $q_B \bar{q}_{\bar{B}}$. Contrary to SM, in STS a meson is similar to a baryon, a meson also is a three-body system that comprises a quark (colour), an antiquark (anti-colour) and a gluon (white). This gluon plays the role of mediator to fasten quark and antiquark together in a meson.

Colour spectral line array of meson is symbolized with $q_i \bar{q}_{\bar{j}} g_k = \left(q_i, \bar{q}_{\bar{j}}, g_k\right)$. The mentioned above is the case of $i = j$, $k = 0$. The discussion about $i \neq j$, $k \neq 0$ will be given later.

In what follows base on **CSDF**, we list the weight diagram **Table 5** of meson octet. Here g_0 is the gluon basic state with $I_3^2(g_0) = 0\hbar$, $A(g_0) = -1$.

7. $\vec{A}(q)\cdot\vec{A}(q)$ Interaction in STS

This paragraph suggests some ideas, similar to spin-spin $\vec{S}_i \cdot \vec{S}_j$ interaction in spin space [3], to disscuss $\vec{A}(q)\cdot\vec{A}(q)$ Interaction in STS. **Table 4** shows that baryons, like quarks (**CSDF**), are marked by *colour spectral line arrays* too, but a slight different from quarks. Actually, there are two kinds of colour spectral line arrays: right-hand colour quantum numbers (*r-h*) (23.1) and left-hand colour quantum numbers (*l-h*) (23.2) *for a given baryon*, which made of quark q^1, q^2 and q^3. Each baryon exists in one of three possible states, labelled with cases: I, II and III in case (23.1) and case (23.2) respectively. The results in **Table 4** are the case of I of (*r-h*) only shown below.

Table 4. Weight diagram for baryon decuplet with $S = +4$ in STS.

$I_3(\Delta^-)=\frac{-3}{2},-4$ (d_R,d_G,d_B) $\left(\frac{-16}{3},\frac{-13}{3},\frac{-7}{3}\right)$	$I_3(\Delta^0)=\frac{-1}{2},-2$ (u_R,d_G,d_B) $\left(\frac{+2}{3},\frac{-13}{3},\frac{-7}{3}\right)$	$I_3(\Delta^+)=\frac{+1}{2},0$ (u_R,d_G,u_B) $\left(\frac{+2}{3},\frac{-13}{3},\frac{+11}{3}\right)$	$I_3(\Delta^{++})=\frac{+3}{2},+2$ (u_R,u_G,u_B) $\left(\frac{+2}{3},\frac{+5}{3},\frac{+11}{3}\right)$
$I_3(\Sigma^{*-})=\frac{-5}{2},-6$ (d_R,s_G,d_B) $\left(\frac{-16}{3},\frac{-31}{3},\frac{-7}{3}\right)$	$I_3(\Sigma^{*0})=\frac{-3}{2},-4$ (u_R,s_G,u_B) $\left(\frac{+2}{3},\frac{-31}{3},\frac{-7}{3}\right)$	$I_3(\Sigma^{*+})=\frac{-1}{2},-2$ (u_R,s_G,u_B) $\left(\frac{+2}{3},\frac{-31}{3},\frac{+11}{3}\right)$	
$I_3(\Xi^{*-})=\frac{-7}{2},-8$ (d_R,s_G,s_B) $\left(\frac{-16}{3},\frac{-31}{3},\frac{-25}{3}\right)$	$I_3(\Xi^{*0})=\frac{-5}{2},-6$ (u_R,s_G,s_B) $\left(\frac{+2}{3},\frac{-31}{3},\frac{-25}{3}\right)$		
$I_3(\Omega^-)=\frac{-9}{2},-10$ (s_R,s_G,s_B) $\left(\frac{-34}{3},\frac{-31}{3},\frac{-25}{3}\right)$			

Note: in Table 4, the value I_3 of every baryon all is half integer, contrary to those what the I_3 might take both half integer and integer (include zero) in case of $J^P = 3/2^+$ in SM.

Table 5. Weight diagram for meson octet with $S = +1$ in STS.

$I_3(k^0)=+1, A(k^0)=+1$ $(d_R,\bar{s}_{\bar{R}},g_0)$ $\left(\frac{-16}{3},\frac{+28}{3},\frac{-3}{3}\right)$		$I_3(k^+)=+2, A(k^+)=+3$ $(u_R,\bar{s}_{\bar{R}},g_0)$ $\left(\frac{+2}{3},\frac{+28}{3},\frac{-3}{3}\right)$
$I_3(\pi^-)=-1, A(\pi^-)=-3$ $(d_R,\bar{u}_{\bar{R}},g_0)$ $\left(\frac{-16}{3},\frac{-8}{3},\frac{-3}{3}\right)$	$I_3(\pi^0,\eta)=0, A(\pi^0,\eta)=-1$ $(u_R,\bar{u}_{\bar{R}},g_0)$ $\left(\frac{+2}{3},\frac{-8}{3},\frac{-3}{3}\right)$ $(d_R,\bar{d}_{\bar{R}},g_0)$ $\left(\frac{-16}{3},\frac{+10}{3},\frac{-3}{3}\right)$	$I_3(\pi^+)=+1, A(\pi^+)=+1$ $(u_R,\bar{d}_{\bar{R}},g_0)$ $\left(\frac{+2}{3},\frac{+10}{3},\frac{-3}{3}\right)$
octet mesons	$I_3(k^-)=-2, A(k^-)=-5$ $(s_R,\bar{u}_{\bar{R}},g_0)$ $\left(\frac{-34}{3},\frac{-8}{3},\frac{-3}{3}\right)$	$I_3(\bar{k}^0)=-1, A(\bar{k}^0)=-3$ $(s_R,\bar{d}_{\bar{R}},g_0)$ $\left(\frac{-34}{3},\frac{+10}{3},\frac{-3}{3}\right)$

Note: in Table 5, the value I_3 of every meson all is integer, contrary to those what the I_3 might take both half integer and integer in case of $J^P = 0^-$ in SM.

There is an amusing equality (23) below among q^1, q^2 and q^3 that is obtained from $q_{\text{RGB}} \equiv (q_{\text{R}}, q_{\text{G}}, q_{\text{B}})$

$$\text{IV}(r\text{-}h).\ q^1 + q^2 + q^3 \equiv \text{I.}\ q_{\text{R}}^1 + q_{\text{G}}^2 + q_{\text{B}}^3 = \text{II.}\ q_{\text{R}}^2 + q_{\text{G}}^3 + q_{\text{B}}^1 = \text{III.}\ q_{\text{R}}^3 + q_{\text{G}}^1 + q_{\text{B}}^2, \quad A = \frac{1}{3}(q^1 + q^2 + q^3) \tag{23.1}$$

$$\text{IV}(l\text{-}h).\ q^2 + q^1 + q^3 \equiv \text{I.}\ q_{\text{R}}^2 + q_{\text{G}}^1 + q_{\text{B}}^3 = \text{II.}\ q_{\text{R}}^3 + q_{\text{G}}^2 + q_{\text{B}}^1 = \text{III.}\ q_{\text{R}}^1 + q_{\text{G}}^3 + q_{\text{B}}^2, \quad A = \frac{1}{3}(q^2 + q^1 + q^3) \tag{23.2}$$

example of p^+

$$\text{IV}(r\text{-}h).\ u^1 + d^2 + u^3 \equiv \text{I.}\ u_{\text{R}} + d_{\text{G}} + u_{\text{B}} = \text{II.}\ d_{\text{R}} + u_{\text{G}} + u_{\text{B}} = \text{III.}\ u_{\text{R}} + u_{\text{G}} + d_{\text{B}}, \quad A = \frac{1}{3}(u^1 + d^2 + u^3) = \frac{0}{3} = 0 \tag{24.1}$$

$$= \frac{+2}{3} + \frac{-13}{3} + \frac{+11}{3} = \frac{-16}{3} + \frac{+5}{3} + \frac{+11}{3} = \frac{+2}{3} + \frac{+5}{3} + \frac{-7}{3}, \tag{24.2}$$

example of n^0

$$\text{IV}(r\text{-}h).\ u^1 + d^2 + d^3 \equiv \text{I.}\ u_{\text{R}} + d_{\text{G}} + d_{\text{B}} = \text{II.}\ d_{\text{R}} + d_{\text{G}} + u_{\text{B}} = \text{III.}\ d_{\text{R}} + u_{\text{G}} + d_{\text{B}}, \quad A = \frac{1}{3}(u^1 + d^2 + d^3) = \frac{-18}{3} = -6 \tag{25.1}$$

$$= \frac{+2}{3} + \frac{-13}{3} + \frac{-7}{3} = \frac{-16}{3} + \frac{-13}{3} + \frac{+11}{3} = \frac{-16}{3} + \frac{+5}{3} + \frac{-7}{3}, \tag{25.2}$$

If array q_{RGB} is defined as a vector (26)

$$q_{\text{RGB}} \equiv (q_{\text{R}}, q_{\text{G}}, q_{\text{B}}) = \vec{A}(q_i) \tag{26}$$

Then the next two tables are constructed from **CSDF**, which may offer some heuristic search for classification of particle mass.

Table 6. Mass Values Comparison between Prediction and Experiment for proton and neutron [3].

		$\text{I}^2+\text{II}^2+\text{III}^2$		Prediction		Experiment
	$\text{I}^2 \propto (+2)^2 + (-13)^2 + (+11)^2 = 294$					
$A^2(p^+) = \vec{A}(p^+)\cdot\vec{A}(p^+)$	$\text{II}^2 \propto (-16)^2 + (+5)^2 + (+11)^2 = 402$	774	$\Vert$	$774 + 1098 = 1872$	$\Leftrightarrow$	$938 + 940 = 1878$
	$\text{III}^2 \propto (+2)^2 + (+5)^2 + (-7)^2 = 78$			$1872/2 = 936$		$1878/2 = 939$
	$\text{I}^2 \propto (+2)^2 + (-13)^2 + (-7)^2 = 222$			$774/936 = 0.827$		$938/939 = 0.999$
$A^2(n^0) = \vec{A}(n^0)\cdot\vec{A}(n^0)$	$\text{II}^2 \propto (-16)^2 + (-13)^2 + (+11)^2 = 546$	1098	$\Vert$	$1098/936 = 1.173$	$\Leftrightarrow$	$940/939 = 1.001$
	$\text{III}^2 \propto (-16)^2 + (+5)^2 + (-7)^2 = 330$			$0.827 + 1.173 = 2$		$0.999 + 1.001 = 2$

Table 7. Comparison between $A^2(q)/A^2(u)$ and $M(q)/M(u)$ of three generations of quarks (ref: diagram **CSDF**).

$A^2(q)=q_Rq_R+q_Gq_G+q_Bq_B$	$A^2(q)/A^2(u)$	$\text{Mev/c}^2(q)\Rightarrow M(q)/M(u)$	q
$A^2(t)\propto(+38)^2+(+41)^2+(+47)^2=5334$	$\Rightarrow 5334/150=35.56$	$173\times10^3\Rightarrow 7521.7$	t
$A^2(c)\propto(+20)^2+(+23)^2+(+29)^2=1770$	$\Rightarrow 1770/150=11.8$	$1.275\times10^3\Rightarrow 554.3$	c
$A^2(u)\propto(+2)^2+(+5)^2+(+11)^2=150$	$\Rightarrow 150/150=1$	$2.3\times10^0\Rightarrow 1$	u
$A^2(d)\propto(-16)^2+(-13)^2+(-7)^2=474$	$\Rightarrow 474/150=3.16$	$4.8\times10^0\Rightarrow 2.1$	d
$A^2(s)\propto(-34)^2+(-31)^2+(-25)^2=2742$	$\Rightarrow 2742/150=18.28$	$95.0\times10^0\Rightarrow 41.3$	s
$A^2(b)\propto(-52)^2+(-49)^2+(-43)^2=6954$	$\Rightarrow 6954/150=46.36$	$4.18\times10^3\Rightarrow 1817.4$	b

In Table 6 and Table 7, $A^2(p^+)$ $A^2(n^0)$ and $A^2(q)$ are the scalar products of $\vec{A}(q_i)$ (26). The masses of particles (both proton, neutron and quarks) are supposed to be proportional to the scalar products from their corresponding **CSDF**.

8. Conclusions

In this paper we have pointed links between flavour quarks and colour quarks in math frame STS, Spin Topological Space: the flavour viewed as a number, named as *flavour quantum number* $A_{j,k}(q_i)$ and the colour viewed as an array, named as *colour spectral line array* q_{RGB} consist of *three colour quantum numbers* q_R, q_G and q_B or $A(q_R)$, $A(q_G)$ and $A(q_B)$. The former is even number, the latter are third-fractions. When one thinks $I_3(q_R)$, $I_3(q_G)$ and $I_3(q_B)$ as three distinct angular momentums respectively, using momentum addtion of three-body, one can construct a variety of baryons.

In contrast to SM, mesons only are made of quark and antiquark, it becomes more complex, as now gluon joins into meson mechanism. In account of what happened in *colour spectral line array* $q_i\bar{q}_{\bar{j}}g_k$ when $i\neq j$, $k\neq 0$, many efforts are needed, after all, so much is not fully understood.

Perhaps **CSDF**, Colour Spectrum Diagram of Flavour is an essential conception for us to realize what flavour and colour of quarks are.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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