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Variational Calculations of Energies of the $(2snl)$ $^{1,3}L^{\pi}$ and $(2pnl)$ $^{1,3}L^{\pi}$ Doubly Excited States in Two-Electron Systems Applying the Screening Constant per Unit Nuclear Charge

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Abstract

In this paper, resonance energies and excitation energies of doubly $2sns\ ^{1,3}S^e$, $2snp\ ^{1,3}P^o$, $2pnp\ ^{1,3}D^e$, $2pnd\ ^{1,3}F^o$ and $2pnf\ ^{1,3}G^e$ excited states of the helium isoelectronic sequence with $Z \leq 10$ are calculated. Calculations are carried out in the framework of the variational procedure of the formalism of the Screening Constant per Unit Nuclear Charge (SCUNC). New correlated wave function of Hylleraas type is used. Precise resonance and excitation energies are tabulated and good agreement is obtained when a comparison is made with available literature values.

Keywords

Doubly Excited States, Helium Isoelectronic Sequence, Screening Constant per Unit Nuclear Charge (SCUNC), Correlated Wave Function, Resonance Energy, Excitation Energies

1. Introduction

Study of Doubly Excited States (DES) of He-like ions remains an active field of investigation due to their importance in the interpretation of astrophysical data [1] [2]. These states were first observed by Madden and Codling [3] [4] in photoabsorption experiments on helium using synchrotron radiation, and further experimental studies have shown their presence in highly charged ions [5]. As shown in various studies, electron correlations play an important role in under-

standing lines atomic species for the diagnosis of astrophysical and laboratory plasma. In addition, DES of the two-electron systems are the most fundamental systems that autoionize. They have been attracting considerable interest, because they are best suited to theoretical study on the resonance phenomena. The understanding of elementary processes in the collisions of electrons with atoms or ions is very important in plasma physics, laser technology, astrophysics and physics of the upper atmosphere.

Experimentally, many of these doubly excited states have been observed in electronic impact experiments by Oda *et al.*, [6] and Hicks and Comer [7]. In their studies, these authors have worked on the energy spectra of ejected electrons from autoionization states in helium excited by electron impact. Other doubly excited states were observed by ion impact by Rudd [8] and by Bordenave-Montesquieu *et al.*, [9]. These DES were also studied by examining the spectra of ejected electrons by Gelabart *et al.*, [10] and by Rodbro *et al.*, [11].

From a theoretical point of view, several ab initio methods have been used. The complex rotation method [12] used in studies of Feshbach-type $^{1,3}D$ resonances in two-electron systems, $Z = 2$ - 10, the variational method [13] [14], the density functional theory [15] was used to calculate the nonrelativistic energies and densities of the doubly excited states of the He-isoelectronics series ($Z = 2$ - 5). The formalism of the Feshbach projection operators [16] was applied for the calculations of energy positions and widths of singlet and triplet (even and odd) resonances of the heliumlike ($Z = 2$ - 10) systems lying between the $n = 2$ and $n = 3$ thresholds, the complex rotation method [17] [18] [19]. The truncated diagonalization method used for calculations of widths for doubly excited states of two-electron systems [20]. The discretization technique [21] applied to the calculation of energies and widths of $^{1,3}S$ resonances of the He isoelectronic series, the semi-empirical procedure of the Screening Constant by Unit Nuclear Charge (SCUNC) method [22] [23] [24]. The time-dependent variation perturbation theory (TDVPT) [25] is employed to study the $Nlnl'^{1}L^{e}$ resonances (with $N = 2$, ···, 5; $n = N$, ···, 5; $l = l' = 0$, 1, 2 and $L = l + l'$) for the ions from $Z = 2$ to $Z = 5$ in the helium isoelectronic sequence, complex rotation combined with discrete finite base sets to accurately describe doubly excited states [26].

Recently, Gning *et al.*, [27] complex rotation method to determine the resonance parameters of the $((2s^2)\ ^1S^e, (2s2p)\ ^{1,3}P^0)$ and $((3s^2)\ ^1S^e, (3s3p)\ ^{1,3}P^0)$ states of helium-like ions with $Z \leq 10$ via a Scilab program. The variational method of Hylleraas was used by Dieng *et al.*, [28] to determine the resonance energies of the doubly excited states $nln'l$ and $nln'l'$ (with $n \leq 3$, $n' \leq 4$ and $l = l'$ or $l \neq l'$) of helium-like ions. Very recently, the resonance parameters of the doubly excited $2sns\ ^{1,3}S^e$, $2snp\ ^{1,3}P^0$, $2pnp\ ^{1,3}D^e$, $3d4d\ ^{1,3}G^e$ states of helium and heliumlike ions are calculated by Sow *et al.*, [29].

In general Most of the theoretical methods mentioned above are based on calculation codes or on tedious and complex mathematical calculation programs and in some cases require very powerful computers. In contrast to these me-

thods, the Screening Constant by Nuclear Unit of Charge (SCUNC) method is a very flexible method and has the advantage of providing very precise resonance energies and excitation energies for very high $n = 10$ of the doubly excited states $(Nln l', {}^{2S+1}L^{\pi})$ without complex mathematical programs or calculation codes. In addition, in the recent past, the variational procedure of the SCUNC method has been successfully applied to calculations of resonance energies of doubly excited states $nln l'$ $(n = 2 - 4)$ in heliumlike ions by Sakho [30] using a special Hylleraas-type wavefunction. The goal of this present work is to extend this study to doubly excited states $Nln l'$ (with $l = l'$ or $l \neq l'$) by reporting precise resonance energies and excitation energies up to $n = 10$. In this work, calculations are devoted to the doubly $2sns\ {}^{1,3}S^e$, $2snp\ {}^{1,3}P^o$, $2pnp\ {}^{1,3}D^e$, $2pnd\ {}^{1,3}F^o$ and $2pnf\ {}^{1,3}G^e$ excited states of He-like ions up to Z = 10 via analytical expressions in contrast with ab initio method requiring computational codes to report accurate resonance data.

Section 2 gives the procedure of the construction of the correlated wavefunctions used along with a brief overview of the establishment of the analytical expressions used in the calculations. Section 3 gives the presentation and the discussion of the results obtained compared to available theoretical and experimental data.

2. Theory

2.1. Hamiltonian and Hylleraas—Type Wavefunctions

The description of the properties of matter at the atomic scale is in principle based on the solution of the time independent Schrödinger equation.

$$\hat{H}\Psi = E\Psi \tag{1}$$

where $\hat{H}$ represents the Hamiltonian operator of the considered system (atom, molecule, solid), Ψ the trial wavefunction and E the associated energy.

The Hamiltonian H of the helium isoelectronic series in given by (in atomic units)

$$H = -\frac{1}{2}\Delta_1 - \frac{1}{2}\Delta_2 - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}} \tag{2}$$

In this equation, Z is the nuclear charge Δ_1 is the Laplacian with reference to the coordinates of the vector radius r_1 which detect the position of the electron 1. Δ_2 Laplacian defines the coordinates of the vector radius r_2 which detect the position of the electron 2 and r_{12} inter-electronic distance.

The exact resolution of Equation (1) is usually far too complicated because of the term $r_{12} = u = |r_1 - r_2|$. It is therefore necessary to implement a rough calculation method using a correlated wavefunction.

In this previous work, Sakho [30] used a special-form Hylleraas correlated wavefunction to calculate the energies of the doubly excited states $nln l'$ $(n = 2 - 4)$ of heliumlike ions. In the present study, we have made modifications to these wavefunctions to extend these calculations to the doubly excited states $(Nln l'$

$^{2S+1}L^{\pi}$). These wavefunctions are defined as follows:

$$\Psi = \sum_{\upsilon=0}^{\upsilon=N-\ell-1} \left(N^2 r_0^2\right)^{\upsilon} \sum_{\upsilon'=0}^{\upsilon'=n-\ell'-1} \left(n^2 r_0^2\right)^{\upsilon'} \left[1+(-1)^S C_0 Z\left(r_1-r_2\right)\right] \times \left(r_1+r_2\right)^j \left(r_1-r_2\right)^k \left|r_1-r_2\right|^m \mathrm{e}^{-\alpha(r_1+r_2)} \tag{3}$$

In this expression, N and n are the principal quantum numbers, l and l' are orbital quantum numbers, r_0 is Bohr radius, S is the total spin of atomic system, α and C_0 are the variational parameters to be determined by minimizing the energy, Z is the nuclear charge number, r_1 and r_2 are the coordinates of electrons with respect to the nucleus, j, k, m are Hylleraas parameters satisfying the double condition (j, k, $m \geq 0$) and $j + k + m \leq 3$. The set of the parameters (j, k, m) define the basis states and then give their dimension D. From the theoretical viewpoint, the Hylleraas variational method is based on the Hylleraas and Undheim theorem [31] according to which, a good approximation of the energy eigenvalue $E(\alpha, C_0)$ is obtained when the minima of the function $(d^2E(\alpha, C_0)/d\alpha dC_0)$ converge with increasing values of the dimension D of the basis states and when the function exhibit a plateau.

Using this theorem, the values of the varitional parameters α and C_0 can be determined by the following conditions:

$$\frac{\partial E(\alpha, C_0)}{\partial C_0} = 0 \tag{4}$$

and

$$\frac{\partial E(\alpha, C_0)}{\partial \alpha} = 0 \tag{5}$$

For all calculations, we fixed the value of $j = 0$ and $k = m = 1$ and this choice has allowed us to obtain:

$$\Psi = \sum_{\upsilon=0}^{\upsilon=N-\ell-1} \left(N^2 r_0^2\right)^{\upsilon} \sum_{\upsilon'=0}^{\upsilon'=n-\ell'-1} \left(n^2 r_0^2\right)^{\upsilon'} \left[1+(-1)^S C_0 Z\left(r_1-r_2\right)\right] \times \left(r_1-r_2\right)\left|r_1-r_2\right| \mathrm{e}^{-\alpha(r_1+r_2)} \tag{6}$$

In the framework of the Ritz' variation principle, the energy $E(\alpha, C_0) = \langle H \rangle (\alpha, C_0)$ is calculated from the relation:

$$E(\alpha, C_0) = \langle H \rangle (\alpha) = \frac{\left\langle \Psi(\alpha, C_0) \middle| H \middle| \Psi(\alpha, C_0) \right\rangle}{\left\langle \Psi(\alpha, C_0) \middle| \Psi(\alpha, C_0) \right\rangle} \tag{7}$$

In this equation, the correlated wavefunctions are given by (6) and the Hamiltonian H of the helium isoelectronic series in given by (2) in atomic units.

Furthermore, the closure relation represents the fact that $|\boldsymbol{r}_1, \boldsymbol{r}_2\rangle$ are continuous bases in the space of the two-electron space, written as follow:

$$\iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 \left|\boldsymbol{r}_1, \boldsymbol{r}_2\right\rangle \left\langle \boldsymbol{r}_1, \boldsymbol{r}_2\right| = 1 \tag{8}$$

Using this relation, according to (7), we obtain:

$$\begin{aligned} &E(\alpha,C_0)\iint \mathrm{d}r_1^3\mathrm{d}r_2^3 \left\langle \Psi(\alpha,C_0)\middle\| \boldsymbol{r}_1,\boldsymbol{r}_2\right\rangle \times \left\langle \boldsymbol{r}_1,\boldsymbol{r}_2\middle\| \Psi(\alpha,C_0)\right\rangle \\ &= \iint \mathrm{d}r_1^3\mathrm{d}r_2^3 \left\langle \Psi(\alpha,C_0)\middle\| \boldsymbol{r}_1,\boldsymbol{r}_2\right\rangle \hat{H} \left\langle \boldsymbol{r}_1,\boldsymbol{r}_2\middle\| \Psi(\alpha,C_0)\right\rangle \end{aligned} \tag{9}$$

By developing this expression (9), we find:

$$\begin{aligned} &E(\alpha,C_0)\iint \mathrm{d}r_1^3\mathrm{d}r_2^3 \Psi(\alpha,C_0)\times \Psi*(\alpha,C_0) \\ &= \iint \mathrm{d}r_1^3\mathrm{d}r_2^3 \Psi(\alpha,C_0)\hat{H}\Psi*(\alpha,C_0) \end{aligned} \tag{10}$$

This means:

$$N*E(\alpha,C_0) = \iint \mathrm{d}r_1^3\mathrm{d}r_2^3 \Psi(\alpha,C_0)\hat{H}\Psi*(\alpha,C_0) \tag{11}$$

With the normalization constant

$$N = \iint \mathrm{d}r_1^3\mathrm{d}r_2^3 \left|\Psi(\alpha,C_0)\right|^2 \tag{12}$$

To make it easier to integrate Equation (11), we operate the variable changes in elliptic coordinates by:

$$s = r_1 + r_2;\ t = r_1 - r_2;\ u = r_{12} \tag{13}$$

On the basis of these variable changes, the elementary volume element

$$\mathrm{d}\tau = \mathrm{d}^3r_1\mathrm{d}^3r_2 = 2\pi^2\left(s^2-t^2\right)u\,\mathrm{d}s\,\mathrm{d}u\,\mathrm{d}t \tag{14}$$

Using these elliptical coordinates, Equation (11) is written as follows

$$\begin{aligned} NE(\alpha,C_0) = &\int_0^\infty \mathrm{d}s\int_0^s \mathrm{d}u\int_0^u \mathrm{d}t\left\{u\left(s^2-t^2\right)\times\left[\left(\frac{\partial\Psi}{\partial s}\right)^2+\left(\frac{\partial\Psi}{\partial t}\right)^2+\left(\frac{\partial\Psi}{\partial u}\right)^2\right]\right. \\ &\left.+2\left(\frac{\partial\Psi}{\partial u}\right)\times\left[s\left(u^2-t^2\right)\times\frac{\partial\Psi}{\partial s}+t\left(s^2-u^2\right)\times\frac{\partial\Psi}{\partial t}-\Psi^2\times\left(4Zsu-s^2+t^2\right)\right]\right\} \end{aligned} \tag{15}$$

With respect to the correlated wave functions given by expression (6), it is expressed as follows

$$\begin{aligned} &\Psi(s,t,u,\alpha,C_0) \\ &= \sum_{\upsilon=0}^{\upsilon=N-\ell-1}\left(N^2r_0^2\right)^\upsilon \sum_{\upsilon'=0}^{\upsilon'=n-\ell'-1}\left(n^2r_0^2\right)^{\upsilon'}\left[1+(-1)^S C_0Zt\right]\times t\,u\exp(-\alpha s) \end{aligned} \tag{16}$$

Furthermore, according to (12), the normalization constant is written in elliptic coordinates as:

$$N = \int_0^\infty \mathrm{d}s\int_0^s \mathrm{d}u\int_0^u \mathrm{d}t\,u\left(s^2-t^2\right)\times\Psi^2 \tag{17}$$

2.2. General Formalism of the SCUNC Method

The Screening Constant by Unit Nuclear Charge (SCUNC) formalism is used in this work to calculate the resonance energies and the excitation energies of the $(2snl)$ $^{1,3}L^\pi$ and $(2pnl)$ $^{1,3}L^\pi$ doubly excited states of the helium-isoelectronic up to Z = 10.

In the framework of the Screening Constant by Unit Nuclear Charge (SCUNC) formalism, resonance energies of the $(Nlnl',\ ^{2S+1}L^\pi)$ doubly excited states are ex-

pressed in Rydberg (*Ry*) as below [22] [23] [32]

$$E\left(N\ell n\ell', {}^{2S+1}L^{\pi}\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\beta\left(N\ell n\ell', {}^{2S+1}L^{\pi}, Z\right)\right]^2\right)Ry \tag{18}$$

In this equation, the principal quantum numbers N and n, are respectively for the inner and the outer electron of the He-isoelectronic series. In this equation, the β-parameters are screening constant by unit nuclear charge expanded in inverse powers of Z and given by

$$\beta\left(N\ell n\ell', {}^{2S+1}L^{\pi}, Z\right) = \sum_{k=1}^{q} f_k\left(\frac{1}{Z}\right)^k \tag{19}$$

where $f_k = f_k\left(N\ell n\ell', {}^{2S+1}L^{\pi}\right)$ are screening constants to be evaluated based on variational predictable using a wavefunction.

Furthermore, in the framework of the Screening Constant by Unit Nuclear Charge formalism, the β-screening constant is expressed in terms of the variational α-parameter as follows:

- For the doubly excited states $(2snl)\ {}^{1,3}L^{\pi}$

$$\beta\left(2sn\ell, {}^{1,3}L^{\pi}, Z, \alpha\right) = \frac{\alpha}{Z^2}\left(1+\frac{L-S+1}{2n+8}\right) \tag{20}$$

- For the doubly excited states $(2pnl)\ {}^{1,3}L^{\pi}$

$$\beta\left(2pn\ell, {}^{1,3}L^{\pi}, Z, \alpha\right) = \frac{\alpha}{Z^2}\left(1+\frac{L-S}{n+S(S+1)+3}\right) \tag{21}$$

In these expressions, N and n, are respectively the principal quantum numbers for the inner and outer electron, L characterizes the quantum state under consideration (S, P, D, F, etc.), S is the total spin of the atomic system and α is the variational parameter.

2.3. Energy Resonances of the $(2snl)\ {}^{1,3}L^{\pi}$ and $(2pnl)\ {}^{1,3}L^{\pi}$ Doubly Excited States of Helium and Heliumlike Ions of Nuclear Charge $Z \le 10$

Using equations (20) and (21), the resonance energies of the doubly excited $(2snl)\ {}^{1,3}L^{\pi}$ and $(2pnl)\ {}^{1,3}L^{\pi}$ states of helium and heliumlike ions of nuclear charge $Z \le 10$ are then expressed as follows in Rydberg (*Ry*):

- For the doubly excited states $(2snl)\ {}^{1,3}L^{\pi}$

$$E\left(2sn\ell, {}^{1,3}L^{\pi}, Z\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\frac{\alpha}{Z^2}\left(1+\frac{L-S+1}{2n+8}\right)\right]^2\right)Ry \tag{22}$$

- For the doubly excited states $(2pnl)\ {}^{1,3}L^{\pi}$

$$E\left(2pn\ell, {}^{1,3}L^{\pi}, Z\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\frac{\alpha}{Z^2}\left(1+\frac{L-S}{n+S(S+1)+3}\right)\right]^2\right)Ry \tag{23}$$

In these equations, only the parameter α is unknown. Considering the $2p3p$

$^1D^e$ level of heliumlike ions (Z = 2 - 10), we calculated the values of the variational parameters α and C_0, the results are presented in Table 1 below.

The Equations (22) and (23) are used to calculate the resonance energies of the $(2snl)\ ^{1,3}L^{\pi}$ and $(2pnl)\ ^{1,3}L^{\pi}$ doubly excited states of helium and heliumlike ions of nuclear charge Z ≤ 10 without a complex calculation program.

3. Results and Discussions

The results obtained in the present study for the resonance energies and the excitation energies of the doubly excited $2sns\ ^{1,3}S^e$, $2snp\ ^{1,3}P^0$, $2pnp\ ^{1,3}D^e$, $2pnd\ ^{1,3}F^0$ and $2pnf\ ^{1,3}G^e$ states with n = 2 - 10 of the helium isoelectronic sequence (Z = 2 - 10) are presented in Tables 1-14 where a comparison between our present results and the experimental and theoretical values available in the literature is made.

Table 1 presents the values of the variational parameters α and C_0 $2 \le Z \le 10$. These parameters are calculated by determining the expression of $E = f(\alpha, C_0)$ from Equation (15) and wavefunction (16) using conditions (4) and (5). All the calculations are performed using a Maxima computer program.

In Table 2 and Table 3, we have listed resonance energies of the $2sns\ ^{1,3}S^e$ and $2snp\ ^{1,3}P^0$ doubly excited states of the helium isoelectronic sequence up to Z = 10. In Table 4, Table 5, Table 6, the present results for the resonance energies of the $2pnp\ ^{1,3}D^e$, $2pnd\ ^{1,3}F^0$ and $2pnf\ ^{1,3}G^e$ doubly excited states up to Z = 10 are presented.

In Table 7, the present resonance energies of doubly $2sns\ ^{1,3}S^e$ (n = 3 - 5) excited states are compared with various calculations. The data of Sow *et al.*, [29] [33] are obtained from variational calculations using wave fubction of Hylleraas type. Ho [18] [19] and Kar and Ho [34] applied the complex coordinate rotation (CCR) method whereas Sakho [24] used the semi-empirical procedure of the SCUNC method. Lipsky *et al.*, [35] used truncated diagonalization method of Ray and Mukherjee [25] applied the Time-Dependent Variation Perturbation Theory (TDVPT) whereas Inanov and Safronova [36] computed double sums over the complete hydrogen spectrum (CHS) to report their data. Roy *et al.*, [15] used the Density-Functional Theory (DFT) formalism and Macias and Riera [21] applied the discretization method. Comparison shows that the present results are generally in good agreement with all the citred literature data up to Z = 10. These agreements point out the validy of the new correlated wave functions used in the present calculations.

Table 1. Values of variational parameters α and C_0 of Helium-like ions (Z = 2 - 10).

Z	2	3	4	5	6	7	8	9	10
α	1.2996	2.0074	2.7157	3.4242	4.1327	4.8414	5.5500	6.2587	6.9674
C_0	0.4433	0.3106	0.2630	0.2385	0.2236	0.2135	0.2063	0.2003	0.1966

Table 2. Energy resonances ($-E$) of doubly excited $2sns\ ^{1,3}S^e$ (n = 3 - 10) states of He-like systems (Z = 2 - 10). The results are expressed in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

	$2sns\ ^1S^e$							
Z	$2s3s\ ^1S^e$	$2s4s\ ^1S^e$	$2s5s\ ^1S^e$	$2s6s\ ^1S^e$	$2s7s\ ^1S^e$	$2s8s\ ^1S^e$	$2s9s\ ^1S^e$	$2s10s\ ^1S^e$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.59450	0.55363	0.53457	0.52414	0.51782	0.51369	0.51086	0.50882
3	1.41465	1.28879	1.23026	1.19834	1.17903	1.16647	1.15783	1.15164
4	2.59511	2.33601	2.21568	2.15014	2.11052	2.08476	2.06707	2.05440
5	4.13627	3.69550	3.49096	3.37961	3.31235	3.26863	3.23862	3.21712
6	6.03848	5.36746	5.05622	4.88685	4.78458	4.71812	4.67251	4.63985
7	8.30160	7.35181	6.91141	6.67183	6.52719	6.43322	6.36873	6.32257
8	10.92578	9.64864	9.05659	8.73458	8.54020	8.41393	8.32730	8.26528
9	13.91114	12.25801	11.49180	11.07512	10.82364	10.66029	10.54822	10.46801
10	17.25749	15.17981	14.21697	13.69342	13.37747	13.17226	13.03148	12.93073
	$2sns\ ^3S^e$							
Z	$2s3s\ ^3S^e$	$2s4s\ ^3S^e$	$2s5s\ ^3S^e$	$2s6s\ ^3S^e$	$2s7s\ ^3S^e$	$2s8s\ ^3S^e$	$2s9s\ ^3S^e$	$2s10s\ ^3S^e$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.60134	0.55701	0.53649	0.52534	0.51862	0.51426	0.51127	0.50913
3	1.42690	1.29483	1.23370	1.20049	1.18047	1.16748	1.15857	1.15219
4	2.61287	2.34476	2.22066	2.15325	2.11261	2.08622	2.06814	2.05520
5	4.15959	3.70699	3.49750	3.38370	3.31508	3.27055	3.24002	3.21818
6	6.06737	5.38169	5.06432	4.89192	4.78797	4.72050	4.67424	4.64115
7	8.33609	7.36880	6.92108	6.67787	6.53123	6.43605	6.37080	6.32412
8	10.96587	9.66838	9.06783	8.74160	8.54490	8.41723	8.32970	8.26709
9	13.95684	12.28051	11.50461	11.08313	10.82899	10.66405	10.55096	10.47007
10	17.30879	15.20507	14.23134	13.70241	13.38347	13.17648	13.03456	12.93304

Table 3. Energy resonances ($-E$) of doubly excited $2snp\ ^{1,3}P^{0}$ (n = 2 - 10) states of He-like systems (Z = 2 - 10). The results are expressed in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

	$2snp\ ^{1}P^{0}$							
Z	$2s2p\ ^{1}P^{0}$	$2s3p\ ^{1}P^{0}$	$2s4p\ ^{1}P^{0}$	$2s5p\ ^{1}P^{0}$	$2s6p\ ^{1}P^{0}$	$2s7p\ ^{1}P^{0}$	$2s8p\ ^{1}P^{0}$	$2s9p\ ^{1}P^{0}$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.69291	0.58789	0.55036	0.53269	0.52296	0.51703	0.51314	0.51045
3	1.74081	1.40265	1.28286	1.22688	1.19622	1.17762	1.16547	1.15710
4	3.28658	2.57761	2.32737	2.21076	2.14705	2.10846	2.08331	2.06601
5	5.33124	4.11321	3.68413	3.48448	3.37556	3.30964	3.26673	3.23723
6	7.87570	6.00985	5.35334	5.04818	4.88182	4.78122	4.71576	4.67078
7	10.91966	8.26738	7.33495	6.90181	6.66582	6.52318	6.43040	6.36667
8	14.46346	10.88596	9.62901	9.04542	8.72759	8.53553	8.41065	8.32490
9	18.50742	13.86572	12.23563	11.47906	11.06715	10.81832	10.65655	10.54549
10	23.05105	17.20646	15.15467	14.20266	13.68447	13.37149	13.16806	13.02842
	$2snp\ ^{3}P^{0}$							
Z	$2s3p\ ^{3}P^{0}$	$2s4p\ ^{3}P^{0}$	$2s5p\ ^{3}P^{0}$	$2s6p\ ^{3}P^{0}$	$2s7p\ ^{3}P^{0}$	$2s8p\ ^{3}P^{0}$	$2s9p\ ^{3}P^{0}$	$2s10p\ ^{3}P^{0}$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.59450	0.55363	0.53457	0.52414	0.51782	0.51369	0.51086	0.50882
3	1.41465	1.28879	1.23026	1.19834	1.17903	1.16647	1.15783	1.15164
4	2.59511	2.33601	2.21568	2.15014	2.11052	2.08476	2.06707	2.05440
5	4.13627	3.69550	3.49096	3.37961	3.31235	3.26863	3.23862	3.21712
6	6.03848	5.36746	5.05622	4.88685	4.78458	4.71812	4.67251	4.63985
7	8.30160	7.35181	6.91141	6.67183	6.52719	6.43322	6.36873	6.32257
8	10.92578	9.64864	9.05659	8.73458	8.54020	8.41393	8.32730	8.26528
9	13.91114	12.25801	11.49180	11.07512	10.82364	10.66029	10.54822	10.46801
10	17.25749	15.17981	14.21697	13.69342	13.37747	13.17226	13.03148	12.93073

Table 4. Energy resonances ($-E$) of doubly excited $2pnp\ ^{1,3}D^e$ (n = 3 - 10) states of He-like systems (Z = 2 - 10). The results are expressed in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

	$2pnp\ ^1D^e$							
Z	$2p3p\ ^1D^e$	$2p4p\ ^1D^e$	$2p5p\ ^1D^e$	$2p6p\ ^1D^e$	$2p7p\ ^1D^e$	$2p8p\ ^1D^e$	$2p9p\ ^1D^e$	$2p10p\ ^1D^e$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.57145	0.54242	0.52824	0.52022	0.51521	0.51188	0.50954	0.50783
3	1.37190	1.26812	1.21866	1.19117	1.17429	1.16316	1.15543	1.14985
4	2.53222	2.30568	2.19870	2.13965	2.10359	2.07994	2.06358	2.05179
5	4.05304	3.65541	3.46853	3.36578	3.30321	3.26227	3.23402	3.21369
6	5.93482	5.31758	5.02833	4.86966	4.77323	4.71023	4.66680	4.63558
7	8.17746	7.29211	6.87805	6.65127	6.51362	6.42378	6.36190	6.31747
8	10.78111	9.57909	9.01774	8.71064	8.52440	8.40295	8.31935	8.25935
9	13.74595	12.17862	11.44746	11.04781	10.80562	10.64776	10.53916	10.46124
10	17.07173	15.09056	14.16713	13.66273	13.35722	13.15819	13.02130	12.92313
	$2pnp\ ^3D^e$							
Z	$2p3p\ ^3D^e$	$2p4p\ ^3D^e$	$2p5p\ ^3D^e$	$2p6p\ ^3D^e$	$2p7p\ ^3D^e$	$2p8p\ ^3D^e$	$2p9p\ ^3D^e$	$2p10p\ ^3D^e$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.58952	0.55108	0.53306	0.52318	0.51716	0.51323	0.51051	0.50856
3	1.40563	1.28417	1.22755	1.19661	1.17785	1.16562	1.15721	1.15116
4	2.58196	2.32928	2.21174	2.14761	2.10880	2.08354	2.06616	2.05371
5	4.11895	3.68665	3.48577	3.37629	3.31009	3.26702	3.23742	3.21622
6	6.01698	5.35647	5.04978	4.88274	4.78178	4.71612	4.67103	4.63873
7	8.27591	7.33868	6.90372	6.66691	6.52384	6.43083	6.36697	6.32122
8	10.89589	9.63337	9.04765	8.72886	8.53631	8.41116	8.32524	8.26372
9	13.87705	12.24059	11.48160	11.06860	10.81920	10.65713	10.54588	10.46623
10	17.21919	15.16024	14.20551	13.68609	13.37249	13.16871	13.02886	12.92873

Table 5. Energy resonances ($-E$) of doubly excited $2pnd^{1,3}F^{o}$ (n = 3 - 10) states of He-like systems (Z = 2 - 10). The results are expressed in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

	$2pnd^{1}F^{o}$							
Z	$2p3d^{1}F^{o}$	$2p4d^{1}F^{o}$	$2p5d^{1}F^{o}$	$2p6d^{1}F^{o}$	$2p7d^{1}F^{o}$	$2p8d^{1}F^{o}$	$2p9d^{1}F^{o}$	$2p10d^{1}F^{o}$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.55847	0.53593	0.52452	0.51787	0.51364	0.51077	0.50873	0.50722
3	1.34648	1.25562	1.21157	1.18674	1.17133	1.16109	1.15393	1.14871
4	2.49403	2.28702	2.18814	2.13309	2.09923	2.07688	2.06136	2.05012
5	4.00193	3.63052	3.45449	3.35706	3.29742	3.25823	3.23108	3.21148
6	5.87075	5.28644	5.01078	4.85878	4.76601	4.70519	4.66313	4.63284
7	8.10035	7.25468	6.85698	6.63822	6.50496	6.41774	6.35752	6.31418
8	10.69095	9.53537	8.99315	8.69541	8.51431	8.39591	8.31424	8.25552
9	13.64274	12.12860	11.41935	11.03041	10.79408	10.63972	10.53332	10.45687
10	16.95544	15.03424	14.13548	13.64314	13.34424	13.14914	13.01474	12.91822
	$2pnd^{3}F^{o}$							
Z	$2p3d^{3}F^{o}$	$2p4d^{3}F^{o}$	$2p5d^{3}F^{o}$	$2p6d^{3}F^{o}$	$2p7d^{3}F^{o}$	$2p8d^{3}F^{o}$	$2p9d^{3}F^{o}$	$2p10d^{3}F^{o}$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.57844	0.54548	0.52981	0.52111	0.51576	0.51223	0.50978	0.50800
3	1.38513	1.27386	1.22158	1.19282	1.17529	1.16381	1.15587	1.15015
4	2.55185	2.31416	2.20300	2.14208	2.10507	2.08089	2.06422	2.05223
5	4.07913	3.66667	3.47423	3.36899	3.30516	3.26353	3.23486	3.21428
6	5.96741	5.33162	5.03543	4.87366	4.77566	4.71179	4.66785	4.63632
7	8.21656	7.30894	6.88656	6.65606	6.51653	6.42565	6.36316	6.31835
8	10.82674	9.59872	9.02766	8.71622	8.52779	8.40513	8.32082	8.26037
9	13.79811	12.20105	11.45879	11.05418	10.80949	10.65025	10.54084	10.46241
10	17.13043	15.11579	14.17988	13.66989	13.36157	13.16098	13.02319	12.92444

Table 6. Energy resonances ($-E$) of doubly excited $2pnf^{1,3}G^e$ (n = 4 - 10) states of He-like systems (Z = 2 - 10). The results are expressed in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

	$2pnf^1G^e$						
Z	$2p4f^1G^e$	$2p5f^1G^e$	$2p6f^1G^e$	$2p7f^1G^e$	$2p8f^1G^e$	$2p9f^1G^e$	$2p10f^1G^e$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.53593	0.52452	0.51787	0.51364	0.51077	0.50873	0.50722
3	1.25562	1.21157	1.18674	1.17133	1.16109	1.15393	1.14871
4	2.28702	2.18814	2.13309	2.09923	2.07688	2.06136	2.05012
5	3.63052	3.45449	3.35706	3.29742	3.25823	3.23108	3.21148
6	5.28644	5.01078	4.85878	4.76601	4.70519	4.66313	4.63284
7	7.25468	6.85698	6.63822	6.50496	6.41774	6.35752	6.31418
8	9.53537	8.99315	8.69541	8.51431	8.39591	8.31424	8.25552
9	12.12860	11.41935	11.03041	10.79408	10.63972	10.53332	10.45687
10	15.03424	14.13548	13.64314	13.34424	13.14914	13.01474	12.91822
	$2pnf^3G^e$						
Z	$2p4f^3G^e$	$2p5f^3G^e$	$2p6f^3G^e$	$2p7f^3G^e$	$2p8f^3G^e$	$2p9f^3G^e$	$2p10f^3G^e$
	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
2	0.54020	0.52672	0.51913	0.51442	0.51127	0.50907	0.50746
3	1.26389	1.21579	1.18914	1.17280	1.16204	1.15457	1.14916
4	2.29940	2.19444	2.13665	2.10140	2.07829	2.06230	2.05078
5	3.64705	3.46288	3.36180	3.30031	3.26009	3.23233	3.21236
6	5.30713	5.02128	4.86470	4.76961	4.70751	4.66470	4.63393
7	7.27957	6.86959	6.64532	6.50928	6.42052	6.35939	6.31549
8	9.56445	9.00787	8.70370	8.51934	8.39915	8.31643	8.25705
9	12.16188	11.43618	11.03988	10.79984	10.64342	10.53582	10.45862
10	15.07172	14.15444	13.65380	13.35071	13.15331	13.01755	12.92018

Table 7. Comparison of the present calculations on resonance energies for the doubly $2sns\,^{1,3}S^e$ (n = 3 - 5) excited states of He-like ions up to Z = 10 with available literature values. All energies are given in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

States										
$2sns\,^{1,3}S^e$	Z	2	3	4	5	6	7	8	9	10
$2s3s\,^1S^e$	$-E^p$	0.59450	1.41465	2.59511	4.13627	6.03848	8.30160	10.92578	13.91114	17.25749
	$-E^a$	0.59924	1.42338	2.59644	4.14121	6.05348	8.32620	10.95973	13.94576	17.30205
	$-E^b$	0.58993	1.41557	2.60205	4.14950	6.05799	8.32750	10.95820	13.94989	17.30269
	$-E^c$	0.59031	1.41479	2.60073	4.14793	6.05631	8.32584	10.95651	13.94829	17.30121
	$-E^d$	0.61164	1.41169	2.59664	4.14303					
	$-E^e$	0.59522	1.40999	2.58534	4.12279					
	$-E^f$					6.05795		10.95814		17.30265
	$-E^g$	0.57698	1.39125	2.56721	4.10436					
	$-E^h$		1.41557	2.60205	4.14950					
	$-E^k$	0.58743	1.41872	2.60771	4.14154	6.06229	8.32902	10.94143	13.93837	17.28455
$2s4s\,^1S^e$	$-E^p$	0.55363	1.28879	2.33601	3.69550	5.36746	7.35181	9.64864	12.25801	15.17981
	$-E^a$	0.55044	1.28504	2.33922	3.69031	5.32084	7.29306	9.58847	12.19147	15.11308
	$-E^i$	0.53529	1.26279	2.30279	3.65529	5.32030	7.29780	9.58780	12.18999	15.10499
	$-E^c$	0.54449	1.27602	2.32017	3.67686	5.34609	7.32782	9.62207	12.22882	15.14806
	$-E^d$	0.54402	1.23704	2.25736	3.67871					
	$-E^g$	0.56179	1.29209	2.33514	3.69129					
	$-E^f$					5.35476		9.63463		15.16445
	$-E^k$	0.53928	1.27497	2.45622	3.84877	5.49894	7.33583	9.90044	12.77848	15.07605
$2s5s\,^1S^e$	$-E^p$	0.53457	1.23026	2.21568	3.49096	5.05622	6.91141	9.05659	11.49180	14.21697
	$-E^e$	0.54682	1.23981	2.21956	3.48607					
$2s3s\,^3S^e$	$-E^p$	0.60134	1.42690	2.61287	4.15959	6.06737	8.33609	10.96587	13.95684	17.30879
	$-E^a$	0.58775	1.42352	2.62004	4.17550	6.04756	8.31893	10.99338	14.02878	17.42454
	$-E^c$	0.60149	1.43840	2.63728	4.19761	6.11922	8.40204	11.04604	14.05119	17.41748
	$-E^d$	0.55931	1.43934	2.63792	4.19769					
	$-E^i$	0.58559	1.41169	2.59899	4.14739	6.05679	8.32739	10.95899	13.95199	17.30599
	$-E^g$	0.59355	1.42376	2.61516	4.16756					
	$-E^h$		1.44020	2.63901	4.19894					
	$-E^k$	0.61136	1.44296	2.62278	4.19909	6.14473	8.41859	11.03398	14.06547	17.36901
$2s4s\,^3S^e$	$-E^p$	0.55701	1.29483	2.34476	3.70699	5.38169	7.36880	9.66838	12.28051	15.20507
	$-E^a$	0.55485	1.26547	2.30884	3.69471	5.39502	7.40835	9.66087	12.25931	15.20525
	$-E^c$	0.54909	1.28559	2.33479	3.69669	5.37119	7.35809	9.65759	12.26996	15.19419
	$-E^k$	0.54491	1.27986	2.31108	3.69755	5.34981	7.35529	9.63054	12.29178	15.13992

$-E^p$: Present work, values calculated from Equation (22); $-E^a$: Sow *et al.* [33]; $-E^b$: Ho [18]; $-E^c$: Sakho [24]; $-E^d$: Lipsky *et al.* [35]; $-E^e$: Ray and Mukherjee [25]; $-E^f$: Kar [34]; $-E^g$: Roy *et al.* [15]; $-E^h$: Ho [19]; $-E^i$: Ivanov and Safronova [36]; $-E^j$: Macias and Riera [21]; $-E^k$: Sow *et al.* [29].

Table 8. Comparison of the present calculations on resonance energies for the doubly $2snp\ ^{1,3}P^0$ (n = 2 - 5) excited states of He-like ions up to Z = 10 with available literature values. All energies are given in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

States										
$2snp\ ^{1,3}P^0$	Z	2	3	4	5	6	7	8	9	10
	$-E^p$	0.69291	1.74081	3.28658	5.33124	7.87570	10.91966	14.46346	18.50742	23.05105
	$-E^a$	0.69309	1.75735	3.31957	5.38010	7.94042	10.99980	14.55934	18.61867	23.17779
	$-E^b$	0.71367	1.77242	3.33060	5.38965	7.94814	11.00679	14.56522	18.62381	23.18293
	$-E^c$	0.69346	1.75809	3.31993	5.38083	7.94116	11.00054	14.56008	18.60250	23.17852
$2s2p\ ^1P^0$	$-E^{d,e}$	0.69383[d]	1.75588[e]							
	$-E^f$	0.69383	1.75515	3.31626	5.37789	7.93895	11.00017	14.56155	18.62308	23.18440
	$-E^g$	0.71362	1.77489	3.36839	5.42760	7.98151	11.09005	14.67334	18.73256	23.26332
	$-E^h$	0.69298	1.75696	3.31809	5.38608	7.94403	10.97484	14.56189	18.57691	23.15271
	$-E^i$	0.69226	1.76471	3.31757	5.37469	7.93602	10.98991	14.56175	18.62290	23.17804
	$-E^p$	0.58789	1.40265	2.57761	4.11321	6.00985	8.26738	10.88596	13.86572	17.20646
$2s3p\ ^1P^0$	$-E^b$	0.58615	1.39611	2.56731	4.09939	5.99271	8.24691	10.86273	13.83905	17.17699
	$-E^f$	0.58615	1.40493	2.58495	4.12621	6.02836	8.29175	10.91638	13.90190	17.24865
	$-E^i$	0.56431	1.35936	2.51307	4.04289	5.91618	8.15979	10.75069	13.71700	17.04579
	$-E^p$	0.55036	1.28286	2.32737	3.68413	5.35334	7.33495	9.62901	12.23563	15.15467
$2s4p\ ^1P^0$	$-E^b$	0.54058	1.26712	2.30565	3.65692	5.32093	7.29731	9.58606	12.18717	15.10103
	$-E^f$	0.54830	1.28255	2.32917	3.68816	5.35989	7.34398	9.64045	12.24928	15.17122
$2s5p\ ^1P^0$	$-E^p$	0.53269	1.22688	2.21076	3.48448	5.04818	6.90181	9.04542	11.47906	14.20266
	$-E^f$	0.53103	1.22596	2.21047	3.48530	5.05009	6.90520	9.04988	11.48453	14.20949
	$-E^p$	0.59450	1.41465	2.59511	4.13627	6.03848	8.30160	10.92578	13.91114	17.25749
	$-E^i$	0.58601	1.40483	2.58477	4.12583	6.02800	8.29129	10.98369	13.90124	17.24789
$2s3p\ ^3P^0$	$-E^k$	0.57942	1.39354	2.57931	4.12121	6.02422	8.28835	10.91359	13.89994	17.24740
	$-E^j$	0.57880	1.39517	2.58219	4.11905	6.02267	8.27916	10.91127	13.83614	17.21117
	$-E^l$	0.58335	1.40295	2.58433	4.12734					
	$-E^p$	0.55363	1.28879	2.33601	3.69550	5.36746	7.35181	9.64864	12.25801	15.17981
$2s4p\ ^3P^0$	$-E^i$	0.54337	1.27326	2.31566	3.67057	5.337978	7.317897	9.610323	12.21526	15.13269
	$-E^l$	0.54220	1.27261	2.31591	3.64826					
	$-E^p$	0.53457	1.23026	2.21568	3.49096	5.05622	6.91141	9.05659	11.49180	14.21697
$2s5p\ ^3P^0$	$-E^i$	0.52473	1.21422	2.19371	3.46321	5.02272	6.87223	9.01175	11.44127	14.16080

$-E^p$: Present work, values calculated from Equation (22); $-E^a$: Ho [17]; $-E^b$: Ivanov and Safronova [36]; $-E^c$: Drake and Dalgarno [2]; $-E^d$: Experimental data, Diehl *et al.* [37]; $-E^e$: Experimental data, Kossmann *et al.* [40]; $-E^f$: Sakho *et al.* [22]; $-E^g$: Biaye *et al.* [38] [39]; $-E^h$: Gning *et al.* [27]; $-E^i$: Sow *et al.* [29]; $-E^j$: Sakho *et al.* [23]; $-E^k$: Ho [18]; $-E^l$: Lipsky *et al.* [35].

Table 9. Comparison of the present calculations on resonance energies for the doubly $2pnp\ ^{1,3}D^e$ (n = 3 - 5) excited states of He-like ions up to Z = 10 with available literature values. All energies are given in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

States										
$2pnp\ ^{1,3}D^e$	Z	2	3	4	5	6	7	8	9	10
	$-E^p$	0.57145	1.37190	2.53222	4.05304	5.93482	8.17746	10.78111	13.74595	17.07173
	$-E^a$	0.57940	1.39115	2.54216	4.03186	5.91564	8.15486	10.74894	13.69746	17.02625
	$-E^b$	0.56907	1.37390	2.53992	4.06708	5.95537	8.20478	10.81531	13.78694	17.11970
	$-E^c$	0.56504	1.36527	2.52749	4.05099					
$2p3p\ ^1D^e$	$-E^d$	0.58390	1.38020	2.53724	4.05599					
	$-E^e$		1.34617	2.54029	4.06777	5.87173	8.10114	10.69138	13.64254	16.95466
	$-E^f$	0.55612	1.34413	2.49002	3.99582					
	$-E^i$	0.56859	1.37868	2.54641	4.06276	6.00418	8.20771	10.84569	13.81923	17.09193
	$-E^p$	0.54242	1.26812	2.30568	3.65541	5.31758	7.29211	9.57909	12.17862	15.09056
	$-E^a$	0.55938	1.27684	2.32878	3.64061	5.29086	7.26753	9.55347	12.17767	15.05129
$2p4p\ ^1D^e$	$-E^b$	0.53645	1.26032	2.29672	3.64562	5.30703	7.28095	9.56736	12.16627	15.07769
	$-E^d$	0.55730	1.27980	2.31544	3.66419					
	$-E^i$	0.53811	1.26503	2.30731	3.64021	5.29988	7.28815	9.59002	12.14703	15.04906
	$-E^p$	0.52824	1.21866	2.21174	3.46853	5.02833	6.87805	9.01774	11.44746	14.16713
$2p5p\ ^1D^e$	$-E^d$	0.54482	1.23381	2.21046	3.47687					
	$-E^g$	0.52222								
	$-E^h$	0.52242								
	$-E^p$	0.58952	1.40563	2.58196	4.11895	6.01698	8.27591	10.89589	13.87705	17.21919
	$-E^a$	0.58080	1.40543	2.58479	4.13099	6.03511	8.30190	10.92526	13.91198	17.25727
	$-E^b$	0.58337	1.40353	2.58541	4.12864	6.03309	8.29874	10.92553	13.91345	17.26251
$2p3p\ ^3D^e$	$-E^c$	0.58498	1.40743	2.59117	4.13589					
	$-E^e$	0.58378	1.40557	2.58834	4.13214	6.03699	8.30293	10.92996	13.91808	17.26731
	$-E^f$	0.58319	1.40401	2.58599	4.12923					
	$-E^i$	0.58546	1.40511	2.58809	4.13378	6.03163	8.29953	10.93380	13.92114	17.26719
	$-E^p$	0.55108	1.28417	2.32928	3.68665	5.35647	7.33868	9.63337	12.24059	15.16024
	$-E^a$	0.53275	1.27584	2.30842	3.64892	5.30830	7.28858	9.57982	12.17055	15.10668
$2p4p\ ^3D^e$	$-E^b$	0.54075	1.27067	2.31355	3.66911	5.33726	7.31797	9.61119	12.21708	15.13522
	$-E^i$	0.54805	1.28091	2.29456	3.65863	5.34968	7.32021	9.61492	12.21952	15.33671

$-E^p$: Present work, values calculated from Equation (23); $-E^a$: Sow *et al.* [33]; $-E^b$: Sakho [22]; $-E^c$: Roy *et al.* [15]; $-E^d$: Ray and Mukherjee [25]; $-E^e$: Ho and Bathia [12]; $-E^f$: Lipsky *et al.* [35]; $-E^g$: Bhatia [41]; $-E^h$: Herrick and Sinanoglu [42]; $-E^i$: Sow *et al.* [29].

Table 8 shows a comparison of the present SCUNC results of resonance energies of the doubly $2snp\,^{1,3}P^{0}$ (n = 2 - 5) excited states of He-like systems up to Z = 10 with the results of Ho [17] who used the complex rotation method, Ivanov and Safronova [36], the results of Drake and Dalgarno [2] from the $1/Z$ expansion perturbation theory, the values of Sakho *et al.*, [22] [23] obtained from the semi-empirical procedure of the SCUNC formalism. Comparison is also done with the experimental data of Diehl *et al.*, [37], the theoretical results of Lipsky *et al.*, [35], Gning *et al.*, [27], Sow *et al.*, [29], Biaye *et al.*, [38] [39] who performed their calculations in the framework of a variational calculations using wave function of Hylleraas type, and with the experimental data of Kossmann *et al.*, [40]. In general, very good agreement is obtained between the present calculations and those of the above-mentioned works for all the states studied for Z = 2 - 10. As underlined above, the present goog agreements between theory and experiments demonstrate the validity of the new correlated wave functions constructed in this work.

Table 10. Comparison of the present calculations on resonance energies for the doubly $2pnd\,^{1,3}F^{0}$ (n = 3 - 5) excited states of He-like ions up to Z = 5 with available literature values. All energies are given in atomic units. 1 a.u. = 2 Ry = 27.211396 eV.

States					
$2pnd\,^{1,3}F^{0}$	Z	2	3	4	5
$2p3d\,^{1}F^{0}$	$-E^{p}$	0.55847	1.34648	2.49403	4.00193
	$-E^{a}$	0.54764	1.32705	2.47020	3.97508
	$-E^{b}$	0.55795	1.34415	2.48636	
$2p4d\,^{1}F^{0}$	$-E^{p}$	0.53593	1.25562	2.28702	3.63052
	$-E^{a}$	0.53377	1.25268	2.28421	3.62815
	$-E^{b}$	0.53209	1.24800	2.27389	3.61152
$2p5d\,^{1}F^{0}$	$-E^{p}$	0.52452	1.21157	2.18814	3.45449
	$-E^{a}$	0.52607	1.21304	2.19005	3.45691
$2p3d\,^{3}F^{0}$	$-E^{p}$	0.57844	1.38513	2.55185	4.07913
	$-E^{a}$	0.56977	1.38243	2.55741	4.09360
	$-E^{b}$	0.56587	1.37371	2.54341	
$2p4d\,^{3}F^{0}$	$-E^{p}$	0.54548	1.27386	2.31416	3.66667
	$-E^{a}$	0.54222	1.270 95	2.312 36	3.66617
	$-E^{b}$	0.53567	1.26072	2.29819	3.64827
$2p5d\,^{3}F^{0}$	$-E^{p}$	0.52981	1.22158	2.20300	3.47423
	$-E^{a}$	0.52954	1.22108	2.20263	3.47402

$-E^{p}$: Present work, values calculated from Equation (23); $-E^{a}$: *Roy et al.* [15]; $-E^{b}$: Lipsky *et al.* [35].

Table 11. Comparison of the present calculations on the variational calculation of the excitation energies of the doubly excited states $2sns\ ^{1,3}S^e$ (n = 3 - 4) of He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in atomic units: 1 a.u. = 2 Ry = 27.211396 eV.

States	Z	2	3	4	5
$2s3s\ ^1S^e$	E^p	2.3092	5.8653	11.0605	17.8947
	E^a	2.3138			
	E^b	2.3139			
	E^c	2.3194			
	E^d	2.3130			
	E^e	2.3267	5.8887	11.0884	17.9266
	E^f	2.3085		11.0721	17.9131
	E^g		5.8643		
	E^h		5.8642	11.0533	17.8812
	E^i		5.8682		
	E^j		5.8649		
	E^k			11.0535	17.8815
$2s4s\ ^1S^e$	E^p	2.3501	5.9911	11.3196	18.3355
	E^f	2.3419			
	E^j	2.3592	6.0039	11.3354	18.3541
	E^m	2.3684	6.0171	11.3528	18.3757
$2s3s\ ^3S^e$	E^p	2.3024	5.8530	11.0427	17.8714
	E^a	2.3011			
	E^e	2.3102	5.8562	11.0404	17.8634
	E^g			11.0193	17.8379
	E^h		5.8397	11.0166	17.8320
	E^i	2.3016	5.8406		
	E^j	2.3022			
$2s4s\ ^3S^e$	E^p	2.3467	5.9851	11.3108	18.3239
	E^j	2.3546	5.9943	11.3208	18.3343
	E^m	2.3625	6.0064	11.3373	18.3343

E^p: Present work; E^a: Burgers *et al.* [44]; E^b: Oza [33]; E^c: Koyama *et al.* [46]; E^d: Experimental value of Hicks and Comer [7]; E^e: Roy *et al.* [15]; E^f: Ray and Mukherjee [25]; E^g: Lipsky *et al.* [35]; E^h: Macias and Riera [21]; E^i: Conneely and Lipsky [20]; E^j: Bhatia [41]; E^k: Ho [18]; E^l: Sakho [24]; E^m: Ivanov and Safronova [36].

Table 12. Comparison of the present calculations on the variational calculation of the excitation energies of the doubly excited states $2snp\,^{1,3}P^{0}$ (n = 3 - 5) of He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in atomic units: 1 a.u. = 2 Ry = 27.211396 eV.

States	Z	2	3	4	5
$2s2p\,^{1}P^{0}$	E^{p}	2.2108	5.5391	10.3689	16.6997
	E^{a}	2.2097		10.3394	16.6529
	E^{b}		5.5249		
	E^{c}		5.5238		
$2s3p\,^{1}P^{0}$	E^{p}	2.3158	5.8773	11.0779	17.9178
	E^{a}	2.3174		11.0707	17.9046
	E^{b}		5.8751		
$2s4p\,^{1}P^{0}$	E^{p}	2.3534	5.9971	11.3282	18.3468
	E^{a}	2.3553		11.3265	18.3427
	E^{b}		5.9975		
$2s5p\,^{1}P^{0}$	E^{p}	2.3710	6.0530	11.4448	18.5465
	E^{a}	2.3725		11.4452	18.5455
	E^{b}		6.0541		
$2s3p\,^{3}P^{0}$	E^{p}	2.3092	5.8653	11.0605	17.8947
	E^{a}	2.3178	5.8751	11.0708	17.9052
	E^{d}		5.8735	11.0663	17.8977
	E^{e}			11.0729	17.9083
$2s4p\,^{3}P^{0}$	E^{p}	2.3501	5.9911	11.3196	18.3355
	E^{a}	2.3604		11.3399	18.3605
	E^{d}		6.0048		
$2s5p\,^{3}P^{0}$	E^{p}	2.3692	6.0497	11.4399	18.5400
	E^{a}	2.3788	6.0658	11.4619	18.5678
	E^{d}		6.0625		

E^{p}: Present work; E^{a}: Sakho [23]; E^{b}: Roy *et al.* [15]; E^{c}: Koyama *et al.* [46]; E^{d}: Kar and Ho [34]; E^{e}: Lipsky *et al.* [35].

Table 13. Comparison of the present calculations on the variational calculation of the excitation energies of the doubly excited states $2pnp\,^{1,3}D^e$ (n = 3 - 4) of He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in atomic units: 1 a.u. = 2 Ry = 27.211396 eV.

States	Z	2	3	4	5
	E^p	2.3323	5.9080	11.1233	17.9779
	E^a	2.3387		11.1281	17.9800
	E^b	2.3345			
	E^c	2.3350			
	E^d	2.3318	5.9105	11.12764	17.9834
$2p3p\,^1D^e$	E^e		5.9058	11.1153	17.9632
	E^f		5.9016		
	E^g		5.9080		
	E^h		5.9060	11.11564	17.9639
	E^i		5.9116		
	E^j			11.1202	17.9799
	E^p	2.3613	6.0118	11.3499	18.3756
$2p4p\,^1D^e$	E^d	2.3692	6.0234	11.3645	18.3929
	E^h	2.3673	6.0196	11.3588	18.3854
	E^p	2.3142	5.8743	11.0736	17.9120
	E^a	2.3187	5.8725	11.0644	17.8951
	E^b	2.3199			
	E^d			11.0792	17.9142
	E^e		5.8743	11.0672	
$2p3p\,^3D^e$	E^f		5.8747		
	E^g		5.8752		
	E^h			11.0702	17.9023
	E^i		5.8759		
	E^k	2.3200			
	E^l			11.0712	17.9064
	E^p	2.3526	5.9957	11.3263	18.3443
$2p4p\,^3D^e$	E^d	2.3641	6.0116	11.3461	18.3679
	E^h	2.3629	6.0092	11.3420	18.3619

E^p: Present work; E^a: Roy *et al.* [15]; E^b: Lindroth [26]; E^c: Oza [45]; E^d: Ivanov and Safronova [36]; E^e: Ho and Bhatia [12]; E^f: Bhatia [41]; E^g: Macias *et al.* [48]; E^h: Sakho [24]; E^i: Conneely and Lipsky [20]; E^j: Ray and Mukherjee [25]; E^k: Bhatia and Temkin [49]; E^l: Lipsky *et al.* [35].

Table 14. Comparison of the present calculations on the variational calculation of the excitation energies of the doubly excited states $2pnd\,^{1,3}P^{e}$ (n = 3 - 5) of He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in atomic units: 1 a.u. = 2 Ry = 27.211396 eV.

States	Z	2	3	4	5
$2p3d\,^{1}P^{e}$	E^{p}	2.3453	5.9334	11.1615	18.0290
	E^{a}	2.3561	5.9529	11.1854	18.0559
	E^{b}	2.3454			
	E^{c}	2.3458			
	E^{d}		5.9338	11.1692	18.0433
	E^{e}		5.9161	11.1483	18.0205
$2p4d\,^{1}P^{e}$	E^{p}	2.3678	6.0243	11.3685	18.4005
	E^{a}	2.3700	6.0272	11.3714	18.4028
	E^{c}	2.3716			
	E^{e}	2.3489	6.0071	11.3536	18.3882
	E^{d}		6.0319	11.3817	18.4195
$2p5d\,^{1}P^{e}$	E^{p}	2.3792	6.0683	11.4674	18.5765
	E^{a}	2.3777	6.0669	11.4655	18.5741
	E^{c}	2.3833			
	E^{d}		6.0761	11.4800	18.5943
	E^{e}	2.3600	6.0495	11.4526	18.5628
$2p3d\,^{3}P^{e}$	E^{p}	2.3253	5.8948	11.1037	17.9518
	E^{a}	2.3340	5.8975	11.0982	17.9374
	E^{b}	2.3375			
	E^{c}	2.3379			
	E^{d}		5.9067	11.1138	17.9611
$2p4d\,^{3}P^{e}$	E^{p}	2.3582	6.0061	11.3414	18.3643
	E^{a}	2.3615	6.0090	11.3432	18.3648
	E^{c}	2.3681			
	E^{d}		6.0197	11.3590	18.3873
$2p5d\,^{3}P^{e}$	E^{p}	2.3739	6.0583	11.4526	18.5567
	E^{a}	2.3742	6.0588	11.4529	18.5570
	E^{c}	2.3815			
	E^{d}		6.0701	11.4692	18.5800

E^{p}: Present work; E^{a}: Roy *et al.* [15]; E^{b}: Lindroth [26]; E^{c}: Conneely and Lipsky [20]; E^{d}: Lipsky *et al.* [36]; E^{e}: Ray *et al.* [25].

Table 9 compares the results for resonance energies of the doubly $2p3p\ ^{1,3}D^e$, $2p4p\ ^{1,3}D^e$ and $2p5p\ ^{1}D^e$ excited states with the theoretical results of Sow *et al.*, [29] [33], Ho and Bathia [12] using the complex roration method, Sakho [24], Lipsky *et al.*, [35], Roy *et al.*, [15], Ray and Mukherjee [25], Bhatia [41] who employed the Feshbach projection Operator (FPO) formalism, Herrick and Sinanoglu [42]. In general comparison show satisfactory agreement between the calculations.

Table 10 lists the present resonance energies of the doubly $2pnd\ ^{1,3}F^0$ (n = 3 - 5) excited states. For these levels, literature data are very scarce. Comparisons show a good agreement between the present calculations and the theoretical results of Roy *et al.*, [15] and of Lipsky *et al.*, [35]. For the $2pnf\ ^{1,3}G^e$ doubly excited states, no literature data were found for comparison. The SCUNC data quoted may be good reference for these doubly excited states.

In **Table 11**, the present SCUNC results for excitation energies of the doubly $2sns\ ^{1,3}S^e$ excited states with n = 2 - 5 and Z = 2 - 5 of He-like systems are listed. The excitation energies are evaluated with respect to the ground state of Frankowski and Pekeris [43]. Comparisons are made with the results from the CCR results [19] and of Burgers *et al.*, [44], the values of the variational algebraic (VA) method of Oza [45], the values of Koyama *et al.*, [46] obtained from the the hyperspherical coordinate approaches, the experimental data of Hicks and Comer [7], the DFT results of Roy *et al.*, [15], the TDVPT results of Ray and Mukherjee [25], the TDM data of Lipsky *et al.*, [35] and of Macias and Riera [21], the diagonalization values of Conneely and Lipsky [20], the FPO values of Bhatia [47], the CCR calculations of Ho [18], the SCUNC method data of Sakho [24], the CHS results of Ivanov and Safronova [36]. Comparisons indicate that the current SCUNC calculations agree very well with all the literature data. This demonstrates again the validity of the present SCUNC variational procedure.

Table 12 shows the present excitation energies of the doubly $2snp\ ^{1,3}P^0$ excited states (n = 2 – 5) of He-like systems. Good agreement is obtained when comparing the SCUNC results to the results of Sakho [23], Roy *et al.*, [15], Koyama *et al.*, [46], Kar and Ho [34], Lipsky *et al.*, [35].

Table 13 presents the SCUNC values for excitation energies of the doubly $2pnp\ ^{1,3}D^e$ (n = 4 - 5) of helium and helium-like ions of nuclear charge $Z \leq 5$. Comparison is done with the DFT data of Roy *et al.*, [15], the CCR results of Lindroth [26], the VA calculations of Oza [45], the CHS results of Ivanov and Safronova [36], the CCR results of Ho and Bathia [12] and of Bhatia [41], the discretization calculations of Macias *et al.*, [48], the semi-empirical SCUNC data of Sakho [24], the TDM values of Conneely and Lipsky [20] and of Lipsky *et al.*, [35], the TDVPT data of Ray and Mukherjee [25], to the results of Bhatia and Temkin [49] and the TDM. Overall, good agreement is obtained.

In **Table 14**, we have listed our results on the calculation of the excitation energies of the doubly excited $2pnd\ ^{1,3}F^0$ (n = 2 - 5) states of He-like systems with $Z \leq 5$. The SCUNC calculations are seen to agree well with the results of Roy *et*

al., [15], Lindroth [26], Conneely and Lipsky, Lipsky *et al.*, [35] and of Ray *et al.*, [25].

Overall, the good agreements between our present calculations and the various experimental and theoretical literature results justify the possibility to use the variationnal procedure of the Screening Constant by Unit Nuclear Charge formalism to calculate precise resonance energies of doubly ($Nlnl'$, ${}^{2S+1}L^{\pi}$) excited states of two electrons systems. It should be mentioned that the present results are obtained from analytical formulae without any code of calculations or a super-powerful computer. In this work, it has been demonstrated that the variational procedure of the Screening Constant by Unit Nuclear Charge (SCUNC) method can be used to compute precise resonance energies and excitation energies of doubly ($Nlnl'$, ${}^{2S+1}L^{\pi}$) excited states of He-like systems. The new results obtained for $2pnf\,{}^{1,3}G^{e}$ doubly excited states may be benchmaked data for theorists focusing their study on the DES of He-lile systems. It should be underlined that, until this date the variational procedure of the SCUNC formalism has not been applied to atomic systems containing more than two electrons such as lithium isoelectronic sequence. The good agreement obtained in this paper open the way for applying the variational procedure of the SCUNC formalism to investigate the properties of complex atomic systems. Study is in such direction.

4. Conclusion

In this work, resonance energies and excitation energies of the doubly $2sns\,{}^{1,3}S^{e}$, $2snp\,{}^{1,3}P^{0}$, $2pnp\,{}^{1,3}D^{e}$, $2pnd\,{}^{1,3}F^{0}$ and $2pnf\,{}^{1,3}G^{e}$ excited states of He-like systems are reported up to $Z = 10$. Calculations are made in the framework of the variational procedure of the Screening Constant by Unit Nuclear Charge (SCUNC) formalism. New correlated wavefunction of Hylleraas-type adapted to the correct description of electron-electron correlation phenomena in the ($Nlnl'$, ${}^{2S+1}L^{\pi}$) doubly excited states of two-electron systems are constructed. Overall, good agreement is obtained with various theoretical and experimental literature data. The adequacy of the present theory in the treatment of the properties of two-electron systems demonstrates the possibilities to extend the variational procedure of the SCUNC formalism to investigate the properties of complex atomic systems beginning by the lithium isoelectronic sequence.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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