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Calculations Energy of the $(nl^2)\ ^1L^\pi$ Doubly Excited States of Two-Electron Systems via the Screening Constant by Unit Nuclear Charge Formalism

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Abstract

In this work, the total energies of doubly excited states $(ns^2)\ ^1S^e$, $(np^2)\ ^1D^e$, $(nd^2)\ ^1G^e$, $(nf^2)\ ^1I^e$, $(ng^2)\ ^1K^e$, and $(nh^2)\ ^1M^e$ of the helium isoelectronic sequence with $Z \leq 10$ are calculated in the framework of the variational method of the Screening Constant by Unit Nuclear Charge (SCUNC). These calculations are performed using a new wavefunction correlated to Hylleraas-type. The possibility of using the SCUNC method in the investigation of high-lying Doubly Excited States(DES) in two-electron systems is demonstrated in the present work in the case of the $(nl^2)\ ^1L^\pi$ doubly excited states, where accurate total energies are tabulated up to $n = 20$. All the results obtained in this paper are in agreement with the values of the available literature and may be useful for future experimental and theoretical studies on the doubly excited $(nl^2)\ ^1L^\pi$ states of two-electron systems.

Keywords

Doubly Excited States, Helium-Like Systems, Screening Constant by Unit Nuclear Charge (SCUNC), Wave Functions Correlated, Total Energy

1. Introduction

Studies of doubly-excited states of helium-like systems remain an active field of research since the early experiments of Madden and Codling [1] [2] concerning the observation of resonant structures in the absorption spectrum of helium using synchrotron radiation. The strong correlation between electrons in the

doubly excited state of two-electron atomic systems has attracted considerable attention from theorists and experimenters as evidenced by the efforts concentrated in the field over the last twenty years. Theoretical investigations in two-electron Doubly Excited States (DES) are of great interest in connection with the understanding of collisional and radiative processes which take place in hot astrophysical and laboratory plasma [3] [4] [5]. In these investigations, great attention has been paid to the study of symmetric (nl^2) DES where the electron-electron correlation effects may be predominant as revealed by the works of Fano [6]. Several experimental and theoretical studies on doubly excited (nl^2) states have been carried out using different methods. Experimentally, many of these doubly excited states have been observed in electronic impact experiments by Oda *et al.* [7] and Hicks and Comer [8]. In their studies, these authors have worked on the energy spectra of ejected electrons from autoionization states in helium excited by electron impact. Other doubly excited states were observed by ion impact by Rudd [9] and by Bordenave-Montesquieu *et al.* [10]. These doubly excited states were also studied by examining the spectra of ejected electrons by Gelabart *et al.* [11] and by Rodbro *et al.* [12]. From a theoretical point of view, several calculation methods have been used, the complex rotation method [13] [14] [15], the variational method of Hylleraas [16], the double sums over the total hydrogen spectrum formalism [17], the density functional theory [18], the formalism of the Feshbach projection operators [19] [20], the discretization technique [21], the truncated diagonalization method [22], the time-dependent variation perturbation theory [23], and the semi-empirical procedure of the screening constant by unit nuclear charge (SCUNC) method [24] [25] [26], to name a few. In all these *ab-initio* methods, energies of (nl^2) doubly-excited states of He isoelectronic sequence can't be expressed in an analytical formula. In addition, most of these preceding methods require large basis-set calculations involving a fair amount of mathematics complexity. But, it's widely believed that there are distinct advantages to viewing problems of physics within the framework of simple analytical models. Contrary to all these methods, the variational procedure of the SCUNC method makes it possible to calculate the energies of the (nl^2) doubly excited states without a complex mathematical program or calculation code but from a simple analytical expression. In addition, in the recent past the variational procedure of the SCUNC method has been successfully applied to calculations of the energies of doubly excited states $nlnl'$ (n = 2 - 4) in helium-like ions by Sakho [27] using a special Hylleraas-type wave function. These reasons sufficiently justify our choice to apply the variational procedure of the SCUNC method in this study. The goal of the present work is to report accurate total energies and excitation energies of the doubly excited states (nl^2) using the variational procedure of the SCUNC formalism but also to show that it is possible to calculate total energies and precise excitation energies of high-lying up to n = 20 of the doubly excited states (nl^2) without any calculation code or complex mathematical program, without a powerful computer but using a sim-

ple analytical expression. Section 2 gives the correlated wave functions and brief overview of the calculation method. Section 3 gives the presentation and the discussion of our results in the case of doubly excited states $(ns^2)\ ^1S^e$, $(np^2)\ ^1D^e$, $(nd^2)\ ^1G^e$, $(nf^2)\ ^1I^e$, $(ng^2)\ ^1K^e$, and $(nh^2)\ ^1M^e$ of the He-like ions up to $Z = 10$ are made. All our results are compared to available theoretical and experimental data.

2. Theory and Calculations

2.1. Hamiltonian and Hylleraas-Type Wave Functions

The time independent Schrödinger equation for the Helium atom, or the positive ions of its isoelectronic sequence, or of the negative Hydrogen ion, is

$$\hat{H}\Psi = E\Psi \tag{1}$$

where $\hat{H}$ represents the Hamiltonian operator of the considered system, Ψ the trial wave function and E the associated energy.

The Hamiltonian H of the helium isoelectronic series in given by (in atomic units)

$$H = -\frac{1}{2}\Delta_1 - \frac{1}{2}\Delta_2 - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}} \tag{2}$$

In this equation, r_1 and r_2 denote the position of the two electrons from the nucleus, Z is the nuclear charge, Δ_1 is the Laplacian with reference to the coordinates of the vector radius r_1 which detect the position of the electron 1. Δ_2 Laplacian defines the coordinates of the vector radius r_2 which detect the position of the electron 2 and r_{12} inter-electronic distance.

Solving Equation (1) is very difficult because of the term $r_{12} = u = |r_1 - r_2|$ representing electron-electron repulsion. It is therefore necessary to implement a rough calculation method using a correlated wavefunction.

The groundbreaking work in this area was conducted by Hylleraas [28] [29] [30]. The simplest Hylleraas wave function is written as follow:

$$\Psi(r_1, r_2, r_{12}) = (1 + br_{12})\exp\left[-\alpha(r_1 + r_2)\right] \tag{3}$$

Since Hyllerass' original work, tremendous efforts have been made to improve upon that work, using larger and larger expansions, adding more complicated terms. In this present work, we have modified this Hylleraas wavefunction in order to adapt it to the study of symmetrical $(nf^2)\ ^1L^\pi$ doubly excited states in two-electron atomic systems. These wave functions are defined as follow:

$$\Psi(r_1, r_2, r_{12}) = \sum_{\upsilon=0}^{\upsilon=n-\ell-1} \left(n^2 r_0^2\right)^\upsilon \times (1 + C_0 Z r_{12})\exp\left[-\alpha(r_1 + r_2)\right] \tag{4}$$

In this expression, n is the principal quantum number; ℓ is orbital quantum number, r_0 is Bohr radius, C_0 and α are the variational parameters to be determined by minimizing the energy, Z is the nuclear charge number, r_{12} represents the term electron-electron repulsion r_1 and r_2 are the coordinates of electrons with respect to the nucleus.

From the theoretical viewpoint, the Hylleraas variational method is based on the Hylleraas and Undheim theorem [31] according to which, a good approximation of the energy eigenvalue $E(\alpha, C_0)$ is obtained when the minima of the function ($\mathrm{d}^2 E(\alpha, C_0) / \mathrm{d}\alpha \mathrm{d}C_0$) converge with increasing values of the dimension D of the basis states and when the function exhibit a plateau.

Using this theorem, the values of the varitional parameters a and C_0 can be determined by the following conditions:

$$\frac{\partial E(\alpha, C_0)}{\partial C_0} = 0 \tag{5}$$

and

$$\frac{\partial E(\alpha, C_0)}{\partial \alpha} = 0 \tag{6}$$

In the framework of the Ritz' variation principle, the energy $E(\alpha, C_0) = \langle H \rangle (\alpha, C_0)$ is calculated from the relation:

$$E(\alpha, C_0) = \langle H \rangle (\alpha) = \frac{\langle \Psi(\alpha, C_0) | H | \Psi(\alpha, C_0) \rangle}{\langle \Psi(\alpha, C_0) | \Psi(\alpha, C_0) \rangle} \tag{7}$$

In this equation, the correlated wave functions are given by (4) and the Hamiltonian H of the helium isoelectronic series in given by (2) (in atomic units).

Furthermore, the closure relation represents the fact that $|\boldsymbol{r}_1, \boldsymbol{r}_2\rangle$ are continuous bases in the space of the two-electron space, written as follow:

$$\iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 |\boldsymbol{r}_1, \boldsymbol{r}_2\rangle \langle \boldsymbol{r}_1, \boldsymbol{r}_2| = 1 \tag{8}$$

Using this relation, according to (7), we obtain:

$$\begin{aligned} & E(\alpha, C_0) \iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 \langle \Psi(\alpha, C_0) || \boldsymbol{r}_1, \boldsymbol{r}_2\rangle \times \langle \boldsymbol{r}_1, \boldsymbol{r}_2 || \Psi(\alpha, C_0)\rangle \\ & = \iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 \langle \Psi(\alpha, C_0) || \boldsymbol{r}_1, \boldsymbol{r}_2\rangle \hat{H} \langle \boldsymbol{r}_1, \boldsymbol{r}_2 || \Psi(\alpha, C_0)\rangle \end{aligned} \tag{9}$$

By developing this expression (9), we find:

$$E(\alpha, C_0) \iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 \Psi(\alpha, C_0) \times \Psi^*(\alpha, C_0) = \iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 \Psi(\alpha, C_0) \hat{H} \Psi^*(\alpha, C_0) \tag{10}$$

This means:

$$N^* E(\alpha, C_0) = \iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 \Psi(\alpha, C_0) \hat{H} \Psi^*(\alpha, C_0) \tag{11}$$

with the normalization constant

$$N = \iint \mathrm{d}r_1^3 \mathrm{d}r_2^3 |\Psi(\alpha, C_0)|^2 \tag{12}$$

To make it easier to integrate Equation (11), we operate the variable changes in elliptic coordinates by:

$$s = r_1 + r_2; \quad t = r_1 - r_2; \quad u = r_{12} \tag{13}$$

On the basis of these variable changes, the elementary volume element

$$\mathrm{d}\tau = \mathrm{d}^3 r_1 \mathrm{d}^3 r_2 = 2\pi^2 \left(s^2 - t^2\right) u \mathrm{d}s \mathrm{d}u \mathrm{d}t \tag{14}$$

Using these elliptical coordinates, Equation (11) is written as follows

$$NE(\alpha,C_0)=\int_0^{\infty}\mathrm{d}s\int_0^{s}\mathrm{d}u\int_0^{u}\mathrm{d}t\left\{u\left(s^2-t^2\right)\times\left[\left(\frac{\partial\Psi}{\partial s}\right)^2+\left(\frac{\partial\Psi}{\partial t}\right)^2+\left(\frac{\partial\Psi}{\partial u}\right)^2\right]\right.$$
$$\left.+2\left(\frac{\partial\Psi}{\partial u}\right)\times\left[s\left(u^2-t^2\right)\times\frac{\partial\Psi}{\partial s}+t\left(s^2-u^2\right)\times\frac{\partial\Psi}{\partial t}-\Psi^2\times\left(4Zsu-s^2+t^2\right)\right]\right\} \tag{15}$$

with respect to the correlated wave functions given by expression (4), it is expressed as follow

$$\Psi(s,t,u,\alpha,C_0)=\sum_{\upsilon=0}^{\upsilon=n-\ell-1}\left(n^2r_0^2\right)^{\upsilon}\times\left(1+C_0Zu\right)\exp(-\alpha s) \tag{16}$$

Furthermore, according to (12), the normalization constant is written in elliptic coordinates as:

$$N=\int_0^{\infty}\mathrm{d}s\int_0^{s}\mathrm{d}u\int_0^{u}\mathrm{d}t u\left(s^2-t^2\right)\times\Psi^2 \tag{17}$$

2.2. General Formalism of the SCUNC Method

The Screening Constant by Unit Nuclear Charge (SCUNC) formalism is used in this work to calculate the total energies of the symmetrical $(n\ell)\ {}^1L^{\pi}$ doubly excited states of the helium-isoelectronic up to $Z=10$.

In the framework of the Screening Constant by Unit Nuclear Charge (SCUNC) formalism, total energies of the $(N\ell n\ell')\ {}^{2S+1}L^{\pi}$ doubly excited states are expressed in Rydberg (Ry) as below [24] [25]

$$E\left(N\ell n\ell',{}^{2s+1}L^{\pi}\right)=-Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\beta\left(N\ell n\ell',{}^{2s+1}L^{\pi},Z\right)\right]^2\right)\mathrm{Ry} \tag{18}$$

In this equation, the principal quantum numbers N and n, are respectively the inner and the outer electron of the helium-isoelectronic series. In this equation, the β-parameters are screening constant by unit nuclear charge expanded in inverse powers of Z and given by

$$\beta\left(N\ell n\ell',{}^{2s+1}L^{\pi},Z\right)=\sum_{k=1}^{q}f_k\left(\frac{1}{Z}\right)^k \tag{19}$$

where $f_k=f_k\left(N\ell n\ell',{}^{2s+1}L^{\pi}\right)$ are screening constants to be evaluated based on variational predicable using a wavefunction.

For the states $(n\ell)\ {}^1L^{\pi}$, $N=n$ and $l=l'$. Hence, the total energy is written as follow:

$$E\left(n\ell^2,{}^1L^{\pi}\right)=-\frac{Z^2}{n^2}\left\{1+\left[1-\beta\left(n\ell^2,{}^1L^{\pi},Z\right)\right]^2\right\}\mathrm{Ry} \tag{20}$$

Furthermore, in the framework of the screening constant by unit nuclear charge formalism, the β-screening constant is expressed in terms of the variational α-parameter as follow

$$\beta\left(n\ell^2,{}^1L^{\pi},Z,\alpha\right)=\frac{\alpha}{Z^2}\left(1+\frac{2L}{2n+4L-3}\right) \tag{21}$$

In this expression, n denotes the principal quantum number, L characterizes the considered quantum state (S, P, D, F etc.) and α is the variational parameter.

Then, using Equation (21), the total energies of the symmetrical $(n\ell^2)\ ^1L^\pi$ doubly excited states in the helium isoelectronic series is expressed in Rydberg (Ry) as below:

$$E\left(n\ell^2, {}^1L^\pi, Z\right) = -\frac{Z^2}{n^2}\left\{1+\left[1-\frac{\alpha}{Z^2}\left(1+\frac{2L}{2n+4L-3}\right)\right]^2\right\}\text{Ry} \tag{22}$$

In this equation, only the parameter α is unknown. Considering the $(2s^2)\ ^1S^e$ state of Helium-like ions (Z = 2 - 10), we calculated the values of the variational parameters α and C_0, the results of which are presented in Table 1.

The Equation (22) is used to calculate the total energies of the $(n\ell^2)\ ^1L^\pi$ doubly excited states of helium-like ions without a complex calculation program.

3. Results and Discussions

The results obtained in the present study for $(ns^2)\ ^1S^e$, $(np^2)\ ^1D^e$, $(nd^2)\ ^1G^e$, $(nf^2)\ ^1I^e$, $(ng^2)\ ^1K^e$, and $(nh^2)\ ^1M^e$ with $n \le 20$ in the helium-like ions up to Z = 10 are listed in Tables 1-16 and compared to various other calculations. Table 1 presents our results on the calculation of the variational parameters α and C_0. These variational parameters are calculated by determining the expression of $E = f(\alpha, C_0)$ from the expression (15) and the wavefunction (16), then according to conditions (5) and (6) we obtained a system of equations whose resolution to give the values of the variational parameters α and C_0 with $2 \le Z \le 10$. All calculations in this work were performed with the calculation program MAXIMA. In Tables 2-7 we have listed our present results E on the calculation of the total energies of the $(n\ell^2)\ ^1L^\pi$ doubly excited states of the helium isoelectronic sequence with $2 \le Z \le 10$ and $2 \le n \le 20$ obtained using Equation (22). Table 2 shows our present results of the $(ns^2)\ ^1S^e$ (n = 2 - 20) doubly excited states of helium - like systems (Z = 2 - 10). Table 3 shows our present results of the $(np^2)\ ^1D^e$ (n = 2 - 20) doubly excited states of helium-like systems (Z = 2 - 10). Table 4 shows our present results of the $(nd^2)\ ^1G^e$ (n = 3 - 20) doubly excited states of helium-like systems (Z = 2 - 10). Table 5 shows our present results of the $(nf^2)\ ^1I^e$ (n = 4 - 20) doubly excited states of helium-like systems (Z = 2 - 10). Table 6 shows our present results of the $(ng^2)\ ^1K^e$ (n = 5 - 20) doubly excited states of helium-like systems (Z = 2 - 10). Table 7 shows our present results of the $(nh^2)\ ^1M^e$ (n = 6 - 20) doubly excited states of helium-like systems (Z = 2 - 10).

Table 8 shows a comparison of the present calculations for the $(ns^2)\ ^1S^e$ states with the results of the semi-empirical procedure of the screening constant by

Table 1. Values of variational parameters α and C_0 of helium-like ions (Z = 2 - 10).

Z	2	3	4	5	6	7	8	9	10
α	0.96105	1.40362	1.93837	2.47394	3.00997	3.54627	4.08276	4.61938	5.15608
C_0	0.24982	0.26539	0.28139	0.29115	0.29771	0.30244	0.30600	0.30878	0.31102

Table 2. Total energy ($-E$) for (ns^2) $^1S^e$ (n = 2 - 20) doubly excited states of helium-like systems (Z = 2 - 10). The energies E are in eV.

Z	2	3	4	5	6	7	8	9	10
ns^2	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
$2s^2$	21.45940	52.42276	96.46002	154.07753	225.28724	310.09472	408.50290	520.51348	646.12749
$3s^2$	9.53751	23.29900	42.87112	68.47890	100.12766	137.81987	181.55684	231.33932	287.16777
$4s^2$	5.36485	13.10569	24.11501	38.51938	56.32181	77.52368	102.12572	130.12837	161.53187
$5s^2$	3.43350	8.38764	15.43360	24.65241	36.04596	49.61515	65.36046	83.28216	103.38040
$6s^2$	2.38438	5.82475	10.71778	17.11973	25.03192	34.45497	45.38921	57.83483	71.79194
$7s^2$	1.75179	4.27941	7.87429	12.57776	18.39080	25.31385	33.34718	42.49090	52.74510
$8s^2$	1.34121	3.27642	6.02875	9.62985	14.08045	19.38092	25.53143	32.53209	40.38297
$9s^2$	1.05972	2.58878	4.76346	7.60877	11.12530	15.31332	20.17298	25.70437	31.90753
$10s^2$	0.85838	2.09691	3.85840	6.16310	9.01149	12.40379	16.34012	20.82054	25.84510
$11s^2$	0.70940	1.73298	3.18876	5.09347	7.44751	10.25107	13.50423	17.20706	21.35959
$12s^2$	0.59609	1.45619	2.67945	4.27993	6.25798	8.61374	11.34730	14.45871	17.94799
$13s^2$	0.50791	1.24078	2.28308	3.64681	5.33224	7.33952	9.66871	12.31985	15.29296
$14s^2$	0.43795	1.06985	1.96857	3.14444	4.59770	6.32846	8.33679	10.62272	13.18628
$15s^2$	0.38150	0.93196	1.71484	2.73916	4.00511	5.51279	7.26227	9.25357	11.48671
$16s^2$	0.33530	0.81911	1.50719	2.40746	3.52011	4.84523	6.38286	8.13302	10.09574
$17s^2$	0.29702	0.72557	1.33509	2.13256	3.11816	4.29197	5.65402	7.20434	8.94294
$18s^2$	0.26493	0.64719	1.19086	1.90219	2.78132	3.82833	5.04325	6.42609	7.97688
$19s^2$	0.23778	0.58086	1.06881	1.70723	2.49626	3.43595	4.52635	5.76746	7.15931
$20s^2$	0.21459	0.52423	0.96460	1.54078	2.25287	3.10095	4.08503	5.20513	6.46127

Table 3. Total energy ($-E$) for the (np^2) $^1D^e$ (n = 2 - 20) doubly excited states of helium-like systems (Z = 2 - 10). The energies E are in eV.

Z	2	3	4	5	6	7	8	9	10
np^2	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
$2p^2$	19.40691	48.98779	91.46705	147.50214	217.11646	300.32082	397.12093	507.52007	631.52025
$3p^2$	8.78089	22.04020	41.04506	66.07698	97.14529	134.25433	177.40638	226.60274	281.84424
$4p^2$	5.00135	12.50334	23.24244	37.37258	54.89863	75.82286	100.14645	127.87008	158.99418
$5p^2$	3.23051	8.05224	14.94822	24.01485	35.25507	48.67024	64.26108	82.02799	101.97124
$6p^2$	2.25935	5.61862	10.41971	16.72838	24.54660	33.87526	44.71484	57.06561	70.92775
$7p^2$	1.66926	4.14359	7.67801	12.32015	18.07141	24.93241	32.90349	41.98486	52.17663
$8p^2$	1.28386	3.18216	5.89260	9.45120	13.85900	19.11648	25.22388	32.18135	39.98897
$9p^2$	1.01824	2.52067	4.66512	7.47977	10.96542	15.12242	19.95098	25.45121	31.62316
$10p^2$	0.82739	2.04609	3.78505	6.06690	8.89227	12.26146	16.17460	20.63180	25.63311
$11p^2$	0.68564	1.69405	3.13258	5.01981	7.35623	10.14210	13.37752	17.06258	21.19731
$12p^2$	0.57748	1.42570	2.63546	4.22227	6.18654	8.52846	11.24814	14.34564	17.82100
$13p^2$	0.49305	1.21645	2.24800	3.60082	5.27527	7.27152	9.58964	12.22970	15.19171

Continued

$14p^2$	0.42590	1.05014	1.94014	3.10718	4.55154	6.27337	8.27274	10.54969	13.10425
$15p^2$	0.37159	0.91576	1.69148	2.70854	3.96718	5.46753	7.20965	9.19357	11.41933
$16p^2$	0.32706	0.80563	1.48776	2.38200	3.48857	4.80759	6.33910	8.08313	10.03971
$17p^2$	0.29008	0.71424	1.31875	2.11115	3.09165	4.26033	5.61723	7.16240	8.89585
$18p^2$	0.25904	0.63757	1.17700	1.88402	2.75882	3.80148	5.01203	6.39051	7.93692
$19p^2$	0.23273	0.57263	1.05694	1.69168	2.47700	3.41297	4.49963	5.73700	7.12510
$20p^2$	0.21024	0.51712	0.95436	1.52736	2.23626	3.08112	4.06199	5.17886	6.43177

Table 4. Total energy ($-E$) for the (nd^2) $^1G^e$ (n = 3 - 20) doubly excited states of helium-like systems (Z = 2 - 10). The energies E are in eV.

Z	2	3	4	5	6	7	8	9	10
nd^2	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
$3d^2$	8.66987	21.84944	40.76532	65.70667	96.68358	133.70073	176.76056	225.86449	281.01340
$4d^2$	4.92028	12.36512	23.04025	37.10534	54.56575	75.42399	99.68137	127.33865	158.39628
$5d^2$	3.17227	7.95352	14.80410	23.82457	35.01823	48.38660	63.93048	81.65034	101.54646
$6d^2$	2.21670	5.54666	10.31481	16.59001	24.37447	33.66919	44.47473	56.79139	70.61936
$7d^2$	1.63727	4.08980	7.59970	12.21693	17.94306	24.77881	32.72457	41.78056	51.94691
$8d^2$	1.25931	3.14100	5.83274	9.37235	13.76099	18.99922	25.08731	32.02543	39.81368
$9d^2$	0.99901	2.48852	4.61839	7.41825	10.88897	15.03099	19.84451	25.32967	31.48654
$10d^2$	0.81206	2.02051	3.74790	6.01801	8.83154	12.18883	16.09005	20.53529	25.52462
$11d^2$	0.67323	1.67337	3.10257	4.98032	7.30720	10.08347	13.30928	16.98469	21.10977
$12d^2$	0.56728	1.40875	2.61088	4.18993	6.14639	8.48047	11.19228	14.28189	17.74935
$13d^2$	0.48458	1.20239	2.22761	3.57401	5.24199	7.23174	9.54334	12.17686	15.13234
$14d^2$	0.41878	1.03834	1.92304	3.08470	4.52364	6.24003	8.23394	10.50542	13.05450
$15d^2$	0.36555	0.90576	1.67701	2.68951	3.94357	5.43931	7.17681	9.15611	11.37723
$16d^2$	0.32189	0.79709	1.47539	2.36575	3.46841	4.78350	6.31106	8.05115	10.00378
$17d^2$	0.28563	0.70689	1.30810	2.09717	3.07430	4.23960	5.59311	7.13489	8.86493
$18d^2$	0.25518	0.63119	1.16777	1.87190	2.74378	3.78351	4.99112	6.36666	7.91012
$19d^2$	0.22936	0.56706	1.04888	1.68110	2.46388	3.39729	4.48139	5.71620	7.10173
$20d^2$	0.20728	0.51223	0.94729	1.51807	2.22474	3.06737	4.04598	5.16061	6.41126

Table 5. Total energy ($-E$) for the (nf^2) $^1I^e$ (n = 4 - 20) doubly excited states of helium-like systems (Z = 2 - 10). The energies E are in eV.

Z	2	3	4	5	6	7	8	9	10
nf^2	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
$4f^2$	4.88462	12.30381	22.95032	36.98626	54.41728	75.24595	99.47366	127.10120	158.12904
$5f^2$	3.14469	7.90632	14.73497	23.73314	34.90428	48.25001	63.77119	81.46828	101.34160
$6f^2$	2.19522	5.51005	10.26127	16.51924	24.28632	33.56358	44.35159	56.65069	70.46106
$7f^2$	1.62030	4.06098	7.55759	12.16131	17.87382	24.69587	32.62789	41.67010	51.82266
$8f^2$	1.24569	3.11795	5.79909	9.32793	13.70571	18.93303	25.01016	31.93731	39.71456

Continued

$9f^2$	0.98792	2.46980	4.59110	7.38224	10.84417	14.97736	19.78201	25.25829	31.40626
$10f^2$	0.80291	2.00511	3.72546	5.98842	8.79474	12.14478	16.03872	20.47668	25.45872
$11f^2$	0.66560	1.66055	3.08390	4.95571	7.27660	10.04685	13.26662	16.93598	21.05500
$12f^2$	0.56085	1.39796	2.59518	4.16925	6.12067	8.44969	11.15644	14.24097	17.70334
$13f^2$	0.47911	1.19322	2.21428	3.55645	5.22017	7.20563	9.51294	12.14215	15.09331
$14f^2$	0.41408	1.03048	1.91163	3.06967	4.50496	6.21768	8.20792	10.47572	13.02111
$15f^2$	0.36150	0.89898	1.66715	2.67654	3.92746	5.42004	7.15437	9.13050	11.34844
$16f^2$	0.31836	0.79119	1.46683	2.35448	3.45441	4.76676	6.29158	8.02891	9.97877
$17f^2$	0.28253	0.70173	1.30062	2.08731	3.06206	4.22496	5.57608	7.11544	8.84307
$18f^2$	0.25245	0.62665	1.16118	1.86323	2.73302	3.77064	4.97615	6.34956	7.89091
$19f^2$	0.22694	0.56304	1.04306	1.67344	2.45436	3.38592	4.46815	5.70109	7.08475
$20f^2$	0.20513	0.50866	0.94211	1.51127	2.21629	3.05726	4.03422	5.14719	6.39618

Table 6. Total energy ($-E$) for the (ng^2) $^1K^e$ (n = 5 - 20) doubly excited states of helium-like systems (Z = 2 - 10). The energies E are in eV.

Z	2	3	4	5	6	7	8	9	10
ng^2	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
$5g^2$	3.12860	7.87865	14.69439	23.67940	34.83728	48.16967	63.67745	81.36112	101.22099
$6g^2$	2.18229	5.48788	10.22878	16.47627	24.23275	33.49937	44.27669	56.56508	70.36472
$7g^2$	1.60979	4.04301	7.53129	12.12654	17.83049	24.64395	32.56734	41.60091	51.74480
$8g^2$	1.23704	3.10320	5.77753	9.29943	13.67022	18.89050	24.96058	31.88065	39.65082
$9g^2$	0.98071	2.45755	4.57320	7.35860	10.81474	14.94209	19.74091	25.21133	31.35343
$10g^2$	0.79685	1.99482	3.71044	5.96858	8.77005	12.11521	16.00426	20.43731	25.41443
$11g^2$	0.66044	1.65182	3.07117	4.93891	7.25569	10.02181	13.23743	16.90264	21.01750
$12g^2$	0.55643	1.39049	2.58428	4.15487	6.10279	8.42829	11.13149	14.21248	17.67130
$13g^2$	0.47529	1.18677	2.20488	3.54406	5.20475	7.18718	9.49144	12.11760	15.06570
$14g^2$	0.41075	1.02488	1.90347	3.05891	4.49158	6.20167	8.18926	10.45441	12.99715
$15g^2$	0.35858	0.89408	1.66002	2.66713	3.91576	5.40604	7.13807	9.11188	11.32750
$16g^2$	0.31579	0.78687	1.46055	2.34621	3.44413	4.75445	6.27724	8.01254	9.96037
$17g^2$	0.28026	0.69791	1.29506	2.08000	3.05297	4.21408	5.56341	7.10097	8.82681
$18g^2$	0.25042	0.62326	1.15624	1.85673	2.72494	3.76097	4.96489	6.33671	7.87646
$19g^2$	0.22513	0.56001	1.03865	1.66763	2.44715	3.37729	4.45810	5.68962	7.07185
$20g^2$	0.20350	0.50594	0.93816	1.50606	2.20982	3.04952	4.02521	5.13690	6.38462

Table 7. Total energy ($-E$) for the (nh^2) $^1M^e$ (n = 6 - 20) doubly excited states of helium-like systems (Z = 2 - 10). The energies E are in eV.

Z	2	3	4	5	6	7	8	9	10
ng^2	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$	$-E$
$6h^2$	2.17365	5.47302	10.20698	16.44740	24.19675	33.45620	44.22633	56.50750	70.29992
$7h^2$	1.60263	4.03075	7.51331	12.10275	17.80083	24.60839	32.52586	41.55350	51.69145

Continued

$8h^2$	1.23105	3.09296	5.76253	9.27960	13.64551	18.86088	24.92603	31.84117	39.60639
$9h^2$	0.97565	2.44891	4.56056	7.34189	10.79392	14.91715	19.71182	25.17808	31.31603
$10h^2$	0.79253	1.98746	3.69968	5.95436	8.75234	12.09399	15.97952	20.40904	25.38263
$11h^2$	0.65672	1.64550	3.06193	4.92670	7.24049	10.00360	13.21620	16.87839	20.99022
$12h^2$	0.55320	1.38501	2.57628	4.14431	6.08964	8.41253	11.11313	14.19150	17.64770
$13h^2$	0.47247	1.18199	2.19790	3.53485	5.19329	7.17345	9.47544	12.09932	15.04514
$14h^2$	0.40827	1.02068	1.89734	3.05082	4.48152	6.18962	8.17522	10.43837	12.97912
$15h^2$	0.35638	0.89037	1.65461	2.66000	3.90688	5.39542	7.12568	9.09773	11.31160
$16h^2$	0.31384	0.78358	1.45575	2.33987	3.43625	4.74502	6.26625	7.99999	9.94626
$17h^2$	0.27851	0.69497	1.29078	2.07435	3.04594	4.20567	5.55361	7.08979	8.81423
$18h^2$	0.24886	0.62062	1.15241	1.85167	2.71865	3.75344	4.95611	6.32669	7.86519
$19h^2$	0.22372	0.55763	1.03520	1.66308	2.44149	3.37052	4.45021	5.68061	7.06172
$20h^2$	0.20222	0.50380	0.93504	1.50195	2.20471	3.04341	4.01809	5.12877	6.37547

Table 8. Comparison of the present calculations on total energies of the doubly (ns^2) $^1S^e$ (n = 2 - 7) excited states of helium-like systems (Z = 2 - 10) with available literature values. All results are expressed in eV.

States $ns^2\ ^1S^e$	Z	2	3	4	5	6	7	8	9	10
$2s^2\ ^1S^e$	$-E^p$	21.45940	52.42276	96.46002	154.07753	225.28724	310.09472	408.50290	520.51348	646.12749
	$-E^a$	21.19004	51.75672	95.92923	153.70728	225.09117	310.08075	408.67588	520.87683	646.68334
	$-E^b$	21.16692	51.86067	96.15032	154.04035	225.53336	310.63042	409.33154	521.63766	647.54894
	$-E^c$	21.19388	51.75719	95.92188	153.72003	225.09205	310.08931	408.67561	520.87758	646.68514
	$-E^d$	21.16678	51.86054	96.15046	154.03995	225.53363	310.62879	409.33289	521.62923	647.54853
	$-E^e$	21.16460	51.86312	96.14556	154.03886	225.53540	310.62852	409.33358	521.63576	647.54377
	$-E^k$	21.19000	52.00000	96.43000	154.45000	226.09000	311.32000	410.17000	522.62000	648.67000
$3s^2\ ^1S^e$	$-E^p$	9.53751	23.29900	42.87112	68.47890	100.12766	137.81987	181.55684	231.33932	287.16777
	$-E^a$	9.64562	23.38097	43.16324	68.99254	100.86877	138.79205	182.76224	232.77934	288.84349
	$-E^c$	9.64065	23.48066	43.29425	69.06061	100.76559	138.39575	181.93676	231.37273	286.68564
	$-E^f$	9.62031	23.40586	43.23344	69.10874	101.02769	138.99573	183.00878	233.07091	289.17806
	$-E^g$	9.41772	23.00300	42.63519	68.31431	100.04059	137.81366	181.63365	231.50082	287.41489
	$-E^d$	9.62017	23.40586	43.23316	69.10847	101.02837	138.99627	183.00918	233.06996	289.17887
	$-E^e$	9.62466	23.40178	43.22527	69.10738	101.02224	138.98212	182.99653	233.05866	289.17125
	$-E^h$	9.58385	22.83852	42.25657	67.85978					
	$-E^k$	9.42000	23.11000	42.86000	68.65000	100.48000	138.37000	182.30000	232.27000	288.30000
$4s^2\ ^1S^e$	$-E^p$	5.36485	13.10569	24.11501	38.51938	56.32181	77.52368	102.12572	130.12837	161.53187
	$-E^a$	5.49139	13.25971	24.42944	39.00059	56.97332	78.34732	103.12276	131.29961	162.87803
	$-E^j$	5.46963	13.25848	24.45079	39.03978	57.02243	78.41957	103.20357	131.40206	162.99422
	$-E^g$	5.29751	12.93915	23.98235	38.42684	56.27276	77.52023	102.16899	130.21932	161.67093

Continued

	$-E^{j}$	5.35000	13.18000	24.49000	38.08000	57.95000	78.43000	103.18000	130.42000	163.20000
	$-E^{k}$	5.30000	13.00000	24.11000	38.61000	56.52000	77.83000	102.54000	130.65000	162.17000
	$-E^{p}$	3.43350	8.38764	15.43360	24.65241	36.04596	49.61515	65.36046	83.28216	103.38040
$5s^2\ ^1S^e$	$-E^{a}$	3.53993	8.52792	15.69281	25.03462	36.55320	50.24884	66.12138	84.17083	104.39706
	$-E^{g}$	3.39040	8.28111	15.34872	24.59325	36.01455	49.61291	65.38816	83.34034	103.46942
	$-E^{j}$	3.22000	8.15000	15.33000	24.28000	36.45000	49.69000	65.29000	83.20000	103.67000
	$-E^{k}$	3.39000	8.32000	15.43000	24.71000	36.17000	49.81000	65.63000	83.62000	103.79000
	$-E^{p}$	2.38438	5.82475	10.71778	17.11973	25.03192	34.45497	45.38921	57.83483	71.79194
$6s^2\ ^1S^e$	$-E^{a}$	2.47025	5.94161	10.92483	17.41965	25.42606	34.94460	45.97447	58.51647	72.57007
	$-E^{k}$	2.35000	5.78000	10.71000	17.16000	25.12000	34.59000	45.57000	58.07000	72.07000
	$-E^{p}$	1.75179	4.27941	7.87429	12.57776	18.39080	25.31385	33.34718	42.49090	52.74510
	$-E^{j}$	1.72000	4.27000	7.82000	12.63000	18.46000	25.39000	33.44000	42.58000	52.83000
$7s^2\ ^1S^e$	$-E^{k}$	1.73000	4.24000	7.87000	12.61000	18.46000	25.41000	33.48000	42.66000	52.95000
	$-E^{a}$	1.821259	4.37559	8.04042	12.81602	18.70239	25.69953	33.80717	43.02557	53.35447

[p]Present work, values calculated from Equation (22), [a]sakho [32], [b]Ho [33], [c]Sow *et al.* [36], [d]Gning *et al.* [37], [e]Konte *et al.* [38], [f]Ho [34], [g]Sakho *et al.* [24], [h]Ray *et al.* [23], [i]Ho [35], [j]Diouf *et al.* [39], [k]Sakho [40].

unit nuclear charge method of Sakho *et al.* [24] [32], with the results of the complex rotation of Ho [33] [34] [35], with the results of Sow *et al.* [36] who used the variational method of the SCUNC formalism, with the complex rotation values of Gning *et al.* [37], with the Konté *et al.* [38] data, with the data from the time-dependent variation perturbation theory of Ray *et al.* [23], with the results of Diouf *et al.* [39] and finally with the data from the modified slater theory of Sakho [40]. The observation of our results shown in this table shows that our present calculations are generally in good agreement with the results of the cited authors up to $Z = 10$.

In **Table 9**, we compare our calculations for $(np^2)\ ^1D^e$ states with the results of the calculations of Badiane *et al.* [41], with the data of Sakho [24] [32] [40], with the results of the complex rotation calculations (CRC) of Ho and Bhatia [13], with the values of Ivanov and Safronova [17], with the results of the variational method calculations of Hylleraas de Biaye *et al.* [16] and finally with those obtained by Roy *et al.* [18] who applied the functional density theory (FDT). Here, the agreements between the calculations are considered good. **Table 10** compares our results for the $(nd^2)\ ^1G^e$ (n = 3 - 10) states with those obtained by Badiane *et al.* [41], sakho [32] [40], Bachau *et al.* [19], Biaye *et al.* [16], Ivanov and Safronova [17], Diouf *et al.* [39], Ray *et al.* [23] and Roy *et al.* [18]. As regards the $(nd^2)\ ^1G^e$-levels, comparison shows also a good agreement up to $Z = 10$. **Table 11** shows the results of our present calculations of the total energies of the doubly excited $(nf^2)\ ^1I^e$ (n = 4 - 10) states of helium-like systems up to $Z = 10$, which we compare with those obtained by Biaye *et al.* [16], Badiane *et al.* [41], Sakho *et al.* [32] [40], Ho [20], Diouf *et al.* [39], and Sow *et al.* [36]. A comparative

Table 9. Comparison of the present calculations on total energies of the doubly (np^2) $^1D^e$ (n = 2 - 5) excited states of helium-like systems (Z = 2 - 10) with available literature values. All results are expressed in eV.

States $np^2\ ^1D^e$	Z	2	3	4	5	6	7	8	9	10
	$-E^p$	19.40691	48.98779	91.46705	147.50214	217.11646	300.32082	397.12093	507.52007	631.52025
	$-E^a$	19.12145	48.19274	90.86974	147.15243	217.04218	300.53626	397.63604	508.34153	632.65271
	$-E^b$	19.12008	48.19002	90.86565	147.14835	217.03537	300.52809	397.62652	508.33201	632.64183
	$-E^c$	19.10104	48.18049	90.81667	147.03678	216.85169	300.26551	397.28229	507.90071	632.12481
$2p^2\ ^1D^e$	$-E^d$	19.41533	48.39138	90.97314	147.16059	216.95238	300.35123	397.35577	507.96601	632.18195
	$-E^e$	19.18403	48.39138	91.14321	147.47352	217.39320	300.90769	398.02245	508.73746	633.05408
	$-E^f$		47.78049	90.54592	146.90888					
	$-E^g$	19.12000	48.19000	90.87000	147.15000	217.03000	300.53000	397.63000	508.33000	632.64000
	$-E^h$	19.12000	48.73000	91.95000	148.77000	219.20000	303.24000	400.88000	512.13000	636.9800
	$-E^p$	8.78089	22.04020	41.04506	66.07698	97.14529	134.25433	177.40638	226.60274	281.84424
	$-E^a$	8.98656	22.56777	42.23345	67.96182	99.74201	137.57401	181.45511	231.38394	287.36187
	$-E^b$	9.19065	22.62219	42.10011	67.62440	99.19642	136.81482	180.48094	230.19344	285.95368
	$-E^c$	9.33759	22.91608	42.54502	68.22033	99.94474	137.71687	181.53675	231.40299	287.31561
$3p^2\ ^1D^e$	$-E^d$	8.98929	22.11198	41.28241	66.49785	97.76238	135.07193	178.43056	227.83422	283.28559
	$-E^e$	8.66955	21.80585	41.03206	66.37676	97.83993	135.44200	179.18160	229.06009	285.07883
	$-E^f$		21.82626	41.09465	66.40669					
	$-E^g$	8.50000	21.42000	40.38000	65.40000	96.46000	133.57000	176.72000	225.92000	281.17000
	$-E^h$	8.50000	21.66000	40.87000	66.12000	97.42000	134.77000	178.17000	227.61000	283.10000
	$-E^p$	5.00135	12.50334	23.24244	37.37258	54.89863	75.82286	100.14645	127.87008	158.99418
	$-E^a$	5.11302	12.82881	23.97187	38.52453	56.48269	77.84364	102.61009	130.77661	162.34727
	$-E^b$	5.383774	13.08187	24.18141	38.68236	56.58609	77.88989	102.59512	130.70314	162.21121
$4p^2\ ^1D^e$	$-E^c$		13.12678	24.26168	38.79937	56.73984	78.08038	102.82642	130.97117	162.51598
	$-E^e$	4.893969	12.23560	22.93921	37.02246	54.49354	75.36060	99.629084	127.30035	158.37441
	$-E^f$		12.37982	23.26166	37.543563					
	$-E^g$	4.78000	12.05000	22.72000	36.79000	54.26000	75.13000	99.41000	127.08000	158.16000
	$-E^h$	4.78000	12.18000	22.99000	37.19000	54.80000	75.81000	100.22000	128.03000	159.24000
	$-E^p$	3.23051	8.05224	14.94822	24.01485	35.25507	48.67024	64.26108	82.02799	101.97124
	$-E^a$	3.28849	8.24777	15.40029	24.73652	36.25238	49.94652	65.81892	83.86824	104.09583
$5p^2\ ^1D^e$	$-E^b$	3.53068	8.51309	15.67240	25.00863	36.52178	50.21047	66.07743	84.12131	104.34209
	$-E^f$		7.95117	14.92953	24.08344					
	$-E^g$		7.71000	14.54000	23.54000	34.72000	48.08000	63.62000	81.33000	101.22000
	$-E^h$	3.06000	7.80000	14.71000	23.80000	35.07000	48.52000	64.14000	81.94000	101.92000

[p]Present work, values calculated from Equation (22), [a]Badiane *et al.* [41], [b]Sakho *et al.* [32], [c]Ho and Bhatia [13], [d]Ivanov and Safronova [17], [e]Biaye *et al.* [16], [f]Roy *et al.* [18], [g]Sakho *et al.* [24], [h]Sakho [40].

Table 10. Comparison of the present calculations on total energies of the doubly (nd^{ℓ}) $^{1}G^{e}$ (n = 3 - 10) excited states of helium-like systems (Z = 2 - 10) with available literature values. All results are expressed in eV.

States $nd^{\ell}\ ^{1}G^{e}$	Z	2	3	4	5	6	7	8	9	10
	$-E^{p}$	8.66987	21.84944	40.76532	65.70667	96.68358	133.70073	176.76056	225.86449	281.01340
	$-E^{a}$	8.31716	20.87794	39.56401	64.34271	95.23988	132.11133	175.08356	224.11306	279.19300
	$-E^{b}$	8.30492	21.07114	39.88374	64.74271	95.64942	132.60385	175.60466	224.65184	279.74676
	$-E^{c}$	8.38927	21.14053	39.89191	64.68148	95.48479	132.35623	175.26860	224.22190	279.18892
$3d^{\ell}\ ^{1}G^{e}$	$-E^{d}$	8.58383	21.54734	40.49464	65.45157	96.431745	133.44332	176.491754	225.57839	280.70732
	$-E^{e}$	8.57703	21.24938	39.97082	64.73727	95.551456	132.41065	175.321664	224.27632	279.27872
	$-E^{f}$	8.50000	21.73000	41.06000	65.52000	96.77000	134.03000	178.740000	227.96000	282.92000
	$-E^{g}$	8.48996	21.15414	39.84021	64.56448					
	$-E^{i}$	8.35000	21.43000	40.55000	65.72000	96.94000	134.21000	177.52000	226.87000	283.28000
	$-E^{p}$	4.92028	12.36512	23.04025	37.10534	54.56575	75.42399	99.68137	127.33865	158.39628
	$-E^{a}$	4.78104	12.03016	22.74328	36.89048	54.45816	75.43951	99.82773	127.62417	158.82476
$4d^{\ell}\ ^{1}G^{e}$	$-E^{b}$	4.85315	12.17709	22.90383	37.03063	54.55884	75.48985	99.82092	127.55342	158.68869
	$-E^{d}$	4.89397	12.23560	22.93785	37.02110	54.49218	75.35787	99.62500	127.29354	158.77713
	$-E^{g}$	5.31166	12.51724	23.33105	37.55445					
	$-E^{i}$	4.70000	12.05000	22.81000	36.97000	54.53000	75.49000	99.85000	127.62000	158.78000
	$-E^{p}$	3.17227	7.95352	14.80410	23.82457	35.01823	48.38660	63.93048	81.65034	101.54646
	$-E^{a}$	3.09394	7.79198	14.71456	23.83718	35.14896	48.64581	64.32502	82.18386	102.22233
$5d^{\ell}\ ^{1}G^{e}$	$-E^{b}$	3.17965	7.92259	14.84245	23.93786	35.21155	48.66214	64.28964	82.09406	102.07538
	$-E^{h}$	3.06672	7.81375	14.73769	23.83854					
	$-E^{f}$	3.04000	7.59000	14.57000	23.600000	34.80000	48.31000	63.87000	81.59000	101.68000
	$-E^{i}$	3.01000	7.71000	14.60000	23.66000	34.90000	48.31000	63.91000	81.67000	101.62000
	$-E^{p}$	2.21670	5.54666	10.31481	16.59001	24.37447	33.66919	44.47473	56.79139	70.61936
$6d^{\ell}\ ^{1}G^{e}$	$-E^{a}$	2.16331	5.45044	10.28591	16.64929	24.53379	33.93533	44.85254	57.28271	71.22855
	$-E^{f}$	2.04000	5.34000	10.12000		24.28000	33.51000	44.31000	56.74000	70.50000
	$-E^{h}$		5.45997	10.28182	16.61392					
	$-E^{i}$	2.09000	5.36000	10.14000	16.43000	24.23000	33.55000	44.38000	56.72000	70.57000
	$-E^{p}$	1.63727	4.08980	7.59970	12.21693	17.94306	24.77881	32.72457	41.78056	51.94691
$7d^{\ell}\ ^{1}G^{e}$	$-E^{a}$	1.59595	4.02456	7.58926	12.27914	18.08469	25.00727	33.04279	42.18991	52.44861
	$-E^{f}$	1.50000	3.93000	7.44000	12.04000	17.87000	24.57000	32.64000	41.68000	51.83000
	$-E^{h}$		3.87490	7.57293	12.23152					
	$-E^{i}$	1.53000	3.94000	7.45000	12.07000	17.80000	24.65000	32.60000	41.67000	51.85000
$8d^{21}G^{e}$	$-E^{p}$	1.25931	3..14100	5.83274	9.37235	13.76099	18.99922	25.08731	32.02543	39.81368
	$-E^{a}$	1.22587	3.09257	5.82867	9.42738	13.88052	19.18810	25.34740	32.35841	40.22114
$9d^{\ell}\ ^{1}G^{e}$	$-E^{p}$	0.99901	2.48852	4.61839	7.41825	10.88897	15.03099	19,84451	25.32967	31.48654
	$-E^{a}$	0.97145	2.44902	4.61641	7.46408	10.98659	15.18531	20.05614	25.60046	31.81826
$10d^{\ell}\ ^{1}G^{e}$	$-E^{p}$	0.81206	2.02051	3.74790	6.01801	8.83154	12.18883	16.09005	20.53529	25.52462
	$-E^{a}$	0.78777	1.98779	3.74565	6.05589	8.91173	12.31587	16.26424	20.75820	25.79638

[p]Present work, values calculated from Equation (22), [a]Badiane *et al.* [41], [b]Sakho *et al.* [32], [c]Bachau *et al.* [19], [d]Biaye *et al.* [16], [e]Ivanov and Safronova [17], [f]Diouf *et al.* [39], [g]Ray *et al.* [23], [h]Roy *et al.* [18], [i]Sakho [40].

Table 11. Comparison of the present calculations on total energies of the doubly (nf^2) $^1I^e$ (n = 4 - 10) excited states of helium-like systems (Z = 2 - 10) with available literature values. All results are expressed in eV.

States $nf^2\ ^1I^e$	Z	2	3	4	5	6	7	8	9	10
	$-E^p$	4.88462	12.30381	22.95032	36.98626	54.41728	75.24595	99.47366	127.10120	158.12904
	$-E^a$	4.88580	12.21518	22.89565	36.94625	54.37377	75.18776	99.38820	126.98190	157.96206
	$-E^b$	4.34429	11.11585	21.49427	35.32989	52.59824	73.28841	97.39225	124.90568	155.82597
$4f^2\ ^1I^e$	$-E^c$	4.40552	11.35530	21.70788	35.46051	52.61456	73.17140	97.12829	124.48662	155.24772
	$-E^d$	4.63682								
	$-E^e$	5.03000	12.12000	23.43000	37.54000	55.20000	76.38000	100.82000	128.65000	159.25000
	$-E^f$	5.02139	12.43411	23.27892	37.66635	55.38959	76.43250	100.78073	128.41943	159.33212
	$-E^g$	4.62000	11.92000	22.63000	36.74000	54.25000	75.16000	99.47000	127.18000	158.30000
	$-E^p$	3.14469	7.90632	14.73497	23.73314	34.90428	48.25001	63.77119	81.46828	101.34160
$5f^2\ ^1I^e$	$-E^b$	2.80685	7.35524	14.16352	23.18954	34.41423	47.82944	63.42973	81.21236	101.17735
	$-E^e$	2.93000	7.59000	14.41000	23.50000	34.69000	48.17000	63.55000	81.41000	100.29000
	$-E^g$	2.95000	7.63000	14.48000	23.51000	34.72000	48.10000	63.66000	81.40000	101.31000
	$-E^p$	2.19522	5.51005	10.26127	16.51924	24.28632	33.56358	44.35159	56.65069	70.46106
$6f^2\ ^1I^e$	$-E^b$	1.97691	5.18241	9.96072	16.27921	24.12697	33.49448	44.38040	56.78062	70.69789
	$-E^e$	2.07000	5.28000	10.04000	16.22000	24.10000	33.41000	44.22000	56.54000	70.38000
	$-E^g$	2.05000	5.30000	10.06000	26.33000	24.11000	33.40000	44.21000	56.53000	70.35000
	$-E^p$	1.62030	4.06098	7.55759	12.16131	17.87382	24.69587	32.62789	41.67010	51.82266
$7f^2\ ^1I^e$	$-E^b$	1.46533	3.83817	7.36884	12.03287	17.81937	24.72289	32.74209	41.87423	52.11931
	$-E^e$	1.50000	3.88000	7.36000	11.95000	17.75000	24.57000	32.48000	41.51000	51.63000
	$-E^g$	1.51000	3.89000	7.39000	11.99000	17.71000	24.54000	32.48000	41.53000	51.69000
$8f^2\ ^1I^e$	$-E^p$	1.24569	3.11795	5.79909	9.32793	13.70571	18.93303	25.01016	31.93731	39.71456
	$-E^b$	1.12791	2.95379	5.66677	9.20697	13.68732	18.98537	25.135152	32.13663	39.99120
$9f^2\ ^1I^e$	$-E^p$	0.98792	2.46980	4.59110	7.38224	10.84417	14.97736	19.78201	25.25829	31.40626
	$-E^b$	0.89389	2.34289	4.49124	7.32666	10.84101	15.03156	19.89560	25.43447	31.64547
$10f^2\ ^1I^e$	$-E^p$	0.80291	2.00511	3.72546	5.98842	8.79474	12.14478	16.03872	20.47668	25.45872
	$-E^b$	0.72654	1.90208	3.64632	5.94568	8.796078	12.19342	16.13771	20.62758	25.66169

[p]Present work, values calculated from Equation (22), [a]Biaye *et al.* [16], [b]Badiane *et al.* [41], [c]Sakho *et al.* [32], [d]Ho [20], [e]Diouf *et al.* [39], [f]Sow *et al.* [36]; [g]Sakho [40].

reading of the results mentioned in this table shows a good agreement between the present SCUNC results and the data found in the literature. For level $4f^2$, it should be noted that comparison with the results of Biaye *et al.* [16] indicates satisfactory agreement for Z = 2 - 10.

In Table 12 and Table 13, we compare the results of our calculations of the total energies of the doubly excited states (ng^2) $^1K^e$ and (nh^2) $^1M^e$ with those of Sakho [40] and Diouf *et al.* [39]. The agreements between the calculations are seen to be generally good. It is worth mentioning that there are not many results

Table 12. Comparison of the present calculations on total energies of the doubly $(ng^2)\ ^1K^e$ (n = 5 - 10) excited states of helium-like systems (Z = 2 - 10) with available literature values. All results are expressed in eV.

States $ng^2\ ^1K^e$	Z	2	3	4	5	6	7	8	9	10
	$-E^p$	3.12860	7.87865	14.69439	23.67940	34.83728	48.16967	63.67745	81.36112	101.22099
$5g^2\ ^1K^e$	$-E^a$	2.93000	7.59000	14.42000	23.44000	34.63000	47.99000	63.54000	81.26000	101.16000
	$-E^b$	2.93000	7.59000	14.41000	23.41000	34.69000	47.90000	63.71000	81.24000	101.28000
	$-E^p$	2.18229	5.48788	10.22878	16.47627	24.23275	33.49937	44.27669	56.56508	70.36472
$6g^2\ ^1K^e$	$-E^a$	2.03000	5.27000	10.02000	16.27000	24.05000	33.33000	44.12000	56.43000	70.25000
	$-E^b$	2.04000	5.23000	10.04000	16.22000	24.10000	33.41000	44.22000	56.54000	70.38000
	$-E^p$	1.60979	4.04301	7.53129	12.12654	17.83049	24.64395	32.56734	41.60091	51.74480
$7g^2\ ^1K^e$	$-E^a$	1.49000	3.87000	7.36000	11.96000	17.67000	24.49000	32.42000	41.46000	51.61000
	$-E^b$	1.50000	3.88000	7.36000	11.95000	17.75000	24.43000	32.48000	41.51000	51.63000
$8g^2\ ^1K^e$	$-E^p$	1.23704	3.10320	5.77753	9.29943	13.67022	18.89050	24.96058	31.88065	39.65082
$9g^2\ ^1K^e$	$-E^p$	0.98071	2.45755	4.57320	7.35860	10.81474	14.94209	19.74091	25.21133	31.35343
$10g^2\ ^1K^e$	$-E^p$	0.79685	1.99482	3.71044	5.96858	8.77005	12.11521	16.00426	20.43731	25.41443

[p]Present work, values calculated from Equation (22), [a]sakho [40], [b]Diouf *et al.* [39].

Table 13. Comparison of the present calculations on total energies of the doubly $(nh^2)\ ^1M^e$ (n = 6 - 10) excited states of helium-like systems (Z = 2 - 10) with available literature values. All results are expressed in eV.

States $nh^2\ ^1M^e$	Z	2	3	4	5	6	7	8	9	10
	$-E^p$	2.17365	5.47302	10.20698	16.44740	24.19675	33.45620	44.22633	56.50750	70.29992
$6h^2\ ^1M^e$	$-E^a$	2.02000	5.25000	9.99000	16.24000	24.01000	33.28000	44.07000	56.37000	70.18000
	$-E^b$	2.04000	5.23000	9.97000	16.22000	24.10000	33.27000	44.06000	56.37000	70.18000
	$-E^p$	1.60263	4.03075	7.51331	12.10275	17.80083	24.60839	32.52586	41.55350	51.69145
$7h^2\ ^1M^e$	$-E^a$	1.48000	3.86000	7.34000	11.93000	17.64000	24.45000	32.38000	41.42000	51.56000
	$-E^b$	1.46000	3.82000	7.36000	11.95000	17.63000	24.43000	32.33000	41.51000	51.63000
$8h^2\ ^1M^e$	$-E^p$	1.23105	3.09296	5.76253	9.27960	13.64551	18.86088	24.92603	31.84117	39.60639
$9h^2\ ^1M^e$	$-E^p$	0.97565	2.44891	4.56056	7.34189	10.79392	14.91715	19.71182	25.17808	31.31603
$10h^2\ ^1M^e$	$-E^p$	0.79253	1.98746	3.69968	5.95436	8.75234	12.09399	15.97952	20.40904	25.38263

[p]Present work, values calculated from Equation (22), [a]Sakho [40], [b]Diouf *et al.* [39].

on the states and the only ones available to our knowledge are those of the authors Sakho [40] and Diouf *et al.* [39]. Moreover for $n > 7$ there are no results available so we think that the results cited up to $n = 20$ in this work may be interesting for future experimental and theoretical studies on these states.

In Tables 14-16 are quoted results for excitation energies of helium-like ions with $Z \leq 10$. Our excitation energies are calculated with respect to the accurate ground state energies of Frankowski and Pekeris [42]. Comparison indicates that our excitation energies for $(ns^2)\ ^1S^e$, $(np^2)\ ^1D^e$ and $(nd^2)\ ^1G^e$ levels agree well

Table 14. Comparison of the present calculations on the variationnal calculation of the excitation energies of the doubly excited states (ns^2) $^1S^e$ (n = 2 - 5) of the He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in eV: 1 Ryd = 13.605698 eV.

States $ns^2\ ^1S^e$	Z	2	3	4	5
$2s^2\ ^1S^e$	E^p	57.5548	145.6737	275.1269	445.4159
	E^a	57.8487	146.2476	275.4882	445.5893
	E^b	58.6460	147.7633	277.6814	448.4165
	E^c	57.8200	146.3400	275.6600	445.7800
$3s^2\ ^1S^e$	E^p	69.4768	174.7975	328.7157	531.0145
	E^b	69.4299	175.2577	329.3314	531.6345
	E^c	69.5900	175.1000	328.9600	531.1800
	E^d	69.3972	174.7026	328.4035	530.5215
$4s^2\ ^1S^e$	E^p	73.6494	184.9908	347.4719	560.9740
	E^a	73.5469	184.8470	347.1820	560.5819
	E^b	73.4218	184.9939	347.5358	561.0989
	E^c	73.7100	185.1600	347.6100	561.0600
$5s^2\ ^1S^e$	E^p	75.5808	189.7089	356.1533	574.8410
	E^b	75.1905	189.4457	355.9523	574.7972
	E^c	75.6200	189.8200	356.2400	574.9000

[p]Present work, [a]Ho [33], [b]Ray and Mukherjee [23], [c]Sakho *et al.* [24], [d]Ho [34].

Table 15. Comparison of the present calculations on the variationnal calculation of the excitation energies of the doubly excited states (np^2) $^1D^e$ (n = 2 - 5) of the He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in eV: 1 Ryd = 13.605698 eV.

States $np^2\ ^1D^e$	Z	2	3	4	5
$2p^2\ ^1D^e$	E^p	59.6074	149.1087	280.1198	451.9913
	E^a	59.8900	149.9100	280.7200	452.3400
	E^b		149.9157	280.7699	452.4575
	E^c	60.3249	150.6749	281.6815	453.8316
	E^d		150.3239	281.0420	452.5854
	E^e		150.0599	280.9522	452.6670
$3p^2\ ^1D^e$	E^p	70.2334	176.0563	330.5418	533.4165
	E^a	70.5100	176.6900	331.2100	534.0900
	E^b		176.4251	330.8008	533.5502
	E^c	70.0693	175.9788	330.1368	533.3216
	E^d		176.2699	330.4933	533.0876
	E^f		176.4441	330.8253	533.5611

Continued

$4p^2\ ^1D^e$	E^p	74.0129	185.5932	348.3444	562.1209
	E^a	74.2300	186.0500	348.8700	562.7000
	E^c	73.5851	185.3858	347.9848	561.5942
	E^d		185.7178	348.3249	561.9479
$5p^2\ ^1D^e$	E^p	75.7838	190.0443	356.6386	575.4786
	E^a	75.9500	190.3900	357.2000	575.9500
	E^c		189.6852	358.2598	575.0067
	E^d		190.1451	356.6570	575.4095

[p]Present work, [a]Sakho *et al.* [24]; [b]Ho and Bhatia [13]; [c]Ray and Mukherjee [23]; [d]Roy *et al.* [18]; [e]Macias *et al.* [21]; [f]Bachau *et al.* [19].

Table 16. Comparison of the present calculations on the variationnal calculation of the excitation energies of the doubly excited states $(nd^2)\ ^1G^e$ (n = 2 - 5) of the He-like systems with some theoretical results available in the literature consulted for Z = 2 - 5. All the results are expressed in eV.

States $nd^2\ ^1G^e$	Z	2	3	4	5
$3d^2\ ^1G^e$	E^p	70.3444	176.2471	330.8215	533.7867
	E^a	70.6245	176.9557	331.6960	534.8128
	E^b	71.0544	177.3529	332.0144	535.0495
	E^c	70.5238	176.9421	331.7477	534.9298
$4d^2\ ^1G^e$	E^p	74.0939	185.7314	348.5466	562.3881
	E^b	74.3089	186.0362	348.7685	562.5167
	E^c	73.7021	185.5789	348.2569	561.9398
$5d^2\ ^1G^e$	E^p	75.8420	190.1429	356.7827	575.6688
	E^b	75.9470	190.2838	356.8475	575.6543
	E^c	75.6667	189.8185	358.3822	575.0611
$6d^2\ ^1G^e$	E^p	76.7975	192.5498	361.2720	582.9034
	E^b	76.8667	192.6376	361.3047	582.8789
$7d^2\ ^1G^e$	E^p	77.3770	194.0067	363.9872	587.2765
	E^b	77.4273	194.0717	364.0150	587.2600

[p]Present work; [a]Bachau *et al.* [19]; [b]Roy *et al.* [18]; [c]Ray and Mukherjee [23].

with results obtained by the authors cited in Tables 14-16. Overall, we find a good agreement between our results and those of these authors. For all the states studied, the results obtained are in good agreement with the theoretical results we have consulted. The small difference noted between our results and those of the authors mentioned above is explained by the used method and the choice of the correlated wavefunction. The actual results presented in these different tables sufficiently show the good agreements between the current calculations and the different *ab-initio* results for the doubly excited states $(ns^2)\ ^1S^e$, $(np^2)\ ^1D^e$, (nd^2)

$^1G^e$, (nl^2) $^1I^e$, (ng^2) $^1K^e$, and (nh^2) $^1M^e$ of the He-like ions up to $Z = 10$. This very good agreement sufficiently justifies the validity of the variational procedure of the SCUNC method to give the precise values obtained directly from an analytical expression, unlike all the *ab-initio* methods cited in this document. Furthermore, the results quoted up to $n = 20$ in this work may be interesting for future experimental and theoretical studies in the doubly excited states (nl^2) $^1L^\pi$. In summary, the manuscript reports on new calculations for key atomic-structure parameters of important fundamental few-body systems (helium and helium-like ions). While not allowing more precision tests of physics due to the neglect of relativistic, spin, and QED effects, such results can still be helpful in the future development of theories to describe more complex atoms, or may be further developed to study the time-dependent evolution of atoms in external (e.g. laser) fields.

4. Conclusion

In this paper, the total energies and excitation energies of the doubly excited (ns^2) $^1S^e$, (np^2) $^1D^e$, (nd^2) $^1G^e$, (nf^2) $^1I^e$, (ng^2) $^1K^e$, and (nh^2) $^1M^e$ states of helium-like ions up to Z =10 are reported. These energies are calculated in the framework of the variationnal procedure of the Screening Constant by Unit Nuclear Charge (SCUNC) formalism. In this present work, a new wavefunction correlated to Hylleraas-type adapted to the correct description of electron-electron correlation phenomena in the (nl^2) doubly excited states of helium-like systems has been constructed. Our results for total energies and the excitation energies are in good agreement with the values cited in the experimental and theoretical literature. Furthermore, for $n > 10$, no theoretical and experimental values from the literature are available for direct comparison. The good precision obtained in this work underlines that the results quoted up to $n = 20$ in this work may be interesting for future experimental and theoretical studies in the doubly excited states (nl^2) $^1L^\pi$. The results presented in this paper show that it is therefore possible to perform an analytical calculation of the total energies of the (nl^2) doubly excited states for helium-like ions, without having to resort to excessively complicated calculations or a computer program.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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