

# A Physical Insight into the Origin of the Corrections to the Magnetic Moment of Free and Bound Electron

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## Abstract

The main goal of the present work is a unitary approach of the physical origin of the corrections to the magnetic moment of free and bound electron. Based on this approach, estimations of lowest order corrections were easily obtained. In the non-relativistic limit, the Dirac electron appears as a distribution of charge and current extended over a region of linear dimension of the order of Compton wavelength, which generates its magnetic moment. The e.m. mass (self-energy) of electron outside this region does not participate to this internal dynamics, and consequently does not contribute to the mass term in the formula of the magnetic moment. This is the physical origin of the small increase of the magnetic moment of free electron compared to the value given by Dirac equation. We give arguments that this physical interpretation is self-consistent with the QED approach. The bound electron being localized, it has kinetic energy which means a mass increase from a relativistic point of view, which determines a magnetic moment decrease (relativistic Breit correction). On the other hand, the e.m. mass of electron decreases at the formation of the bound state due to coulomb interaction with the nucleus. We estimated this e.m. mass decrease of bound electron only in its internal dynamics region, and from it the corresponding increase of the magnetic moment (QED correction). The corrections to the mass value are at the origin of the lowest order corrections to the magnetic moment of free and bound electron.

## Keywords

Magnetic Moment of Dirac Electron, Electromagnetic Self-Energy, Physical Origin of the Corrections to the Magnetic Moment of Free and Bound Electron

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### 1. Introduction

In the non-relativistic limit, the Dirac electron appears as a distribution of charge and current extended over a region of linear dimension of about reduced Compton wavelength ( $\lambda_e$ ), a subject largely treated in the literature [1]-[7]. This explains the appearance of the Darwin term and the appearance of an interaction term ( $-\mu H$ , spin-orbit interaction) characteristic to the presence of a magnetic moment of the electron equal to  $\frac{e\hbar}{2m}$ .

But in the Schwinger approximation (lowest order QED correction) the magnetic moment of free electron is [8] [9]:

$$\mu = \frac{e\hbar}{2m}(1 + \alpha/2\pi) \tag{1}$$

A simple physical insight into the origin of this correction ( $\alpha/2\pi$ ) to the magnetic moment of free electron was proposed in [10] [11]. Based on [1]-[7], it was assumed for the Dirac electron that the dynamics (distribution of charge and current) which generates the magnetic moment takes place inside a spherical region of diameter equal to Compton wavelength ( $\lambda_e$ ). The electron self-energy (e.m. mass) outside this region does not participate to this “internal” dynamics and in consequence does not contribute to the magnetic moment of electron.

The electrostatic self-energy (mass) in the electric field generated by the electron charge in the exterior of a sphere of diameter  $2r$ , which enclosed the electron charge, is:

$$U_e = \int_r^\infty u_e 4\pi r^2 dr = \frac{e^2}{2r} \tag{2}$$

where:

$$u_e = \frac{E^2}{8\pi} = \frac{e^2}{8\pi r^4}$$

is the e.m. energy density in the electric field generated by electron.

This means that the electron mass (self-energy) outside the sphere of diameter  $\lambda_e$ , which does not participate to the “internal” dynamics, is:

$$\delta m_e c^2 = e^2/\lambda_e \tag{3}$$

It results a straightforward calculus of the anomalous magnetic moment of the electron [10] [11]:

$$\mu = \frac{e\hbar}{2(m - \delta m_e)} = \frac{e\hbar}{2m \left(1 - \frac{\delta m_e c^2}{mc^2}\right)} \tag{4}$$

But:

$$\frac{\delta m_e c^2}{mc^2} = \frac{e^2/\lambda_e}{mc^2} = \frac{e^2}{2\pi\lambda_e * mc^2} = \frac{e^2}{2\pi(\hbar/mc) * mc^2} = \frac{\alpha}{2\pi} \tag{5}$$

where  $\lambda_e = \hbar/mc$ .

If we introduce relation (5) in relation (4) it results the Schwinger approximation:

$$\mu = \frac{e\hbar}{2m\left(1 - \frac{\alpha}{2\pi}\right)} \cong \frac{e\hbar}{2m}\left(1 + \frac{\alpha}{2\pi}\right) \quad (6)$$

J. Schwinger has written in [8]: “The e.m. self-energy of a free electron can be ascribed to an e.m. mass, which must be added to the mechanical mass of the electron. The only meaningful statements of the theory involve the sum of these two masses, which is the experimental mass of a free electron... However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which means that the e.m. mass must be a small correction  $\alpha m_0$  to the mechanical mass  $m_0$ ”.

This weak interaction supports the idea that the e.m. mass outside the region of diameter  $\approx \lambda_e$  does not contribute to the “internal” dynamics, which generates the magnetic moment.

Section 2 gives more arguments that this physical interpretation of the origin of the anomalous magnetic moment of free electron is self-consistent with the QED approach. An alternative QED approach based on self-energy diagram is presented.

In Section 3, the same line of reasoning is applied to bound electron. Estimations of the lowest order corrections to the magnetic moment (gyromagnetic factor) of bound electron in H-like ions are presented.

## 2. An Alternative QED Approach

There is only the vertex graph (**Figure 1**) one has to consider in QED, in the lowest order (second order) of approximation, to calculate the anomalous magnetic moment of electron [12] [13] [14] [15]. A virtual photon is emitted by the electron before the interaction and is reabsorbed after the interaction.

In [13] it is written: “The ‘dressed’ electron wave function already contains its quantum corrections (interactions with virtual photons). These virtual photons carry off some of the mass of the electron while leaving its charge unaltered and this can affect the magnetic moment generated by the electron during interactions”. In other words in the moment of interaction some of the mass of the electron is carried off by virtual photons. This points to an explanation of the origin of the correction to the magnetic moment of free electron, similar to that given in Section 1.

But the mass of the electron carried off by the virtual photons can be estimated in QED taking into account the self-energy diagram (**Figure 2**).

In [14], it was derived the following expression for the electron self-energy:

$$\delta m_e = \frac{2m\pi^2 e^2}{(2\pi)^4} \int_0^1 dx (1+x) \ln \frac{m^2 x^2 + \mu^2 (1-x)}{m^2 x^2} \quad (7)$$

where  $\mu$  is the energy cut off.

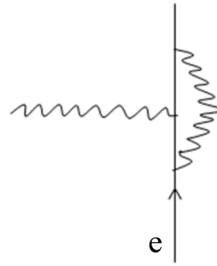


Figure 1. Vertex diagram.

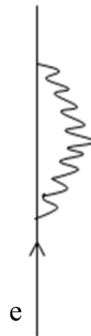


Figure 2. Self-energy diagram.

For a wave number cut off equal to that used in relation (5),  $r_\mu = \frac{\lambda_e}{2}$ , this means  $\frac{\mu}{m} = \frac{\lambda_e}{\lambda_e/2} = \frac{1}{\pi}$ , it results from relation (7), by numerical calculation, that:

$$\frac{\delta m_e}{m} = 0.87 \frac{\alpha}{2\pi} \tag{8}$$

This value is relatively close to that obtained in relation (5):  $\frac{\delta m_e}{m} = \frac{\alpha}{2\pi}$ , which explain the correction to the magnetic moment. We note here that in [14] it is proved that Equation (7), for a cut off at small energy ( $\mu \rightarrow 0$ ), gives the same result for the self-energy like formula (2).

If we take into consideration the formula (7.28) of self-energy from [15] and assuming that the photon mass is equal to zero, it results:

$$\delta m_e = \frac{3m_0\alpha}{2\pi} \left( \log_{10} \frac{\Lambda}{m_0} + \frac{1}{4} \right) \tag{9}$$

where  $m_0$  is the mechanical mass, which at low energy cut off is lower with only about  $\alpha m_0$  than the electron mass [8]. For  $\frac{\Lambda}{m_0} = 1$  this means the wave number cut off is at the reduced Compton wavelength  $\lambda_e$ , it results:

$$\frac{\delta m_e}{m_0} = \frac{3 \alpha}{4 2\pi} \tag{10}$$

again a value relatively close to that obtained in relation (5). In relation (9) the exact value of correction ( $\alpha/2\pi$ ) is obtained for a wave number cut off at  $0.8\lambda_e$ .

Because the self-energy cannot be separated by the mechanical mass, in [14] it is written that “the calculation of the electron self-energy by itself does not have any direct experimental implications”. And in [12] it is written that “it is not clear what exactly one would measure to test whatever result we find by evaluating the self-energy”.

The direct connexion established here between the correction to the magnetic moment of electron and the e.m mass (self-energy) which does not participate to the “internal” dynamics of electron (weak coupling with the mechanical mass) could represent a test for the self-energy formula derived in QED, at low energy cut off.

A possible limitation of this approach: it is probable that there is not a sharp frontier (a cut off) but a transition region between the “internal” dynamics region and the e.m. mass “outside” this dynamics region.

### 3. On the Physical Origin of the Corrections to the Magnetic Moment of Bound Electron

The expression of the magnetic moment of free electron  $\left(\frac{e\hbar}{2m}\right)$  is presented usually as follows:  $\mu = g_s \frac{e}{2m} S$ , where the spin  $S = \hbar/2$  and the gyromagnetic factor  $g_s = 2$ .

The magnetic moment anomaly  $(\alpha/2\pi)$  of free electron, analysed in Sections 1 and 2, is included in the expression of the gyromagnetic factor:  $g_s = 2\left(1 + \frac{\alpha}{2\pi}\right)$ .

Let’s analyses the physical origin of the corrections to the magnetic moment of electron bound into the atom. On one hand, being localized in the atom, the electron has kinetic energy, this means its mass, in particular its mechanical mass, increases from a relativistic point of view. We note this increase  $\delta m_{kin} c^2$ .

On the other hand, due to electron-nucleus Coulomb interaction the e.m. mass (self-energy) of bound electron decreases as compared to unbound electron. We note this decrease  $\delta m_e^{in} c^2$ .

Following the same line of reasoning as in Section 1 (see relation 4), the expression of the magnetic moment of the bound electron is:

$$\begin{aligned} \mu &= \frac{e\hbar}{2(m - \delta m_e + \delta m_{kin} - \delta m_e^{in})} = \frac{e\hbar}{2m\left(1 - \frac{\delta m_e c^2}{mc^2} + \frac{\delta m_{kin} c^2}{mc^2} - \frac{\delta m_e^{in} c^2}{mc^2}\right)} \quad (11) \\ &\cong \frac{e\hbar}{2m} \left(1 + \frac{\delta m_e c^2}{mc^2} - \frac{\delta m_{kin} c^2}{mc^2} + \frac{\delta m_e^{in} c^2}{mc^2}\right) = \frac{e\hbar}{2m} \left(1 + \frac{\alpha}{2\pi} - \frac{\delta m_{kin} c^2}{mc^2} + \frac{\delta m_e^{in} c^2}{mc^2}\right) \end{aligned}$$

where we have used relation (5) for  $\frac{\delta m_e c^2}{mc^2}$ .

From (11) it results that the gyromagnetic factor  $g^*$  of the bound electron is:

$$g^* = 2\left(1 + \frac{\alpha}{2\pi} - \frac{\delta m_{kin} c^2}{mc^2} + \frac{\delta m_e^{in} c^2}{mc^2}\right) \quad (12)$$

The gyromagnetic factor in our calculations is with asterisk.

From the literature [16] [17] [18] [19] [20] it is well known that the gyromagnetic factor of the bound electron is:

$$g = g_D + \Delta g_{QED} \tag{13}$$

where  $g_D$  is the point-nucleus Dirac value (relativistic Breit correction) and  $\Delta g_{QED}$  is the QED correction. The corrections due to nuclear recoil, nuclear size, and nuclear polarization are not taken into account.

For ns states of H-like ions the point-nucleus Dirac value is [19]:

$$g_D = 2 \left( 1 + \frac{2}{3} \frac{E - m}{m} \right) = 2 \left( 1 - \frac{1}{3} \frac{(\alpha Z)^2}{n^2} + \dots \right) \tag{14}$$

where  $E$  is the Dirac energy:

$$E = mc^2 \left[ 1 - \frac{(\alpha Z)^2}{2n^2} + \dots \right]$$

The QED correction of the gyromagnetic factor of electron in ns states of H-like ions is [19]:

$$\Delta g_{QED} = 2 \left( \frac{\alpha}{2\pi} + \frac{\alpha}{2\pi} \frac{(\alpha Z)^2}{6n^2} + \dots \right) \equiv 2 \left( \frac{\alpha}{2\pi} + \frac{\alpha}{6\pi} \frac{(\alpha Z)^2}{2n^2} + \dots \right) \tag{15}$$

The first term,  $\frac{\alpha}{2\pi}$ , is the correction to the magnetic moment of free electron.

In the present work, from Equation (12) it results that the corrections are:

$$g_D^* = 2 \left( 1 - \frac{\delta m_{kin} c^2}{mc^2} \right) \tag{16}$$

and

$$\Delta g_{QED}^* = 2 \left( \frac{\alpha}{2\pi} + \frac{\delta m_e^{in} c^2}{mc^2} \right) \tag{17}$$

### 3.1. The Estimation of Relativistic Breit Correction to the Gyromagnetic Factor of Bound Electron

To estimate  $g_D^*$  (Equation (16)) we take into account the Virial Theorem. We can write for any level of an atom that the mean kinetic energy  $\langle E_{kin} \rangle$  of the electron bound into the atom is equal, in absolute value, with half the mean potential energy  $\langle V \rangle$  and consequently with the binding energy  $E_{bind}$ :

$$\delta m_{kin} c^2 = \langle E_{kin} \rangle = \frac{1}{2} |\langle V \rangle| = |E_{bind}| = mc^2 \frac{(\alpha Z)^2}{2n^2} \tag{18}$$

Replacing this expression in Equation (16), one obtains:

$$g_D^* = 2 \left( 1 - \frac{\delta m_{kin} c^2}{mc^2} \right) = 2 \left( 1 - \frac{mc^2 \frac{(\alpha Z)^2}{2n^2}}{mc^2} \right) = 2 \left( 1 - \frac{1}{2} \frac{(\alpha Z)^2}{n^2} \right) \tag{19}$$

This result is similar with the Breit correction, Equation (14), less the factor 1/2 which appears in the second term in brackets instead of factor 1/3 which appears in Equation (14).

Let's take into account the interaction of the bound electron with an external magnetic field. In the Larmor theorem it is shown that the application of an external magnetic field to an atom has like effect an additional rotation movement of the bound electron on his atomic orbit around the nucleus, this means without the change of the radius of the orbit [21]. This Larmor rotation motion is added or is subtracted to the orbital motion of the bound electron, resulting in the change of rotation frequency and in consequence of energy of that atomic level, as presented in the classical treatment of Zeeman effect [21].

This splitting of energy levels in an external magnetic field is used to determine the gyromagnetic factor of bound electron. The expectation value  $\langle r \rangle$  of the electron-nucleus distance is the relevant quantity for the bound electron magnetic moment [18] [20]. For ns states of H-like ions:

$$\langle r \rangle_n = \frac{3}{2} a_0 \frac{n^2}{Z} \tag{20}$$

The potential energy  $V^*$  at this relevant distance gets:

$$\langle V^* \rangle = -\frac{e^2 Z}{\langle r \rangle_n} = -\frac{2 e^2 Z^2}{3 a_0 n^2} = -2mc^2 \frac{(\alpha Z)^2}{3n^2}$$

and relation (18) gets:

$$\delta m_{kin}^* c^2 = \langle E_{kin}^* \rangle = \frac{1}{2} \langle V^* \rangle = mc^2 \frac{(\alpha Z)^2}{3n^2} \tag{21}$$

Replacing this expression in Equation (16) it results a correction which is identical with the Dirac value (relativistic Breit correction) of the gyromagnetic factor of bound electron in ns states of H-like ions, Equation (14):

$$g_D^* = 2 \left( 1 - \frac{\delta m_{kin}^* c^2}{mc^2} \right) = 2 \left( 1 - \frac{mc^2 \frac{(\alpha Z)^2}{3n^2}}{mc^2} \right) = 2 \left( 1 - \frac{1}{3} \frac{(\alpha Z)^2}{n^2} \right) \tag{22}$$

### 3.2. The Estimation of QED Correction to the Gyromagnetic Factor of Bound Electron

We analyze  $\Delta g_{QED}^*$ , Equation (17). The first term in brackets, as in formula (15), is the correction  $(\alpha/2\pi)$ , for the free electron already discussed in Sections 1 and 2.

The second term is due to the e.m. mass lost (released) by the electron at the formation of the bound state due to coulomb interaction with the nucleus. But the e.m. mass of the electron outside the region of diameter  $\approx \lambda_e$ , does not participate in the "internal" dynamics, which is at the origin of the magnetic moment of free electron (see Sections 1 and 2). Therefore we must estimate the e.m.

mass lost by the bound electron only in its close vicinity (inside the region of dimension  $\approx \lambda_e$ ), which influences its “internal” dynamics and by consequence its magnetic moment.

Let’s start our analysis on a simplified model of a hydrogen atom: the electron cloud is concentrated on a spherical surface of radius  $r$ . The electric field generated by the proton in the region of the electron cloud cancels the electric field, of opposite sign, generated by electron cloud. More exactly the electric field in the exterior of the electron cloud is zero, because the total electric charge (proton plus electron charge) enclosed in the sphere of radius  $r$  is zero. Consequently the self-energy of electron gets zero in the exterior of spherical surface of radius  $r$ . We must estimate the self-energy (mass) which is lost by the electron only in its close vicinity ( $\approx \lambda_e$ ), this means between the spherical surfaces of radius  $r$  and  $r + \lambda_e$ . This is the term  $\delta m_e^{in} c^2$  in relations (11), (12) and (17).

The same reasoning applies to the self-energy lost by electron in its close vicinity due to the coulomb interaction with the nucleus in H-like ions.

The e.m. mass of electron cloud in the region between the spherical surface of radius  $r$  and spherical surface of radius  $r + \lambda_e$ , where  $\lambda_e \ll r$ , is given by Equation (2):

$$U_e = \int_r^{r+\lambda_e} u_e(r) 4\pi r^2 dr = \frac{e^2 \lambda_e}{2r(r + \lambda_e)} \cong \frac{e^2}{2r^2} \lambda_e \tag{23}$$

Using the uncertainty principle ( $p \sim \hbar/r$ ) and the Virial Theorem (equation 18) one obtains:

$$\left\langle \frac{p^2}{2m} \right\rangle \cong \left\langle \frac{\hbar^2}{2mr^2} \right\rangle = |E_{bind}| \tag{24}$$

It results for H-like ions in ns states:

$$\left\langle \frac{1}{r^2} \right\rangle = \frac{2m}{\hbar^2} |E_{bind}| = \frac{2m}{\hbar^2} mc^2 \frac{(\alpha Z)^2}{2n^2} \tag{25}$$

Introducing this expression in Equation (23) it results that the e.m. mass lost by the bound electron only in its close vicinity, at the formation of H-like ions, due to the interaction with the nucleus is:

$$\delta m_e^{in} c^2 = U_e = \frac{e^2}{2} \frac{2m}{\hbar^2} mc^2 \frac{(\alpha Z)^2}{2n^2} \lambda_e = e^2 \frac{m}{\hbar^2} mc^2 \frac{(\alpha Z)^2}{2n^2} \frac{\hbar}{mc} = \alpha mc^2 \frac{(\alpha Z)^2}{2n^2} \tag{26}$$

Replacing  $\delta m_e^{in} c^2$  from Equation (26) in Equation (17), it results the following estimation of the QED correction of gyromagnetic factor of bound electron in ns states of H-like ions,;

$$\Delta g_{QED}^* = 2 \left( \frac{\alpha}{2\pi} + \frac{\delta m_e^{in} c^2}{mc^2} \right) = 2 \left( \frac{\alpha}{2\pi} + \alpha \frac{(\alpha Z)^2}{2n^2} \right) \tag{27}$$

This expression is in accord with the lowest order correction calculated in QED (Equation (15)), less a factor  $1/6\pi$  which multiplies the second term in brackets in Equation (15).



Let's compare the ratio of self-energy to electron mass in our calculus (Section 1) to that calculated in QED, for a cut-off at  $\lambda_e$ . From relation (5), one obtains that this ratio is equal to:

$$\frac{\delta m_e}{m} = \frac{e^2/\lambda_e}{mc^2} = \frac{e^2}{mc^2} \frac{mc}{\hbar} = \alpha \quad (28)$$

But in QED calculus (Equation (10)), for the same cut-off at  $\lambda_e$ , the same ratio is  $\frac{3}{4} \frac{\alpha}{2\pi}$ . This means that the value of self-energy of electron in its close vicinity in our calculus (Equation (26) and second term in brackets in Equation (27)) should be multiplied by a factor  $\frac{3}{4} \frac{1}{2\pi} = \frac{3}{8\pi}$ . This is only about 2 times larger than the factor  $1/6\pi$  which appears in the second term in brackets in QED calculus (Equation (15)).

#### 4. Conclusions

The corrections to the gyromagnetic factor, and implicitly to the magnetic moment, of free and bound electron are calculated in QED with very high precision. The main goal of the present work was a unitary approach of the physical origin of these corrections. Based on this approach, estimations of lowest order corrections were relatively easily obtained.

In the non-relativistic limit, the Dirac electron appears as a distribution of charge and current extended over a region of linear dimension  $\approx \lambda_e$  and this explains the appearance of the magnetic moment of electron equal to  $\frac{e\hbar}{2m}$ . The e.m. mass (self-energy) of electron outside this region is weak coupled to the “internal” dynamics and therefore does not contribute to the mass term which appears in the expression of the magnetic moment. It results a small increase of the magnetic moment (gyromagnetic factor), which represents the lowest order correction to the magnetic moment of free electron.

This physical origin of the correction is self-consistent with the significance of the vertex diagram, which in the lowest order is the only graph to consider in QED to calculate the anomalous magnetic moment of free electron. In the vertex diagram the virtual photons carry off some of the mass of the electron in the moment of interaction with the magnetic field, while leaving its charge unaltered. But this mass carried off by virtual photons can be calculated in QED taking into account the self-energy diagram. For a wave number cut-off at  $\lambda_e/2$  in the self-energy formula from [14] and at  $\lambda_e$  in self-energy formula from [15] one obtains values of e.m mass relatively close to that obtained in our estimation of the e.m. mass which does not contribute to the magnetic moment, resulting in the lowest order correction to the magnetic moment. By this alternative QCD approach a direct relation was established between electron self-energy calculated in QED for low energy cut off and the correction to the magnetic moment of free electron, which could represent a test for the self-energy formula.

The changes of the same parameter mass due to electron binding in atom were analyzed to determine the physical origin of the corrections to the magnetic moment of bound electron in ns states of H-like ions. On one hand due to its localization into the atom, electron has kinetic energy, this means its mass increases from a relativistic point of view and consequently its magnetic moment decreases (relativistic Breit correction). It is interesting to note that a similar effect, due to mass increase, was observed for free electrons. In an experiment made with polarized electrons, of Larmor rotation of spins in a magnetic field, by Louisell, Pidd and Crane in 1954 [21]. It was proved that the correct value of the gyromagnetic ratio in the laboratory frame of reference is obtained for relativistic electron mass value.

On the other hand, due to electron-nucleus Coulomb interaction, the e.m. mass (self-energy) of bound electron decreases at the formation of the atom, with a value of the order of binding energy. Only a small fraction of this decrease must be taken into account: the e.m. mass decrease of electron which takes place “inside” its dynamics region, this means in its close vicinity ( $\leq \lambda_e$ ) Only this e.m. mass decrease influences the “internal” dynamics and determines a small increase of the magnetic moment of electron. An estimation of this mass decrease of electron, of the order of  $\alpha E_{bind}$ , was obtained on a simplified model of H-like ions. The correction to the magnetic moment of bound electron due to this e.m. mass decrease is in agreement with lowest order correction calculated in QED, up to a numerical multiplicative factor. This increase of magnetic moment is in addition to the increase calculated for free electron.

The corrections to the mass value are at the origin of the lowest order corrections to the magnetic moment (gyromagnetic factor) of free and bound electron.

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### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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