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Friedmann-Like Cosmological Equations for the Accelerated Expansion of the Universe

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Abstract

In recent papers [1] [2] [3], we framed suitable axioms for Space called Super Space by Wheeler [4]. Using our axioms in Newtonian formalism and considering the density of the universe to be constant in time, we showed in the above references that at $t = 0$ the radius of the universe need not be zero. And thus, we avoided the problem of singularity. We further showed that the Hubble factor is no longer constant in time and goes on decreasing as confirmed by experiments. We pointed out in the above references that Space is the source of dark energy which is responsible for the accelerated expansion of the universe. With a view to improving the above-mentioned results quantitatively, in this paper, we are discussing the consequences of our axioms using Einstein's field equations of general theory of relativity. Friedmann-like Cosmological equations with Dark Energy built-in are derived. This derivation is obtained using Robertson-Walker line element and by introducing a suitable expression for Energy-Momentum tensor in terms of matter and Dark energy contents of the universe. The solutions of our cosmological equations obtained here, show that the radius of the universe cannot reach zero but has a minimum value and there is also maximum value for the radius of the universe. The inflationary expansion of the very early universe emerges from our theory.

Keywords

Axioms for Space, Dark Energy, Seed of the Universe, Einstein's Field Equations, Space (Vacuum) Energy, Accelerated Expansion

1. Introduction

As instruments and techniques improve in sensitivity and accuracy, new observations emerge in science and they challenge the existing concepts and theories and motivate us to find better and more accurate explanations. The experimental

discovery of the accelerated expansion of the universe by Adam G. Riess *et al.,* [5] is a case in hand. Such an accelerated expansion of the universe is clearly beyond the scope of the Big Bang theory and hence provides a strong motivation to go in for a new cosmological theory where acceleration follows as a natural consequence of the theory. The acceleration suggests the presence of an all-pervading anti-gravity force, now called in the literature Dark Energy. In the existing Big Bang theory, the source of Dark Energy which produces the acceleration is completely unknown and this situation is a severe limitation of the theory. In our work reported here, we identify Dark energy as waves in Space whose cause we have extensively discussed. Big Bang theory does not explain how the "cosmic egg" which exploded came into existence. In sharp contrast to this, we have shown that the universe expands from a seed of finite parameters and we have also explained how the seed came into existence. We give below the various opinions of other research workers about the cause and the functional aspect of dark energy and the accelerated expansion of the universe.

In a recent paper, Paolo Christillin, [6] considers the universe as an expanding black hole. To quote Christillin "Backed up also by the consideration of the black body self-energy, the post Big Bang temperature decrease is consistent with particle creation if energy conservation applies at every scale. This process is shown to provide a gravitational repulsive force which can counter balance gravitational attraction, thus allowing the possibility of a steady non-inflationary expansion. That seems to provide an alternative coherent scheme for our picture of the universe evolution, disposing of the cosmological term, of dark energy and of the bulk of dark matter".

According to Gregory Ryskin [7], we have: "Observational data suggest that the universe is spatially flat. Its content dominated by Dark energy which makes up about 73% of the total. The rest is the non-relativistic matter (Including Dark matter); the energy density of relativistic matter and radiation is negligible, except in the early universe. With dark energy is associated negative pressure equal and opposite to its density: this negative pressure acts as a source of anti-gravity, with the result that expansion of the universe is accelerating. The nature of Dark energy remains unexplained".

According to Wheeler [4]: "As we start, we recognise that the arena of Einstein's geometrodynamics is not space, not even spacetime, but super space. Space, to be sure, is the dynamical object. And a spacetime is one classical history of that dynamical object. But the arena in which the dynamics unrolls is super space".

It is highly significant that Wheeler considers the super space as a source of particles. To quote him: "First, we will accept Einstein's general relativity or 'geometrodynamics' in its standard 1915 form, translated of-course the appropriate quantum version. Second, we accept as tentative working hypothesis the picture of Clifford and Einstein that particles originate from geometry; that there is no such thing as a particle immersed in geometry, but only a particle built out

of geometry.

The above paragraphs show that there are diverse views on Dark energy and the link between the particles and the Space. Our views on the link between particles and Space are expressed in our axioms given below for Space.

1) Space is all-pervading and is endowed with potential energy. It has the property of constant self-compression and continually exerting compressive pressure on every system in it.

2) Self-compression results in the formation of infinitesimal spinning quanta of Space, called "formative dust". Due to the surrounding compressive pressure of Space, dust are forced into formation of discrete groups we know as fundamental particles; every group of dust formed by the surrounding compressive pressure of Space has a spin and hence becomes a source of a radial field with a repulsive force at every space-time point.

Explaining our axioms, we state the following: The self-compression of Space results in the infinitesimal spinning quanta of space called dust. A group of dust becomes a fundamental particle with spin. From a spinning fundamental particle, energetic constituent dust are thrown out. Each such thrown out dust produces an outgoing wave in Space. These waves produced by every fundamental particle, constitute the Dark energy. As gravity, there is no place where Dark Energy is not there. The role of Dark Energy is anti-gravity.

The concept of self-compression of Space has a similarity with the concept of action of gravitational field upon itself introduced by Merrison [8]. The above axioms give us a possible methodology for the formation of primordial seed which by its expansion has become the present universe. As already stated, the self-compression of space results in the formation of innumerable dust. Out of the dust emerge fundamental particles, each particle being a group of certain number of dust. Further, certain number of fundamental particles is grouped by the compressive pressure of Space to form a structure. With more and more particles pushed into the structure by the Space, the formation of the structure will continue until the density of the particles in the structure attains homogeneity and isotropy. Inside the Structure the Dark Energy wave from one particle will be pushing every other particle away. Once this anti-gravity force gains an upper hand over the compressive thrust on the structure due to Space, the structure as a whole start expanding. Let β be the radius of the structure when it started expanding. This structure with finite size and density, we consider as the primordial seed of the universe.

When we talk about the primordial seed of the universe the following concept of Vladimir S. Netchitailo [9] is worth considering: "Before the beginning of the World, there was nothing but an External Universe. About 14.2 billion years ago the World was started by a fluctuation in the External Universe and the Nucleus of the World which is a four-dimensional ball was born. An extrapolated nucleus radius at the beginning was equal to basic unit of size, a".

2. Friedmann-Like Cosmological Equations

From our axioms, it is clear that there are two forces operating at the cosmic level. One is the compressive thrust on the universe due to the Space surrounding the universe (Or in other words a kind of gravitational collapse due to gravity). This compressive thrust is a measure of the mass-energy content (W) of the universe. The other force is due to the waves in Space (Dark energy) trying to push the boundary of the universe away. Let E_D be the total Dark energy content of the universe. The dynamics of the universe is controlled by both the mass-energy of the universe (W) and dark energy content (E_D) of the universe. Both W and E_D are functions of time. It must be held in our mind that there is a continuous flow of spinning particles from Space into the Universe. Since every incoming particle carries a mass and becomes a source of waves in Space, W and E_D keep changing. In this context the following observation of Sean M. Carroll [10] of University of Chicago is worth noting: "In a universe with both matter and vacuum energy there is a competition between the tendency of the vacuum energy to cause acceleration and the tendency of matter to cause deceleration, with the ultimate fate of the universe depending on the precise amounts of each component."

Combining Einstein's field equation, R-W line element and the Energy – Momentum tensor $T^{\alpha\beta}$ given as,

$$T^{\alpha\beta} = (\rho + p)u^{\alpha}u^{\beta} - pg^{\alpha\beta}$$

Friedmann [11] obtained his famous cosmological equations which predicted the expansion of the universe. We obtain here similar cosmological equations which take into account the role of Dark energy.

By Einstein's field equations, we have,

$$R_{\nu}^{\mu} - (1/2)Rg_{\nu}^{\mu} = T_{\nu}^{\mu} \tag{1}$$

Our expression for the Energy – momentum tensor is of the form,

$$\begin{aligned} T_{\nu}^{\mu} = &-\left\{\sigma\rho_e/E_D - \alpha/W\,\rho_m + (E_D + W - E_S)/3H\right\}U^{\mu}U_{\nu} \\ &+ g_{\nu}^{\mu}(E_D + W - E_S)/3H \end{aligned} \tag{2}$$

The R-W metric is of the form,

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\mathrm{d}r^2/(1 - kr^2) + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right] \tag{3}$$

Here H denotes the Hubble Factor and E_S represents the energy manifested from Space to form the universe. U^{μ} is the four velocity of a galaxy with the conditions, $U^{\mu}U_{\nu}$ equals 1 when $\mu = \nu = 0$ and 0 otherwise. Here α and σ are factors independent of time. ρ_m (mass-energy density), ρ_e (Dark energy density) are considered as constants. $W = (4\pi/3)s^3\rho_m$ and $E_D = (4\pi/3)s^3\rho_e$. We take the unit c (velocity of light) = 1.

It can be verified that the above expression for T_{ν}^{μ} satisfies the divergence condition

$$T_{\nu;\mu}^{\mu} = 0 \tag{4}$$

implying,

$$E_S = (W + E_D) \tag{5}$$

The above equation is the grand conservation law of Energy between Space and the universe.

In this regard the following observation of Auguste Meesen [12] is worth our consideration: "Since our universe was in a zero-energy state, it is reasonable to assume it arose from vacuum fluctuations. The resulting state was stabilized, but gave then rise to an amazing sequence of transformations and a highly astonishing evolution at different levels of complexification. It began with the conversion of the primeval photon into many material elementary particles, according to the conservation law for u-quantum numbers. These particles interacted with one another, which lead to the formation of compound particles in negative energy state. The total amount of negative energy increased as well as the total amount of positive energy, but energy conservation implies that our universe is still in a zero-energy state." One can find a very mild comparison between the concept of vacuum fluctuations of Auguste Meesen and our concept of particles emerging from Space.

From Equation (1), Equation (2) and Equation (5) we obtain the Friedmann-like cosmological equations as

$$3\left(\dot{s}^2 + k\right)\big/S^2 = \left(-\sigma\rho_e/E_D + \alpha/W\rho_m\right) \tag{6}$$

$$2\ddot{s}/S + \left(\dot{s}^2 + k\right)\big/S^2 = 0 \tag{6a}$$

If the R.H.S of the above Equation (6) is positive, then the acceleration will be negative. With negative acceleration the seed cannot grow. For the seed to grow the acceleration has to be positive. So, for the formation of the universe the R.H.S of the Equation (6) has to be negative.

During the early universe, the average spin of the fundamental particles (the source of Dark energy) is very high and therefore a negative pressure due to dark energy remains extremely high when compared to the inward thrust on the universe due to the Space surrounding the universe. Hence the negative term $(-\sigma\rho_e/E_D)$ associated with dark energy would have been a dominating factor for the early universe. Hence, retaining the dominating term only, we write for the early universe

$$3\left(\dot{s}^2 + k\right)\big/S^2 = -\sigma\rho_e/E_D$$

Taking $k = -1$ in the RW metric element, we obtain

$$\dot{s}^2 = \left(1 - \beta/S\right) \quad \text{Where} \quad \beta = \sigma/4\pi \tag{7}$$

Equation (7) shows that for $S < \beta$, $\dot{s}^2$ becomes negative and hence $\dot{s}$ becomes imaginary. The physical meaning of this situation is that expansion starts from $S = \beta$

Solving the above relations, we get

$$S = \left(\beta/2\right)\left(1 - \cosh\phi\right), \text{ with } S > \beta \tag{8}$$

$$t = (\beta/2)(\phi - \sinh\phi) \tag{9}$$

when $S = \beta$, $\phi = \pi$, $\dot{s} = 0$ and $t = (\beta\pi)/2$.

Equation (7) can be written as,

$$H^2 = (1/s^2 - \beta/s^3) \tag{10}$$

It can be easily verified that H^2 will have maximum value when

$$S = 3\beta/2 \tag{11}$$

Replacing S by $3\beta/2$ in the Equation (10), we get

$$H_m^2 = 4/(27\beta^2) \tag{12}$$

where H_m is the maximum value of H

When the expansion started, H^2 goes on increasing, reaching a maximum and then decreasing. This shows that the initial expansion was inflationary. Thus, the inflationary expansion of the early universe follows from our theory. It is highly gratifying to note that the inflationary expansion of the early universe and the accelerating expansion of the present universe come out as a consequence of our theory

From the Equation (6a) and Equation (6), we get

$$\ddot{s} = \beta/2s^2 \tag{13}$$

The acceleration $\ddot{s}$ goes on decreasing with distance but remains always positive. This decrease is understandable. As time evolves, the average spin of the fundamental particles in the universe goes on decreasing and therefore the power of the dark energy waves produced by the fundamental particles decreases leading to less and less acceleration. Gradually as the power of the dark energy decreases, the universe enters Space or Matter dominating era, where the compressive thrust on the universe exceeds the repulsive thrust due to the dark energy. Let S_r be the radius of the universe when it enters the matter dominating era. In this case, the nature of curvature of Space changes and k becomes zero making the acceleration negative.

For this era, we have,

$$3\dot{s}^2/S^2 = \alpha/W\rho_m \quad \text{with} \quad S > S_r$$

$$\dot{s}^2 = \alpha S^2/(3W\rho_m)$$

$$\dot{s}^2 = \gamma/S \quad \text{where} \quad \gamma = \alpha/4\pi\rho_m^2 \tag{14}$$

The above Equation (14) can be solved to get,

$$S = (2/3\sqrt{\gamma})^{2/3} t^{2/3} \quad \text{and}$$

$$\ddot{s} = -(2/9)(2/3\sqrt{\gamma})^{2/3} t^{-4/3}$$

Thus, when $S > S_r$, k becomes zero and acceleration becomes negative. As time further increases, the compressive thrust on the universe due to Space becomes enormously high. Consequently, at a point of time the nature of curvature inside

the universe changes in such a way that k becomes +1. For the era when $k = +1$ we have,

$$3\left(\dot{s}^2 + 1\right)\Big/S^2 = \alpha\left(W\rho_m\right) \quad \text{with} \quad S \gg S_r \tag{15}$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2\left(t\right)\left[\mathrm{d}r^2\Big/\left(1 - r^2\right) + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right]$$

Now the Friedmann-like cosmological equation becomes,

$$\dot{s}^2 = \left(\gamma/S\right) - 1 \quad \text{where} \quad \gamma = \alpha\Big/\left(4\pi\rho_m^2\right) \tag{16}$$

The solution for the above equation during the Space dominating era can be written as

$$S = \left(\gamma/2\right)\left(1 - \cos\theta\right) \quad \text{with} \quad S > S_r \quad \text{and}$$

$$t = \left(\gamma/2\right)\left(\theta - \sin\theta\right)$$

During the matter dominating era, the acceleration becomes negative, and hence the expansion of the universe will have deceleration and eventually will stop expanding at a later time. The dynamical behaviour of the universe after its expansion stops at the distance $S = \gamma$, is under investigation.

3. Conclusions

The cosmological model given in this paper, which we call "Space Transformation Model", is based on our two axioms for Space. These two axioms are condensed versions of the views of Vethathiri Maharishi on Space [13].

The primordial seed which expanded into the present universe comes from and of the Space. Due to the dominating dark energy, the seed started expanding when $S = \beta$ with positive acceleration. The acceleration remains positive but went on decreasing. When the radius of the universe reaches the value S_r, the velocity of the expansion will be $(1 - \beta/S_r)^{1/2}$. Beyond S_r, the compressive thrust on the universe due to space will dominate the repulsive force due to the dark energy and hence the curvature of Space inside the universe keeps changing so that from -1, k changes to 0 and then to +1. In both cases when $k = 0$ and +1, the acceleration of the expansion becomes negative. Hence, the velocity of the expansion will start decreasing, finally becoming zero when $S = \gamma$. In the Big Bang model, during the evolution of the universe, the factor k in R-W metric remains either -1 or 0 or +1 throughout the evolution of the universe. But in our theory presented here, the universe during its expansion assumes all the three phases $k = -1$, 0 and 1 one after the other. Once expansion velocity becomes zero at $S = \gamma$, a natural question will be: "what will be the dynamics of the universe after that?" Our theoretical investigation regarding this question will be published elsewhere in future.

Inflationary expansion of the early universe when it started expanding and the acceleration of the expansion of the universe at present turn out as the consequences of our theory.

The Hubble constant which was once considered as a universal constant, as

recent observations show, is no longer a constant. We have obtained here a relation between H and S in Equation (10) of this paper which gives the nature of variation of Hubble factor with radius of the universe. Such a variation of the Hubble constant has been obtained also by Mishra and Vadrevu [14].

By introducing the term Es in our Energy-Momentum tensor, we established that Space is the source of both the matter and the dark energy contents of the universe.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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