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Most Intense X-Ray Lines of the Helium Isoelectronic Sequence for Plasmas Diagnostic

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Abstract

We report accurate wavelengths for the three most intense lines (resonance line: $1s^2\ {}^1S0$ - $1s2p\ {}^1P1$, intercombination line: $1s^2\ {}^1S0$ - $1s2p\ {}^3P1$ and forbidden line: $1s^2\ {}^1S0$ - $1s2s\ {}^3S1$) along with wavelengths for the $1s^2\ {}^1S0$ - $1snp^1P1$ and ${}^1S0$ - $1snp^3P2$ ($2 \le n \le 25$) transitions in He-like systems ($Z = 2 - 13$). The first spectral lines that belong to the above transitions are established in the framework of the Screening Constant per Unit Nuclear Charge method. The results obtained agree excellently with various experimental and theoretical literature data. The uncertainties in wavelengths between the present calculations and the available literature data are less than 0.004 Å. A host of new data listed in this paper may be of interest in astrophysical and laboratory plasmas diagnostic.

Keywords

He-Like Systems, Semi-Empirical, Screening Constant Per Unit Nuclear Charge, Excited States, X-Ray Spectra

1. Introduction

The helium-like isoelectronic series emit strong X-ray wavelengths. The most intense lines of these systems are the resonance line designated by ω (also labelled r: $1s^2\ {}^1S_0$ - $1s2p^1P_1$), the intercombination lines ($x + y$) (or i: $1s^2\ {}^1S_0$ - $1s2p^3P_{2,1}$) and the forbidden line z (or f: $1s^2\ {}^1S_0$ - $1s2s\ {}^3S_1$). These three lines correspond to the transitions between the $n = 2$ excited shell and the $n = 1$ ground state shell. The determination of these lines is of great interest because the line ratios f/i and $(f + i)/r$ provided respectively electrons density ($n_e \sim 10^8$ - 10^{13} cm^{-3}) and electrons temperature ($T_e \sim 1$ - 10 MK) as first shown by Gabriel and Jordan [1] and are widely used for collisional solar plasma diagnostics [1] [2] [3]. On the other hand, these line ratios enable also to determine the prevailed ionization processes (photo-ionization and/or collisional ionization) in the plasma [4] [5] [6]. Traditionally, He-like ions *f*/*i* line ratios have been used to derive electron densities of X-ray line-emitting regions since the populations of the 2^3P level are controlled by collisional excitation from the 2^3S level [1]. At low density, the $n = 2$ states are populated by electron excitation and then decay radiatively. Then, the relative intensities of the three lines are independent of the density [7]. Above the critical density (n_{crtit}): ($n_{crit}\, C \sim A$, *C being the rate coefficient for collisional excitation* $2^3S \rightarrow 2^3P$ *and A denotes the radiative transition probability of* $2^3S \rightarrow 1^1S$), the 2^3S upper level of the forbidden line becomes to be depleted by collision to the 2^3P upper levels of the intercombination line. As a result, when the electron density increases, the intensity of the forbidden line decreases strongly whereas that of the intercombination line increases. However, in the case of a strong UV radiation, the photo-excitation 2^3S → 2^3P becomes no negligible. Subsequently, the ratio (*f*/*i*) of the forbidden line on the intercombination line is no longer an electron density diagnostic. As concern the ratio $(f + i)/r$, it is sensitive to electron temperature as the dependence of the collisional excitation rates with the temperature for the resonance line is not the same for the forbidden and intercombination lines. In short, for plasma dominated by photo-ionization and recombination, the forbidden line (or the intercombination line at high density) becomes much stronger than the resonance line. In the case of plasma dominated by collisional ionization and excitation, the resonance line is stronger or comparable to the forbidden line and the intercombination line [4]. The following considerations indicate that the determination of the most intensive lines of Helium-like ions in the X-ray range is of great interest in laboratory and astrophysical plasma diagnostics.

On the experimental side, high-precision measurements of the energy difference between S and P levels in the helium isoelectronic series were made three decades ago. Robinson [7] presents measurements of the $1s^2\,{}^1S_0$ - $1snp\,{}^1P_1$ series of the Helium isoelectronic sequence for Be III, B IV and C V. Since that time, many experiments have been improved. Twelve lines in the region 20 - 100 Å belonging to the resonance series of Be III, B IV, C V and O VII are remeasured by Svensson [8] using spectrograms. Beiersdorfer *et al.* [9] use the tokamak plasmas from the Princeton Large Torus (PLT) high-resolution Johann spectrometer to report the $n = 2 \rightarrow 1$ X-ray transitions of Helium-like potassium, scandium, titanium, vanadium, chromium, and iron ($Z = 19$ - 26) along with wavelengths belonging to the $1s^2\,{}^1S_0$ - $1snp\,{}^1P_1$ ($n = 3$ - 5) transitions. Furthermore, Engtröm and Litzén [10] generate spectra of C, N and O simultaneously by focusing 1 GW laser pulses on targets made of either ammonium hydrogen carbonate or ammonium oxalate and then determine the wavelengths of the $1s^2\,{}^1S$ - $1snp\,{}^1P$ ($n = 2$ - 4) resonance lines in N VI and O VII (17 - 30 ÅÅ) with uncertainties ranging from 0.2 to 0.7 mÅÅ. Bartnik *et al.* [11] measure the wavelengths of the $1snp\,{}^1P$ - $1s^2\,{}^1S$ ($n = 4$ - 10) transitions in He-like O VII in laser-produced gas puff plasmas with an accuracy measurement ranging between (1.5 - 3.0 mÅ).

On the theoretical side, many techniques are presented. Acaad *et al.* [12] construct wave function expanded in a triple series of Laguerre polynomials of the *perimertric* coordinates to study the S and P states of the helium isoelectronic sequence and report nonrelativistic wavelengths and total wavelengths including mass polarization relativistic and, the Lamb shift corrections for $Z = 2$ - 9 belonging to the 1s*n*p ^{1}P - 1s^{21}S ($n = 2$ - 5) transitions. In addition, Safronova *et al.* [13] apply the MZ code through a perturbation theory based on hydrogen-like functions to compute wavelengths of highly charged He-like ions ($Z = 6$ - 54) for both satellite lines (1s2*l'nl* - 1s^2*n'l'*, *n*, *n'* = 2, 3) and (1s*n*p ^{1,3}P - 1s^2, $n = 2$, 3 and 1s2s ^{1,3}S - 1s^2) transitions. Additionally, the plasma simulation code CLOUDY is used by Porter [14] to present wavelengths of the UV, intercombination, forbidden, and resonance transitions oh He-like ions for $Z = 6$ - 14 and for $Z = 16$, 18, 20, and 26. But, as far as we know, the wavelengths cannot be directly determined within a single analytical formula for a whole members of He-like ions using one of the preceding method or one of the other existing computational techniques. Then, analytical spectral lines in two-electron systems such as the Balmer or the Lyman spectral lines of the hydrogen-like systems are not yet established. In this paper, we intend to present analytical spectral lines belonging to the resonance line: 1s^2 $^1S_0$ - 1s2p $^1P_1$ and intercombination line: 1s^2 $^1S_0$ - 1s2p $^3P_{2,1}$ along the 1s^2 $^1S_0$ - 1s*n*p $^1P_1$ ($n \leq 10$) transitions in the helium isoelectronic sequence. In our study, we use the Screening Constant per Unit Nuclear Charge (SCUNC) method suitable in the analysis of atomic spectra [15] [16]. All the results obtained in the present work compared very well to the available experimental and theoretical literature data. A host of data listed in this paper may be of interest in astrophysical and laboratory plasmas diagnostic.

In section 2, we present the theoretical procedure adopted in this work. In section 3, the presentation and the discussion of the results are made. A comparison of our results with available experimental and theoretical results is also made.

2. Theory

2.1. Brief Description of the SCUNC Formalism

In the framework of Screening Constant per Unit Nuclear Charge formalism, total energy of $(N\ell, n\ell')\,^{2S+1}L^{\pi}$ excited states are expressed in the form (in rydberg units)

$$E\left(N\ell n\ell';\,^{2S+1}L^{\pi}\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\beta\left(N\ell n\ell';\,^{2S+1}L^{\pi};Z\right)\right]^2\right). \quad (1)$$

In this equation, the principal quantum numbers N and n, are respectively for the inner and the outer electron of He-isoelectronic series. In this equation, the β-parameters are screening constant by unit nuclear charge expanded in inverse powers of Z and given by

$$\beta\left(N\ell n\ell';\,^{2S+1}L^{\pi};Z\right) = \sum_{k=1}^{q} f_k \times \left(\frac{1}{Z}\right)^k. \quad (2)$$

where $f_k = f_k\left(N\ell n\ell';\,^{2S+1}L^{\pi}\right)$ are parameters to be evaluated empirically.

2.2. Energies for the Ground State

For the ground state, Equations (1) and (2) give

$$E\left(1s^2;{}^1S_0\right)=-Z^2\left(1+\left\{1-\frac{f_1}{Z}-\frac{f_2}{Z^2}-\frac{f_3}{Z^3}\right\}^2\right). \quad (3a)$$

Using the experimental total energy of He I, Li II and Be III respectively (in eV) −79.01 [17], −198.09 [18] and −371.60 [18], the screening constants in Equation (4) are evaluated by use of the infinite rydberg energy 1 Ryd = 13.605698 eV. We find then

$$E\left(1s^2;{}^1S_0\right)=-Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right). \quad (3b)$$

2.3. Spectral Lines of the 1^1S_0 - $1s2p\ ^1P_1$ Resonance Transition

During the $1s^2\ ^1S_0$ - $1s2p\ ^{1,3}P_1$ transitions, the energy of the system varies as

$$\Delta E=\frac{hc}{\lambda}=E\left(1s2p;{}^{1,3}P_1\right)-E\left(1s^2;{}^1S_0\right). \quad (4)$$

Using Equations (1) and (3b), we obtain from Equation (4)

For $2 \le Z \le 15$

$$\begin{aligned}\frac{hc}{\lambda}=&Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)\\&-Z^2\left(1+\frac{1}{4}\left\{1-\frac{f_1}{Z}-\frac{f_2\times(Z-Z_0)}{Z^2}-\frac{f_1^2\times(Z-Z_0)^2\times(Z-Z_0')}{Z^3}\right.\right.\\&\left.\left.-\frac{f_1^2\times(Z-Z_0)^2\times(Z-Z_0')^2}{Z^4}-\frac{f_1\times(Z-Z_0)^2\times(Z-Z_0')^2}{Z^5}\right\}^2\right)\end{aligned} \quad (5a)$$

In these equations, Z_0 and Z_0' denote the nuclear charge of the helium-like systems used in the empirical determination of the f_i'—screening constants. On the basis of $h=6.626276\times10^{-34}$ J·s, $c=2.99792458\times10^{8}$ m/s and $e=1.602189\times10^{-19}$ C and using for $1s^2\ ^1S_0$ - $1s2p\ ^3P_1$ the experimental wavelengths of He I ($Z_0=2$) and that of Li II ($Z_0'=3$) respectively 584.3339 Å [19] and 199.280 Å [7], Equation (5a) gives $f_1=1.004778731$ and $f_2=0.026277861$. We obtain then explicitly

$$\begin{aligned}\frac{1}{\lambda}=&Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)\\&-Z^2\left(1+\frac{1}{4}\left\{1-1.004778731\frac{1}{Z}-0.026277861\frac{Z-2}{Z^2}\right.\right.\\&-0.000690525\frac{(Z-2)^2\times(Z-3)}{Z^3}-0.000690525\frac{(Z-2)^2\times(Z-3)^2}{Z^4}\\&\left.\left.-0.026277861\frac{(Z-2)^2\times(Z-3)^2}{Z^5}\right\}^2\right)\times10973644.9\end{aligned} \quad (5b)$$

In Equation (5b), wavelengths are expressed in meters (m) and the infinite rydberg energy 1 Ryd = 13.605698 eV is used along with 1 eV =1.602189 × 10^{-19} J. So Ryd/hc = 10973644.9 (m).

2.4. Spectral Lines of the $1s^2\,{}^1S_0$ - $1s2p\,{}^3P_1$ Intercombination Transition

Using Equations (1) and (3b), Equation (4) yields for the $1s^2\,{}^1S_0$ - $1s2p\,{}^3P_1$ intercombination transition

$$\frac{hc}{\lambda}=Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)$$
$$-Z^2\left(1+\frac{1}{4}\left\{1-\frac{f_1''}{Z}-\frac{f_2''\times(Z-Z_0)}{Z^2}-\frac{f_2''^2\times(Z-Z_0)\times(Z-Z_0')^2}{Z^3}\right.\right.$$
$$\left.\left.-\frac{f_2''^2\times(Z-Z_0)^2\times(Z-Z_0')^2}{Z^4}-\frac{f_2''^2\times(Z-Z_0)^2\times(Z-Z_0')^3}{Z^5}\right\}^2\right) \quad (6a)$$

Here again, Z_0 and Z_0' denote the nuclear charge of the helium-like systems used in the empirical determination of the f_i'' —parameters. For $1s^2\,{}^1S_0$ - $1s2p\,{}^3P_1$, the experimental wavelengths of He I (Z_0 = 2) and that of B IV ($Z_0' = 5$) are respectively equal to 591.4121Å [19] and 61.0880 Å [9] as quoted in Ref. [12], we obtain from Equation (6a) $f_1''= 0.967951498$ and $f_2 = -0.06781546$.
Equation (6a) becomes then

$$\frac{1}{\lambda}=Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)$$
$$-Z^2\left(1+\frac{1}{4}\left\{1-0.967951498\frac{1}{Z}+0.06781546\frac{Z-2}{Z^2}\right.\right.$$
$$-0.004598936\frac{(Z-2)\times(Z-5)^2}{Z^3}-0.004598936\frac{(Z-2)^2\times(Z-5)^2}{Z^4} \quad (6b)$$
$$\left.\left.-0.004598936\frac{(Z-2)^2\times(Z-5)^3}{Z^5}\right\}^2\right)\times 10973644.9$$

2.5. Spectral Lines of the $1s^2\,{}^1S_0$ - $1s2s\,{}^3S_1$ Forbidden Transitions

For the $1s^2\,{}^1S_0$ - $1s2s\,{}^3S_1$ forbidden transitions, the spectral lines are given by

$$\frac{hc}{\lambda}=Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)$$
$$-Z^2\left(1+\frac{1}{4}\left\{1-\frac{f_1''}{Z}-\frac{f_2''\times(Z-Z_0)}{Z^2}-\frac{f_2''^2\times(Z-Z_0)\times(Z-Z_0')}{Z^3}\right.\right.$$
$$-\frac{f_2''^2\times(Z-Z_0)^2\times(Z-Z_0')}{Z^4}-\frac{f_2''^2\times(Z-Z_0)^2\times(Z-Z_0')^3}{Z^5} \quad (7a)$$
$$\left.\left.-\frac{f_2''^2\times(Z-Z_0)\times(Z-Z_0')^3}{Z^6}\right\}^2\right)$$

For $1s^2\ ^1S_0$ - $1s2s\ ^3S_1$, the experimental wavelengths from NIST [20] for He I ($Z_0 = 2$) and for Li II ($Z_0' = 3$) are respectively equal to 625.563 Å and 210.069 Å. Equation (7a) provides then $f_1'' = 0.816109425$ and $f_2'' = -0.079252785$. Equation (7a) becomes explicitly

$$\frac{hc}{\lambda} = Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)$$
$$-Z^2\left(1+\frac{1}{4}\left\{1-\frac{0.816109425}{Z}-\frac{0.079252785\times(Z-2)}{Z^2}\right.\right.$$
$$-\frac{0.006281003\times(Z-2)\times(Z-3)}{Z^3}-\frac{0.006281003\times(Z-2)\times(Z-3)}{Z^3} \tag{7b}$$
$$-\frac{0.006281003\times(Z-2)^2\times(Z-3)^2}{Z^4}-\frac{0.006281003\times(Z-2)^2\times(Z-3)^3}{Z^5}$$
$$\left.\left.-\frac{0.006281003\times(Z-2)\times(Z-3)^3}{Z^6}\right\}^2\right)\times 10973644.9$$

2.6. Spectral Lines of the $1s^2\ ^1S_0$ - $1snp\,^1P_1$ Transitions

Following the same reasoning above, we express from Equations (1) and (2) total energies belonging to the $1snp\,^1P_1$ levels

$$E\left(1snp;\,^1P_1\right) = -Z^2\left(1+\frac{1}{n^2}\left\{1-\frac{f_1}{Z(n-1)}-\frac{f_2}{Z}-\frac{f_3\times(Z-Z_0)}{Z^2n^2}\right.\right.$$
$$\left.\left.-\frac{f_3\times(Z-Z_0)^2\times(Z-Z_0')}{Z^3}-\frac{f_3\times(Z-Z_0)^2\times(Z-Z_0')^2}{Z^4}\right\}^2\right) \tag{8a}$$

For the $1s^2\ ^1S_0$ - $1snp\,^1P_1$ transitions, we get

$$\frac{hc}{\lambda} = Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)$$
$$-Z^2\left(1+\frac{1}{n^2}\left\{1-\frac{f_1}{Z(n-1)}-\frac{f_2}{Z}-\frac{f_3\times(Z-Z_0)}{Z^2n^2}\right.\right. \tag{8b}$$
$$\left.\left.-\frac{f_3\times(Z-Z_0)^2\times(Z-Z_0')}{Z^3}-\frac{f_3\times(Z-Z_0)^2\times(Z-Z_0')^2}{Z^4}\right\}^2\right)$$

For $1s^2\ ^1S_0$ - $1s3p\ ^3P_1$ and $1s^2\ ^1S_0$ - $1s4p\ ^3P_1$ transitions, the corresponding experimental wavelengths of Li II ($Z_0 = 3$) are respectively equal to 178.014 Å and 171.575 Å [7]. In addition, for Be III ($Z_0' = 4$), the wavelength for to the $1s^2\ ^1S_0$ - $1s3p\ ^3P_1$ transition is 88.314 Å [7]. Using these wavelengths, we get from Equation (8b) $f_1 = 0.011679205$, $f_2 = 1.003675341$, and $f_3 = 0.008177868$. The spectral lines belonging to the $1s^2\ ^1S_0$ - $1snp\,^1P_1$ transitions is then in the shape.

$$\frac{1}{\lambda}=Z^2\left(1+\left\{1-\frac{0.625085938}{Z}-\frac{0.031315676}{Z^2}-\frac{0.059849712}{Z^3}\right\}^2\right)$$
$$-Z^2\left(1+\frac{1}{n^2}\left\{1-0.011679205\frac{1}{Z(n-1)}-1.003675341\frac{1}{Z}\right.\right.$$
$$-0.008177868\frac{Z-3}{Z^2n^2}-0.008177868\frac{(Z-3)^2\times(Z-4)}{Z^3} \tag{8c}$$
$$\left.\left.-0.008177868\frac{(Z-3)^2\times(Z-4)^2}{Z^4}\right\}^2\right)\times 10973644.9$$

Before presenting and discussing the results obtained in this work, let us first move on explaining how electron-electrons and relativistic effects are accounted in the present SCUNC formalism. As mentioned previously [16] in the framework of the SCUNC formalism, all the relativistic corrections in many electron systems are incorporated in the β-parameters. To enlighten this point, let us move on considering the main relativistic terms in the Hamiltonian operator of Q-electron systems. For Q-electron systems, the Hamiltonian can be expressed as follows

$$H=H_0+W\,. \tag{9}$$

In this expression, H_0 denotes the nonrelativistic Hamiltonian and W is the sum of the perturbation operators which includes mainly correction to kinetic energy (W_{kin}), the Darwin term (W_{D}), mass polarization (W_{M}), spin-orbit corrections (W_{so}), spin-other orbit corrections (W_{soo}) and spin-spin corrections (W_{ss}). For Q-electron systems, the non-relativistic Hamiltonian and the perturbation operators are explicitly the following

$$H_0=\sum_{i=1}^{Q}\left[-\frac{1}{2}\nabla_i^2-\frac{Z}{r_i}\right]+\sum_{\substack{i,j=1\\ i\neq j}}^{Q}\frac{1}{r_{ij}};\quad W_{\text{kin}}=-\frac{\alpha^2}{8}\sum_{i=1}^{Q}\boldsymbol{p}_i^4\,;\quad W_{\text{D}}=\frac{3\pi\alpha^2}{2}\sum_{i=1}^{Q}\delta(\boldsymbol{r}_i)\,.$$

$$W_{\text{M}}=-\frac{1}{M}\sum_{\substack{i,j=1\\ i\neq j}}^{Q}\nabla_i\cdot\nabla_j\,;\quad W_{\text{so}}=\frac{Z}{2c^2}\sum_{i=1}^{Q}\frac{\boldsymbol{l}_i\cdot\boldsymbol{s}_i}{r_i^3}\,;$$

$$W_{\text{soo}}=-\frac{1}{2c^2}\sum_{\substack{i,j=1\\ i\neq j}}^{Q}\left[\frac{1}{r_{ij}^3}(\boldsymbol{r}_i-\boldsymbol{r}_j)\times\boldsymbol{p}_i\right]\cdot(\boldsymbol{s}_i+2\boldsymbol{s}_j)\,.$$

$$W_{\text{ss}}=\frac{1}{c^2}\sum_{\substack{i,j=1\\ j>i}}^{Q}\frac{1}{r_{ij}^3}\left[\boldsymbol{s}_i\cdot\boldsymbol{s}_j-\frac{3(\boldsymbol{s}_i\cdot\boldsymbol{r}_{ij})(\boldsymbol{s}_j\cdot\boldsymbol{r}_{ij})}{r_{ij}^2}\right].$$

In these expressions, α denotes the fine structure constant and M is the nuclear mass of the Q-electron systems. The energy value of the Hamiltonian (9a) is in the form

$$E=E_0+w\,. \tag{9b}$$

with

$$w=\langle W_{\text{kin}}\rangle+\langle W_{\text{D}}\rangle+\langle W_{\text{M}}\rangle+\langle W_{\text{so}}\rangle+\langle W_{\text{soo}}\rangle+\langle W_{\text{ss}}\rangle\,. \tag{9c}$$

For a-given Nl_1, nl_2 configuration of He-like ions where N, n, and l_1, l_2, are respectively principal and orbital quantum numbers, the total energy is given by

$$E = -\frac{Z^2}{N^2} - \frac{Z^2}{n^2}\left[1-\beta\left(Nl_1nl_2;\,^{2S+1}L^{\pi};Z\right)\right]^2. \tag{9d}$$

Developing Equation (9d), we obtain

$$E = -\frac{Z^2}{N^2} - \frac{Z^2}{n^2} + \frac{Z^2}{n^2}\beta\left(Nl_1nl_2;\,^{2S+1}L^{\pi};Z\right)\left[2-\beta\left(Nl_1nl_2;\,^{2S+1}L^{\pi};Z\right)\right]. \tag{9e}$$

Equation (9e) can be rewritten in the form

$$E = -\frac{Z^2}{N^2} - \frac{Z^2}{n^2} + \sum_{i=1}^{2}\frac{Z^2}{v_i^2}\beta_i \times \left[2-\beta_i\right].$$

This equation can be expressed in the same shape than Equation (9b)

$$E = E_0 + w.$$

where

$$\begin{cases} E_0 = -\dfrac{Z^2}{N^2} - \dfrac{Z^2}{n^2} \\ w = \displaystyle\sum_{i=1}^{2}\frac{Z^2}{v_i^2}\beta_i \times \left[2-\beta_i\right] \end{cases} \tag{10}$$

Using (9c) and the last equation in (10), we find

$$\sum_{i=1}^{2}\frac{Z^2}{v_i^2}\beta_i \times \left[2-\beta_i\right] = \langle W_{\text{kin}}\rangle + \langle W_{\text{D}}\rangle + \langle W_{\text{M}}\rangle + \langle W_{\text{so}}\rangle + \langle W_{\text{soo}}\rangle + \langle W_{\text{ss}}\rangle. \tag{11}$$

Equation (11) indicates clearly that, in the framework of the SCUNC-formalism, all the relativistic corrections are incorporated in the β-screening constants per unit nuclear charge. In the structure of the independent particles model disregarding all the relativistic effects, total energy is given by E_0. Subsequently $w = 0$. This involves automatically $\beta = 0$. Then, all relativistic effects are accounted implicitly in general Equation (1) via the β-parameters expanded in inverse powers of Z as shown by Equation (2) where the $f_k = f_k\left(Nlnl';\,^{2S+1}L^{\pi}\right)$—screening constants are evaluated empirically using experimental data incorporating all the relativistic effects and all electrons-electrons effects in many electron systems.

3. Results and Discussions

The present SCUNC wavelengths predictions for the wavelengths belonging to the $1s^2\ ^1S_0 \rightarrow 1snp\ ^1P1$ ($3 \le n \le 13$) transitions in He-like (Z = 3 - 38) ions are quoted in Table 1. Table 2 Presents a comparison between theoretical and experimental wavelengths of the $1\ ^1S_0 \rightarrow np\ ^1P_1$ ($1s^2\ ^1S_0 \rightarrow 1snp\ ^1P_1$) transitions of helium-like ions up to Z = 8. The present SCUNC calculations values, are compared to the experimental data of Robinson [7], Svensson [8], Bartnik *et al.* [11] and to the experimental data of Engtröm and Litzén [10]. For the resonance $1\ ^1S_0 \rightarrow 2p\ ^1P_1$ transition, it is seen that the current SCUNC results compared very well to the experimental values. Here, the $\Delta\lambda/\lambda$ percentage deviations with

Table 1. Present wavelengths (λ, in Å) of the $1s^2\ {}^1S_0 \rightarrow 1snp\ {}^1P_1$ transitions in He-like (Z = 3 - 15) ions.

1S-$n\ {}^1$P	Li II λ	Be III λ	B IV λ	C V λ	N VI λ	O VII λ	F VIII λ	Ne IX λ	Na X λ	Mg XI λ	Al XII λ	Si XIII λ	P XIV λ
1S-3^{1}P	178.0140	88.3140	52.6852	34.9749	24.9012	18.6283	14.4588	11.5474	9.4344	7.8524	6.6374	5.6841	4.9222
1S-4^{1}P	171.5750	84.7502	50.4334	33.4271	23.7736	17.7709	13.7853	11.5474	9.4344	7.8524	6.6374	5.6841	4.9222
1S-5^{1}P	168.7422	83.1934	49.4536	32.7553	23.2850	17.3999	13.4942	11.0046	8.9877	7.4785	6.3199	5.4110	4.6850
1S-6^{1}P	167.2401	82.3706	48.9368	32.4014	23.0278	17.2047	13.348	10.7701	8.7949	7.3171	6.1829	5.2933	4.5827
1S-7^{1}P	166.3466	81.8821	48.6302	32.1916	22.8754	17.0891	13.2504	10.5738	8.6335	7.1821	6.0683	5.1948	4.4972
1S-8^{1}P	165.7714	81.5679	48.4332	32.0568	22.7775	17.0148	13.1922	10.5270	8.5950	7.1499	6.0409	5.1713	4.4768
1S-9^{1}P	165.3792	81.3539	48.2990	31.9651	22.789	16.9643	13.1525	10.4951	8.5688	7.1280	6.0223	5.1553	4.4629
1S-10^{1}P	165.0997	81.2015	48.2035	31.8997	22.6635	16.9284	13.1243	10.4724	8.5501	7.1124	6.0091	5.1440	4.4531
1S-11^{1}P	164.8934	81.0890	48.1331	31.8516	22.6285	16.9018	13.836	10.4557	8.5364	7.1009	5.9993	5.1356	4.4458
1S-12^{1}P	1647369	81.0037	48.0796	31.8150	22.6020	16.8817	13.0878	10.4430	8.5259	7.0922	5.9919	5.1292	4.4403
1S-13^{1}P	164.6153	80.9374	48.0381	31.7867	22.5814	16.8661	13.0756	10.4331	8.5178	7.0854	5.9862	5.1243	4.4360
1S-14^{1}P	164.5187	80.8849	48.0052	31.7642	22.5651	16.8537	13.0659	10.4253	8.5114	7.0801	5.9816	5.1204	4.4326
1S-15^{1}P	164.4410	80.8425	47.9787	31.7461	22.5519	16.8438	13.0580	10.4190	8.5063	7.0757	5.9780	5.1172	4.4298
1S-16^{1}P	164.3775	80.8079	47.9570	31.7312	22.5412	16.8356	13.0517	10.4139	8.5020	7.0722	5.9750	5.1147	4.4276
1S-17^{1}P	164.3248	80.7793	47.9391	31.7190	22.5323	16.8289	13.0464	10.4096	8.4985	7.0693	5.9725	5.1125	4.4258
1S-18^{1}P	164.2808	80.7553	47.9240	31.7087	22.5248	16.8232	13.0419	10.4061	8.4956	7.0668	5.9704	5.1107	4.4242
1S-19^{1}P	164.2435	80.7349	47.9113	31.7000	22.5185	16.8184	13.0382	10.4030	8.4931	7.0648	5.9687	5.1092	4.4229
1S-20^{1}P	164.2117	80.7176	47.9005	31.6926	22.5131	16.8144	13.0350	10.4005	8.4910	7.0630	5.9672	5.1079	4.4218
1S-21^{1}P	164.1843	80.7027	47.8911	31.6862	22.5085	16.8109	13.0322	10.3983	8.4892	7.0615	5.9659	5.1068	4.4208
1S-22^{1}P	164.1605	80.6898	47.8831	31.6807	22.5045	16.8078	13.0299	10.3963	8.4876	7.0602	5.9648	5.1059	4.4200
1S-23^{1}P	164.1399	80.6785	47.8760	31.6759	22.5010	16.8052	13.0278	10.3947	8.4862	7.0590	5.9638	5.1050	4.4193
1S-24^{1}P	164.1217	80.6686	47.8698	31.6717	22.4979	16.8028	13.0260	10.3932	8.4850	7.0580	5.9629	5.1043	4.4186
1S-25^{1}P	164.1057	80.6599	47.8644	31.6679	22.4952	16.8008	13.0243	10.3919	8.4840	7.0571	5.9622	5.1036	4.4181

Table 2. Theoretical and experimental wavelengths of the $1\ {}^1S_0 \rightarrow np\ {}^1P_1$ ($1s^2\ {}^1S_0 \rightarrow 1snp\ {}^1P_1$) transitions of helium-like ions up to Z = 8.

1S-$n\ {}^1$P	Li II			Be III			B IV		
	λ^P	$\lambda exp^{(a)}$	$\Delta\lambda/\lambda$	λ^P	$\lambda exp^{(a)}$	$\Delta\lambda/\lambda$	λ^P	$\lambda exp^{(a)}$	$\Delta\lambda/\lambda$
1S-2^{1}P	199.2800	199.280	0.0000%	80.2522	80.254	0.0018%	60.390	60.313	0.0033%
1S-3^{1}P	178.0140	178.014	0.0000%	88.3140	88.314	0.0000%	52.6852	52.679	0.098%
1S-4^{1}P	171.5750	171.575	0.0000%	84.7502	84.758	0.0092%	50.4334	50.435	0.0032%
1S-5^{1}P	168.7421			83.1934	83.202	0.083%	49.4536	49.456	0.0048%
1S-6^{1}P	167.2401			82.3706	82.377	0.0198%	48.9368		
1S-7^{1}P	166.3466			81.8821	81.891	0.089%	48.6302		
1S-8^{1}P	165.7714			81.5679			48.4332		
1S-9^{1}P	165.3792			81.3539			48.2990		
1S-8^{1}P	165.0997			81.2015			48.2035		

Continued

1S-n ^{1}P	CV			NVI			OVII		
	λnr^p	$\lambda exp^{(a, b)}$	$\Delta\lambda/\lambda$*	λnr^p	$\lambda exp^{(d)}$	$\Delta\lambda/\lambda$	λnr^p	$\lambda exp^{(a, c)}$	
1S-2^{1}P	40.2647	40.268^b	0.0082%	28.7857	28.787	0.0045%	21.6021	21.602^a	0.0005%
1S-3^{1}P	34.9749	34.973$^{a, b}$	0.0054%	24.9012	24.898	0.0128%	18.6283		
1S-4^{1}P	33.4271	33.426$^{a, b}$	0.0033%	23.7736	23.771	0.089%	17.7709		
1S-5^{1}P	32.7553	32.754$^{a, b}$	0.0039%	23.2850	23.281	0.0172%	17.3999		
1S-6^{1}P	32.4014	32.399^b	0.0074%	23.0278	23.024	0.0165%	17.2047	17.199^c	0.0331%
1S-7^{1}P	32.1916			22.8754			17.0891	17.083^c	0.0357%
1S-8^{1}P	32.0568			22.7775			17.0148	17.008^c	0.0399%
1S-9^{1}P	31.9651			22.789			16.9643	16.957^c	0.0431%
1S-8^{1}P	31.8997			22.6635			16.9284	16.924^c	0.0230%

Here, λ^p denotes the present SCUNC calculations values, λexp represents the experimental values and $\Delta\lambda/\lambda$ stands for the percentage deviations with respect to the experimental value of the corresponding system. (a), experimental data of Robinson [7]; (b), experimental data of Svensson [8]; (c), experimental data of Bartnik *et al.* [11]; (d), experimental data of Engtröm and Litzén [10]. Wavelengths are in angstroms.

respect to the experimental values of the corresponding system are less than 0.009%. The slight discrepancies can be explained by the fact that the present formalism disregards explicitly mass polarization, relativistic and QED corrections. For the transitions 1 $^1S_0 \rightarrow np\ ^1P_1$ ($n \geq 3$), comparison with the quoted experimental data indicates again good agreements. For these levels, the percentage deviations with respect to the experimental value of the corresponding system are less than 0.05%. Here, the discrepancies may be imputed mainly to mass polarization corrections which are not taken into account in the present calculations. In fact, and as well mentioned by Beiersdorfer *et al.* [9], the $n \geq 3$ levels are less affected by electron-electron interactions, relativistic and QED corrections. Then, for $n \geq 3$ states, the ratio m/M (m and M respectively the electron and nuclear masses) becomes important while increasing the Z-charge number. Nevertheless, the present SCUNC semi-empirical formulas may be considered as good representative of experimental data when electron-electron interactions, relativistic and QED corrections are disregarded. In **Table 3**, the SCUNC predictions for the wavelengths belonging to the $1s^2\ ^1S_0 \rightarrow 1s2p\ ^{1,3}P_1$ transitions in He-like ions are compared to the *ab initio* calculations of Acaad *et al.*, [12] using wave function expanded in a triple series of Laguerre polynomials of the *perimertric* coordinates, the computational results of Safronova *et al.*, [13] applying the MZ code through a perturbation theory based on hydrogen-like functions and with the data of Porter [14] using the plasma simulation code CLOUDY. The overall agreement between the calculations is reasonably gratifying. Here, the $|\Delta\lambda_{theo}|$ differences in wavelengths between the present calculations and the theoretical literature data [12] [13] [15] have never overrun 0.003 Å for the $1s^2\ ^1S_0 \rightarrow 1s2p\ ^1P_1$ resonance line and 0.008 Å for the $1s^2\ ^1S_0 \rightarrow 1s2p\ ^3P_1$ intercombination line up to $Z = 22$. This may point out

Table 3. Theoretical wavelengths for $1s^2\ ^1S_0 \rightarrow 1s2p\ ^{1,3}P_1$ for He-like ions ($2 \le Z \le 22$).

Z	$1s^2\ ^1S_0 \rightarrow 1s2p\ ^1P_1$ (resonance line: r)					$1s^2\ ^1S_0 \rightarrow 1s2p\ ^3P_1$ (intercombinaison line: i)				
	λp	λtheo^{a,b}	λtheo^{c}	$\lvert\Delta\lambda$theo$\rvert^{a,b}$	$\lvert\Delta\lambda$theo$\rvert^{c}$	λp	λtheo^{a,b}	λtheo^{c}	$\lvert\Delta\lambda$theo$\rvert^{a,b}$	$\lvert\Delta\lambda$theo$\rvert^{c}$
2	584.3339	584.3343^{a}		0.0004		591.4121	591.499^{a}		0.0002	
3	199.2800	199.2791^{a}		0.0009		202.2252				
4	80.2522	80.2535^{a}		0.0013		81.6677				
5	60.390	60.3135^{a}		0.0020		61.0880	61.0882^{a}		0.0002	
6	40.2647	40.2671^{a}	40.2680	0.0024	0.0033	40.7302	40.7299^{a}	40.7310	0.0003	0.0008
7	28.7857	28.7867^{a}	28.7870	0.0010	0.0013	29.0818	29.0840^{a}	29.0840	0.0022	0.0022
8	21.6021	21.6012^{a}	21.6020	0.0009	0.0001	21.7988	21.8033^{a}	21.8070	0.0008	0.0045
9	16.8088	16.8061^{a}	16.8070	0.0027	0.0018	16.9438	16.9496^{a}	16.9470	0.0045	0.0082
8	13.4514		13.4470		0.0044	13.5464		13.5530		0.0066
9	9.0050		9.0030		0.0020	9.0880		9.0830		0.0060
12	9.1689		9.1688		0.0001	9.2310		9.2312		0.0062
13	7.7568		7.7573		0.0005	7.8044		7.8070		0.0026
14	6.6475		6.6480		0.0005	6.6847		6.6883		0.0036
15	5.7701					5.7898				
16	5.0387	5.0386^{b}	5.0387	0.0002	0.0000	5.0667	5.0667^{b}	5.0665	0.0000	0.0002
17	4.4445	4.4445^{b}		0.0002		4.4682	4.4681^{b}		0.0001	
18	3.9491	3.9492^{b}	3.9488	0.0001	0.0003	3.9694	3.9695^{b}	3.9691	0.0001	0.0003
19	3.5318	3.5319^{b}		0.0002		3.5493				
20	3.1771	3.1772^{b}	3.1772	0.0001	0.0001	3.1924	3.1928^{b}	3.1928	0.0004	0.0004
21	2.8731	2.8731^{b}		0.0000		2.8866	2.8871^{b}		0.0005	
22	2.684	2.685^{b}		0.0001		2.6226	2.6230^{b}		0.0004	

Here, λ^p denotes the present SCUNC calculations, λtheo represents the theoretical values and $|\Delta\lambda$theo$|$ stands for the difference in wavelengths between the present calculations and the other theoretical ones (λtheo^a or λtheo^b). (a): calculations of Accad *et al.*, [12], (b): calculations of Safronova *et al.* [13]; (c): calculations of Porter [14]. Wavelengths are in angstroms.

the good agreement between the calculations. The discrepancies with respect to the accurate *ab initio* computations are due to the present none-relativistic formalism. Table 4, shows a comparison of the present wavelengths for the forbidden $1s^2\ ^1S_0 \rightarrow 1s2s\ ^3S_1$ transitions of He-like systems ($Z = 2$ - 15) with the NIST compiled data. Excellent agreement is obtained between the SCUNC predictions and the NIST data. Except for $Z = 8$, the maximum shift in wavelengths with respect to the NIST values is at 0.003 Å. In Table 5, the present theoretical wavelengths for the $1snp\ ^1P1 \rightarrow 1s^2\ ^1S0$ ($2 \le n \le 5$) transitions of the helium-like ions up to $Z = 9$ are compared to the λnrel—nonrelativistic wavelengths values and to the λ_{tot}—total wavelengths (including mass polarization, relativistic corrections and the Lamb-shift correction for the $1\ ^1S$ level) computed by Accad *et al.* [12]. For the $1s^2\ ^1S_0 \rightarrow 1s2p\ ^1P_1$ resonance line, the uncertainties between the present calculations and the λ_{tot}—total wavelengths

Table 4. Comparison of the SCUNC predictions with the NIST data the wavelengths belonging to the forbidden $1s^2\ ^1S_0 \rightarrow 1s2s\ ^3S_1$ transitions in He-like (Z = 2 - 15) systems. Wavelengths are in angstroms.

Z	λSCUNC	λNIST	$\|\Delta\lambda\|$*
2	625.563	625.563	0.000
3	210.069	210.069	0.000
4	104.547	104.548	0.001
5	62.439	62.440	0.001
6	41.469	41.472	0.003
7	29.531	29.534	0.003
8	22.094	22.101	0.007
9	17.149	-	
10	13.696	13.699	0.003
11	11.190	11.192	0.002
12	9.313	-	
13	7.872	-	
14	6.741	6.740	0.001
15	5.838	-	

*$|\Delta\lambda| = |\lambda^{SCUNC} - |\lambda^{NIST}|$.

Table 5. Theoretical wavelengths for the $1s^2\ ^1S_0 \rightarrow 1snp\ ^1P_1$ ($2 \le n \le 5$) transitions in He-like (Z = 3 - 9) ions. Here, λ denotes the present SCUNC calculations, λnrel denotes the nonrelativistic wavelengths and λtot the theoretical wavelengths of Accad *et al.* [12] including mass polarization, relativistic corrections and the Lamb-shift correction for the 1S level. Wavelengths are in angstroms.

System	Transition	Theory			Comparison	
		Present λ	Accad *et al.* λnrel	Accad *et al.* λtot	$\|\lambda - \lambda\text{nrel}\|$	$\|\lambda - \lambda\text{tot}\|$
Li II	1S-2^{1}P	199.2800	199.2813	199.2791	0.0013	0.0009
	1S-3^{1}P	178.0140	178.0162	178.0143	0.0022	0.0003
	1S-4^{1}P	171.5750	171.5776	171.5757	0.0026	0.0007
	1S-5^{1}P	168.7421				
Be III	1S-2^{1}P	80.2522	80.2600	80.2535	0.0078	0.0013
	1S-3^{1}P	88.3140	88.3134	88.3075	0.0006	0.0065
	1S-4^{1}P	84.7502	84.7588	84.7532	0.0086	0.0030
	1S-5^{1}P	83.1934	83.2044	83.1989	0.090	0.0055
B IV	1S-2^{1}P	60.390	60.3224	60.3135	0.094	0.0025
	1S-3^{1}P	52.6852	52.6876	52.6800	0.0024	0.0052
	1S-4^{1}P	50.4334	50.4408	50.4335	0.0074	0.0001
	1S-5^{1}P	49.4536	49.4621	49.4549	0.0085	0.0013
	1S-2^{1}P	40.2647	40.2774	40.2671	0.0127	0.0024

Continued

C V	1S-3^{1}P	34.9749	34.9811	34.9723	0.0062	0.0026
	1S-4^{1}P	33.4271	33.4343	33.4259	0.0072	0.0012
	1S-5^{1}P	32.7553	32.7622	32.7540	0.0069	0.0013
	1S-2^{1}P	28.7857	28.7980	28.7867	0.0123	0.0010
N VI	1S-3^{1}P	24.9012	24.9098	24.9002	0.0086	0.0010
	1S-4^{1}P	23.7736	23.7806	23.7714	0.0070	0.0022
	1S-5^{1}P	23.2850				
	1S-2^{1}P 1S-3^{1}P	21.6021	21.6133	21.6012	0.092	0.0009
O VII		18.6283	18.6381	18.6280	0.0098	0.0003
	1S-4^{1}P	17.7709	17.7777	17.7680	0.0068	0.0029
	1S-5^{1}P	17.3999	17.4051	17.3957	0.0052	0.0042
	1S-2^{1}P	16.8088	16.8188	16.8061	0.080	0.0027
F VIII	1S-3^{1}P	14.4588	14.4690	14.4584	0.082	0.0004
	1S-4^{1}P	13.7853				
	1S-5^{1}P	13.494 2				

results [12] are less than 0.003 Å. As far as comparison with the λ_{nrel}—nonrelativistic wavelengths values are concerned, it is seen that the uncertainties are about 0.01 Å for $Z = 5$ - 9. This points out that, the present SCUNC results are most accurate than the λ_{nrel}—nonrelativistic wavelengths obtained by Accad *et al.* [12] when increasing the nuclear charge. For $n \geq 3$ states, it can also be seen that the present SCUNC wavelengths values are most accurate than that of Accad *et al.* [12]. Here, the uncertainties with respect to the λ_{tot}—total wavelengths are less than 0.005 Å for all the entire series considered ($Z = 2$ - 9) whereas the uncertainties with respect to the λ_{nrel}—nonrelativistic wavelengths increase up to 0.01 Å for $Z = 9$. This may point out again that, in the SCUNC formalism, relativistic effects are implicitly incorporated in the f_i—screening constants evaluated from experimental data. Besides, it should be mentioned that the λ_{tot}—total wavelengths equal to 88.3075 Å for the $1s^2\ ^1S_0 \rightarrow 1s3p\ ^1P_1$ transition of Be III may be probably lower as the corresponding high precision measurement is at 88.3140 Å [7] to be compared to the present prediction at 88.3140 Å.

4. Conclusion

The Screening Constant per Unit Nuclear Charge method has been applied to inaugurate the first spectral lines for the three most intense lines (resonance line $1s^2\ ^1S_0$ - $1s2p^1P_1$ intercombination line $1s^2\ ^1S_0$ - $1s2p\ ^3P_1$ and forbidden line $1s^2\ ^1S_0$ - $1s2s\ ^3S_1$ and for the $1s^2\ ^1S_0$ - $1snp^1P_1$ transitions in the helium isoelectronic sequence. In our knowledge, only the spectral lines of the Hydrogen-like ions have determined empirically in the past. At present hour, the possibilities to calculate easily the most intense lines of helium-like systems in the X-ray range in connection with plasma diagnostic are demonstrated in this work. All the results obtained in the present paper compared very well to various experimental and theoretical literature data. It should be underlined the merit of the SCUNC formalism providing accurate results via simple analytical formulas without needing to use codes of simulation. The accurate results obtained in this work point out the possibilities to investigate highly charged He-positive like ions in the framework of the SCUNC method.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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