Scientific Research Publishing

Journal of Modern Physics, 2024, 15, 796-803
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Standard Model Fermion Masses and Charges from Holographic Analysis

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How to cite this paper: Mongan, T.R. (2024) Standard Model Fermion Masses and Charges from Holographic Analysis. *Journal of Modern Physics*, **15**, 796-803.
https://doi.org/10.4236/jmp.2024.156035

Received: April 18, 2024
Accepted: May 19, 2024
Published: May 22, 2024

Abstract

The Standard Model of particle physics involves twelve fundamental fermions, treated as point particles, in four charge states. However, the Standard Model does not explain why only three fermions are in each charge state or account for neutrino mass. This holographic analysis treats charged Standard Model fermions as spheres with mass 0.187 g/cm^2 times their surface area, using the proportionality constant in the holographic relation between mass of the observable universe and event horizon radius. The analysis requires three Standard Model fermions per charge state and relates up quark and down quark masses to electron mass. Holographic analysis specifies electron mass, to six significant figures, in terms of fundamental constants $\alpha, \hbar, G, \Lambda$ and Ω_Λ. Treating neutrinos as spheres and equating electron neutrino energy density with cosmic vacuum energy density predicts neutrino masses consistent with experiment.

Keywords

Electron Mass, Up Quark Mass, Down Quark Mass, Neutrino Masses

1. Introduction

This holographic analysis

1) Explains why three Standard Model fermions are in each charge state $e, \frac{2}{3}e$ and $-\frac{1}{3}e$;

2) Relates electron mass to up and down quarks masses;

3) Specifies electron mass in terms of fine structure constant α, Planck's constant $\hbar$, gravitational constant G, cosmological constant Λ, and vacuum energy fraction Ω_Λ.

2. Background for Holographic Analysis

Holographic analysis is based on quantum mechanics, general relativity, thermodynamics, and Shannon information theory. Bousso's [1] review of holographic analysis indicates only about 5 × 10^{122} of bits of information on the event horizon will ever be available to describe physics in our universe where the cosmological constant [2] is $\Lambda = 1.088 \times 10^{-56}\ \text{cm}^2$.

The radius of the event horizon is $R_H = \sqrt{3/\Lambda} = 1.66 \times 10^{28}\ \text{cm}$. With Hubble constant [2] $H_0 = 67.4\ \text{km}/(\sec \cdot \text{Mpc})$, critical energy density $\rho_{crit} = \frac{3H_0^2}{8\pi G}$, gravitational constant [2] $G = 6.67430 \times 10^{-8}\ \text{cm}^3/(\text{g} \cdot \sec^2)$, and vacuum energy fraction [2] $\Omega_\Lambda = 0.685$, the mass of the observable universe within the event horizon is $M_H = \frac{4}{3}\pi(1-\Omega_\Lambda)\rho_{crit} R_H^3 = (0.187\ \text{g}/\text{cm}^2) R_H^2$. So M_H is the total mass of the bits of information necessary to describe all physics within the event horizon, indicating the bits of information describing a particle with definite mass m within the universe are available on a spherical surface around the particle with radius $r = \sqrt{\frac{m}{M_H}} R_H$.

3. Input Data

Discussion of charged Standard Model fermion masses must begin with PDG [2] 2023 experimental data on those masses. PDG [2] 2023 lists electron mass $m_e = 0.511\ \text{MeV}/c^2 = 9.11 \times 10^{-28}\ \text{g}$, up quark mass $m_{up} = 2.16^{+.49}_{-.26}\ \text{MeV}/c^2$, and down quark mass $m_{down} = 4.67^{+.49}_{-.26}\ \text{MeV}/c^2$ where c is the speed of light. Up and down quark holographic radii $r_{up} = 2r_e$, $r_{down} = 3r_e$ and quark masses $m_{up} = 4m_e$, $m_{down} = 9m_e$ used in this analysis are consistent with PDG data. **Table 1** then lists masses and holographic radii of the nine charged Standard Model fermions as used in this analysis.

Table 1. Charged standard model fermion masses and holographic radii.

Fermion	Charge	Mass (MeV/c^2)	Holographic radius r (fm)
electron	$-e$	0.510999	0.698031
muon	$-e$	105.7	10.04
tau	$-e$	1777	41.16
up quark	$2e/3$	2.04340	1.39606
charm quark	$2e/3$	1270	34.80
top quark	$2e/3$	173,000	406
down quark	$-e/3$	4.59899	2.09409
strange quark	$-e/3$	93.4	9.44
bottom quark	$-e/3$	4180	6.31

4. Charged Standard Model Fermion Structure and Masses

Standard Model fermions described as spheres with holographic radius r, mass $m=\left(0.187\text{ g/cm}^2\right)r^2$, and matter density $\rho = m\Big/\left(\frac{4}{3}\pi r^3\right)$ have a surface mass component and an axial mass component along their rotation axis. Holographic analysis is based in part on general relativity, and general relativity is not valid at distances less than the Planck length $l_P=\sqrt{\frac{\hbar G}{c^3}}=1.61625\times10^{-33}\text{ cm}=1.61625\times10^{-20}\text{ fm}$, so fermion surface mass components are treated as spherical shells with mass m_S, thickness l_P, and matter density $\rho_S l_P$ per unit area, while axial mass components with mass m_A are treated as cylinders with diameter l_P and matter density $\rho_A l_P^2$ per unit length. A cubic equation for fermion holographic radius in each charge state is

$$\frac{4}{3}\pi\rho r^3 = m_S + m_A = \rho_S l_P 4\pi r^2 + \rho_A \pi l_P^2 (2r)$$

or

$$\rho r^3 - 3\rho_S l_P r^2 - \frac{3}{2}\rho_A l_P^2 r = ar^3 + br^2 + cr = 0$$

with $a=\rho$, $b=-3\rho_S l_P$ and $c=-\frac{3}{2}\rho_A l_P^2$. Discriminant $b^2c^2-4ac^3$ of the cubic is positive and three real roots of the equation correspond to holographic radii of three fermions per charge state. Parameters ρ_S and ρ_A (determined by holographic radii r_1, r_2, and r_3) using Nickalls' solutions [3] to cubic equations involving

$$r_N=\frac{r_1+r_2+r_3}{3} \quad\text{and}\quad \delta^2=\frac{(r_1-r_N)^2}{4}+\frac{(r_2-r_3)^2}{12}$$

are $\rho_S l_P = r_N\rho$, from $r_N=-b/3$, and $\rho_A l_P^2 = 2\rho\left(r_N^2-\delta^2\right)$, from $\delta^2=\frac{b^2-3ac}{9a^2}$.

Tangential velocity v_T of points on fermion surfaces are found from $I\omega=\hbar/2$, where fermion moment of inertia $I=I_S+I_A$, with shell moment of inertia $I_S=\frac{2}{3}m_S r^2$, axial moment of inertia $I_A=\frac{m_A}{2}\left(\frac{l_P}{2}\right)^2$, and $I_S \gg I_A$. The electron is the only Standard Model fermion with $v_T=\frac{\hbar}{4mr_N}>c$, but points on the electron surface are not particles and cannot send signals, so no particle or signal travels with speed $>c$.

5. Electron, Up Quark, and Down Quark Masses

All persistent structures in the universe are composed of electrons, protons, and neutrons. Protons and neutrons are composed of up and down quarks. So the lowest mass Standard Model fermions in each charge state (electrons, up quarks, and down quarks) are constituents of all persistent structures in the universe. Holographic analysis then provides succinct explanations relating lowest mass

Standard Model fermions in each charge state.

Electrons, the only Standard Model fermions that can persist in isolation in the universe, have the smallest mass and holographic radius of the nine charged Standard Model fermions. Up quarks, with twice the electron holographic radius, have four times the electron mass. Down quarks, with three times the electron holographic radius, have nine times the electron holographic mass.

Protons are composed of two up quarks and one down quark, and neutrons are composed of two down quarks and one up quark. Isolated neutrons decay to protons, so up quarks must have lower mass than down quarks, consistent with experimental data.

6. Electron Mass from $\alpha, \hbar, G, \Lambda$ and Ω_Λ

Using electron holographic radius $r_e = \sqrt{\frac{m_e}{M_H}} R_H$, holographic analysis specifies electron mass to six significant figures in terms of fundamental constants $\alpha, \hbar, G, \Lambda$ and Ω_Λ.

Our universe is so large it is almost flat, and Friedmann's equation $H_0^2 = \frac{8\pi G}{3}\rho_{crit} + \frac{\Lambda c^2}{3}$ identifies $\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$. Since $M_H = \frac{4}{3}\pi(1-\Omega_\Lambda)\rho_{crit} R_H^3$, $M_H = \frac{(1-\Omega_\Lambda)c^2}{2G\Omega_\Lambda}\sqrt{\frac{3}{\Lambda}}$ is constant in time.

Electrostatic potential energy of electron charge e and positron charge $-e$ separated by $2r_e$ is $V = -\frac{e^2}{2r_e} = -\frac{\alpha\hbar c}{2r_e}$, with Planck's constant [2] $\hbar = 1.05457\times10^{-27}\ \text{g}\cdot\text{cm}^2/\text{sec}$. Adjacent spheres with holographic radii r_e, a precursor for electron-positron pair production, have total energy $E = 2m_e c^2 - \frac{\alpha\hbar c}{2r_e} = 0$ when $r_e = \frac{\alpha\hbar c}{4m_e c^2}$.

Two equations for r_e give $\frac{\alpha\hbar c}{4m_e c^2} = \sqrt{\frac{m_e}{M_H}} R_H$ and electron mass

$$m_e = \left[\left(\frac{\alpha\hbar^2}{32}\right)\left(\frac{1-\Omega_\Lambda}{G\Omega_\Lambda}\sqrt{\frac{\Lambda}{3}}\right)\right]^{1/3}.$$

If $\Lambda = 1.08800\times10^{-56}\ \text{cm}^2$ and $\Omega_\Lambda = 0.6853855$ (within PDG [2] 2023 error bars) electron mass is specified to six significant figures, since gravitational constant G is only known to six significant figures.

7. Neutrino Masses

Each lepton has a corresponding neutrino, but neutrinos oscillate between mass states when propagating through space and are not persistent structures in the universe. So neutrinos are not consistently related to holographic radii. Characteristic lengths of neutrinos are Compton wavelengths $\lambda = \frac{\hbar}{mc}$. Electron neutrinos with radius $\frac{1}{4}\frac{\hbar}{mc}$ and the lowest energy density in the universe (cosmic vacuum energy density $\rho_v = 5.83 \times 10^{-30}\ \text{g/cm}^3$) have mass $m_1 = \left[\frac{\pi}{6}\rho_v\left(\frac{\hbar}{c}\right)^3\right]^{\frac{1}{4}} = 2.02 \times 10^{-36}\ \text{g} = 0.0013\ \text{eV}$. Neutrino oscillation data [4] predict $m_2 = \sqrt{m_1^2 + 7.37 \times 10^{-5}\ \text{eV}^2} = 0.00866\ \text{eV}$ and $m_3 = \sqrt{0.5(m_1 + m_2)^2 + 2.5 \times 10^{-3}\ \text{eV}^2} = 0.0505\ \text{eV}$. Neutrino mass sum 0.0603 eV is then 50% of Vagnozzi's [5] experimental upper limit of 0.12 eV on neutrino mass sum.

8. Results

This holographic analysis based on quantum mechanics, general relativity, thermodynamics, and Shannon information theory:

1) Requires three Standard Model fermions in each charge state $e, \frac{2}{3}e$, and $-\frac{1}{3}e$;

2) Relates masses of the electron, up quark, and down quark constituents of all permanent structures in the universe to electron mass;

3) Specifies electron mass in terms of fundamental constants $\alpha, \hbar, G, \Lambda$ and Ω_Λ;

4) Accounts for observed matter dominance in the universe, if the universe is closed.

9. Conclusion

This analysis, specifying electron, up quark, down quark, and three neutrino masses, reduces by six the number of free parameters in the Standard Model of particle physics. These results can provide insight to develop physical theories to supplement the Standard Model.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: Matter Dominance in a Closed Universe

Charged bits on Standard Model fermion surfaces must be at the rotation axis, to avoid radiation from accelerated charge. Bits of information on the horizon can indicate presence of a charged Standard Model fermion somewhere along the axis between diametrically opposed bits of information on opposite hemispheres of the horizon, so charge $\pm e/6$ must be associated with each bit of information. One, two, or three bit pairs on opposite surfaces of spherical Standard Model fermions at their rotation axis specify charge $-e/3$ or $2e/3$ quarks, or charge e leptons.

A closed universe beginning by a quantum fluctuation from nothing [6] must be charge neutral, with equal numbers of $e/6$ and $-e/6$ bits. Regardless of details of how bits of information specify protons or anti-protons, configurations specifying protons differ in 6 bits from configurations specifying anti-protons. In any physical system, energy is transferred to change bits from one state to another, and $e/6$ bits with lower energy than $-e/6$ bits result in more matter than anti-matter in a closed universe, as discussed below.

Temperature at time of baryon formation was $T_B = \frac{2m_p c^2}{k} = 2.18\times10^3\ \text{K}$, with Boltzmann constant $k = 1.38\times10^{-16}\left(\text{g}\cdot\text{cm}^2/\text{sec}^2\right)/\text{K}$ and proton mass $m_p = 1.67\times10^{-24}\ \text{g}$. Radius of the universe at baryogenesis was [7] $R_B = \left(\frac{2.725}{2.18\times10^3}\right)R_0 \approx 10^{15}\ \text{cm}$, where 2.725 K is today's microwave background temperature and radius of the universe today is $R_0 \approx 10^{28}\ \text{cm}$. Baryogenesis time t_B in seconds after the end of inflation is determined by Friedmann's equation $\left(\frac{\text{d}R}{\text{d}t}\right)^2 - \left(\frac{8\pi G}{3}\right)\varepsilon(R)\left(\frac{R}{c}\right)^2 = -\kappa c^2$. After inflation, in a closed universe so large it is almost flat, curvature parameter $\kappa \approx 0$. Energy density $\varepsilon(R) = \varepsilon_r\left(\frac{R_0}{R}\right)^4 + \varepsilon_m\left(\frac{R_0}{R}\right)^3 + \varepsilon_v$, where $\varepsilon_r, \varepsilon_m$ and ε_v are today's radiation, matter, and vacuum energy densities. Matter energy density $\varepsilon_m \approx 9\times10^{-9}\left(\text{g}\cdot\text{cm}^2\cdot\text{sec}^{-2}/\text{cm}^3\right)$, and vacuum energy density was negligible in the early universe, so radiation dominated when $R \ll 10^{-5}R_0$ before radiation/matter equality. Integrating $\left(\frac{\text{d}R}{\text{d}t}\right)^2 - \left(\frac{8\pi G}{3c^2}\right)\frac{\varepsilon_r R_0^4}{R^2} = \left(\frac{\text{d}R}{\text{d}t}\right)^2 - \frac{A^2}{R^2} = 0$ where $A = \sqrt{\frac{8\pi G\varepsilon_r R_0^4}{3c^2}}$, from the end of inflation at $t = 0$ to t, determines $At = \frac{1}{2}\left(R(t)^2 - R_i^2\right)$, where R_i is radius of the universe at the end of inflation and $R_B \gg R_i$. So $t_B = \frac{R_B^2 - R_i^2}{2A} \approx \frac{R_B^2}{2A} \approx 10^{-7}$ seconds. Distance from any point in the universe at baryogenesis to the horizon for that point [8] is

$$d_B = c\int_0^{t_B}\frac{\text{d}t'}{R(t')} = \frac{cR_B}{A}\left[\sqrt{R_i^2 + 2At_B} - R_i\right] \approx c\frac{R_B^2}{A} \approx 10^4\ \text{cm}.$$

Surface gravity on the horizon at baryogenesis is $g_{HB} = G\frac{4\pi}{3}\frac{\varepsilon(R_B)}{c^2}d_B \approx \frac{4\pi G}{3c}\varepsilon_r\frac{R_0^4}{AR_B^2}$, and associated horizon temperature [9] is $T_{HB} = \frac{\hbar}{2\pi ck}g_{HB} \approx 6\times10^{-7}\text{ K}$. Occupation probabilities of bits on the horizon at baryogenesis are proportional to their Boltzmann factors. If energy of $e/6$ bits is $E-\Delta$ and energy of $-e/6$ bits is $E+\Delta$, proton-antiproton ratio at baryogenesis is $\left(\mathrm{e}^{-\frac{E-\Delta}{kT_{HB}}}\Big/\mathrm{e}^{-\frac{E+\Delta}{kT_{HB}}}\right)^6 = \mathrm{e}^{\frac{12\Delta}{kT_{HB}}} \approx 1+\frac{12\Delta}{kT_{HB}}$ and proton excess is $\frac{12\Delta}{kT_{HB}}$. Energy released when a $-e/6$ bit on the horizon changes to an $e/6$ bit raises another bit from $e/6$ to $-e/6$, ensuring charge conservation. Energy to change the state of bits on the horizon must be transferred by massless quanta with wavelength related to the scale of the horizon, and the only macroscopic length characteristic of the horizon at baryogenesis is the circumference $2\pi R_B$. If energy 2Δ to change the state of bits on the horizon (and corresponding bits within the universe) is energy of massless quanta with wavelength characteristic of a closed Friedmann universe with radius R_B at baryogenesis, $2\Delta = \frac{\hbar c}{R_B}$. Substituting from above, proton excess at baryogenesis is $\frac{12\Delta}{kT_{HB}} = \left(\frac{24\pi c^2}{R_0}\right)\left(\frac{2.725}{T_B}\right)\sqrt{\frac{3}{8\pi G\varepsilon_r}} \approx 1.8\times10^{-9}$. WMAP [10] found (baryon density)/(microwave background photon density) = 6.1×10^{-10}. At baryogenesis, the number of protons, anti-protons, and photons were approximately equal. When almost all protons and anti-protons annihilated to two photons, the baryon to photon ratio became $\frac{1}{3}\left(1.8\times10^{-9}\right) = 6\times10^{-10}$, in agreement with WMAP results.