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Critical Analysis of the Origins of Heisenberg's Uncertainty Principle

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Abstract

Analysis of the initial stages of the logical process followed by Louis de Broglie in establishing the electron phase wave equation in his 1924 thesis, which triggered the development of Wave Mechanics when Erwin Schrödinger formalized this concept with his vectorial wave equation. This development was soon followed by Quantum Mechanics, when Schrödinger proved that the Matrix Mechanics independently developed by Werner Heisenberg was equivalent to Wave Mechanics, with both theories leaving room for some degree of uncertainty as to the physical localization of the moving electron. This is what led Heisenberg to also formalize the Uncertainty Principle to take this situation into account. This principle was soon regarded as a fundamental axiomatic principle that seemed to make further exploration of the subatomic level of magnitude appear impossible to most researchers. We will analyze in this article the reason why the phase-wave velocity established by de Broglie generated this uncertainty in the localization of the moving electron in light of the current state of knowledge on the behavior of the electron in motion, in view of establishing the relevance of maintaining the Uncertainty Principle in the study of the subatomic level of magnitude.

Keywords

Phase Wave Velocity, Wave Mechanics, Matrix Mechanics, Quantum Mechanics, Uncertainty Principle

1. Introduction

It seems that Heisenberg formalized this principle because he considered that his personal defeat in trying to understand the subatomic level more clearly than was possible with his Matrix Mechanics method [1], apparently confirmed by the problem arising from the velocity calculated for the de Broglie *phase wave*,

established, from his point of view, an objective impossibility for mankind as a whole to ever understand the subatomic level any more clearly. Thus, given his immense status as a major physicist of the 20th century, his attitude unduly discouraged further research in this field, which severely hampered fundamental research aimed at better understanding the nature of elementary particles over the course of the past century:

"*There can be no visual description of the structure of the atom, because such a description—precisely because it's visual—would have to use the concepts of classical physics, concepts that no longer allow us to grasp the phenomena. You understand that, in attempting to create a theory of this kind, we're undertaking an a priori impossible task. We need to say something about atomic structure, but we have no language in which to make ourselves understood.*" Werner Heisenberg (1969) [2]

This unjustified certainty about such a postulated inability of humankind to ever understand more clearly the subatomic level eventually became so deeply anchored into the thinking processes of even top-level contributors to fundamental physics that some even openly discouraged this type of research. This could not be better illustrated than by this comment from one of the major physicists in his 1949 seminal paper:

"*In many problems, for example, the close collisions of particles, we are not interested in the precise temporal sequence of events. It is of no interest to be able to say how the situation would look at each instant of time during a collision and how it progresses from instant to instant.*" Richard Feynman (1949) ([3]: p. 771)

As it turns out, it was a choice made by the scientific community in 1907, during the process of adoption of the theory of Special Relativity, to ignore a confirmed experimental finding about the behavior of electrons in acceleration processes at the subatomic level, which seemed innocuous at the time and was already all but forgotten 20 years later in theoretical physics circles, after having aroused too few objections—mainly by Max Abraham and Walter Kaufmann for a while—that seems to have been the direct cause of the development of this Principle, that actually acted as a compensatory fuzzy replacement for this now forgotten but clearly defined characteristic of accelerating electrons, that, strangely, the engineering community nevertheless constantly used in all applications that required guiding free moving electrons ever since the Lorentz force equation used by Kaufmann to establish this characteristic proved it to be correct for application to elementary particles motion, such as the electron.

This 1907 decision was made by the community in light of this remark by Einstein that most scientists of the era were apparently in agreement with:

"*Herr Kaufmann has determined the relation between* [*electric and magnetic deflection*] *of β-rays with admirable care. ... Using an independent method, Herr Planck obtained results which fully agree with Kaufmann. ... It is further to be noted that the theories of Abraham and Bucherer yield curves which fit the observed curve considerably better than the curve obtained from relativity theory.*

However, in my opinion, these theories should be ascribed a rather small probability because their basic postulates concerning the mass of the moving electron are not made plausible by theoretical systems which encompass wider complexes and phenomena." Albert Einstein (1907) ([4]: p. 159)

Due to the neglect of taking into account this proven feature of electron motion in the fundamental physics community, that concerns their electromagnetic behavior as analyzed in Reference [5], the resulting uncertainty about the location of the electron in the hydrogen atom remained unresolved at the time and remained endemic in the theoretical physics community even before Heisenberg formalized the concept.

This information about the 1907 decision came to light only in 1982 with the publication of Einstein's biography by Abraham Pais [4] that regrouped references to most correspondence and articles that Einstein authored over the course of his life. To this author's knowledge, no other source in scientific literature ever mentioned this decision.

By 1923, quantum theory had reached a dead end because, while the Bohr model of the hydrogen atom satisfactorily explained its energy spectrum, it encountered extreme difficulties in explaining the behavior of the two electrons of the helium atom, among other issues.

The first significant advances beyond Bohr's idealized model, in understanding the mechanical behavior of the electron in the hydrogen atom, were made by Louis de Broglie in 1924 with his doctoral thesis [6] [7], and independently in 1925 by Werner Heisenberg with his Matrix Mechanics [1]. In 1926, particularly impressed from his own admission by de Broglie's *phase-wave* theory, *considered in a region of zero potential energy*, Erwin Schrödinger demonstrated that these two theories were equivalent by developing his vectorial wave equation based on this *phase wave* concept, involving the amount of momentum kinetic energy of the electron, as defined by de Broglie for the hydrogen atom [8]:

$$i\hbar\frac{\partial}{\partial t}\Psi\left(\vec{r},t\right)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V\left(\vec{r},t\right)\right]\Psi\left(\vec{r},t\right) \tag{1}$$

which is a wave-mechanics vectorial version of the classical-mechanics representation of the electron *resonance state* that de Broglie discovered and represented by this equation:

$$m_0\oint v\cdot \mathrm{d}l=2\pi R m_0 v=nh \quad n=1,2,3,\cdots \tag{2}$$

and that he illustrated with the following metaphor:

"*The notion of phase wave will allow us to provide an explanation of Einstein's condition. It results from the considerations of Chapter II that the trajectory of the mobile is one of the rays of its phase wave, the latter must run along the trajectory with a constant frequency* (*since the total energy is constant*) *and a variable velocity whose value we have learned to calculate. Propagation is therefore analogous to that of a liquid wave in a channel closed on itself and of variable depth. It is physically evident that to have a stable regime, the length of the*

channel must be in resonance with the wave. In other words, the wave portions that follow each other at a distance equal to an integer multiple of the length l of the channel and that are therefore at the same point in the channel, must be in phase. The resonance condition is $l = n\lambda$ *if the wavelength is constant and* $\oint (v/V)dl = n$ (*integer*) *in the general case.*" Louis de Broglie (1924) ([7]: p. 51)

With symbol λ, de Broglie was referring here to the constant length of the ground state orbit of the Bohr model, that we will clearly identify in this article as $\lambda_{\text{deBroglie}}$ to represent the $2\pi R_1$ element of Equation (2), to clearly distinguish it from the same λ symbol used to represent the wavelength of electromagnetic photons moving at the speed of light as in the familiar equation $c = \lambda\nu$.

Given that this younger generation of theoreticians knew nothing of Kaufmann's discovery, since it was mentioned nowhere in their textbooks nor by their professors, for which the overwhelming subject of discussion was the Special Relativity Theory and the more recent General theory of Relativity, Kaufmann's confirmed discovery was therefore not taken into account in any of the three models that defined Quantum Mechanics, which resulted in the observed blurring of the representation of the moving electron in all of them.

But before delving deeper into the early 1900 experimental results, whose neglect resulted in the development of the Uncertainty Principle, let us have a look at the manner in which the behavior of electrons in the hydrogen atom was analyzed in the 1920s.

2. Establishment of the Phase Wave Velocity *w*, of the Wave Group Velocity *g* and of the Controlled Particle Velocity *v*

From the very clear account of Reference [9], which is possibly the most comprehensive and trustable undergrad textbook on Quantum Physics currently available, we will follow a careful step-by-step description of this development, detailing on the way many issues that will appear trivial at first glance and that are generally taken for granted, but whose detailed correlation as a whole will shed new light on many currently obscured aspects of the true origins of Quantum Mechanics and of the Uncertainty Principle.

From the Reference [9] account, the development of Quantum Mechanics is grounded on Albert Einstein's condition referred to in his first 1905 paper [10], that was established by Max Planck in 1899 [11] from his analysis of the data collected by Wilhelm Wien in 1893 that confirmed the quantized nature of electromagnetic radiation [12]:

$$E = h\nu \tag{3}$$

as correlated with the following Louis de Broglie's relation established in his 1924 Thesis [6] [7]:

$$p = \frac{h}{\lambda} \quad \text{Reformulated as} \quad p = \frac{h}{\lambda_{\text{deBroglie}}} \tag{4}$$

The first step of the development of Quantum Mechanics hinged on the

meaning that was assigned to the wavelength $\lambda_{\text{deBroglie}}$ of de Broglie's *matter wave* and to the definition of its *phase velocity* w, as represented in Equation (3.8) of Reference [9], reproduced here as Equation (5):

$$w = \lambda \nu \quad \text{Rewritten as} \quad w = \lambda_{\text{deBroglie}} \nu \tag{5}$$

The remaining variable ν—Greek letter "nu" not to be confused with very similar italicized Latin letter v often used in textbooks as a symbol for "velocity"—is the electromagnetic "frequency" of an elementary energy quantum obtained by dividing its amount of energy by Planck's constant h as defined with Planck's Equation (3).

Briefly summarized, in context of de Broglie's analysis of the behavior of the electron, all terms such as $\lambda_{\text{deBroglie}}$, *phase wave*, *wave group*, *wave packet*, *matter wave*, *pilot wave*, all specifically pertain to the *momentum energy* of the moving electron, with its value on the ground state orbit of the electron in the idealized Bohr atom taken as a guiding numerically resolvable reference example.

Equation (5) is quite different from the well known very similar Equation (6) because for the same frequency ν of a given amount of momentum energy in both equations, Equation (5) will provide the velocity of a related moving massive electromagnetic particle, such as the electron, while Equation (6) will provide the invariant speed of light c for the same amount of energy moving in the form of a free moving electromagnetic photon, in other words, an amount of energy that does not propel any massive electromagnetic particle:

$$c = \lambda \nu \tag{6}$$

For example, for the frequency ν = 6.579683918E15 Hz of the amount of energy induced in an electron in the ground state of the Bohr atom—which is accounted for by the number of times the electron travels the Bohr orbit per second at its classical velocity, as we'll see later—the *phase velocity* of its momentum energy component will be established with Equation (5) as w = 2187691.252 m/s (first erroneously estimated to be only half this velocity w = 1093845.625 m/s due to having involuntarily used a frequency of only ν = 3.289841958E15 Hz as we will see further on), while it will be c = 299792458 m/s with Equation (6), *i.e.* the speed of light.

The difference lies in the length of the wave, $w/\nu = \lambda_{\text{deBroglie}}$ = 3.32491846E−10 m, which is the length of the Bohr orbit $\lambda_B = 2\pi a_0$ = 3.32491846E−10 m for Equation (5), while the wavelength in Equation (6) is c/ν = 4.556335255E−8 m, which is the electromagnetic wavelength of a free moving photon of energy $E = h\nu$ = 4.359743806E−18 j, *i.e.* the same amount of energy as the carrier energy of the captive electron in the ground-state orbital of the hydrogen atom, half of which K = 2.179871903E−18 j constitutes its momentum energy.

The generalized use in the literature of the same symbol λ to often indifferently refer to the $\lambda_{\text{deBroglie}}$ momentum energy wavelengths and to electromagnetic λ wavelengths of free moving electromagnetic photons has been the source of much confusion.

In other words, the $\lambda_{\text{deBroglie}}$-wavelength is the distance covered by an electron during one cycle of the frequency ν of its total energy at the velocity at which its momentum energy propels it, which will vary with the total amount of energy induced in the electron at any given moment. On its part, the λ-wavelength of a free moving electromagnetic photon of the same frequency ν will always provide the distance covered at the speed of light during one cycle of this frequency.

In harmony with Bohr's condition "*The only trajectories that are stable are circular trajectories for which the momentum is an integral multiple of h*/2π, *h being Planck's constant*", de Broglie established with Equation (2) that the length of the ground state orbit $\lambda_{\text{deBroglie}} = 2\pi R_1$ of the Bohr atom matches exactly with the amount of energy of Planck's constant h, that is, to exactly one cycle of the frequency ν of the total amount of energy induced in the electron by the Coulomb force at the Bohr radius distance from the proton.

$$2\pi R_1 m_0 v = 2\pi a_0 m_0 v = \lambda_B m_0 v = h = 6.626068757\text{E}-34 \tag{7}$$

In which R_1 is the radius of the ground state orbit in the Bohr atom, often referred to as constant a_0 = 5.291772083E−11 m in the literature, that when multiplied by 2π corresponds to the de Broglie wavelength $\lambda_B = \lambda_{\text{deBroglie}}$ = 3.32491846E−10 m on the Bohr orbit. So, the number of times per second that the electron theoretically runs this ideal λ_B orbit at the well established theoretical classical velocity of the electron on this orbit will provide the exact number of cycles of the electromagnetic frequency ν of the energy induced at the Bohr radius according to the Coulomb law interaction between the negative unit charge e_1 = 1.602176462E−19 Coulomb of the electron and the equal and opposite unit charge e_2 of the central proton as a function of the inverse of the distance a_0 separating the Bohr orbit from the central proton.

Before proceeding further, let us recall at this point that electric field $\boldsymbol{E}$ was defined by Gauss by simply removing one of the two charges from the Coulomb equation, and by conceptualizing the remaining charge as being an idealized mathematical point-charge from which the intensity of a potential electric $\boldsymbol{E}$-field diminishes omnidirectionally from a theoretical infinite intensity level at the location of this point charge as an inverse function of the square of the increasing distance from this point. This means that whenever an electron is related to such a potential $\boldsymbol{E}$-field, the Coulomb equation is re-established in linear interaction between the 2 charges, as in the coming equation, and that the energy that it induces in the interacting particles can be calculated.

A note of interest in this regards is that this definition by Gauss of the potential electric $\boldsymbol{E}$-field is what allowed the establishment in Reference [13] of the vector field common to both kinematic and electromagnetic mechanics.

Before the advent of Quantum Mechanics, the velocity of an electron was calculated by first equating the Coulomb equation $F = e\boldsymbol{E}$ and the fundamental acceleration equation $F = ma$ to relate the velocity parameter with the Coulomb equation, as mentioned by Einstein in his 1910 paper [14] [15] and analyzed in Reference [5], and by then expanding them to their first level forms in order to

isolate the velocity variable v of the particle as a second step:

$$F = e\boldsymbol{E} = ma = \frac{|e_1||e_2|}{4\pi\varepsilon_0 r^2} = m_0 \frac{v^2}{r} \tag{8}$$

Simplifying these combined equations and isolating the velocity parameter v, the classical velocity of the electron at the Bohr radius is obtained:

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_0 a_0}} = 2187691.252 \text{ m/s} \tag{9}$$

It can be observed from page 12 of Reference [7] that de Broglie did not use this equation to determine the classical velocity of the electron on the Bohr orbit, nor is there any mention in his thesis of Einstein's establishment of the relation $m\mathrm{d}^2x/\mathrm{d}t^2 = e\boldsymbol{E}_x$—*i.e.* $ma = e\boldsymbol{E}$, that is Equation (8)—on page 143 of his 1910 paper [14], nor is it used in Reference [9].

With this classical velocity 2187691.252 m/s of the electron on the Bohr orbit, and with Equation (7), revealing that Planck's constant h is directly related to the length of this orbit $\lambda_{\text{deBroglie}} = 2\pi R_1 = 3.32491846\text{E}{-10}$ m, each orbit taking 1.59186E−16 second to be completed, this adds up to a number of orbits traveled per second exactly equal to the exact frequency $\nu = 6.579683918\text{E}15$ Hz of the energy induced in the electron by the Coulomb force at Bohr radius distance from the proton, as put to light in References [13] [16] [17] [18]:

$$\nu_{R_1} = \frac{v}{\lambda_{R_1}} = \frac{2187691.252}{3.32491846\text{E}-10} = 6.579683918\text{E}15 \text{ Hz} \tag{10}$$

Making use now of Equation (3) established by Planck and multiplying this frequency ν by Planck's constant h provides the total amount of energy $E = 4.359743806\text{E}{-18}$ j involved:

$$E_B = h\nu_{R_1} = 4.359743806\text{E}-18 \text{ j} \tag{11}$$

Let us note that de Broglie did not numerically resolve this equation on page 12 of his thesis [7], no doubt assuming that all physicists would use the proper frequency value ν in resolving it, and the same goes for Reference [9].

We will now observe that the Coulomb equation applied to the interaction at Bohr ground state radius $R_1 = a_0 = 5.291772083\text{E}{-11}$ m distance between the electron and the proton in the Bohr atom provides the very same amount of energy as Equation (11) by means of Planck's constant h:

$$E = a_0 \cdot F = a_0 \cdot e \cdot \boldsymbol{E} = \frac{|e_1||e_2|}{4\pi\varepsilon_0 a_0} = m_0 v^2 = 4.359743804\text{E}-18 \text{ joules} \tag{12}$$

Let us note here that the classical velocity obtained with Equation (9) is calculated for the *rest mass* $m_0 = 9.10938188\text{E}{-31}$ kg of the electron from the total amount of energy $E = 4.359743806\text{E}{-18}$ joules confirmed with Equations (11) and (12), which is twice the amount of energy $K = 2.179871903\text{E}{-18}$ joules which is the classical momentum energy traditionally related to this classical velocity, which is confirmed by using the velocity calculated with Equation (9)

from the total amount of energy induced in the electron at the Bohr orbit a_0, *i.e.* $E = h\nu$ = 4.359743806E−18 j, to recuperate half of this total amount of energy specifically corresponding to the exact amount of momentum energy traditionally related to this classical velocity:

$$K = \frac{m_0 v^2}{2} = \frac{(9.10938188\text{E}-31)(2187691.252)^2}{2} = 2.179871902\text{E}-18 \text{ j} \quad (13)$$

But why this apparent dichotomy? Why does the Coulomb equation combined with the fundamental acceleration equation $F = ma$ provide the correct classical velocity of the electron with Equation (9) from twice the amount of momentum energy classically related to this velocity?

Before addressing this issue, that directly concerns the confirmed electromagnetic behavior of the electron that the theoreticians community preferred to exclude from the development of the Special Relativity Theory in 1907, let us move on with a few more steps of the logical development provided in Reference [9], that led to the establishment of Schrödinger's equation and of the Uncertainty Principle.

Now, after having posed Equation (5), the procedure explained in Reference [9] involves, in the first of a two step process, relating the de Broglie momentum Equation (4) $p = m_0 v$ to the frequency ν of the total energy induced in the electron on the Bohr orbit, obviously calculated with Equation (11), by substituting ν for E/h from this equation in Equation (14) in order to ultimately relate the *phase velocity* w of the *matter wave* to the particle velocity v in the second step to come. Let us remember at this point that de Broglie did not numerically resolve Equation (11) on page 12 of his thesis [7]:

$$w = \lambda_{\text{deBroglie}} \nu = \frac{h}{p}\frac{E}{h} = \frac{E}{p} \quad (14)$$

But strangely, the authors—as well as de Broglie himself, as can be seen on page 12 of his thesis [7]—replace E with $m_0 v^2/2$ in coming Equation (15), that we know from Equation (13) provides only half of the energy K = 2.179871902E−18 J that E represents in Equation (11) that was used to introduce the value of the energy in Equation (14).

Following the logic presented in Reference [9], Equation (14) is then considered assuming that the electron is moving at non-relativistic velocity v, *in a region of zero potential energy*, with $p = m_0 v$ and E = 4.359743806E−18 j being evaluated in terms of the rest mass m_0 = 9.10938188E−31 kg of the electron, of its non-relativistic velocity v = 2187691.252 m/s and of the amount of momentum kinetic energy $K = m_0 v^2/2$ = 2.179871903E−18 j from Equation (13), which we can observe is obviously erroneously substituted to the total energy E = 4.359743806E−18 j from Equation (11) that should be used, which of course resulted in only half the velocity v of the particle being obtained:

$$w = \frac{E}{p} = \frac{m_0 v^2/2}{m_0 v} = \frac{v}{2} = 1093845.625 \text{ m/s} \quad (15)$$

which was erroneous without anyone in the community realizing it, due, as we shall see further on, to a widespread habit in most reference works of too often not numerically resolving familiar equations, unintentionally assuming that the correct values are always taken into account for each parameter. The same state of confusion also arose with regard to the equality between the Coulomb force and the gravitational force, as analyzed in Reference [19].

And of course, following this calculation, the authors of Reference [9] justifiably remark:

"*This result seems disturbing because it appears that the matter wave w would not be able to keep up with the particle whose motion it controls*." Robert Eisberg & Robert Resnick (1974) ([9]: p. 69)

We will understand further on why de Broglie obviously used frequency ν = 3.289841958E15 Hz instead of ν = 6.579683918E15 Hz in resolving equation $E = h\nu$ on page12 of Reference [7], which led him—and Schrödinger himself later [8]—to logically replace E by $m_0v^2/2$ in Equation (15), and why he, as well as the whole new generation of theoreticians of the 1920's, was not aware that the true frequency of the energy induced at the Bohr radius is ν = 6.579683918E15 Hz and not ν = 3.289841958E15 Hz, as we shall soon see.

Considering again Equation (14) from which Equation (15) was established in Reference [9], we observe that the substitution of frequency ν by E/h can originate from no other source but from Planck's Equation (3), then numerically resolved as Equation (11) for the case of the frequency ν = 6.579683918E15 Hz of the energy induced at Bohr radius distance a_0 from the proton, *i.e.*, an amount of energy that can be confirmed by four mutually confirming methods, fully supported by the data collected by Kaufmann 25 years earlier, which was immediately confirmed as valid by the analyses of all the leading physicists of the first decade of the 20th century.

(1) The Coulomb Equation (12);

(2) The fundamental acceleration Equation (8) $F = ma$;

(3) By multiplying Planck's constant h by the number of times that the electron theoretically travels the Bohr orbit during 1 second at the classical velocity related to its momentum energy—Equation (10)—*i.e.* ν = 6.579683918E15 Hz, that amounts to a total of E = 4.359743806E−18 joules.

(4) By means of the Lorentz force equation $F = e(\boldsymbol{E} + v\boldsymbol{B})$ confirmed by the Kaufmann data that also confirms that the same frequency is related to this velocity as established in Reference [20] and whose calculation we will briefly examine.

So we now have at our disposal 4 different methods that all confirm the correct amount of energy and electromagnetic frequency of the energy induced in the electron at the mean Bohr radius distance from the proton in the hydrogen atom, which is double the amount that was historically related to the *phase wave* velocity, with method (4) even directly providing the relativistic velocity of the particle.

Indeed, unbeknownst to the theoreticians of the fundamental theoretics community of the 1920s, the applied physics and engineering community—two communities that don't communicate much with each other, except when new experimental discoveries are made, as when Lorentz confirmed Kaufmann's findings 20 years earlier—had been continuously using the Lorentz force equation over the course of the 20 previous years to control free-moving electrons on utterly precise trajectories in numerous experiments and applied developments, using the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields of the Lorentz Equation in the process, which can only be established from the correct electromagnetic wavelength $\lambda = c/\nu = 4.556335255\text{E}{-8}$ m established with Equation (6) corresponding to the correct frequency $\nu = 6.579683918\text{E}15$ Hz of the total amount of energy actually induced in the ground state of the hydrogen atom.

But how could the fundamental physics theoreticians of the 1920's even suspect that the frequency of the amount of kinetic energy of the momentum on the electron moving on the Bohr ground state orbit was only half that of the real amount of energy induced at mean ground state distance from the proton, given that they knew nothing of the Kaufmann discoveries made 20 years earlier on the one hand, and that every spectral ray on record for the hydrogen atom matched "exactly" this frequency of $\nu = 3.289841958\text{E}15$ Hz of the amount of kinetic energy emitted as a bremsstrahlung electromagnetic photon when an electron is captured by a proton to form a hydrogen atom? And the same for the spectral rays of the complete Lyman series that all could be related to the respective momentum energy amounts of each orbital!

The reason why only the momentum energy of an electron is released as a bremsstrahlung photon that can be recorded as such in spectra recordings each time it stabilizes in its rest orbital in an atom, that is, half the total energy induced, is analyzed and explained in References [21] [22].

Here are the detailed equations of the local composite $\boldsymbol{E}$-field and $\boldsymbol{B}$-field of the moving electron established in Reference [20] making use of the electron electromagnetic Compton electromagnetic wavelength $\lambda_c = 2.46310215\text{E}{-12}$ m of the electron rest mass m_0 and of the electromagnetic wavelength $\lambda = c/\nu = 4.556335255\text{E}{-8}$ m of the energy induced at the Bohr radius to calculate the related velocity:

$$\boldsymbol{E} = \frac{\pi e}{\varepsilon_0 \alpha^3} \frac{\left(\lambda^2 + \lambda_C^2\right)\sqrt{\lambda_C\left(4\lambda + \lambda_C\right)}}{\lambda^2 \lambda_C^2 \left(2\lambda + \lambda_C\right)} \qquad \boldsymbol{B} = \frac{\pi \mu_0 ec\left(\lambda^2 + \lambda_C^2\right)}{\alpha^3 \lambda^2 \lambda_C^2} \tag{16}$$

And here is the standard electromagnetic equation that emerges from the Lorentz force equation to establish the relativistic velocity of the electron, as was first proven by Kaufmann in the first decade of the 20th century:

$$v = \frac{\boldsymbol{E}}{\boldsymbol{B}} = \frac{1.813341121E13 \text{ j/C} \cdot \text{m}}{8.289000246E13 \text{ j} \cdot \text{s/C} \cdot \text{m}^2} 10^7 = 2187647.566 \text{ m/s} \tag{17}$$

This is one of the velocity values that belongs to the curve that Abraham and Bucherer established from the Kaufmann data, about which Einstein had the

following comment in 1907, as also previously quoted: "*It is further to be noted that the theories of Abraham and Bucherer yield curves which fit the observed curve considerably better than the curve obtained from relativity theory.*" ([4]: p. 159).

But let's come back for a moment to the lower *phase wave velocity w* obtained from Equation (15)—Equation (3)-(9) in Reference [9]—and see how the problem was historically resolved by making the *phase wave*, that was calculated to travel at velocity $w = v/2$, supposedly meet the apparently illogical and impossible challenge of catching up with the particle that it controls, that would itself be moving in the same direction at twice this velocity, *i.e.* the classical velocity $v =$ 2187691.250 m/s of the electron on the fundamental Bohr orbit.

It is at this point that the *matter wave* concept was introduced by de Broglie [6] [7], and according to Reference [9], designated by symbol $\Psi_{(x,t)}$, and corresponding to a *group of waves* spread out in the direction of the controlled particle, parallel to an x-axis, and whose theoretically assumed plot would look like **Figure 1**—Figures 3-7 in Reference [9].

The authors of Reference [9] explain:

"*The amplitude of the matter wave must be modulated in such a way that its value is nonzero only over some finite region of space in the vicinity of the particle. This is necessary because the matter wave must somehow be associated in space with the particle whose motion it controls. The matter wave is in the form of a group of waves and, as time passes, the group surely must move along the x axis with the same velocity of the particle.*" Robert Eisberg & Robert Resnick (1974) ([9]: p. 70)

The use of the qualifying expression "*surely must*" in this last sentence suggests that the authors seemed to have some doubts as to the real ability for the *matter wave*, that would be moving at only velocity $w = v/2$, to really continue as time passes, to remain in the vicinity of the particle moving at velocity v that it controls. Well, who wouldn't? Indeed it seems even physically impossible!

Indeed, such an enormous difference between the *calculated* velocity of the electron momentum energy, now represented by the *group wave*, and that of the particle proper that it was known to *propel*, to *catch up with* and to *control*, the smallest quantity of which was now understood to be the apparently indivisible energy of Planck's quantum h, must have cast doubt on the very velocity calculated for the electron itself, or at least cast doubt on the then prevailing view that the electron could really be a localized solid massive particle, as conceived in classical mechanics, and gave credence to the idea that the electron itself could

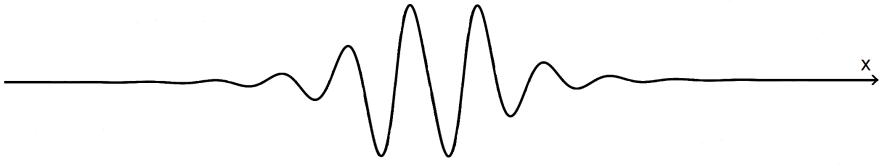

Figure 1. The idealized *matter wave*.

be a *group wave*, or *wave packet*, whose velocity could vary locally within the range that de Broglie's calculations seemed to accredit, which ended up being the conclusion adopted by the community of theoreticians, as formalized in accordance with Heisenberg's Matrix Mechanics [1], Schrödinger's wave equation [8], and eventually by Feynman's Path Integral [3], all three methods firmly grounded on Heisenberg's Uncertainty Principle now perceived as a fundamental axiomatic principle!

Even Einstein had his doubt about the ability of the *group wave* method to correctly represent the stationary resonance oscillation of the electron in the hydrogen atom. Here is his comment in this regard as formulated in the introduction of a book published in France in 1952, to which Einstein, Schrödinger, Pauli, Rosenfeld, Heisenberg, Yukawa, Davisson and de Broglie, to name only the most famous contributors, jointly collaborated to provide a general overview of the state of quantum physics in 1952, highlighting in its historical context the contribution of Louis de Broglie [23].

Einstein contributed the Introduction to this work in German, translated by the editors to French on the facing pages, the remainder of the book being in French. It is in fact a 500 pages work summarizing the scientific thoughts of the best physicists of the 20th century, never translated into English to be made available to the international scientific community.

"*Ich will dem zusammen mit Frau B. Kaufman verfassten Beitrag zu diesem Bande einige Worte vorausschicken in der einzigen Sprache, in der ich mich mit einige Leichtigkeit ausdrücken kann. Es sind Worte der Entschuldigung. Sie sollen zeigen, warum ich, trotzdem ich De Broglie visionäre Entdeckung des inneren Zusammenhanges zwischen diskreten Quantenzuständen und Resonanzzuständen in relativ jungen Jahren bewundernd miterlebt habe, doch unablässig nach einem Wege gesucht habe, das Quantenrätsel auf anderem Wege zu lösen oder doch wenigstens eine Lösung vorbereiten zu helfen.*" Albert Einstein (1952) ([23]: p. 4)

"*I will begin my contribution prepared for this book in collaboration with Mrs. Kaufman with a few words in the only language in which I can express myself with any ease. They are words to express regret. They are meant to show why—although I observed admiringly in my years of relative youth the genial discovery by Louis de Broglie of the intimate relation between the discrete quantum states and resonance states—I nevertheless ceaselessly searched for some manner to resolve the enigma of quanta by some other means, or at least help in preparing such a solution.*"

We find in Chapter II of de Broglie's thesis ([7]: p. 17), that de Broglie refers to in the previous quote from his thesis ([7]: p. 51) specifically titled "*II. Phase Velocity and Group Velocity*" the actual explanation that de Broglie provided to explain how the *group of wave*s can be expected to catch up with the particle that it controls, an explanation related to *the beat velocity of the superposed phase waves of the group*, which is not specifically mentioned in Reference [9].

"*If waves of a very similar frequencies propagate in the same direction Ox with velocities V that we will call phase propagation velocities, by their superposition, these waves will give rise to beat phenomena if velocity V varies with frequency v. These phenomena have been studied in particular by Lord Rayleigh in the case of dispersive media… If we designate by U the propagation speed of the beat, or speed of the group of waves, we find…that the speed of the group of phase waves is equal to the speed of the moving body.*"

In other words, he concluded that it is the propagation velocity *of the beat* within the wave group that will catch up with the velocity of the moving body.

But what must be understood of de Broglie's discovery of the state of resonance in standing mode of the electron in the ground state of the hydrogen atom, seems to be in reality that it would be the electron itself, that we now know from its electromagnetic structure [13] to be permanently localized, which would be captive of a stationary action resonance trajectory in the hydrogen atom at the velocity provided by its momentum kinetic energy "*when in regions of null potential energy*" as quoted from Reference [9], and as completely analyzed in References [24] [25]—see also **Figure 2**—and not specifically that only its momentum

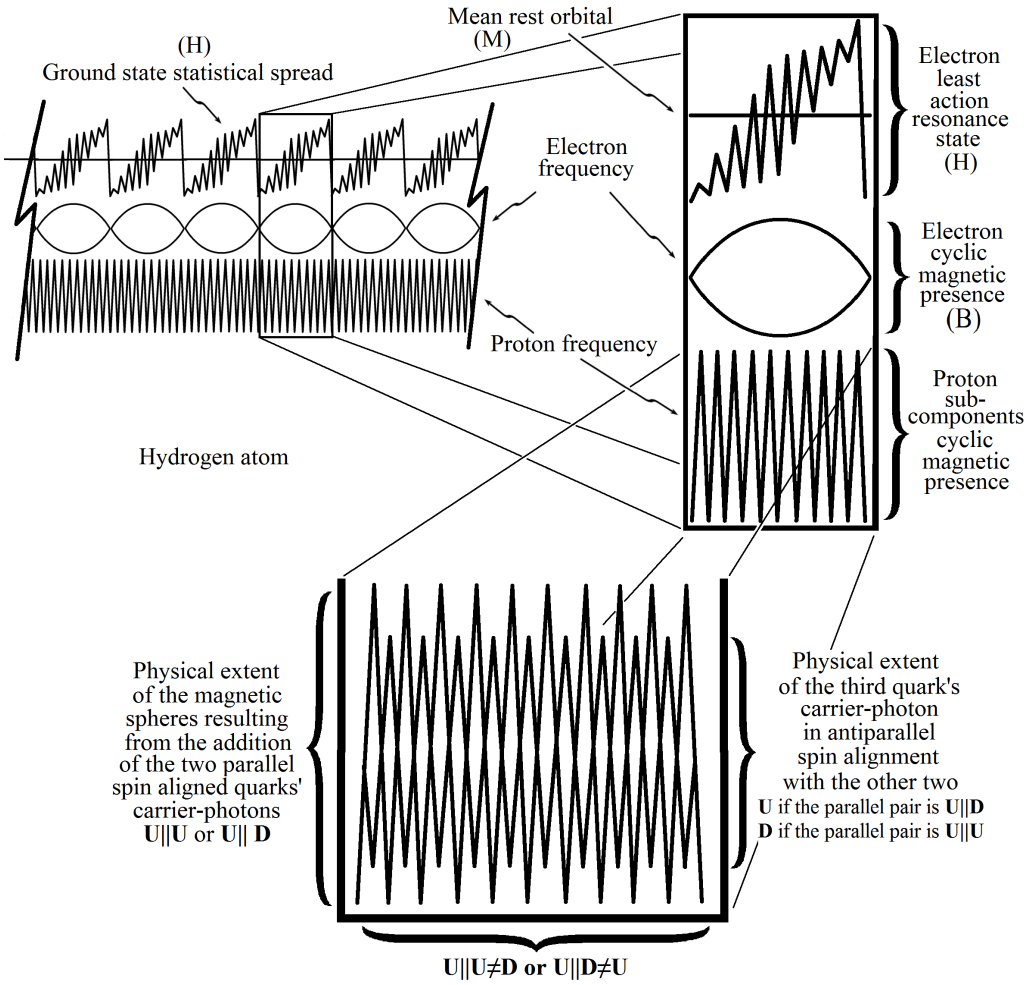

Figure 2. Establishment of the stationary action resonance state of the electron in the hydrogen atom.

energy needed to be dealt with by means of the *phase wave* calculated as moving at half the velocity of the particle that it controls, and of the *group wave* meant to catch up with it, as the concept ended up being dealt with in Quantum Mechanics.

The *phase wave*/*group wave*/*matter wave*/*pilot wave* concept is an ingenious method that de Broglie suggested to solve the velocity problem created by the only too-low frequency v that he and his contemporaries were in a position to calculate from the only amount of kinetic energy they knew could be related to the electron—its momentum energy—since this seemed at the time to be the only way out of this apparent dead end to succeed in associating with the electron the stationary resonance characteristic he had discovered from his study of the hydrogen atom spectrum.

But we now observe that when using the correct frequency of the real amount of energy induced in the ground state of the hydrogen atom, the concept of *wave group* or *wave packet* was not needed to explain the difference between the presumed but erroneous calculated velocity of the *phase wave* and that of the particle that it controls, since a calculation with the correct frequency clearly demonstrates that there is no difference in velocity between the momentum energy of the electron and that of the electron itself, and that the *phase wave* can only move at the same velocity as the particle whose motion it controls, which means *that there remains no uncertainty* as to the location of the electron with respect to its momentum energy.

Let's note that this unintentional error on de Broglie's part—as we'll see further on—about the frequency of the energy induced at the Bohr radius, which was echoed in all reference works up to Reference [9], in no way invalidates the logical sequence of reasoning leading to the establishment of the quantum wave equation. Under the assumption that the momentum energy of the electron has to behave like a spatially distributed *wave group* for it to catch up with the electron that it propels, it only renders superfluous the part of the logic that was meant to prove that the *phase wave velocity w*, erroneously calculated to be half the particle velocity v with Equation (15), is identical to *wave group velocity g* and to the particle velocity v, as confirmed with Equation (17) and also by Equation (3)-(13b) of Reference ([9]: p. 72) that confirms the equality of the velocities of both the *phase wave* and the *wave group*.

Let us now go back to the related issue of why combining the Coulomb equation with the fundamental acceleration equation $F = ma$ as Equation (9) allows calculating the same classical velocity of the electron on the Bohr orbit v = 2187691.252 m/s from twice the amount of related momentum energy E = 4.359743806E−18 j, while the classical mechanics Equation (13) relates this velocity to only the momentum energy K = 2.179871902E−18 j, and why the four different methods (1), (2), (3) and (4) listed previously unanimously confirm the actual presence of this double amount of energy as being induced at ground state orbital distance in the hydrogen atom, of which the momentum energy makes

up only half.

The key to understanding the origin of this extra amount of energy revealed by all four methods (1), (2), (3) and (4) previously mentioned, as being induced in addition to the momentum energy, that was known ever since these equations were developed, but that had remained unexplained in the case of methods (1), (2) and (3), and which is equal to, but is nevertheless different from, the momentum energy provided by classical mechanics Equation (13), resides in the proof of its separate existence brought by method (4) in the data collected by Walter Kaufmann during his experiments of the early 1900s [26] [27] [28] [29], as analyzed and confirmed by H. A. Lorentz in his 1904 paper [30].

The Kaufmann experiments were the first in history to involve precisely guided free moving electron beams in a bubble chamber by means of carefully calibrated $\boldsymbol{E}$ and $\boldsymbol{B}$ fields in view of collecting the data of their trajectories. These electrons, that were deemed at the time in context of classical mechanics to be massive in the same sense as macroscopic masses, were made to accelerate on curved trajectories to highly relativistic velocities, which allowed recording data about their velocity related behavior in their direction of motion as well as perpendicularly to this direction, revealing a behavior that never was observed in any experiment carried out with macroscopic masses.

H. A. Lorentz analyzed this data in his famous paper of 1904 [31] and confirmed this unexpected behavior of electrons from the data collected by Kaufmann, who guided these electron beams according to method number (4), which Lorentz himself had proposed in 1895 [32]. With clear experimental support, he concluded:

"*...in processes in which acceleration occurs in the direction of motion, the electron behaves as if it had mass* m_1*, and in acceleration in a direction perpendicular to the motion, it behaves as if it had mass* m_2*. These quantities* m_1 *and* m_2 *are therefore appropriately named the 'longitudinal' and 'transverse' electromagnetic masses. I will assume that, in addition, there is no 'real' or 'material' mass.*"

Deep analysis of this data and of more data subsequently collected by Planck, Bucherer and Neumann, confirmed the conclusion drawn by Lorentz, also separately confirmed by Abraham, Poincaré and Einstein himself, as put in historical perspective in Reference [5].

Summarily described, Kaufmann accelerated these electrons on curved trajectories in his bubble chamber by means of $\boldsymbol{E}$- and $\boldsymbol{B}$-fields calculated according to the Lorentz force equation:

$$F = e\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right) \tag{18}$$

the first term of which $e\boldsymbol{E}$ is the Coulomb equation as expanded in Equation (8), related to the Coulomb force that induces energy in all charged particles as a function of the inverse of the distances that separates each elementary charged particle from all other elementary charged particles:

$$F = e\boldsymbol{E} = \frac{eQ}{4\pi\varepsilon_0 R^2} \quad \rightarrow \quad E_{\text{Induced}} = R \cdot F = \frac{eQ}{4\pi\varepsilon_0 R} \tag{19}$$

in which Q represents the resultant of all elementary charges in the environment that interacts with the electron and define the intensity of their common relative $\boldsymbol{E}$-field, and R representing the mean distance at which the charges represented by Q are from the electron.

The Kaufmann data revealed that in the direction of its motion, the electron has an inertia m_1 that represents the sum of the inertia of the rest mass m_0 of the electron plus that of the total amount of energy induced by the Coulomb force, that increased with velocity according to the Lorentz γ-factor:

$$\text{Forward inertia of} \quad E_{\text{m}_1} = E_{\text{m}_0} + E_{\text{Induced}} \tag{20}$$

In which

$$E_{\text{Induced}} = \frac{\gamma m_0 v^2}{2} + \frac{\gamma m_0 v^2}{2c^2} \tag{21}$$

On its part, m_2 represents the sum of the inertia of the rest mass m_0 of the electron plus half the inertia of the energy induced by the Coulomb force, that Paul Marmet discovered in 2003—see further on—was the same amount of energy that also increases non-rectilinearly with velocity according to the Lorentz γ-factor, and that accounts simultaneously for both the instantaneous velocity related Δm mass increment and $\Delta\boldsymbol{B}$-field increment of the accelerating electron:

$$\text{Transverse inertia of} \quad E_{\text{m}_2} = E_{\text{m}_0} + \frac{E_{\text{Induced}}}{2} \tag{22}$$

In which

$$\frac{E_{\text{Induced}}}{2} = \frac{\gamma m_0 v^2}{2c^2} \tag{23}$$

Consideration of the difference between the inertia of m_1 with respect to that of m_2 led Lorentz and all leading researchers who confirmed this interpretation to the conclusion that half of the energy induced by the Coulomb force converts to a velocity related mass increment $\Delta m = \gamma m_0 v^2/2c^2$ which is measurable longitudinally as well as transversely, just like the electron rest mass m_0, while the inertia of the momentum energy $\Delta K = \gamma m_0 v^2/2$—that propels the electron rest mass m_0 plus this Δm mass increment—is measurable only longitudinally, allowing Equations (20) and (22) to be rewritten as follows:

Total energy defining the measurable forward inertia of a moving electron:

$$E = \Delta K + \Delta m c^2 + m_0 c^2 \tag{24}$$

and total energy defining the measurable transverse inertia of a moving electron:

$$E = \Delta m c^2 + m_0 c^2 \tag{25}$$

But despite the acceptance by all leading physicists of the era of the reality of this behavior of electrons at relativistic velocities at the subatomic level, doubts

remained as to its possible reality for macroscopic masses, because the experimental velocities possible for macroscopic masses were way too low below the relativistic range to make any confirmation possible for masses at the macroscopic level.

Given that simultaneously with these discoveries and discussions about the behavior of the electron mass at the subatomic level, the Special Relativity theory proposed only two years earlier in 1905 by Einstein was deemed to primarily apply to the motion of macroscopic masses in the universe at the astronomical range, doubts were sufficient in the community to justify the decision taken in 1907 not to take in consideration this confirmed mass increase of electrons at relativistic velocities, in context of the establishment of the Special Relativity theory, as witnessed by the quote of Einstein's 1907 comment on this issue presented in the Introduction.

Born in 1892, Louis de Broglie was only 15 years old in 1907, which means he had no direct knowledge of the discussions going on in the theorists' community, since at that time such discussions were only epistolary on a personal level between theoreticians or on the occasion of personal visits, and he may not even have heard of them later on during his studies, only reaching doctoral level almost 20 years later with his thesis in 1924. On his part, Schrödinger graduated to formal status only in 1914, 7 years after these discussions were over, no trace of which appeared in the reference works of the time, given the rejection of these conclusions for application to macroscopic masses.

It is also obvious from de Broglie's 1924 thesis that all of his thinking processes were grounded on both the Special and the General Relativity theories that never refer to nor integrate in any way the conclusions drawn from the Kaufmann data, which explains why no mention whatsoever is made in his thesis of this confirmed behavior of the electron that was measurable, even at the low relativistic velocity possible with the energy induced at the mean Bohr radius of the hydrogen atom.

So, instead of calculating the total energy induced in the electron from Equation (12) which indeed provides the total amount of energy induced in a charged particle from the first term of Equation (18) of method number (4) that Kaufmann used to control his electron beams and identify the increase in electron mass at relativistic velocities, and which all engineers have been using ever since in their applications, it can be observed on page 12 of de Broglie's thesis that he calculated the frequency of the total amount of energy at the Bohr radius, not from the actual total amount of energy induced at this distance from the proton as calculated with the *Coulomb*/*acceleration* Equation (12), but only from the momentum energy of the electron K = 2.179871903E−18 j—that is, half the actual total amount energy involved—with the classical mechanics *kinetic* Equation (13), which when divided by h gives the frequency $E/h = \nu$ = 3.289841958E15 Hz, obviously leading to the wrong *phase wave* velocity of Equation (15).

He calculated this momentum energy with Equation (13) to obtain the non-

relativistic momentum energy K = 2.179871903E−18 j leading to classical velocity v = 2187691.252 m/s, and also with relativistic momentum kinetic energy equation:

$$K = m_0 c^2 \left(\gamma - 1\right) = 2.179873\text{E} - 18 \text{ j} \tag{26}$$

The latter energy value leads to relativistic velocity v = 2187647.561 m/s—also directly obtained from electromagnetic Equation (17)—that reveals, as we now understand, that the actual mass of the electron at this velocity has to be slightly higher than its rest mass m_0 = 9.10938188E−31 kg to explain this slightly lower velocity and higher momentum energy of the electron, as analyzed in Reference [21] [22] and put in perspective with Equation (24).

3. Establishment by de Broglie of the Conditions of Existence of the Localized Photon

De Broglie's discovery of the state of resonance of the electron in the hydrogen atom was not his only major contribution to fundamental physics. In a work published in 1937 [33], he identified the fundamental condition that localized electromagnetic photons, as postulated by Einstein in his first paper of 1905 [10], must obey in order to perfectly explain the photoelectric effect, while remaining fully consistent with the properties of Dirac's theory of complementary corpuscle symmetry, and obey the Bose-Einstein statistic as required by the precision of Planck's blackbody law. This fundamental condition is that the existence of the localized photon can only be explained if it involves two particles, or half-photons, of spin 1/2.

According to him, such a complementary couple of particles of spin $h/4\pi$ is likely to annihilate at the contact of matter by relinquishing all of its energy and finally that this model of the photon allows the definition of an electromagnetic field linked to the probability of annihilation of the photon, a field that obeys Maxwell's equations and has all the characteristics of electromagnetic light waves.

He also concluded that it would not be possible to accurately represent elementary particles in continuous 3D space, and that he expected this question to be resolved eventually.

"*...la non-individualité des particules, le principe d'exclusion et l'énergie d'échange sont trois mystères intimement reliés: ils se rattachent tous trois à l'impossibilité de représenter exactement les entités physiques élémentaires dans le cadre de l'espace continu à trois dimensions* (*ou plus généralement de l'espace-temps continu à quatre dimensions*). *Peut-être un jour, en nous évadant hors de ce cadre, parviendrons-nous à mieux pénétrer le sens, encore bien obscur aujourd'hui, de ces grands principes directeurs de la nouvelle physique*." Louis de Broglie 1937 ([33]: p. 273)

"*...the non-individuality of particles, the exclusion principle and exchange energy are three closely related enigmas: they are all linked to the impossibility of accurately representing elementary physical entities within the framework of*

three-dimensional continuous space (*or, more generally formulated, four-dimensional continuous space-time*). *Perhaps one day, by escaping from this framework, we'll be able to better penetrate the meaning, still quite obscure today, of these great guiding principles of the new physics*."

This issue was effectively resolved in the form of an expansion of the quaternion vector coordinate system, initially summarily described in the form of a trispatial vector geometry in a presentation at the Congress-2000 event held at St. Petersburg State University in July 2000 [34], and formally analyzed in Reference [35].

This expanded vectorial geometry then allowed the identification of the properties that the fundamental *energy substance* must have, of which quantized photons must be made, for them to obey the complete set of conditions identified by de Broglie, allowing the establishment of the trispatial LC equation for photons as well as its equivalence equation for localized $\boldsymbol{E}$- and $\boldsymbol{B}$-fields, as analyzed in reference [36] [37].

The discovery published by Paul Marmet in 2003 [38] that the energy of the magnetic field of the accelerating electron that increases with velocity was the same energy that accounts for its mass increase with velocity brought attention back to Kaufmann's experiments of the early 1900's, which then allowed understanding that the energy induced in electrons had the same electromagnetic structure as that of the double-particle de Broglie photon, half of which being its momentum energy that can be calculated with Equations (13) and (26), while the other half, accounted for in addition to the momentum energy by the Coulomb equation or the Fundamental acceleration equation $F = ma$, or their combination as Equation (8), and also by the Lorentz Equation (18), oscillates in stationary mode on a plane perpendicular to the direction of application of the momentum energy, between the $\boldsymbol{E}$- and $\boldsymbol{B}$-fields states—or between L- and C-states, depending on the mathematical representation mode used—at the frequency ν related to the total amount of this carrying energy.

This discovery allowed in turn the development of an integrated series of complementary mechanical processes explaining the cause of electromagnetic frequencies, the relationship between the electron's spin and its magnetic aspect, the decoupling of massless electromagnetic photons of sufficient energy into massive electron-positron pairs, the existence of the invariant electric charges for the electron and positron and of the fractional charges of the scatterable internal sub-components of protons and neutrons, the stability of the proton and the instability of the isolated neutron, etc., as summarized in Reference [35].

4. The Hydrogen Atom Resonance States

When de Broglie discovered the resonance state of the electron in the hydrogen atom, it was not yet understood that the electron is an electromagnetic particle, *i.e.*, that its rest mass is made of a localized amount of stabilized *electromagnetic energy* oscillating in least action standing mode, nor that the resonance state of

the electron that he had discovered was due to the electromagnetic nature of the energy induced in it by the Coulomb restoring force in addition to its rest mass, half of which is its momentum energy that kinematic mechanics accounts for, and that it is the second half of this induced energy, the existence of which he was not aware of, provided via the Lorentz force equation or via the fundamental acceleration equation $F = ma$, which is at the origin of its $\nu = 6.579683918\text{E}15$ Hz oscillating frequency in standing mode between local $\boldsymbol{E}$- and $\boldsymbol{B}$-field states, as analyzed in References [36] [37].

Nor was it yet understood that the central proton is not an elementary particle, but is rather a system of elementary scatterable charged and massive electromagnetic particles, each having characteristics of mass and charge similar to those of the electron, just like the solar system is not a heavenly body, but a system of heavenly bodies, which revealed that these inner charged and massive scatterable subcomponents of the proton are also stabilized in various stationary electromagnetic resonance states oscillating in standing mode between local $\boldsymbol{E}$- and $\boldsymbol{B}$-states as a function of each their local frequencies.

The confirmed physical existence of the multi-particle scatterable inner structure of the proton—and of the neutron—was experimentally established only much later in the second half of the 1960s, in the first years of operation of the Stanford Linear Accelerator (SLAC), which was the first accelerator able to accelerate electrons or positrons with sufficient energy to breach the nucleon volume—against which they rebound when insufficiently energized—to non-destructively scatter in a highly inelastic manner against their inner charged and massive subcomponents, henceforth named Up and Down quarks, in accordance with the Murray Gell-Mann and George Zweig theory, as accounted for in Reference [39] and as analyzed in Reference [40]. The major difference between destructive and non-destructive scattering is analyzed in References [17] [18].

The reader may be highly interested in knowing that a project is currently underway at the Brookhaven National Laboratory [41] to build a new collider entirely dedicated to further investigate the inner scatterable structures of nucleons by means of non-destructive collisions, thus renewing with the lower energy method used for a short while at the SLAC facility in the 1960s. All aspects of the establishment of this new facility, the Electron-Ion Collider (EIC), and of the various programs that are in process of being defined, are described in Reference [42]. It is to be noted that further exploration of deep inelastic scattering events as reported at the SLAC facility with Reference [39] will be a major objective of the EIC project.

Another issue was the manner in which electrons organize in layers in atoms which was also not yet clearly understood in the early stages of Quantum Mechanics development, related to the observation that electrons stabilize only in pairs in electronic orbitals in atoms. Once two electrons are paired in an atom, no more electrons can occupy this orbital.

Given that the Schrödinger equation directly explains and measures only the

momentum energy of moving electrons, they were then assumed to rotate perpendicularly to their expected direction of motion according to the direction of motion of the electron. Since only clockwise and counterclockwise mechanical rotations are possible about a given rotation axis, Dirac then proposed that only an electron rotating counterclockwise could pair up with a clockwise rotating electron to fill an orbital, which gave rise to the definition of a *quantum number* having only two values. Clockwise rotation was assigned the value +1/2, and counterclockwise rotation was assigned the value −1/2 and this is the origin of the term "spin" given to this characteristic of electron behavior in atoms.

But we know better now that it is understood that electrons are electromagnetic in nature, and that they consequently each have a local magnetic field, meaning that they interact as small magnets when in sufficiently close proximity, despite their mutually repulsive same sign electric charges. So pairing is now more logically explained by anti-parallel magnetic alignment of the magnetic fields of two electrons. Like poles repel (repelling parallel magnetic orientation, corresponding to two +1/2 particles interacting in parallel magnetic orientation, or two −1/2 particles interacting in parallel magnetic orientation), and unlike poles attract (attractive antiparallel magnetic orientation, corresponding to a pair of particles interacting in +1/2 −1/2 magnetic orientation), as established in Reference [43] and completely analyzed in Reference [13].

Even if as it stands, although the Schrödinger equation and even the Dirac equation involving the Hilbert space allow accounting for the complete complement of ΔK momentum energy of the electron in motion or captive in atomic orbitals, they are unable to mechanically account for the electron ΔZ zitterbewegung as stemming from the magnetic interaction between the electron $\boldsymbol{B}$-field and the $\Delta\boldsymbol{B}$ element of its carrying energy as analyzed in References [24] [25], both fields oscillating at different frequencies, the $\boldsymbol{B}$-field of the rest mass of the electron oscillating at the fixed frequency $\nu = m_0c^2/h = 1.235589976\text{E}20$ Hz, while the oscillating frequency of the $\Delta\boldsymbol{B}$-field of its carrying energy varies with the amount of energy induced in it at any given instant, the resulting beat frequency of their combination varying with velocity, resulting from their interaction at these different frequencies, is now understood as being the direct cause of the zitterbewegung motion of the electron, which is now understood as not being stochastic, but as a clearly defined multi-frequency beat.

Indeed, the classical mechanics foundation of the Schrödinger equation, which is incompletely related to electromagnetism by means of its complex plane relation, as put in perspective in Reference [35], does not allow the reverse engineering of any of the electromagnetic resonance characteristics of the electron from its wave function characteristics, which is the disconnect that Feynman observed in 1964 that prevented Quantum Mechanics from being completely synchronized with electromagnetism [44]:

"*There are difficulties associated with the ideas of Maxwell's theory which are not solved by and not directly associated with quantum mechanics…when elec-*

tromagnetism is joined to Quantum Mechanics, *the difficulties remain*." Richard Feynman (1964) ([44]: Vol. II, p. 28-1)

All the more so since as we have seen, the wave equation does not represents the electron in motion itself, but only the volume of space that the *wave group* representing its momentum kinetic energy visits over the course of time while in the ground state orbital of the hydrogen atom.

To get an idea of the challenge that the community was confronted with when trying to discover via reverse engineer the cause of the resonance state of the electron in the hydrogen atom, let's examine how the nature of the resonator generating a well understood resonance volume in classical mechanics can easily be understood by means of reverse-engineering.

Who has not observed how a guitar string that was just picked practically *disappears* from sight while transversely *visiting*, so to speak, a very characteristic volume of space, which is its *resonance volume*, that can also be represented by a wave equation?

In this case, we obviously know in advance that the resonator is a continuous elastic string tied at both ends, because we can actually see the string when at rest, and even though it seems to practically disappear when vibrating, we also know that the string still physically exists even if we barely see it as it momentarily oscillates transversely too fast for us to clearly see.

We can also conceive that someone having never seen a guitar nor any other string instrument, but expert in mathematics, being shown the very characteristic wave equation describing the stationary resonance volume of the string could well be able, after carefully observing the symmetrically diminishing toward zero of the amplitude of the resonance volume on either side of its maximum value, to deduce that this resonance volume could only have been produced by a continuous elastic string anchored in fixed positions at both end, thus discovering and understanding the nature of a resonator that he knew nothing about previously.

But no such luck with the Schrödinger's wave equation, since the electromagnetic resonance anchoring points of the resonator responsible of the resonance volume considered are not conveniently located outside this resonance volume as in the case of the guitar string, but inside this volume, which provides no clue whatsoever that could help to even recognize the very existence of the resonator and consequently its relation to electromagnetism. This is why the only possible starting point to reverse-engineer the relationship between the volume of space defined by the Schrödinger and Dirac equations, on the one hand, and electromagnetism, on the other, is the set of confirmed electromagnetic characteristics of the energy of which the rest mass of the electron and of its carrying energy are made.

As put in perspective in Reference [35], In the case of the electron captive within the resonance volume that it visits when stabilized in the isolated hydrogen atom fundamental state, for example, it was suggested in 2013 in Reference

[43] to restrict the probabilistic spread of its possible locations in the ground state to the limits of the cylindrical volume defined by the Schrödinger equation about the mean Bohr orbit on the complex plane [35], instead of extending it to + and −∞, to account for the limits imposed by its inertia during the transverse acceleration and deceleration sequences that it is subjected to as it oscillates about the mean Bohr radius, with the set of most probable locations averaging out at the Bohr radius:

$$\int_{r}^{R}\left|\psi_{(x,t)}\right|^{2}\mathrm{d}V\mathrm{d}t=1 \tag{27}$$

Of course, due to constant interactions with surrounding matter, this cylindrical ring volume is likely in reality to spread at the limit to a 3D volume circumscribed by the surfaces of two concentric spheres whose inner and outer radii could be labelled r and R on either side of the Bohr radius. Consequently, it is exclusively to this volume located at distance r from the central proton, accounted for by the product of its complex function with its conjugate $\Psi^{*}\Psi$ in Equation (27), that the normalization condition should apply, any other localization in space becoming physically impossible for the electron while in the ground state of the hydrogen atom, due to its inertia.

As analyzed in Reference [45], it was only after it was understood that the trispatial electromagnetic structure of the rest mass of the electron is identical to that of the double-particle de Broglie photon, but entirely rotated by 90˚ to stabilize completely on the YZ-*complex plane* within the Y and Z *complex spaces* by comparison with the photon trispatial electromagnetic structure, which is stabilized on the YX-plane with only its momentum energy residing within *normal* X-*space*, as put in final perspective in Reference [35], that it became possible to understand the mechanics of the resonance state of the electron in the ground state of the hydrogen atom, as summarized with **Figure 2**.

In 1998, an experiment was carried out to study the interaction between magnets whose two poles coincide with the geometric center of each magnet, based on the hypothesis that these magnets would behave magnetically when interacting with each other in a manner similar to electrons interacting magnetically with each other, given that the point-like location in space of the *center-of-presence* of each electron during all scattering experiments implies that the two poles of each of their magnetic fields must coincide by structure with this *center-of-presence*.

The data collected in the experiment and the resulting conclusions were published in 2013 [43], and revealed many surprising features, the first being that such magnets interact as a function of the inverse cube of the distance separating them, which was confirmed in the case of electrons less than a year later, in 2014, by the publication of the report of an experiment carried out with real electrons by the team of Kotler *et al.* [46].

The most surprising observation however was the experimental confirmation that for magnets in which both poles coincide with their geometric center, which also applies by structure to electrons, their poles can be present only in alter-

nance, contrary to bar magnets, for example, in which both poles are statically located at some distance from each other. A comparative study of both types of magnets is provided in Reference [13].

In the particular case of electrons and of all other elementary electromagnetic particles whose *center-of-presence* can be established by direct scattering, such as Up and Down quarks, the implication is that their magnetic field physically oscillates between states of relative attraction and repulsion at the frequency related to the energy of which their rest mass is made up. Given that their carrier-photons are also known to be electromagnetic in nature, since Marmet's discovery in 2003 [38] of the relationship between the accelerating electron's magnetic field increment and the mass increment provided by its carrier energy, this condition also applies to them.

With regard to the possible configurations of the magnetic fields of the three quarks of the proton, given that their odd number always guarantees that when one is in repelling mode, the other two will be in attracting mode by structure, and the reverse, which is a condition analyzed and explained in Reference [43], this also means that if the magnetic field of the electron captive in a hydrogen atom is not repelled by the single magnetic field of the odd quark out, it will be repelled by the other two, and the reverse, which is what is illustrated in **Figure 2**.

After it was revealed by experiment [43] that the interaction between the magnetic fields of the inner subcomponents of the proton could only be in permanent default oscillating predominantly repelling parallel magnetic spin alignment with respect to that of the electron as represented in **Figure 2**, it became obvious that it was only the pressure constantly applied by the momentum energy of the electron, oriented towards the proton, that maintained the electron captive at mean Bohr radius distance from the proton, and that it was the interplay between this momentum energy pressure against the mutually repelling oscillating magnetic fields of the electron and those of the proton subcomponents that kept it in the resonance state discovered by de Broglie, within the volume defined by the Schrödinger equation. The mechanics of this resonance state is analyzed in detail in References [24] [25].

5. The Stability of the Hydrogen Atom

In **Figure 2**, the central sequence "B" symbolically represents an arbitrary sample of 6 occurrences of the intensity variation of the spherical presence of the electron magnetic energy as a function of its frequency. In a simplified manner, each of these 6 occurrences is confronted in the lower sequence by the more than 600 occurrences of the intensity variation of the spherical presence of the magnetic energy of the 3 carrier-photons of the three scatterable electromagnetic subcomponents of the proton as a function of their own frequencies.

The stationary action orbital equilibrium state of the electron is consequently established by the fact that the ΔK momentum energy half-quantum of its carri-

er-photon is alternately hindered in its forward motion toward the proton, when the magnetic interaction becomes repulsive between the magnetic energy spheres of the electron and one of the proton magnetic energy spheres, and is then freed from this counter-pressure while the magnetic interaction becomes attractive between the magnetic spheres involved.

As illustrated in Figure 2, during each of the 600 magnetic cycles of a proton subcomponent's carrier-photon, the electron magnetic sphere "B" will be axially repelled away from the proton by distance Δd during half of the proton subcomponent carrier-photon magnetic presence cycle during which their spin alignment is parallel thus repulsive, and since the electron will be farther away from the proton as the relation becomes antiparallel attractive for the same duration, there will be a physical impossibility for it to be axially brought back all the way by distance $-\Delta d$ before the following proton magnetic reversal cycle initiates, given that the magnetic inverse cube force will be weaker at this farther location from the proton at the beginning of the antiparallel attractive phase than it was at the beginning of the previous parallel repulsive phase.

Therefore, and by structure, given the more weakly acting inverse cube attraction function of the increased distance at the beginning of attractive phase, the electron can be axially brought back only to distance $-(\Delta d - \Delta(\Delta d))$ before the following proton magnetic reversal cycle initiates, which will cause it to progressively move away from the proton at each relative magnetic spins polarity reversal sequence between the electron magnetic field and those of the proton for each of the 600 or so occurrences of the magnetic cycles of the facing proton subcomponents, until the electron magnetic energy presence "B" momentarily falls to zero, moment during which only the electron carrier-photon ΔK momentum energy will be active, now causing the electron to freely move as close to the proton as the Coulomb force inverse square law function of the distance will bring it, until the next magnetic presence "B" cycle of the electron initiates again and that the whole predominantly repulsive magnetic sequence "B" versus the magnetic fields of the proton inner subcomponents is initiated again, as represented with Figure 2.

Of course the actual resonance state of the electron in the stationary action orbital of the hydrogen atom or in any other atom will be much more complex than hinted at with this limited example, which is only meant to summarily describe the fundamental mechanics of the magnetic interaction between the electron magnetic energy sequence "B" and its ΔK momentum energy on one hand, and the predominantly repulsive by default magnetic energy of the carrier-photons of the scatterable subcomponents of the proton, on the other hand. Obviously, the exact resonance volume within which each elementary electromagnetic massive particle in the hydrogen atom will be circumscribed, which are one electron, one down quark and two up quarks, can eventually be determined only by a careful study of all electromagnetic interactions between them and their carrier-photons.

Given that the mean equilibrium distance that this process forces the electron

in motion to stabilize at in the hydrogen atom coincides with the densest area of probability distribution of Heisenberg's statistical method, it would seem that the axial trajectory of the electron about this mean distance within the volume that the electron can thus visit as a function of its varying relativistic mass and related inertia at any given instant, it should directly correspond with Heisenberg's probability distribution of all of the possible instantaneous locations that the electron can be calculated to be localized at when repeatedly theoretically collapsing the wave function in its current form [27] [31], and whose quantized axial beat can no doubt be related to the regularities of the fine structure of the hydrogen spectrum, that Sommerfeld first associated to a hypothetical elliptical orbit that the electron would follow, in his attempt to explain the fine splitting of the main spectral lines ([9]: p. 114).

So the very limited resonance volume described with Equation (27) within which the electron is kept on this precise zigzagging trajectory is determined in all circumstances by the constant interplay between its ΔK momentum constantly pushing it towards the proton, and the precise back-pressure sequence of permanent mutual repulsion between the predominantly default repulsive parallel spin-aligned magnetic fields between the electron and the internal subcomponents of the proton, as established during the foundational experiment of the trispatial model [43].

It must be quite obvious that this type of resonance interaction, involving that the ΔK momentum energy of the electron is permanently oriented towards the proton, that is, not along the idealized Bohr trajectory about the proton as still assumed at the beginning of the 20th century, involves in no way that the electron has to be moving on a closed orbit about the proton to be permanently maintained at some distance from the proton—at the mean a_0 Bohr radius in the isolated hydrogen atom in the present case. This obviously means that this interaction will remain true even for two electrons joined in a covalent bound uniting two hydrogen atoms, even if the electrons are restricted to oscillate locally midway between the two protons involved, and in all other stable relations into which electrons are involved with other electromagnetic particles.

It must be noted here that the projection recorded during the Aneta Stodolna *et al.* experiment [47] involved tens of thousands of orbits of the electron about the immobilized nucleus, and that it is then logical that an apparent *probabilistic cloud* would seem to appear on the recording within the r and R limits of the *probability ring*, due to the 8.734668247E−14 m displacement of the beginning of the 138th electron magnetic cycle of each successive orbit, as analyzed in Reference [43], and also for the other metastable orbitals to which the electron fleetingly moved before returning to the rest orbital, as a result of parasitic stochastic energy excitation from the environment.

But despite the apparent complexity of the thousands of superposed recorded traces of the recording of the Stodolna *et al.* Experiment [47], it does not seem unrealistic to think, based on the stationary action electromagnetic equilibrium

revealed by the trispatial model, that it could become eventually possible to calculate with great precision all future physically possible locations of a localized electron in the QM statistical distribution in an isolated hydrogen atom, with as a starting location any point arbitrarily chosen on the electron zigzagging orbit within the boundaries set by its inertia and its transverse accelerations and decelerations sequence for a single orbit, thus putting an end to the unconditional reign of the Heisenberg Uncertainty Principle.

All the more so since that contrary to the mantra of the Copenhagen interpretation of Quantum Mechanics to the effect that if the location of an electron is determined via a "collapse" of the *wave function*, it would not be possible to simultaneously know its momentum energy, because, given that when the distance from the central proton that an electron is even momentarily determined to be on its axially zigzagging trajectory as previously described, then its momentum energy at this distance can easily be calculated via the Coulomb Equation (8), since the invariant charges of both the electron and of the proton are known.

6. Conclusions

The long-standing and unjustified assumption by the theoreticians community that correct numerical values are always used when solving well-established equations has been the source of much confusion throughout history, as can be seen in the case of the *phase wave* velocity as analyzed in **Section 2**, and also in the case of the relationship between the Coulomb force and the gravitational force as analyzed in reference [19].

This confusion has lasted for a full century in the case of *phase-wave* velocity, contributing to the establishment of the Uncertainty Principle as an ontological axiomatic principle, thus enormously limiting the scope of research projects in fundamental physics, at least for the subatomic level of magnitude, and for much longer in the case of the relationship between the Coulomb force and the gravitational force, unduly delaying any coherent research into gravitation.

It is to be hoped that the detection of these previously undetected wrong values in two crucial derivation sequences of fundamental physics by simply carrying out a careful numerical verification of all parameters involved will encourage theoreticians to carry out such a complete numerical verification of all the parameters used in future developments in fundamental physics.

In conclusion, although the theoreticians of the 1920s had no doubt that the frequency of the energy induced in the electron when in the ground state of the hydrogen atom, was only ν = 3.289841958E15 Hz—*i.e.*, half the real frequency that we now know to be involved—this cannot be perceived as an error on their part, because there was no way for them to even suspect otherwise from the experimental data that was at their disposal. And the same goes for the whole community, in the absence of any reference to the Kaufmann experiments in any reference work.

Even the information to the effect that the data collected by Kaufmann in the early years of the twentieth century had been excluded from the development of the theory of Special Relativity became available only very discreetly and indirectly in 1982 via a seemingly innocuous comment from Einstein, quoted on page 159 of his biography by Abraham Pais [4], and has been documented nowhere else in the formal literature.

Moreover, the pertinence of this data in experimentally establishing that the actual amount of energy induced in charged particles by the Coulomb interaction is double the amount of measurable momentum kinetic energy, came to light only in 2003 with Marmet's discovery ([38], Equation (23)) that the increasing energy corresponding to the magnetic field increase measurable for accelerating electrons could only be the same energy that accounts for its relativistic mass increase with velocity, as put in perspective in References [36] [37].

This wrong frequency value on which de Broglie, Heisenberg, Schrödinger and then also Feynman grounded their contributions to Quantum Mechanics as well as all other contributors to Quantum theory did not prevent them from leaving us with a wonderful inheritance of mathematical tools that will continue to be useful. Schrödinger's equation, in particular with its addition of the complex plane to his solution paved the way to a clearer harmonization to eventually be established between quantum theory and electromagnetism, as put in perspective in Reference [35].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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