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The Theoretical Value of Mass of the Light η-Meson via the Quarks' Geometric Model

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Abstract

Highlighting a golden triangular form in u and d quarks (Quark Geometric Model), we build the geometric structures of light meson η and individualize its decays and spin. By the structure equations describing mesons, we determine a mathematic procedure to calculate the theoretical value of the mass of light mesons η.

Keywords

Quark, Structure Equation, Geometric Structure, Golden Number, Massive Coupling, Interpenetration, IQuO, Pion, Meson, Photon

1. Introduction

One of the more problematic aspects in Hadronic literature [1] [2] is that one in which only a small fraction of the hadron mass (see proton) seems to be associated with the (bare) mass of the elementary quarks. The same occurs in a pion or, generally, in mesons. Nowadays, one of the answers to this mystery of mass origin in hadrons is the fact that the remaining mass fraction is due to the force (gluons) binding the quarks within hadrons, *i.e.*, Quantum Chromo Dynamics (QCD) [3] [4]. These mass values of bound quarks are obtained [5] directly in lattice simulations and are corresponding to a particular discretization of potentials formulated in QCD. These values together with some values of experimental masses [5] are used as input (see the pion mass) to improve the mass values and coupling parameters in the interactions and so to calculate the masses of hadrons [6]. Yet, the procedures of calculation are very complicated and supported by processed programs of calculation with computers.

In a precedent set of studies [7] [8] for solving these problems and to calculate the mass values of hadrons (light mesons), we have used another path. Instead to incorporate the binding energy of the quarks in appropriate potentials of the binding quanta, see the QCD, we incorporated it into the quarks' mass which compose the pion. Nevertheless, we do not consider the bare masses of quarks, but appropriate values of them masses derived from a "geometric structure" of pions [7] [8] and of same quarks. These mass values are very greater than that of free quarks. This way, the bonded quarks inside the hadron would have enough mass to reach the hadron mass value they compose, provided that we correctly sum them up. Since the pion, the lightest meson, is made of two bonded states of quarks, its mass could be placed as base mass to determine the mass of the most massive mesons and hadrons. We will then prove in two separate articles that all light mesons, following the pion, are elaborated structures of pion compositions and a $\{d, \underline{d}\}$-lattice of d-quarks. In essence, we refer to this as to a hypothesis of hadronic lattice. This hypothesis induces us to admit an appropriate structure equation [8], which describes all hadrons and thus also mesons. By this equation, we can calculate the masses of light mesons and also nucleons. Moreover, the structure equation allows us to identify the possible decays of a particle and its spin value. Nevertheless, all of this will be possible only if we affirm [7] [8] the quarks having a well-defined *geometric structure*. One can achieve the idea of a geometric structure of quarks if we assume a quark made by a *non-separable* set of coupled quantum oscillators with "A*urea*" (golden) geometric structure [7] [8]. We will refer to this as the Aureum Geometric Model (AGM) of quarks. Thanks to the quarks geometric hypothesis, it is possible to explain fundamental issues such as the origin of the mass of the hadrons, the role of gluons in constructing bonded quarks and the existence of *molecules of pions*. We can achieve all of this without turning to the QCD and its calculations by colour potentials.

Nevertheless, for obtaining the masses (and, in the following study, the nucleons ones as well), we need to introduce a new idea of mass calculation ($\underline{\otimes}$-operation). This idea of calculation takes into account both interactions between quarks and a possible *interpenetration* [8] of the quarks. The latter may exist if we admit that two different structures (states) of coupled quantum oscillators (or quarks) can overlap without exchanging energy. The global mass of hadron must so take into account all the possible configurations of quark components, both the ones with interpenetration and the ones with interactions. So, the hypothesis of the quark geometrical structure introduces a *"new paradigm"* [7] [8] in the phenomenology of hadronic interactions. Thanks to a few primary hypotheses, the new paradigm allows to describe the hadronic phenomenology in a more structured and straight forward way of that of QCD. The first evidence is the spectrum of light mesons with mass values very close (if not even equal) to the experimental ones, which we will calculate in two paper. In this first paper, exactly, we will calculate by theoretical physics aspects and mathematical procedure only the mass of η-meson, because that of the pion is placed as basic mass which value is the experimental one, see the ref. [8].

2. The Hypothesis of Structure

2.1. The "Golden" Hypothesis of Particles

As is well known in literature [6] [9], the protons are composed of three quarks: three centres of diffusion positioned in a triangular form in diffusion experiments with "bullet" electrons. These centres indicate the proton could have an internal *geometric* structure. Since between the Compton's wave lengths of the Planck (Δ_{pl}) particle and the one of proton (Δ_p) there is a *golden* relation, at less than a scale factor, then we have, see the pentagon in **Figure 1**.

The vertices (A, B, D) are three quantum oscillators with which photon couples in the electromagnetic interactions. In pentagonal structure, it is evident that the diagonals [(AD), (BD)] are in a golden ratio with the base AB: ($\phi \approx 1.618$) → [(AD/AB) = ϕ]. Where (ϕ) is the "*aureus*" (golden) number. This rapport also implies quarks (u, d) are *aureus* triangles with [($\lambda_u = \lambda_\gamma$), ($\lambda_d = \lambda_\beta$)] and $\left[\left(\lambda_u/\lambda_d\right) = \left(m_d/m_u\right) = \phi \approx 1.618\right]$ with ($m_d > m_u$). Besides it is [$\lambda_p = k_p \lambda_u$], where ($\lambda_u$) is the Compton wavelength of "free" quark, while (k_p) is a coefficient of "*elastic adaptation*", when (u, d) quarks reciprocally bind for origin the proton. Just k_p can be in relation with binding gluons of the (u, d) quarks into proton; we will point out $\left[V\left(r\right)_{\text{QCD}} \leftrightarrow k_p\right]$, where $V(r)$ is colour potential in QCD theory [3] [10] [11]; so in this theory, or AGM, an elastic tension k replaces the potential $V(r)$. We notice that this structure can be realizable only through "particular" quantum oscillators: the IQuO (Intrinsic Quantum Oscillator), which have a "*sub-structure*" made two sub-oscillators, where there are half-quanta see [7] [12] [13]. Furthermore, we recall that a geometric property of the golden triangles (u or d) is that one each triangle is composed in its turn of two golden triangles (u_1, d_1), see **Figure 2** and ref. [7] [8].

This configuration occurs, see a-configuration, when u-quark ($\underline{u}$) in a meson could attach themselves to IQuO-chain of the diagonal (AC) belonging to $\underline{d}$-quark (d). These two features may origin a new *strange* state in quarks, which becomes so the *strange quark* (s).

2.2. The Structure of Charged $\pi^{\pm}$

We suggested that mesons [8] are geometric structures realized by coupling between the golden quarks (see **Figure 3**).

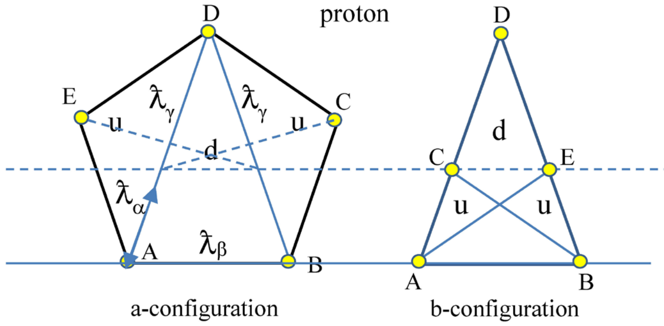

Figure 1. Geometric structure at quark of the proton.

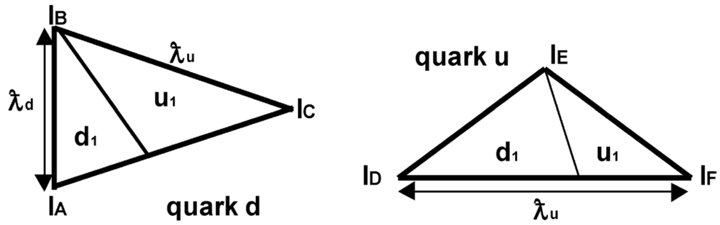

Figure 2. Golden triangles (u_1, d_1) into golden triangle (u, d).

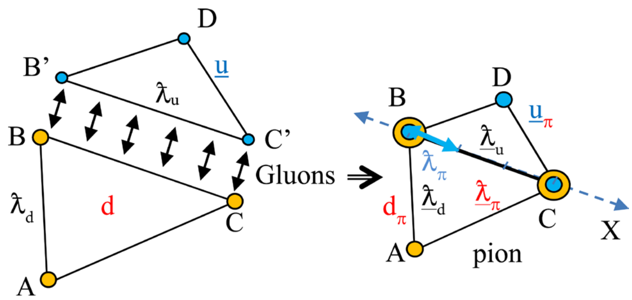

Figure 3. Geometric form of pion.

The first attempt of a formal "*structure equation*" is: $\left[\left(\pi^{+}\right)=\left(u \oplus \underline{d}\right),\left(\pi^{-}\right)=\left(\underline{u} \oplus d\right)\right]$.

Where the components (π; u, d) are matrices with elements expressed by wave functions in the representation of quantum oscillators of the field [10] [11], see ahead. The sign ($\oplus$) point out formally dynamics coupling between quarks. Then, we have the configuration illustrated in **Figure 3**.

Where $\underline{u}$ is the u antiquark. The bonding (see gluons) between two free quarks ($u \leftrightarrow d$) increases the elastic tension between IQuO components of quarks $\left[\left(k_u,k_d\right)\rightarrow\left(\underline{k}_u,\underline{k}_d\right)\right]$, which, in turn, increases the "free" frequencies $\left[\left(\omega_u,\omega_d\right)\rightarrow\left(\underline{\omega}_u,\underline{\omega}_d\right)\right]$ or mass $\left[\left(m_u,m_d\right)_{free}\rightarrow\left(\underline{m}_u,\underline{m}_d\right)_{bounds}\right]$, see ref. [8] Equations (1) and (2); thus, the Compton wavelength decreases ($\underline{\lambda}_u<\lambda_u$), ($\underline{\lambda}_d<\lambda_d$)], shrinking u and d quarks (see second drawing in **Figure 3**). The π-structure oscillates with only one frequency (ω_π) and period (τ_π), to which we associate (Δ_π, m_π). Speaking of elastic tensions in bound quarks, we can admit that in ($\underline{k}_i$) are contained the mass defects (recall that k replaces the potential $V(r)$), so that we have $\left[m_\pi=\underline{m}_u+\underline{m}_d\right]$. $V(r)$, in turn, is related to coupling ($\oplus$) between IQuO, where $\left[\oplus=\left(\oplus_g+\oplus_{em}\right)\right]$ with $\oplus_g$ indicating the coupling by gluons and $\oplus_{em}$ indicating the electromagnetic coupling. Therefore, even along sides, there are gluons (see the Joining-IQuO of **Figure 3**) connecting the vertex oscillators. This aspect is consistent with QCD [4]. In [8], we calculated the mass values of quarks inside the pion (u_π, d_π) by an equations' system with solutions:

$$\left[m\left(u_\pi\right)=(53.31)\,\text{MeV},m\left(d_\pi\right)=(86.26)\,\text{MeV}\right] \quad (1)$$

We assign these mass values to the physical system composed by quarks (u, d) with gluons that form the pion, m ($\pi^{\pm}$) = (139, 57) MeV. Note the u_π and d_π quarks with their gluons are called "dressed" quarks.

3. The Quarks' Interpenetration

3.1. The Quarks' Interpenetration and Spin

In the pion, we note different configurations related to the X-junction axis, Figure 4: two quarks have such a relative motion around X-axis because the vertices (A, D) are free of rotating [8]. These configurations, or orientations, can induce us to think about the *spin* of the pion. To individual quark, seen like Fermionic particle, one associates an *intrinsic spin* of quarks (s_q), while now we associate also an *orbital spin* (s_l) to rotations of u-quark (or d-quark) around the X-axis (see the experimental observations about proton spin [14]). The hypothesis of structure does not conflict with experiments [15] of CERN (COMPASS), SLAC, and DESY (HERMES) and one can demonstrate the total spin: $\left[s_\pi = s_q + s_l + s_g\right]$.

As noted, different relative orientations between *u* quarks and *d* quarks imply a relative rotation (spin) of one quark around the other quark, suggesting a mutual crossing of the quarks (see the overlapping of $[I_{DC}]_u$ and $[I_{AC}]_d$ in Figure 4. This crossing can have the meaning of a reciprocal *interpenetration* (⊗) between quarks ($q_i \otimes q_j$). During this mutual crossing, quarks not exchange energy in those parts that interpenetrate, while, instead, there is an exchange of binding quanta in the diagonal BC of Figure 4. Note that the interpenetration between quarks implies a reciprocal *intrinsic movement.* This internal movement of components has a kinetic energy form, called by us *interpenetration kinetic energy* [8], which must be counted in the mass defect.

The interpenetration could explain the zero value of pion's spin. In ref. [8] we showed that quarks have the orbital spins ($s_l(u)$, $s_l(d)$) in a relative way opposite rotations: $\left[s_l(u) = -s_l(d)\right]_\pi \to \left[s_l(u) + s_l(d)\right]_\pi = 0$.

This opposite orbital rotation drags the bounding gluons of two quarks (g_u, g_d), which so have opposite orbital spins:
$\left[s_l(g_u) = -s_l(g_d)\right]_\pi \to \left[s_l(g_u) + s_l(g_d)\right]_\pi = 0$.

The same occurs in gluons propagating along the IQuO of side BC; recall gluons as bosons with spin (s_g = 1), then would may be that the gluons in IQuO_{BC} (Figure 4) have opposite spins:
$\left[s_g(u) = -s_g(d)\right]_\pi \to \left[s_g(u) + s_g(d)\right]_\pi = 0$.

The same could happen at the wave function (spinor) of quarks along the side BC:

$$\left[s_q(u) = -s_q(d)\right]_\pi \to \left[s_q(u) + s_q(d)\right]_\pi = 0$$

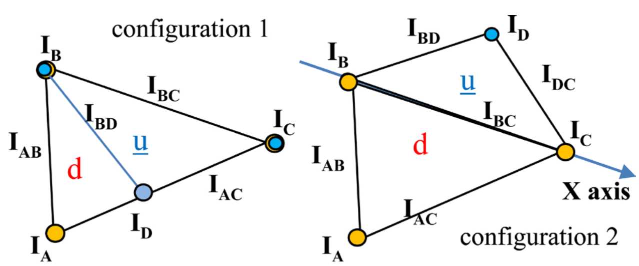

Figure 4. Pion configurations.

Then, the global spin of a pion is zero.

3.2. The $\underline{\otimes}$-Operation

In the interpenetration of quarks and their dynamics interactions, we used [8] a new $\underline{\otimes}$-*operation* of combination (or coupling) of quarks and particles: ($a_i \underline{\otimes}\, b_j$). The new operation ($\underline{\otimes}$) indicates a composition of two operations (⊗, ⊕) or [$\underline{\otimes}$≡ (⊗ U ⊕)], where ⊗-operation describes the proper interpenetration of the quarks and follows, in algebraic calculations, the properties of multiplication. Instead, ⊕-operation describes dynamics interactions and follows, in algebraic calculations, the properties of the sum (see Table 1).

3.3. The Structure Equation of Pions

The operation of combination (⊕) could be used for express dynamics couplings ($\underline{u} \leftrightarrow d$) (or ($u \leftrightarrow \underline{d}$)) in charged pion π (Figure 3). Nevertheless, the different relative orientations of quarks (u, d) induce us to admit interpenetrations between two quarks. Thus, we should write $\left[\pi^{\pm} = \left(u \underline{\otimes} d\right)\right]$.

In AGM, (u) and (d) represent the wave function associated with quark [8] [11] [13] in the representation of operators of annihilation and creation (a, a^+), relative to IQuO or quantum oscillators at *semi-quanta*:

$$\Psi_q\left(\vec{x},t\right) = \left(\frac{1}{\sqrt{Vol}}\right)\sum_{\vec{k}} q_{\vec{k}}\left(t\right)\exp\left(i\vec{k}\cdot\vec{x}\right)$$

$$\left[q_{\vec{k}}\left(t\right)\right] \equiv \begin{pmatrix} a^+_{\vec{k}}\left(t\right) \\ a_{\vec{k}}\left(t\right)\end{pmatrix}_{\text{IQuO}} = \left\{\begin{pmatrix}\left\|u^+_{\vec{k}}\right\|\left(t\right) \\ \left\|u_{\vec{k}}\right\|\left(t\right)\end{pmatrix}_{\text{IQuO}}, \begin{pmatrix}\left\|d^+_{\vec{k}}\right\|\left(t\right) \\ \left\|d_{\vec{k}}\right\|\left(t\right)\end{pmatrix}_{\text{IQuO}}\right\}$$

where $\|u\|,\|d\|$ are matrices which express the structure of quantum oscillators (or IQuO) of quarks (u,d) in the IQuO-representation, see [7] [8] [12] [13].

We conjectured, see [8], that π^0 neutral pions is originated by all the possible combinations of quarks' couplings using both the ⊗-operation and ⊕-operation or ($\underline{\otimes}$). Then, we defined (1-property Table 1)

$$\left(\pi^0\right) = \left[\left(\pi^+\right)\underline{\otimes}\left(\pi^-\right)\right] = \left[\left(u \oplus \underline{d}\right)\underline{\otimes}\left(\underline{u} \oplus d\right)\right] \tag{2}$$

And we demonstrated [8] that it is:

$$\begin{aligned}\Psi\left(\pi^0\right)_{(\underline{\otimes})} &= \left[\left(\pi^-\right)\underline{\otimes}\left(\pi^+\right)\right] \\ &= \left\{\left[\left(u \oplus \underline{d}\right)\otimes\left(\underline{u} \oplus d\right)\right]\oplus\left[\left(u \oplus \underline{u}\right)\otimes\left(\underline{d} \oplus d\right)\right]\right\}_{\underline{\Psi}} \\ &= \left\{\left[\left(\pi^-\right)\otimes\left(\pi^+\right)\right]\oplus\left[\left(u \oplus \underline{u}\right)\otimes\left(\underline{d} \oplus d\right)\right]\right\}_{\underline{\Psi}}\end{aligned} \tag{3}$$

Table 1. The $\underline{\otimes}$ composition of two operation (⊗, ⊕).

$\underline{\otimes} \equiv [\otimes, \oplus]$ ↔ composed operation
1) If $\left\{\left[A = (a \oplus b), B = (c \oplus d)\right]\right\}$ → $(A \underline{\otimes} B) = (a \oplus b) \otimes (c \oplus d)$
2) $(a \oplus b) \otimes (c \oplus d) = (a \otimes c) \oplus (a \otimes d) \oplus (b \otimes c) \oplus (b \otimes d)$

This equation represents the *structural equation* of neutral pion (π^0) in no-local state ($\underline{\Psi}$). Note $\left\{\left(\pi^0\right)=\left[\left(\pi^+\right)\underline{\otimes}\left(\pi^-\right)\right]\right\}\neq\left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right]$ and thus $\left\{m\left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right]\neq m\left(\pi^0\right)\right\}$.

The interpenetration between two charged pions allows of assuming:

$$m\left[\left(\pi^-\right)\otimes\left(\pi^+\right)\right]=m\left(\pi^\pm\right)^0=m\left(\pi^\pm\right) \tag{4}$$

The $\left(\pi^\pm\right)^0=\left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right]$ is called a *virtual neutral pion*.

In graphic mode, by ref. [8] we conjectured the following configurations (**Figure 5**).

Note (see **Figure 5**) there are two possible configurations: $[(\pi^0)_a, (\pi^0)_b]$.

There are so different possibilities of configuration of quarks around the axes of propagation, with consequent rotations (*spin*). If the spins of ($u \oplus \underline{d}$) and ($\underline{u} \oplus d$) are respectively zero, then one can think the spin of neutral pion is zero.

3.4. The F_m Function of Partial Mass and $F_{\Delta m}$ Mass Defect

The coupling between two quarks involves the combination of all possible configurations of these two quarks. In this way, the total mass ($m_{tot}\equiv m_{\underline{\otimes}}$) is the sum of all masses associated with each combination of interpenetration ($m_\otimes$) and interaction ($m_\oplus$) [8]. It follows: $\left[m_{\underline{\otimes}}=m_\otimes+m_\oplus\right]$.

In general, for every *X*-system (composed by more particles), we use a partial mass Function (F_m) applied to *structure equation* of *X*-system, with $X\left[\left(A_1,A_2,\cdots,A_n\right)_\otimes;\left(B_1,B_2,\cdots,B_n\right)_\oplus\right]$, where $\left[\left(A_i\right)_\otimes,\left(B_j\right)_\oplus\right]$ are the "base components" of the structure [8]. The (F_m) is an application on the structure components (A, B), which gives us the mass values (m_i) of these components of base. The (F_m) operates on X, in the following way:

$$\begin{aligned}F_m\left(X\right)&=\left\{\sum_{(i,j)=1}^{n}F_m\left[\left(A_i\right)_{(\otimes)},\left(B_j\right)_{(\oplus)}\right]\right\}=\left[\sum_{i=1}^{n}m\left(A_i\right)_{(\otimes)}\right]_A+\left[\sum_{j=1}^{m}m\left(B_j\right)_{(\oplus)}\right]_B\\&=\left[m\left(a\otimes b\right)_{A_1}+\cdots+m\left(w\otimes z\right)_{A_n}\right]_A+\left[m\left(a\oplus b\right)_{B_1}+\cdots+m\left(w\oplus z\right)_{B_m}\right]_B\end{aligned} \tag{5}$$

We will have the following applications:

$$\begin{cases}F_m\left(A_{(\otimes)}\right)=F_m\left[\left(a\right)\otimes\left(b\right)\right]_A=\left\langle m\left(a,b\right)\right\rangle=\left\langle m\left(a\right),m\left(b\right)\right\rangle\\F_m\left(B_{(\oplus)}\right)=F_m\left[\left(a\right)\oplus\left(b\right)\right]_B=m\left(\left(a\right)\oplus\left(b\right)\right)=m\left(a\right)+m\left(b\right)\end{cases} \tag{6}$$

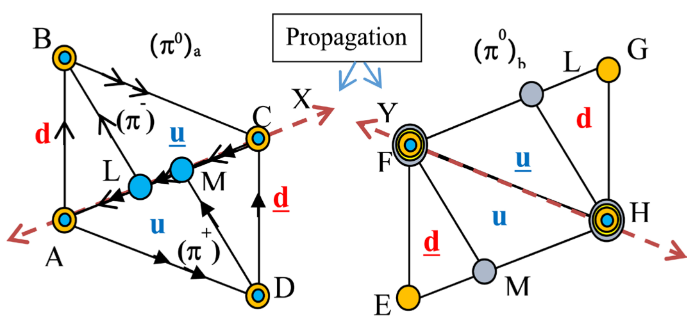

Figure 5. Configurations of neutral pion.

To obtain the total mass of a structure, it needs to add eventual (m_{kin}) relativistic kinetic mass and mass defects (Δm). To the exception of some cases (which we will specify) $(m_{kin}) \ll m_0$, therefore we will have: $\left[m_{tot} = m_{part} \pm \Delta m_i\right]$. The mass defect is: $\Delta m = \Delta m_g + \Delta m_{em}$. Here Δm_g is the mass defect due to binding by gluons. Nevertheless, the Δm_g has been englobed in masses of the charged pionor in quark (u_π, d_π)); therefore, we consider only electromagnetic mass defect: $\Delta m = \Delta m_{em}$.

To obtain the mass defects ($\Delta m > 0$, $\Delta m < 0$) we use a Function ($F_{\Delta m}$) of mass defect applied to structure equation so defined:

$$
\begin{aligned}
&F_{\Delta m}\left(A_1, A_2, \cdots, A_n\right) \\
&= \left\{\sum_{(i,j)=1}^{n} F_{\Delta m}\left[\left(A_i\right)_{(\otimes)}, \left(B_j\right)_{(\oplus)}\right]\right\} \\
&= \left[\sum_{i=1} \Delta m\left(A_i\right)_{(\otimes)} + \sum_{j=1} \Delta m\left(B_i\right)_{(\oplus)}\right] + \left[\sum_{(i=1,j=1)}^{n} \Delta m\left(A_i \oplus B_j\right)\right]_{(\text{I}^\circ\text{-degree})} \\
&\quad + \left[\sum_{\substack{i=1,j=1,k=1 \\ i \neq j \neq k}}^{n} \Delta m\left(A_i \oplus \left(A_j \oplus A_k\right)\right)\right]_{(\text{II}^\circ\text{-deg.})} \\
&\quad + \left[\sum_{\substack{i=1,j=1,k=1 \\ i \neq j \neq k}}^{n} \Delta m\left(B_i \oplus \left(B_j \oplus B_k\right)\right)\right]_{(\text{II}^\circ\text{-deg.})}
\end{aligned}
\tag{7}
$$

It needs to consider that:

$$
\begin{aligned}
&\Delta m\left[(a) \otimes (b)\right]_{A_i} = \left\{\begin{array}{c} 0 \\ \Delta m(a,b)_{(a \cap b)} \neq 0 \end{array}\right\} \\
&\Delta m\left[(a) \oplus (b)\right]_{B_i} = \Delta m(a,b)_{\text{interaction}}
\end{aligned}
\tag{8}
$$

where the (a, b) point out "base particles" of the (A_i, B_j)-component, as *i.e.* the pions π or the quarks q. Note mass defect is zero if there is the only interpenetration between the two particles (a, b) without interacting parts. Instead, the mass defect cannot be zero if there are some parts of (a, b) dynamically interacting ($a \cap b$), see the neutral pion in diagonal (AC or FH in **Figure 5**), where the d-quark and u-quark, exchange energy quanta along the diagonal. Now, we can consider the properties of **Table 1** in the space of the functions (F_m, $F_{\Delta m}$) to obtain the properties of the operations ($\otimes$, $\oplus$) for calculating the masses and mass defects of mesons (see **Table 2**).

3.5. The Matrix of Dynamics Couplings

Equivalent two ways can treat the binding energy between quarks inside pions. The function $F_{\Delta m}$ of mass defects expresses these two possibilities.

The structure equation contains the considerations on the various dynamics couplings and interpenetrations: by this equation, we obtain the mass value if we consider the binding energies. The Equation (7) expresses all possible couplings

Table 2. The fundamental operations of functions (F_m, $F_{\Delta m}$).

($\otimes$-operation) in (F_m, $F_{\Delta m}$) representations	$\oplus$-operation in (F_m, $F_{\Delta m}$) representations
1) $F_m(a_i \otimes a_j) = \langle m(a_i, a_j)\rangle = \langle m(a_i), m(a_j)\rangle$ $F_m(a_i \otimes a_i) = F_m(a_i) = m(a_i)$	1) $F_m(a_i \oplus a_j) = F_m(a_i) + F_m(a_j) = m(a_i) + m(a_j)$ $F_m(a_i \oplus a_i) = F_m(a_i) + F_m(a_i) = m(a_i) + m(a_i) = 2m(a_i)$
2) $F_{\Delta m}(a_i \otimes a_j) = 0$ $F_{\Delta m}(a_i \otimes a_j)_{\oplus} \neq 0$ with $(a_i \otimes a_j)_{\oplus} = (a_i \otimes a_j) \oplus a_k$	2) $F_{\Delta m}(a \oplus b \oplus c)$ $= F_{\Delta m}(a) + F_{\Delta m}(b) + F_{\Delta m}(c) + F_{\Delta m}(a \underline{\oplus} b)_{ab}$ $+ F_{\Delta m}(a \underline{\oplus} c)_{ac} + F_{\Delta m}(b \underline{\oplus} c)_{bc}$
3) $F_{\Delta m}(a_i \otimes a_j)_{\oplus} = F_{\Delta m}(a_i)$ $F_{\Delta m}((a_i \otimes a_j) \oplus a_k) = F_{\Delta m}((a_i) \oplus a_k)$	3) $F_{\Delta m}[(a_i \oplus a_j) \oplus a_k]$ $= F_{\Delta m}[(a_i \oplus a_k) \oplus (a_j \oplus a_k)]$

between pions (quarks): component Δm (A) takes into account the eventual dynamics coupling in the interpenetration between two pions, the Δm (B) the specific dynamics coupling and Δm ($A \oplus B$) takes in consideration the coupling between pairs of pions or quarks. Into evolving the calculations on the mass defects we can use two procedures. The first way is considering without dynamical couplings all the interpenetrations between two pions, but not the ones between two pairs of pions. Vice versa, the second way is considering without dynamical couplings all the interpenetrations between two pairs of pions, but not the ones between two pions. This last aspect implies that all interactions of the different pairs are attributable to the individual couplings between two pions. In this second case, we can represent all possible dynamics couplings by a matrix: $\|A_{ij}\| = \left[\left(\pi_i^{\pm} \otimes \pi_j^{\pm}\right)_{\oplus}\right]$.

We will show the two possible ways in the calculations of meson masses.

4. The Light Mesons

4.1. The Neutral Pion Mass

By structure equation [8] the (m^*) partial mass of π^0 is:

$$m(\pi^0)^* = F_m\left(\left\{\left[(\pi^-) \otimes (\pi^+)\right]_{A_1} \oplus \left[(u \oplus \underline{u}) \otimes (\underline{d} \oplus d)\right]_{A_2}\right\}\right) = \left\langle m(\pi^-, \pi^+)\right\rangle_{A_1} + \left\langle m\left((u\underline{u})_{\oplus}, (\underline{d}d)_{\oplus}\right)\right\rangle_{A_2} \tag{9}$$

We have

$$\left\langle m(\pi^+, \pi^-)\right\rangle_{A_1} = \left[\frac{m(\pi^+) + m(\pi^-)}{2}\right]_{A_1}$$
$$\left\langle m\left((u\underline{u})_{\oplus}, (\underline{d}d)_{\oplus}\right)\right\rangle_{A_2} = \left[\frac{2m(u) + 2m(d)}{2}\right]_{A_2} \tag{10}$$

The values of Δm mass defects (see equation (7) and $2_{\oplus}$-property in **Table 2**) would be:

$$\begin{aligned}\Delta m\left(\pi^0\right)^* &= F_{\Delta m}\left(\left\{\left[\left(\pi^-\right)\otimes\left(\pi^+\right)\right]_{A_1}\oplus\left[\left(u\oplus\underline{u}\right)\otimes\left(\underline{d}\oplus d\right)\right]_{A_2}\right\}\right)\\ &= F_{\Delta m}\left(\left[\left(\pi^-\right)\otimes\left(\pi^+\right)\right]_{A_1}\right)+F_{\Delta m}\left(\left[\left(u\oplus\underline{u}\right)\otimes\left(\underline{d}\oplus d\right)\right]_{A_2}\right)\\ &\quad+F_{\Delta m}\left(\left[\left(\pi^-\right)\otimes\left(\pi^+\right)\right]_{A_1}\underline{\oplus}\left[\left(u\oplus\underline{u}\right)\otimes\left(\underline{d}\oplus d\right)\right]_{A_2}\right)_{(B=A_1\oplus A_2)}\end{aligned}\tag{11}$$

where the $\underline{\otimes}$-operation point out the coupling between two no-separated components (A_1, A_2) and with

$$\begin{aligned}&\text{a) } F_{\Delta m}\left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right]_{A_1}=0\\ &\text{b) } F_{\Delta m}\left[\left(u\underline{u}\right)_\oplus\otimes\left(\underline{d}d\right)_\oplus\right]_{A_2}=\Delta m\left(\left(u\underline{u}\right)_\oplus,\left(\underline{d}d\right)_\oplus\right)\neq 0\\ &\text{c) } F_{\Delta m}\left\{\left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right]_{A_1}\underline{\oplus}\left[\left(u\underline{u}\right)_\oplus\otimes\left(\underline{d}d\right)_\oplus\right]_{A_2}\right\}_{(B=A_1\oplus A_2)}\neq 0\end{aligned}\tag{12}$$

In [8] we demonstrated that the global mass value of pion is:

$$\begin{aligned}m\left(\pi^0\right) &= m\left(\pi^0\right)^*-\Delta m\\ &= m\left(\pi^\pm\right)-(1/2)\left[\varepsilon_u(\gamma)+\varepsilon_d(\gamma)\right]_{(u,d)_0}\\ &= m\left(\pi^\pm\right)-\varepsilon_\gamma(\pi)_{(u,d)_0}\end{aligned}$$

where (ε_γ) is annihilation energy of pairs $\left[\left(u\oplus\underline{u}\right),\left(d\oplus\underline{d}\right)\right]$, with $\left[\Delta m_\gamma=\varepsilon_\gamma=(4.59)\,\text{MeV}\right]$. This energy, as already it has been said, is coincident with mass at rest of free quarks (see the bare mass):

$$\left[m_b(u)=(3.51)\,\text{MeV},\,m_b(d)=(5.67)\,\text{MeV}\right]\tag{13}$$

These values [8] are inside the range anticipated by literature [16] [17].

It is so evident that the values of masses of quarks, both bare and dressed inside pion, could be used to obtaining the mass spectrum of light mesons. To make this, it needs so to admit the presence of a lattice of bound "virtual pions" $\{\pi^+, \pi^-\}$, with an elementary cell having the form of the $\left\{\left(\pi_r^0\right)=\left(\pi^+\oplus\pi^-\right)\right\}$, which represents a *"molecule"* of pions, you see **Figure 4**. In this way, both the $\left\{\pi_r^0\right\}$-lattice and $\{d, d\}$ participating in building the mass spectrum of light mesons. By Equations (5) and (7) we will show the calculations to find the mass value of light mesons with notable precision as also the mass value of fundamental nucleons (a work in preparation). We will use the equations of **Table 3**, which represent the application of the properties of **Table 1** and **Table 3**.

4.3. The Pseudo-Scalar Mesons (η) (547.86 MeV)

It is physically possible that the pions [π^0, π^+, π^-] may overlap simultaneously, then they can give origin to massive mesons if the internal quanta are sufficient in number. The pion triplet will be called "meson of basic level" or first-level of structure.

As is well known, the first massive meson, after (π^0), is the meson η. The

η-meson places to the 2-level" of structure. We can conjecture a possible state of not-separated overlapping of mesons [π^0; (π^+, π^-)]. In the first moment, we can consider the following combination or "structure equation":

$$\eta = \left[\left(\pi^+ \oplus \pi^-\right)_1 \otimes \left(\pi^+ \oplus \pi^-\right)_2\right] = \left[\left(\pi^0\right)_{r1} \otimes \left(\pi^0\right)_{r2}\right]$$

where $\left(\pi^0\right)_r = \left(\pi^+ \oplus \pi^-\right)$ is a *"molecule"* of pions, composed by 4 quarks. This is built by a dynamics coupling $\left[\oplus = \oplus_g + \oplus_{em}\right]$ of two charged pions; the electric charge is zero and mass $\left\{m\left(\pi^0\right)_r = m\left[\left(\pi^+\right) \oplus \left(\pi^-\right)\right] = m\left[\left(\pi^+\right) + \left(\pi^-\right)\right] = 2m\left(\pi^\pm\right)\right\}$, see **Table 3**.

Immediately, note there are two double pairs $\left[\left(u_1 \underline{d}_1\right)_{\pi^+}, \left(\underline{u}_1 d_1\right)_{\pi^-}\right]$, $\left[\left(u_2 \underline{d}_2\right)_{\pi^+}, \left(\underline{u}_2 d_2\right)_{\pi^-}\right]$.

This structure of couplings can be somewhat unstable; if the quarks (u_2, $\underline{u}_2$) fix their respective vertices (G, H) on the diagonal of quarks (d_2, $\underline{d}_2$), then the structure can have more stability. The fixing (see section 2.1 and **Figure 2**) gives origin to configuration with *strange quarks* (s, $\underline{s}$): if quanta number is sufficient, then two new pairs of quarks are created [(u_3, d_3), ($\underline{u}_3$, $\underline{d}_3$)]$_{s\underline{s}}$. We point out these quarks with a denomination of *sub-quarks* (u, d). The representative figure of η-meson is (**Figure 6**).

Recall, by literature, the wave function of η-meson: $\left[\eta = c_1\left(u\underline{u} + d\underline{d}\right) + c_2\left(s\underline{s}\right)\right]$.

Note that in the structure equation, the quarks (s, $\underline{s}$) are "hidden". Moreover, the fixing in (G, H) determines a zero value of the *interpenetration kinetic energy* (K_{int}) of the d-quarks and sub-quarks. There is a (K_{int}) in pair (u_1, $\underline{u}_1$) for the rotation of u-quarks around two Y-axis, but this energy could be minimal (K_{kin}) $\ll$ m_0c^2). The spin of pair (u_1, $\underline{u}_1$) is zero like that of neutral pion because this is to base of η-meson (see its structure). Processing the "structure equation" one has:

Table 3. Applications of the function (F_m, $F_{\Delta m}$) to the pions.

Properties of pions in (⊗) operations	Properties of pions in (⊕) operations
1) $\left[\pi^\pm = \left(u \underline{\otimes} d\right)\right]$	4) $m\left(\pi^+ \oplus \pi^-\right) = m\left(\pi^+\right) + m\left(\pi^-\right)$
2) $(\pi)^0 = \left[\left(\pi^+\right) \underline{\otimes} \left(\pi^-\right)\right] = \left[\left(u \oplus \underline{d}\right) \underline{\otimes} \left(\underline{u} \oplus d\right)\right] = \left[\left(\pi^+\right) \otimes \left(\pi^-\right)\right] - \left[\left(u \oplus \underline{u}\right) \otimes \left(d \oplus \underline{d}\right)\right]$	5) $\left\{\left(\pi_r^0\right) = \left(\pi^+ \oplus \pi^-\right)\right\}; m\left(\pi_r^0\right) = 2m\left(\pi^\pm\right)$
3) $\left(\pi^\pm\right)^0 = \left[\left(\pi^+\right) \otimes \left(\pi^-\right)\right]$ virtual neutral pion	6) $F_m\left[\left(\pi^+ \otimes \pi^-\right) \oplus \left(\pi^+ \otimes \pi^-\right)\right] = F_m\left[2\left(\pi^\pm\right)^0\right]$
4) $m\left[\left(\pi^+\right) \otimes \left(\pi^-\right)\right] = \left(\pi^\pm\right)^0 = m\left(\pi^\pm\right)$ Equation (11)	7) $\Delta m\left[\left(\pi_1^\pm \oplus \pi_2^\mp\right) \otimes \pi^0\right] = \Delta m\left[\left(\pi_1^\pm \oplus \pi_2^\mp\right)_{\pi^0}\right]$
5) $m\left(\pi^+ \otimes \pi^+\right) = m\left(\pi^+\right)$	
6) $m\left(\pi^+ \otimes \pi^-\right) = \left\langle m\left(\pi^+\right), m\left(\pi^-\right)\right\rangle$	
7) $\left\{\Delta m(\pi) = \left[m\left(\pi^\pm\right) - m\left(\pi^0\right)\right]\right\} = (4.59)\,\text{MeV}$	
8) $\left[\left(\pi_1^\pm \otimes \pi_2^\pm\right) \oplus \pi_3^\pm\right] \rightarrow \Delta m\left(\pi_i^\pm \otimes \pi_j^\pm\right)_\oplus = \Delta m\left(\pi_j^\pm\right)$	

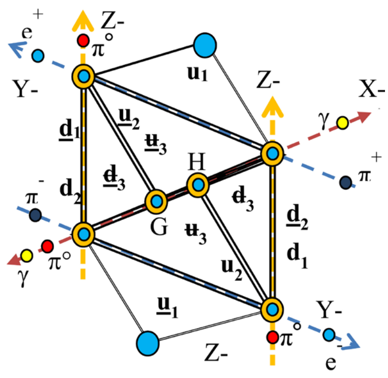

Figure 6. Configuration of (η)-meson.

$$
\begin{aligned}
\eta &= \left[\left(\pi_1^0\right)_r \underline{\otimes} \left(\pi_2^0\right)_r \right] = \left[\left(\pi^+ \oplus \pi^-\right)_1 \otimes \left(\pi^+ \oplus \pi^-\right)_2 \right] \\
&= \left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^+ \otimes \pi_2^-\right) \oplus \left(\pi_1^- \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right) \right]
\end{aligned} \tag{14}
$$

The indices (1, 2) point out the two molecules of pions $(\pi^0)_r$.

To obtain the partial mass, we will apply the mass Function (F_m) to the structural equation of the η-system. In processing the structure equation, we use the properties of $\otimes$-operation and that of mass function F_m (see **Table 2**). Then the partial mass m^* (see Equations (5) and (6) is:

$$
\begin{aligned}
m(\eta)^* &= F_m \left\{ \left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^+ \otimes \pi_2^-\right) \oplus \left(\pi_1^- \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right) \right] \right\} \\
&= F_m \left\{ \left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^+ \otimes \pi_2^-\right) \right]_{B_1} \right\} \oplus F_m \left\{ \left[\left(\pi_1^- \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right) \right]_{B_1} \right\} \\
&= F_m \left\{ \left[\left(\pi_1^+ \otimes \pi_2^-\right) \oplus \left(\pi_2^+ \otimes \pi_1^-\right) \right]_{B_1} \right\} + F_m \left\{ \left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right) \right]_{B_2} \right\} \\
&= m \left\{ \left[\left(\pi_1^+ \otimes \pi_2^-\right) + \left(\pi_2^+ \otimes \pi_1^-\right) \right]_{B_1} \right\} + m \left\{ \left[\left(\pi_1^+ \otimes \pi_2^+\right) + \left(\pi_1^- \otimes \pi_2^-\right) \right]_{B_2} \right\} \\
&= m \left(2\left(\pi^+ \otimes \pi^-\right)_{12} \right)_{A_1} + m \left(2\left(\pi^\pm \otimes \pi^\pm\right)_{12} \right)_{A_2} \\
&= 2\left(\left\langle m\left(\pi^+, \pi^-\right) \right\rangle \right) + 2\left(\left\langle m\left(\pi^\pm, \pi^\pm\right) \right\rangle \right) \\
&= 2m\left(\pi^\pm\right) + 2m\left(\pi^\pm\right) = (558.28)\,\text{MeV}
\end{aligned} \tag{15}
$$

where $\left[F_m\left(B_1 \oplus B_2\right) = m\left(B_1 \oplus B_2\right) \right]$, see Equation (6), and $\left(B_i\right)_\oplus \to \left(A_i\right)_\otimes$.

Note, see Equation (10) and **Table 2**, that $F_m\left(A \otimes B\right) = \left\langle m(A), m(B) \right\rangle$; if ($A$ = B), then $F_m\left(A \otimes A\right) = \left\langle m(A), m(A) \right\rangle = m(A)$, thus it follows:

$$F_m\left(A \otimes A\right) = F_m(A) \leftrightarrow \left(A \otimes A\right)_{F_m} = m(A).$$

4.4. The Processing of the Mass Defect Scalar in η

To obtain the mass defects, we should use the Function ($F_{\Delta m}$) (see Equation (7)) applied to the following structure equation, with $\oplus$-operation of dynamics coupling:

$$
\begin{aligned}
\eta &= \left[\left(\pi_1^0\right)_r \underline{\otimes} \left(\pi_2^0\right)_r \right] = \left[\left(\pi^+ \oplus \pi^-\right)_1 \otimes \left(\pi^+ \oplus \pi^-\right)_2 \right] \\
&= \left[\left(\pi_1^+ \otimes \pi_2^+\right)_a \oplus \left(\pi_1^+ \otimes \pi_2^-\right)_b \oplus \left(\pi_1^- \otimes \pi_2^+\right)_c \oplus \left(\pi_1^- \otimes \pi_2^-\right)_d \right]
\end{aligned} \tag{16}
$$

where we have used the 2-property of Table 1. Note that this equation explicates all dynamic couplings. Then, the equation of mass defect should be:

$$\Delta m(\eta) = F_{\Delta m}\left(\left[\left(\pi_1^+ \otimes \pi_2^+\right)_a \oplus \left(\pi_1^+ \otimes \pi_2^-\right)_b \oplus \left(\pi_1^- \otimes \pi_2^+\right)_c \oplus \left(\pi_1^- \otimes \pi_2^-\right)_d\right]\right)$$

To obtain the global mass defect, we should consider all possible dynamic couplings (⊕) between the four components (*a*, *b*, *c*, *d*), see Table 4.

It is possible to calculate all mass defects of these couplings; note that into mass values of pions are contained the binding energies of gluons; therefore, mass defects will be relative to electromagnetic coupling ($\oplus_{em}$). The mass defect most relevant is that relative to coupling $\left[\left(\pi^+ \oplus \pi^-\right)\right]$ in neutral pion (π^0):

$$\left[\Delta m(\pi) = \left[m\left(\pi^\pm\right) - m\left(\pi^0\right)\right] = (4.59)\,\text{MeV}\right] \text{ or } \left[\Delta m\left(\pi^+ \oplus \pi^-\right) = (4.59)\,\text{MeV}\right]$$

All the combinations of pairs $\left[\left(\pi_1^\pm \otimes \pi_2^\pm\right)_i \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_j\right]$ have values of mass defect that are sub-multiples of $\Delta m(\pi)$:

$$\Delta m\left(A_i \oplus A_j\right) = \left(\frac{\Delta m(\pi)}{n}\right) \tag{17}$$

where n is an integer, $n \geq 1$; the number of quarks or pions involved in couplings can determine the number n, because the binding gluons are distributed on the n "sides" of coupling between quarks. Here it is necessary a selection rule of various couplings, because mesons are couplings two by two of quark-antiquark; then, we consider only couplings of the first degree (I˚) and, some cases, of the second (II˚).

If we omit superior degrees (III˚ or IV˚) of ⊕-coupling, the same, we can obtain values of the mass defect with excellent approximation because the mass defects of combinations of degrees (III˚, IV˚) are very smalls. We point out with ($O_{\Delta m}$) the values of mass defects omitted, which

$$O_{\Delta m} = \left(\frac{\Delta m(\pi)}{\underline{n}^r}\right) \tag{18}$$

where ($\underline{n}$, r) are integers ($r > 1$, $\underline{n} > 1$); r is correlated to the degree of coupling between pion pairs, while n is correlated to pion pairs: for pion pairs ($\underline{n} = 2$) and ($r = 2$), it is:

$$O_{\Delta m} = \left(\frac{\Delta m(\pi)}{2^2}\right) = \left(\frac{(4.59)\,\text{MeV}}{4}\right) = (1.15)\,\text{MeV} \tag{19}$$

Table 4. All possible combinations of couplings between pairs of pions.

Combinations	Coupling
$\left[\left(\pi_1^\pm \otimes \pi_2^\pm\right)_i \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_j\right]$	coupling of I˚ degree
$\left\{\left(\pi_1^\pm \otimes \pi_2^\pm\right)_i \oplus \left[\left(\pi_1^\pm \otimes \pi_2^\pm\right)_j \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_k\right]\right\}$	coupling of II˚ degree
$\left[\left(\pi_1^\pm \otimes \pi_2^\pm\right)_i \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_j\right] \oplus \left[\left(\pi_1^\pm \otimes \pi_2^\pm\right)_k \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_r\right]$	coupling of III˚ degree
$\left\{\left(\pi_1^\pm \otimes \pi_2^\pm\right)_i \oplus \left[\left(\pi_1^\pm \otimes \pi_2^\pm\right)_j \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_k \oplus \left(\pi_1^\pm \otimes \pi_2^\pm\right)_r\right]\right\}$	coupling of IV˚ degree

Then the mass defect is, see Equation (7):

$$\begin{aligned}\Delta m(\eta) &= F_{\Delta m}\left\{(\pi_1^+\otimes\pi_2^-)_{A_1}\oplus(\pi_1^-\otimes\pi_2^+)_{A_2}\oplus(\pi_1^+\otimes\pi_2^+)_{A_3}\oplus(\pi_1^-\otimes\pi_2^-)_{A_4}\right\}\\ &= \Delta m(A_1)+\Delta m(A_2)+\Delta m(A_3)+\Delta m(A_4)+\Delta m(A_{12})\\ &\quad+\Delta m(A_{13})+\Delta m(A_{14})+\Delta m(A_{23})+\Delta m(A_{24})+\Delta m(A_{34})\end{aligned} \tag{20}$$

If we take into account the Equation (12a), it derives that:

$$\left[\Delta m(A_1)=\Delta m(A_2)=\Delta m(A_3)=\Delta m(A_4)=0\right]$$

By this hypothesis, one admits that into interpenetration between two pions there is not a mass defect if between these there is not dynamics couplings (2-property Table 2):

$$\Delta m(\pi_i^\pm\otimes\pi_j^\pm)=\Delta m(\pi_i^\pm\otimes\pi_j^\mp)=0$$

Therefore, mass defect will be obtained by:

$$\Delta m(\eta)=\Delta m(A_{12})+\Delta m(A_{13})+\Delta m(A_{14})+\Delta m(A_{23})+\Delta m(A_{24})+\Delta m(A_{34})$$

With

$$\Delta m(A_{13})=\Delta m(A_{23});\Delta m(A_{14})=\Delta m(A_{24})$$

$$\Delta m\left(\left[(\pi_1^+\otimes\pi_2^-)_{A_1}\oplus(\pi_1^+\otimes\pi_2^+)_{A_3}\right]\right)=\Delta m\left(\left[(\pi_1^-\otimes\pi_2^+)_{A_2}\oplus(\pi_1^+\otimes\pi_2^+)_{A_3}\right]\right)$$

$$\Delta m\left(\left[(\pi_1^+\otimes\pi_2^-)_{A_1}\oplus(\pi_1^-\otimes\pi_2^-)_{A_4}\right]\right)=\Delta m\left(\left[(\pi_1^-\otimes\pi_2^+)_{A_2}\oplus(\pi_1^-\otimes\pi_2^-)_{A_4}\right]\right)$$

Nevertheless, note that two interpenetrated pions with equal charge $\left[(\pi_1^\pm,\pi_2^\pm)\right]$, but interacting with other pions $\left[(\pi_1^\pm\otimes\pi_2^\pm)\oplus\pi_3^\pm\right]$, can be an only one particle (see boson properties): $\Delta m\left((\pi_i^\pm\otimes\pi_j^\pm)_\oplus\right)=\Delta m(\pi_i^\pm)$

See 3-property in Table 2 and 8-property in Table 3. It follows:

$$\begin{aligned}\Delta m(A_{34}) &= \Delta m\left(\left[(\pi_1^+\otimes\pi_2^+)_{A_3}\oplus(\pi_1^-\otimes\pi_2^-)_{A_4}\right]\right)\equiv\Delta m\left(\left[(\pi_{12}^+)_\oplus\oplus(\pi_{12}^-)_\oplus\right]\right)\\ &= \Delta m\left(\left[(\pi^+)\oplus(\pi^-)\right]\right)=(4.59)\,\text{MeV}\end{aligned}$$

In Equation (20), we observe that $\Delta m\,(A_{34})$ admits four possible combinations of exchange ($\pi_i\leftrightarrow\pi_j$); then we determine four equivalent combinations (S_1, S_2, S_3, S_4) using the operation of exchange S which acts on the pions (bosons):

$$\begin{aligned}\hat S_{(\pi_1^-\leftrightarrow\pi_1^+)}(A_{34}) &= \hat S_{(\pi_1^-\leftrightarrow\pi_1^+)}\left[(\pi_1^+\otimes\pi_2^+)\oplus(\pi_1^-\otimes\pi_2^-)\right]_{A_{34}}\\ &= \left[(\pi_1^-\otimes\pi_2^+)\oplus(\pi_1^+\otimes\pi_2^-)\right]_{S_1}\end{aligned}$$

$$\begin{aligned}\hat S_{(\pi_1^+\leftrightarrow\pi_2^-)}(A_{34}) &= \hat S_{(\pi_1^+\leftrightarrow\pi_2^-)}\left[(\pi_1^+\otimes\pi_2^+)\oplus(\pi_1^-\otimes\pi_2^-)\right]_{A_{34}}\\ &= \left[(\pi_2^-\otimes\pi_2^+)\oplus(\pi_1^-\otimes\pi_1^+)\right]_{S_2}\end{aligned}$$

$$\begin{aligned}\hat S_{(\pi_2^+\leftrightarrow\pi_1^-)}(A_{34}) &= \hat S_{(\pi_2^+\leftrightarrow\pi_1^-)}\left[(\pi_1^+\otimes\pi_2^+)\oplus(\pi_1^-\otimes\pi_2^-)\right]_{A_{34}}\\ &= \left[(\pi_1^+\otimes\pi_1^-)\oplus(\pi_2^+\otimes\pi_2^-)\right]_{S_3}\end{aligned}$$

$$\hat{S}_{(\pi_2^+ \leftrightarrow \pi_2^-)}(A_{34}) = \hat{S}_{(\pi_2^+ \leftrightarrow \pi_2^-)}\left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right)\right]_{A_{34}}$$
$$= \left[\left(\pi_1^+ \otimes \pi_2^-\right) \oplus \left(\pi_1^- \otimes \pi_2^+\right)\right]_{S_4}$$

Each of these combinations (S_1, S_2, S_3, S_4) must contribute to overall mass defect (quantum aspect). Note that

$$\Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right) \oplus \left(\pi_1^- \otimes \pi_2^+\right)\right]\right) \Rightarrow \Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right)\right]\right)_{S_4}$$

Where have exchanged of place ($\pi_1^+ \leftrightarrow \pi_1^-$); then we can think that:

$$\Delta m\left(\left[\left(\pi_1^- \otimes \pi_2^+\right) \oplus \left(\pi_1^+ \otimes \pi_2^-\right)\right]\right)$$
$$= \left(\frac{1}{4}\right) \Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right)\right]\right) = \left(\frac{1}{4}\right)(4.60)\,\text{MeV} = (1.15)\,\text{MeV} \quad (21)$$

Thus, in η-meson, it is:

$$\Delta m(A_{12}) = \Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^+\right)_{A_2}\right]\right)_1 = (1.15)\,\text{MeV}$$
$$\Delta m(A_{13}) = \Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^+ \otimes \pi_2^+\right)_{A_3}\right]\right)_2 = (1.15)\,\text{MeV}$$
$$\Delta m(A_{14}) = \Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^-\right)_{A_4}\right]\right)_3 = (1.15)\,\text{MeV}$$
$$\Delta m(A_{23}) = \Delta m\left(\left[\left(\pi_1^- \otimes \pi_2^+\right)_{A_2} \oplus \left(\pi_1^+ \otimes \pi_2^+\right)_{A_3}\right]\right)_4 = (1.15)\,\text{MeV} \quad (22)$$
$$\Delta m(A_{24}) = \Delta m\left(\left[\left(\pi_1^- \otimes \pi_2^+\right)_{A_2} \oplus \left(\pi_1^- \otimes \pi_2^-\right)_{A_4}\right]\right)_5 = (1.15)\,\text{MeV}$$
$$\Delta m(A_{34}) = \Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^+\right)_{A_3} \oplus \left(\pi_1^- \otimes \pi_2^-\right)_{A_4}\right]\right)_6 = (4.60)\,\text{MeV}$$

Then we can have:

$$\Delta m(\eta) = F_{\Delta m}\left(\left\{\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^+\right)_{A_2} \oplus \left(\pi_1^+ \otimes \pi_2^+\right)_{A_3} \oplus \left(\pi_1^- \otimes \pi_2^-\right)_{A_4}\right\}\right)$$
$$= F_{\Delta m}\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^+\right)_{A_2}\right] \oplus \left[\left(\pi_1^+ \otimes \pi_2^+\right)_{A_3} \oplus \left(\pi_1^- \otimes \pi_2^-\right)_{A_4}\right]\right)$$
$$= F_{\Delta m}\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^+\right)_{A_2}\right] \oplus \left[\left(\pi_{12}^+\right)_{A_3} \oplus \left(\pi_{12}^-\right)_{A_4}\right]\right) \quad (23)$$
$$= F_{\Delta m}\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^+\right)_{A_2}\right] \oplus \left[\left(\pi_{12}^+ \oplus \pi_{12}^-\right)_{A_{34}}\right]\right)$$
$$= F_{\Delta m}\left(\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^- \otimes \pi_2^+\right)_{A_2} \oplus \left(\pi_{12}^+ \oplus \pi_{12}^-\right)_{A_3}\right)$$

It needs determining the $\Delta m(A_{13})$ and $\Delta m(A_{23})$. Note that if

$$\Delta m\left(\left[\left(\pi_1^- \otimes \pi_2^+\right) \oplus \left(\pi_1^+ \otimes \pi_2^-\right)\right]\right) = (1.15)\,\text{MeV}$$
$$\Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^+\right) \oplus \left(\pi_1^- \otimes \pi_2^-\right)\right]\right) = \Delta m\left(\left[\left(\pi_1^+\right) \oplus \left(\pi_2^-\right)\right]\right) = (4.6)\,\text{MeV}$$

Then we can think:

$$\Delta m\left(\left[\left(\pi_1^- \otimes \pi_2^+\right) \oplus \left(\pi_1^+ \oplus \pi_2^-\right)\right]\right) = \left(\frac{\Delta m(\pi)}{n}\right)_{(n=2)} = (2.3)\,\text{MeV} \quad (24)$$

The overall mass defect will be:

$$\begin{aligned}\Delta m(\eta) &= \Delta m(A_1)+\Delta m(A_2)+\Delta m(A_3)+\Delta m(A_{12})+\Delta m(A_{13})+\Delta m(A_{23})\\ &= \left[(0)_{A_1}+(0)_{A_2}+(4.6)_{A_3}+(1.15)_{A_{12}}+(2.3)_{A_{13}}+(2.3)_{A_{23}}\right]\text{MeV}\\ &= (10.35)\,\text{MeV}\end{aligned}$$

The global mass is (omitting ($O_{\Delta mi}$)):

$$m(\eta)\approx m(\eta^*)-\Delta m_\eta = \left[(558.28)-(10.35)\right]\text{MeV} = (547.95)\,\text{MeV} \quad (25)$$

This value is very next to that experimental, see ref. [16] where $m(\eta)$ = (547,86) MeV.

Since any coupling $(A_i \oplus A_j)$ between pion pairs is always attributable to the couplings between individual pions, $(\pi_i^\pm \otimes \pi_j^\pm)$, we can treat the mass defect taking in account only reciprocal dynamics couplings between two pions and no between pairs. Therefore, we could consider

$$\Delta m(A_{12})=\Delta m(A_{13})=\Delta m(A_{14})=\Delta m(A_{23})=\Delta m(A_{24})=\Delta m(A_{34})=0$$

$$\left[\Delta m(A_1),\Delta m(A_2),\Delta m(A_3),\Delta m(A_4)\right]\neq 0$$

We build the matrix A_{ij} of all couplings with interpenetration between pions $\left[(\pi_i^\pm \otimes \pi_j^\pm)_\oplus\right]$, which compose the structure equation in η-meson. The matrix, see **Table 5**, is symmetric $A_{ij} = A_{ji}$, therefore we consider only half elements, the ones in colour:

We pose, see Equation (22), the binding energy $\left[(\pi_i^\pm \otimes \pi_i^\mp)_\oplus\right]^G = (4.6)\,\text{MeV}$ in pions belonging to the same molecule (Green colour). Besides, we will have the binding energy $\left[(\pi_i^\pm \otimes \pi_j^\mp)_\oplus\right]^Y = (2.3)\,\text{MeV}$, because in pions belonging to two different molecules the binding is weaker (Yellow colour). Instead, there is repulsive energy between pions with same charge (Blue colour): $\left[(\pi_i^\pm \otimes \pi_j^\pm)_\oplus\right]^B = -(1.15)\,\text{MeV}$.

The repulsive action is weaker than that attraction because η-meson is a binding state until to decay. Finally, to all elements of the diagonal (Red colour), we also assign an overall mass defect value given by the coupling term that follows

$$\begin{aligned}&\bigcup_k\left(\Delta m(\pi_i^\pm \otimes \pi_i^\pm)\right)_k\\ &= F_{\Delta m}\left\{(\pi_1^+\otimes\pi_1^+)\underline{\otimes}(\pi_2^+\otimes\pi_2^+)\underline{\otimes}(\pi_1^-\otimes\pi_1^-)\underline{\otimes}(\pi_2^-\otimes\pi_2^-)\right\}^R\end{aligned}$$

Table 5. The matrix of dynamics couplings with mass defect between pions.

η	π_1^+	π_2^+	π_1^-	π_2^-
π_1^+	$(\pi_1^+,\pi_1^+)_\oplus^R$	$(\pi_1^+,\pi_2^+)_\oplus$	$(\pi_1^+,\pi_1^-)_\oplus$	$(\pi_1^+,\pi_2^-)_\oplus$
π_2^+	$(\pi_2^+,\pi_1^+)_\oplus^B$	$(\pi_2^+,\pi_2^+)_\oplus^R$	$(\pi_2^+,\pi_1^-)_\oplus$	$(\pi_2^+,\pi_2^-)_\oplus$
π_1^-	$(\pi_1^-,\pi_1^+)_\oplus^G$	$(\pi_1^-,\pi_2^+)_\oplus^Y$	$(\pi_1^-,\pi_1^-)_\oplus^R$	$(\pi_1^-,\pi_2^-)_\oplus$
π_2^-	$(\pi_2^-,\pi_1^+)_\oplus^Y$	$(\pi_2^-,\pi_2^+)_\oplus^G$	$(\pi_2^-,\pi_1^-)_\oplus^B$	$(\pi_2^-,\pi_2^-)_\oplus^R$

Being all repulsive terms, we will assign an overall mass defect with a negative value:

$$\bigcup\left(\left[\left(\pi_i^{\pm}\otimes\pi_i^{\pm}\right)_{\oplus}\right]^R\right)=-(1.15)\,\text{MeV}$$

In synthesis, we will have

$$\begin{aligned}
&\left[\Delta m\left(\left(\pi_1^{-}\otimes\pi_1^{+}\right)_{\oplus}\right)=\Delta m\left(\left(\pi_2^{-}\otimes\pi_2^{+}\right)_{\oplus}\right)\right]_G=(4.6)\,\text{MeV}\\
&\left[\Delta m\left(\left(\pi_2^{-}\otimes\pi_1^{+}\right)_{\oplus}\right)=\Delta m\left(\left(\pi_2^{+}\otimes\pi_1^{-}\right)_{\oplus}\right)\right]_Y=(2.3)\,\text{MeV}\\
&\left[\Delta m\left(\left(\pi_2^{+}\otimes\pi_1^{+}\right)_{\oplus}\right)=\Delta m\left(\left(\pi_2^{-}\otimes\pi_1^{-}\right)_{\oplus}\right)\right]_B=-(1.15)\,\text{MeV}\\
&\left[\bigcup_k\Delta m\left(\left(\pi_i^{\pm}\otimes\pi_i^{\pm}\right)_{\oplus}\right)_k\right]_R=-(1.15)\,\text{MeV}
\end{aligned}\tag{26}$$

The indexes (G, Y, B, R) are the colours of elements of matrix $\|A_{ij}\|$. The total mass defect is

$$\Delta m(\eta)=\sum_{\left((i,j)\in\|A_{ij}\|\right)}\Delta m\left(\pi_i\otimes\pi_j\right)_{\oplus}$$

Summing we will have:

$$\begin{aligned}
\Delta m(\eta)=&\left[\Delta m\left(\left(\pi_1^{-}\otimes\pi_1^{+}\right)_{\oplus}\right)+\Delta m\left(\left(\pi_2^{-}\otimes\pi_2^{+}\right)_{\oplus}\right)\right]_G\\
&+\left[\Delta m\left(\left(\pi_2^{-}\otimes\pi_1^{+}\right)_{\oplus}\right)+\Delta m\left(\left(\pi_2^{+}\otimes\pi_1^{-}\right)_{\oplus}\right)\right]_Y\\
&-\left[\Delta m\left(\left(\pi_2^{+}\otimes\pi_1^{+}\right)_{\oplus}\right)+\Delta m\left(\left(\pi_2^{-}\otimes\pi_1^{-}\right)_{\oplus}\right)\right]_B-\left[\Delta m\left(\left(\pi_i^{\pm}\otimes\pi_i^{\pm}\right)_{\oplus}\right)\right]_R\\
\Delta m(\eta)=&\left\{\left[2(4.6)_{(i,i)G}+2(2.3)_{(i,j)Y}\right]_{(\pm,\mp)}-\left[2(1.15)_{(i,j)B}+(1.15)_{(i,i)R}^{(\pm,\pm)}\right]\right\}\text{MeV}\\
&=(10.35)\,\text{MeV}
\end{aligned}\tag{27}$$

It follows:

$$m(\eta)\approx m\left(\eta^{*}\right)-\Delta m_{\eta}=\left[(558.28)-(10.35)\right]\text{MeV}=(547.95)\,\text{MeV}$$

We obtained the same result of the procedure with $F_{\Delta m}$. In this way, we can say that the two procedures are equivalent to calculate the mass defect.

Note in **Figure 6** there is a principal axis (axis of propagation) and two secondary. There are four possible particles (π^+, π^-, π°, γ) with the following combination or channels:

1) $(\pi^+, \pi^-, \pi^\circ)_{(23\%)}$, 2) $(\pi^0, \pi^0, \pi^0)_{(33\%)}$, 3) $(\gamma, \gamma)_{(39\%)}$, 4) $(\pi^+, \pi^-, \gamma)_{(5\%)}$, 5) $((e^- + e^+), \gamma)$, 6) (π^+, π^-)

So, see structure Equation (15), the possible decays (see even **Figure 6**) of η, will be (talk [16] about the probabilities):

1) only principal X-axis $\left[(\eta)\rightarrow(\gamma+\gamma)\right]_{(\text{B12})}$; this channel is possible only if the quarks ((u_1, $\underline{u}_1$)) are attached in diagonal along X-axis (see b-configuration in **Figure 5**)

2) with principal axis and secondary

- (X-axis, Z-axis) $\left[(\eta)\rightarrow\left(3\pi_1^0\right)\right]_{(\text{B1})+(\text{B12})}$.

- (X-axis, Y-axis) $\left[(\eta)\to\left(\pi^{+}+\pi^{-}\right)+\left(\pi^{\circ}\right)\right]_{(\mathrm{B2})+(\mathrm{B1})}$.
- (X-axis, Y-axis) $\left[(\eta)\to\left(\pi^{+}+\pi^{-}\right)+(\gamma)\right]$.
- (X-axis, Y-axis) $\left[\eta\to\left(\mathrm{e}^{-}+\mathrm{e}^{+}\right)+\gamma\right]$.

(see the Dalitz decay) and other possible secondary decays.

Note that a decay $\left[\eta\to\left(\pi^{+}+\pi^{-}\right)+2\pi^{\circ}\right]$ is not admitted by the energy conservation (see the mass defects) in rest frame referent of (η) because

$$\left[m\left(\pi^{+}\right)+m\left(\pi^{-}\right)+2m\left(\pi^{\circ}\right)\right]>m(\eta).$$

Note that a decay $\left[\eta\to\left(\pi^{+}+\pi^{-}\right)\right]$ is not possible for the presence of three axis (X, Y, Z).

5. Conclusions

In this paper, we have shown that quarks, mesons and proton have a geometric structure. The phenomenology of interactions points out that particles transform in other particles: we ask us how it is possible that coupling photons create quarks' pairs, see the reaction ($e^{-}+e^{+}\to q+\underline{q}$). A comprehensive answer could then be to assign a geometric structure (of coupled quantum oscillators) to all elementary particles (that is not composed of sub-particles) and that it is possible to transform a structure into another. Thus there would be a mechanism of topological transformations on geometrical structures which would transform the one into any other. In this way, we introduce a new paradigm in the phenomenology of particles and the interactions, which opens up new perspectives for resolving the various problems of the Standard Model.

The first perspective is the one of the "internal structure" of quarks that is realizable only through "particular" quantum oscillators. In a particle-structure, the vertex-oscillators and junction oscillators must have the "hooks" to the extremity [7] [8]. This induces us to talk about a "sub-structure" into a quantum oscillator which is highlighted only into quantum oscillator coupled to other oscillators [12] [13]. Not only but more components in an oscillator encourages us to believe that the energy of the "quanta" should be distributed between these oscillating components. The presence of more components in an oscillator causes the splitting of its quanta of energy into two, and more, sub-oscillators: this introduces the idea of half-quanta ("semi-quanta") or individually half-quantum ("semi-quantum").

A quantum oscillator with a sub-structure constituted by sub-unit of oscillation, or "sub-oscillators", and "semi-quanta" is an oscillator of type "IQuO" [7] [12] [13]. To treat the particles as IQuO structures can explain the origin of some fundamental physics greatness as the electric charge, spin, isospin and colour charge [18].

The second perspective is the one that a particle more massive can be structured by particles of base, see the η-meson. In the next study we will show that all mesons more massive than pion are composite structure of pion molecules.

Not only, but we think that also the nucleons and other hadrons are geometric structures of coupled quantum oscillators. The mathematics procedure which has allowed of calculating the mass of η-meson, the same will allows us of calculating the mass of other mesons (ρ, ω, ϕ) which make, with the pions, the light component but strangeness of the octet of fundamental mesons.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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