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A Dark Energy Hypothesis II

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Abstract

The article develops a cosmological model based on a hypothesis that dark energy is a cosmological variable rather than a constant. A companion paper (DEH I) derives a formula for this variable cosmological parameter as well as an argument that the early universe produces it and dark matter. The developed model leads to a series of self-consistent results including a prediction that provides a test for it. The results include comparisons of the DEH and the ΛCDM theory.

Keywords

Dark Energy, Dark Matter, Cosmological Constant, Coupling of Space and Time

1. Introduction

The dark energy parameter is

$$\Lambda = \frac{1}{\eta^2 a^2} = \kappa\varepsilon \tag{1}$$

In this formula, "a" is the scale factor, η the conformal time, $a\mathrm{d}\eta = c\mathrm{d}t$, κ the Einstein gravitational constant,

$$\kappa = \frac{8\pi G}{c^4}$$

And ε the dark energy density. The companion paper argues that this parameter is produced in the early universe by the uncoupling of two comoving coordinates, $x^0 = \eta$ and $x^1 = \chi$, the cosmic latitude. A co-product of the uncoupling is a line element

$$\mathrm{d}s^2 = a^2\left[\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - f(\chi)\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2\right)\right]$$

The function $f(\chi)$ depends on the curvature. Since this is an element of curved spacetime and curved spacetime is gravity, and since gravity is associated with matter, then the second part of the DEH is that this is the line element of dark matter, whose nature will remain mysterious. Hence, dark energy and dark matter, by hypothesis, are co-products of the decoupling.

The task now is to develop the model and then to show how the model leads to the following results.

1) The fields of dark energy and dark matter couple.

2) Hyperbolic space necessarily leads to dark energy dominance.

3) The ratio of dark matter to baryonic matter varies systematically on the backward light cone. This is the prediction mentioned in the abstract.

4) The DEH is consistent with observations of type Ia supernovae.

5) The Gaussian curvature, particle horizon distance, and cosmological parameter are inter-related—the numerical value of one determines the numerical values of the other two.

6) A four-fold comparison of the DEH and ΛCDM theory concludes the results.

Development of the Dark Energy Hypothesis

Development means combining expansion with energy conservation. The expansion of the universe is governed by the Friedmann-Lemaître equation [1]:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} \tag{2a}$$

Conformal time is a more convenient independent variable than cosmic time, t:

$$\left(\frac{c}{a}\right)^2 \left(\frac{a'}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} \tag{2b}$$

where the prime on the scale factor means $\mathrm{d}a/\mathrm{d}\eta$. The second term on the left contains the Gaussian curvature: $K = k/a^2$, with $k = -1, 0, +1$ as the curvature constant. The density term on the right-hand side may be due to both matter and radiation: $\rho = \rho_m + \varepsilon_r/c^2$. The equation holds for both constant and variable Λ.

The Friedmann-Lemaître equation may also be written as an algebraic equation. Since each term has the units of 1/second^2, each may be written as the product of a dimensionless parameter Ω times H^2, where H is the Hubble parameter.

$$1 = \Omega_r + \Omega_m + \Omega_\Lambda - \Omega_k \tag{2c}$$

The subscripts refer to the parameters for radiation, matter, dark energy, and curvature, resp.

The conservation law is the first law of thermodynamics: $\mathrm{d}U = \mathrm{d}Q - p\mathrm{d}V$. The expansion is adiabatic: $\mathrm{d}Q = 0$. The galaxies are a pressure-less dust and dark energy is also pressure-less as argued later in this article. Hence, $\mathrm{d}U = \mathrm{d}(\varepsilon V) = 0$. There are two energy densities, one due to matter and one due to dark energy.

$$\frac{U}{V} = \rho c^2 + \frac{\Lambda}{\kappa}$$

The gravitational term is

$$V\rho c^2 = \left[M(dm) + M(b)\right]c^2$$

Designating the two kinds of mass, dark matter and baryonic. With $V = a^3$, the dark energy term is

$$U_\lambda = \frac{a}{\kappa\eta^2}$$

The term a/η^2 is appropriately called the dark energy length because of its units. The conservation law becomes

$$U = U_\lambda + \left[M(dm) + M(b)\right]c^2$$

Subsequent work will argue that the sum of dark energy and dark matter is conserved, and that baryonic matter is separately conserved, so that the total energy is conserved. The Einstein gravitational constant has the units of length/energy, so multiplying it through the preceding equation converts it into a length, $\Gamma = \kappa U$.

The task now is to incorporate energy conservation into Equation (2b) by multiplying through by $a^4/c^2 = aV/c^2$, giving

$$aV\frac{8\pi G\rho}{c^2} = a\kappa Mc^2$$

and

$$aV\Lambda = aV\kappa\varepsilon = a\kappa U_\lambda$$

The Friedmann-Lemaître equation becomes

$$\left(\frac{\mathrm{d}a}{\mathrm{d}\eta}\right)^2 + ka^2 = \frac{a\Gamma}{3} \tag{3}$$

To integrate for the curved spaces, complete the square, make a trigonometric or hyperbolic substitution as appropriate, and apply the boundary condition that $a = 0$ when $\eta = 0$. Then find the time-dependence from $c\mathrm{d}t = a\mathrm{d}\eta$.

The integrated equations are:

$$\begin{gathered}k = +1,\quad a = \frac{\Gamma}{6}\left(1 - \cos(\eta)\right) \;\;\&\;\; ct = \frac{\Gamma}{6}\left(\eta - \sin(\eta)\right)\\ k = -1,\quad a = \frac{\Gamma}{6}\left(\cosh(\eta) - 1\right) \;\;\&\;\; ct = \frac{\Gamma}{6}\left(\sinh(\eta) - \eta\right)\\ k = 0,\quad a = \frac{\Gamma}{12}\eta^2 \;\;\&\;\; ct = \frac{\Gamma}{36}\eta^3\end{gathered} \tag{4}$$

Perhaps readers will identify these equations as the cycloid, hyperbolic, and Einstein-de Sitter equations, but those are for $\Lambda = 0$ with only a variable gravitational field. Now there is the cosmological parameter and two variable fields, gravitational and dark energy.

The Hubble parameter can be found from

$$H = \frac{ca'}{a^2} \tag{5}$$

A dimensionless form of the conservation law will be useful. Define a dimensionless dark energy parameter λ by

$$k=+1,\quad \lambda=\frac{1-\cos\eta}{6\eta^2}$$
$$k=-1,\quad \lambda=\frac{\cosh(\eta)-1}{6\eta^2} \tag{6}$$
$$k=0,\quad \lambda=\frac{1}{12}$$

It follows that $\Gamma\lambda=\kappa U_\lambda$. In the same manner, define a dimensionless mass parameter by $\chi(m)=\chi(dm)+\chi(b)$ by $\Gamma\chi(m)=\kappa Mc^2$. Then the conservation law becomes

$$\lambda+\chi(dm)+\chi(b)=1 \tag{7a}$$

According to the ΛCDM theory, at the present epoch the preceding equation reads

$$\frac{7}{10}+\frac{1}{4}+\frac{1}{20}=1$$

To consider only dark energy and dark matter, the conservation law is

$$\lambda+\chi(dm)=0.95 \tag{7b}$$

This will be the point of departure for numerical work.

In sum, dark energy and matter may be represented by a dimensionless parameter or by a length. If by a length, to get an energy divide by the Einstein constant, $\kappa = 2.076\times10^{-43}$ m/J or a mass divide by $\kappa c^2 = 1.866\times10^{-26}$ m/kg.

2. Results of the Dark Energy Hypothesis

1) The dark energy and dark matter fields are coupled.

Equations (4) show that dark energy changes with time in curved spaces but remains constant in flat space. This statement is reinforced by considering the scalar integral [2],

$$S=\int\frac{a}{\eta^2}\sqrt{-g}\,\mathrm{d}^4x$$

where g is the determinant of the metric tensor. The integral diverges for spherical and hyperbolic space but converges for flat space, meaning that the dark energy length is invariant in flat space, but not in the two curved spaces.

In curved spaces, dark energy must have a source or sink if energy is to be conserved; the default candidate is dark matter, since baryonic matter is the product of the cooling of thermal radiation during the first three minutes of cosmological history. Thus, the dark energy and dark matter fields are coupled.

2) Hyperbolic space necessarily evolves into a state of dark energy dominance.

For $\eta = 0$, $\lambda = 1/12$ and $\chi(m) = 11/12$ for all three spaces. Flat space is stuck in this state of matter dominance, and spherical space must evolve to even greater matter dominance. Hence result 2) follows immediately and with it the notion that dark energy dominance implies that space is hyperbolic. The numerical work in this section illustrates trends that are consistent with the 70/25/5 energy distribution including the assumption that the baryonic term is fixed.

If $\lambda_0 = 7/10$, then it follows readily from Equation (6) that the conformal time of the current epoch is $\eta_0 = 5.571$. Given that the value of the Hubble parameter in the current epoch is $H_0 = 2.20 \times 10^{-18}\ \text{s}^{-1}$ [3], then from the formula for the Hubble parameter, Equation (5)

$$\frac{\Gamma}{6} = 1.051 \times 10^{24}\ \text{m}$$

This is a pivotal result because Γ is the total cosmological energy expressed as a length. With this result, the age of the universe is $t_0 = 13.97$ Gyr by Equation (4).

The two short **Table 1** and **Table 2** of numbers follow from the equations of the Development section using what in effect is the energy equation. For example, $M = \chi(m)\Gamma/\kappa c^2$. The four values of λ are for the early universe, for a later date of matter dominance, for the current epoch, and for the future epoch when dark matter has disappeared.

Table 1. DEH parameters vs dark energy, λ.

λ	η	t(Gyr)	a(m)/10^{26}	M(kg)/10^{50}	$\Lambda(\text{m}^{-2})/10^{-54}$
1/10	1.493	0.0688	0.0141	3.04	2.27×10^5
3/10	4.116	3.12	0.328	2.37	53.5
7/10	5.571	14.0	1.37	1.01	1.72
19/20	6.033	22.5	2.18	0.169	0.578

The dimensionless parameters, Ω, illustrate evolutionary trends. They are relative energy densities.

Table 2. DEH dimensionless parameters vs dark energy, λ.

λ	Ω_r	Ω_m	Ω_Λ	Ω_k
1/10	0.389	0.450	0.0500	−0.111
3/10	0.0038	0.0421	0.0180	−0.942
7/10	1.5×10^{-4}	0.00453	0.0106	−0.985
19/20	6×10^{-5}	0.000477	0.0090	−0.993

Certain features are common to other models. At early epochs, space looks flat; at $t = 3$ min, for example, $\Omega_k \sim -10^{-10}$ in the DEH, but the actual curvature manifests itself as time elapses. Radiation is unimportant in the energy balance of the late universe.

Other features belong to the DEH model. For example,

$$\frac{\Omega_m}{\Omega_\Lambda} = \frac{\chi(m)}{\lambda} \quad \text{and} \quad \frac{K}{\Lambda} = \frac{\Omega_k}{\Omega_\Lambda} = k\eta^2$$

Another is that the early universe has a high mass density vis-à-vis that of the ΛCDM theory, meaning that gravitational collapse and structure formation might begin at early epochs. A third is that the disappearance of dark matter would have major implications for the stability of spiral galaxies and galactic clusters.

3) The ratio of dark matter to baryonic matter increases on the backward light cone.

Given that dark matter is disappearing while the quantity of baryonic matter remains constant, this conclusion follows immediately and is illustrated in the following graph (**Figure 1**).

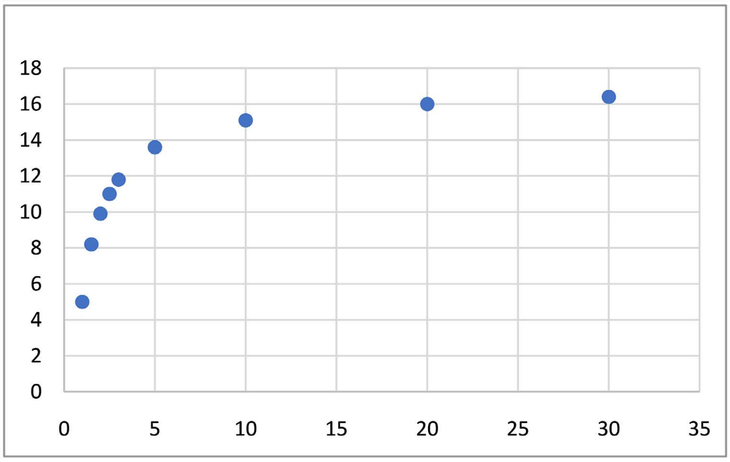

Figure 1. Ratio of dark matter to baryonic matter vs Doppler shift.

The initial ratio is 5 and the limit is 52/3. The algorithm for constructing the graph is straight forward. From the definition of the cosmological Doppler and redshift,

$$D = z + 1 = \frac{a_0}{a} = \frac{\cosh(\eta_0) - 1}{\cosh(\eta) - 1} = \frac{130.35}{\cosh(\eta) - 1}$$

Then

$$\eta = \cosh^{-1}\left[1 + (130.35/D)\right]$$

is the conformal time corresponding to the Doppler shift. Equation (6) gives the dark energy λ at this time, Equation (7b) gives $\chi(dm)$ and the ratio is $20\chi(dm)$.

4) The hypothesis is consistent with observations of Type Ia supernovae.

An apparent magnitude/red shift analysis based on DEH parameters matches the values observed by Perlmutter *et al.* [4]. The working equation for the DEH model is

$$m - M = 5\log_{10}\left(\frac{d_L}{1\,\text{Mpc}}\right) + 25$$

In this equation, m is the apparent magnitude, $M = -19$ is the absolute magnitude of a type Ia supernova, and d_L is the luminosity distance measured in megaparsecs:

$$d_L = a_0 \sinh(\chi_e)(z+1)$$

As usual, a_0 is the value of the scale factor in the current epoch, and χ_e is the comoving coordinate of the supernova [3]:

$$\chi_e = \frac{z a_H}{a_0}\left[\frac{2 - z(1+q_0)}{2}\right]$$

The current value of the Hubble length is designated by a_H and q_0 is the current value of the deceleration parameter, $1/(1+\cosh(\eta_0))$. The preceding formula becomes

$$\chi_e = 0.993z[1 - 0.504z]$$

The data set of Perlmutter *et al.* consists of forty-two observations divided into two sets, low and high z. The following calculation refers to the first number listed in each set (Table 3).

Table 3. Apparent magnitude vs redshift.

z	χ_e	$\sinh(\chi_e)$	d_L(Mpc)	m(calc)	m(obs)
0.030	0.0293	0.0293	1.34	16.6	16.3
0.458	0.350	0.357	2.31×10^3	22.8	23.1

The differences between calculated and observed values are 1% - 2%.

The graph below is for eighteen of the forty-two data points. The eighteen data points were those redshifts whose third digit was a zero, an objective, arbitrary choice. The blue circles are DEH calculated values and the orange observed values. There were three instances in the observed values where two supernovae had the same redshift but slightly different apparent magnitudes, which produced the "knots" in the graph (Figure 2).

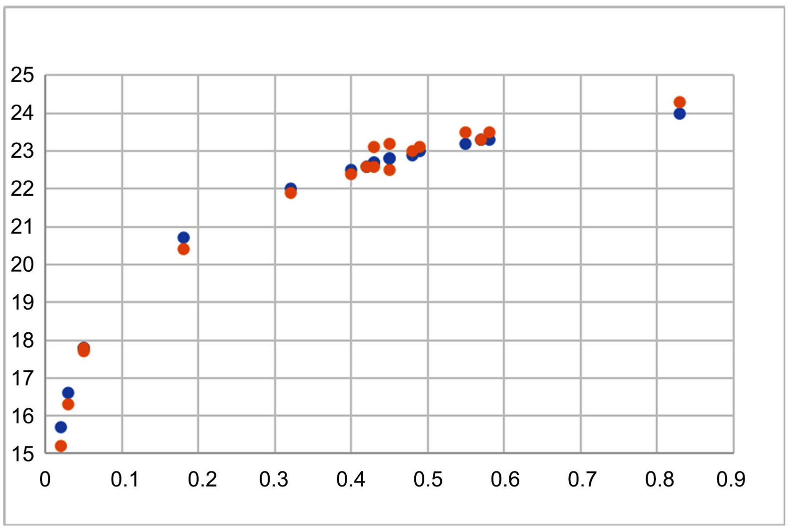

Figure 2. Apparent magnitude vs redshift.

5) The Gaussian curvature, particle horizon distance, and the cosmological parameter are interdependent, that is, the numerical value of one determines the numerical values of the other two.

A galaxy of comoving coordinate χ_e emits a radial light ray at time η_e that is received at $\chi = 0$ and time η_0. The location of the galaxy at the time of reception is

$$a_0\left(\eta_0 - \eta_e\right) = a_0\chi_e$$

If the light emission occurs at the earliest possible time, $\eta_e = 0$, then the galaxy is on the particle horizon. Hence, the particle horizon distance is

$$d_{PH}\left(t_0\right) = a_0\eta_0$$

But this is simply related to the cosmological parameter:

$$d_{PH}\left(t_0\right) = \Lambda_0^{-1/2}$$

It is also evident that the Gaussian curvature is

$$K_0 = k\eta_0^2\Lambda_0$$

These relationships hold for any epoch of reception.

From the tables in § 2,

$$\Lambda_0 = 1.72\times10^{-54}\ \text{m}^{-2}$$

$$d_{PH}\left(t_0\right) = 7.62\times10^{26}\ \text{m} = 5.77ct_0$$

$$K_0 = -5.34\times10^{-53}\ \text{m}^{-2}$$

6) A four-fold comparison of the ΛCDM theory and the DEH.

a) On mass and energy.

In the ΛCDM theory mass is conserved. The dark energy density is constant because Λ is constant. The total dark energy then is

$$U_\lambda = a^3\varepsilon = \frac{\Lambda a^3}{\kappa}$$

Hence, the total dark energy grows as the universe expands and it seems that the total energy is not constant: $U = U_\lambda + Mc^2$. Cosmologists have noted that the constancy of dark energy density is troublesome in this model [5].

In the DEH model neither dark energy nor dark matter are separately conserved, but their sum is.

$$U = \left[M\left(dm\right)c^2 + U_\lambda\right] + M\left(b\right)c^2$$

$$U = U_D + M\left(b\right)c^2$$

Both terms on the right-hand side are conserved and hence their sum also.

ΛCDM. Mass is conserved but total energy is not.

DEH. Total energy is conserved but mass is not.

b) On pressure and dark energy.

The equation of state is $p = w\varepsilon$, where ε is the dark energy density and w is a constant to be determined. The first law of thermodynamics with $\mathrm{d}Q = 0$ is $\mathrm{d}U =$

$-p\mathrm{d}V$, leading to $\mathrm{d}\varepsilon = -\varepsilon(1 + w)\mathrm{d}V/V$.

ΛCDM. The left-hand side vanishes because the dark energy density is constant, which requires that $w = -1$. Pressure is a negative energy density.

DEH. This requires $w = 0$ so that $\varepsilon V = U_\lambda$. Dark energy is pressure-less.

If $p > 0$, pressure contributes to gravity, that is, it acts like an attractive "force" in Newtonian terms. If $p < 0$ as in the ΛCDM theory, then dark energy opposes gravity, acting like a repulsive "force", which is why Einstein introduced the cosmological constant into his field equations in the first place [6] [7]. If $p = 0$ as in the DEH model, dark energy has nothing to do with gravity; dark energy is simply a sink for dark matter.

c) On acceleration and deceleration.

In the ΛCDM theory, the initial expansion decelerates until some epoch before the current epoch, then begins to accelerate, eventually reaching a de Sitter state of unending acceleration

$$a = a^* \exp\left(ct\sqrt{\frac{\Lambda}{3}}\right)$$

With a deceleration parameter $q = -1$. This so-called "big rip" is a consequence of vanishing gravity with $p < 0$ or, equivalently, of the non-conservation of total energy.

The expansion continuously decelerates in the DEH model, which is indicated by the deceleration parameter, $0 < q < 1/2$. As $\eta \to \infty$, $q \to 0$ and $\dot{a} \to c$.

d) On a difference in point of view.

If radiation is negligible, then the curvature parameter is given by an

$$\Omega_k = \Omega_m + \Omega_\Lambda - 1$$

In the analysis of supernovae data, Perlmutter *et al.* find $\Omega_k = 0.300 + 0.700 - 1 = 0$, so space is flat. In the DEH model, $\Omega_\Lambda/\Omega_m = 0.700/0.300$ at the present epoch, and as $\eta \to \infty$, $\Omega_k \to 0 + 0 - 1 = -1$, so space is hyperbolic as illustrated in § 2 above.

3. Conclusions

If the prediction of § 3 is confirmed, then there is reason to give serious consideration to the Dark Energy Hypothesis. If not, then *c'est la vie*.

If the DEH gets serious consideration, then perhaps there's a case for thinking that space is hyperbolic and that the expansion is not accelerated. As to the 70/25/5 distribution of energies, for which ΛCDM seems to provide no explanation, it's an evolved state according to the DEH.

If not, then perhaps the line of investigation employed here into the nature of dark energy will prove fruitful in other hands.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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