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# Newtonian Gravitational Radiation and Waves

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## Abstract

In this paper, we review historical Maxwell's equation for gravity and recent studies on the lack of curvature of linear dipole gravitational waves. The extended Newton's gravity necessarily has the continuity equation for the conservation of mass, and with the Gauss' equation associated to gravitational time depending field **R**, bring about a new field **W** which resembles the magnetic field in Electrodynamics. Although this field has not been found yet, its existence comes from a strong mathematical statement, and it is shown that linear dipole gravitational waves have their origin in extended Newton theory of gravity. This is a direct mathematical consequence of Gauss' law and the continuity equation for the density of mass and current, and as a direct result of this, any accelerated mass will emit mainly dipole gravitational radiation. Then, one concludes that dipole gravitational waves can have its origin on the extended Newton's gravity equations.

## Keywords

Gravitational Field, Maxwell's Equations, Gravitational Waves, Gravitational Radiation Reaction

## 1. Introduction

Gravitational field is one of the most important fields created by Nature which has (so far) pure attractive effect among anything with mass or energy [1, 2], and its associated force between two objects is radial, proportional to the product of the masses of the objects, and inverse proportional to the square of the separation of the objects. In addition, this law does not change due to the presence of a third object [3]. This Newton's finding brought about the concept of gravitational field **R**, the Gauss' law associated to it [4], $\nabla\cdot\mathbf{R}=4\pi\rho$ being $\rho$ the density of mass, and the potential theory for the solution of Poisson's equation [5], $\nabla^2\Phi=-4\pi\rho$ with $\mathbf{R}=-\nabla\Phi$. One hundred years later and after huge success of Maxwell's equations in Electrodynamics [6], Heaviside [7–9] postulated, without any mathematical or experimental justification, a field **h** to force gravitational equations of the form Maxwell's equations. It was Einstein [10] who though about gravity in different way, as a deformation of the space-time (manifolds) formed by the presence of mass, and ripples on this manifold due to gravitational quadrupole waves, and its theory was called General Relativity (GR). Since Heaviside's approach lacks of mathematical justification and was done for a very particular case (fluid), it can not be considered as a deduction of Maxwell's equation for gravity. However, the idea settled there was that Maxwell-like equations for gravity maybe could be arisen without GR theory.

In general relativity, using perturbation theory [11, 12], or the decomposition of the Weyl and Maxwell tensors in electric and magnetic parts [13–20], the so called Gravito-Electromagnetism (GEM) emerges, as a result of these approaches, which is the expression of the equations to describe the gravitational field as a Maxwell-like type of equations. In a similar axiomatic form, Yaroslav [21] has recently show the deduction of Maxwell-like equations for gravity without GR, where perihelion of Mercury and bending of light by massive object are presented. It also worths to mention works based on Hodge theory and differential forms [22], where a similarity with the electromagnetic permittivity and permeability was found. The idea of having a magnetic-like gravitational field in GR was reinforced due to Lense-Thirring effect [23, 24], where the angular momentum of a rotating body can be interpreted as a gravitational magnetic field, at large distances. Then, one could have the wrong conclusion that GR is needed in order to see a gravitational field like GEM. On this paper, it is used the mathematical deduction made by López [25] of the Maxwell's equations for gravity on flat space-time, which arises as a consequence of the continuity equation for the mass density and the usual Gauss' theorem, and it is pointed out that linear gravitational waves have its origin in the extended Newton's theory of gravity.

## 2. Maxwell's Equation for Gravity

In reference [25], it is shown that due to Gauss' theorem for the gravitational field $\mathbf{R}$, $\nabla\cdot\mathbf{R} = 4\pi\rho$, and the continuity equation for the mass density, $\rho$, and density current $\mathbf{J}$, $\partial\rho/\partial t + \nabla\cdot\mathbf{J} = 0$, for explicitly time depending variables, there must exist an additional gravitational field $\mathbf{W}$ such that the Maxwell's equation appears naturally for gravity

$$\nabla\cdot\mathbf{R} = 4\pi G\rho \tag{1}$$

$$\nabla\times\mathbf{R} = -\frac{1}{\lambda}\frac{\partial\mathbf{W}}{\partial t} \tag{2}$$

$$\nabla\cdot\mathbf{W} = 0 \tag{3}$$

$$\nabla\times\mathbf{W} = \frac{4\pi G}{\lambda}\mathbf{J} + \frac{1}{\lambda}\frac{\partial\mathbf{R}}{\partial t}, \tag{4}$$

where $\lambda$ is the speed of gravity propagation, and $\mathbf{J}$ is the density of current ($G \approx 6.674\times10^{-11}\ \mathrm{m}^3/\mathrm{Kg}\cdot\mathrm{s}^2$ is the gravitational constant). Now, given $\rho$ and $\mathbf{J}$, the resulting decoupled equations for $\mathbf{R}$ and $\mathbf{W}$ are the inhomogeneous wave equations

$$\nabla^2\mathbf{R} - \frac{1}{\lambda^2}\frac{\partial^2\mathbf{R}}{\partial t^2} = 4\pi G(\nabla\rho) + \frac{4\pi G}{\lambda^2}\frac{\partial\mathbf{J}}{\partial t} \tag{5}$$

and

$$\nabla^2\mathbf{W} - \frac{1}{\lambda^2}\frac{\partial^2\mathbf{W}}{\partial t^2} = -\frac{4\pi G}{\lambda}\nabla\times\mathbf{J}, \tag{6}$$

where one can see the $\lambda$ is effectively the speed of gravity propagation. As in ordinary Maxwell's equation for electrodynamics, the fields $\mathbf{R}$ and $\mathbf{W}$ can be written in terms of an scalar potential $\Phi$ and vector potential $\mathbf{A}$ as

$$\mathbf{W} = \nabla \times \mathbf{A}, \qquad \text{and} \qquad \mathbf{R} = -\nabla\Phi - \frac{1}{\lambda}\frac{\partial \mathbf{A}}{\partial t}, \tag{7}$$

and with the usual Lorentz' norm ( $\nabla \cdot \mathbf{A} + \partial\Phi/\partial\lambda t = 0$), one gets that $\Phi$ and $\mathbf{A}$ satisfy inhomogeneous wave equations

$$\nabla^2\Phi - \frac{1}{\lambda^2}\frac{\partial^2\Phi}{\partial t^2} = -4\pi G\rho, \quad \text{and} \quad \nabla^2\mathbf{A} - \frac{1}{\lambda^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi G}{\lambda}\mathbf{J}, \tag{8}$$

emphasizing the wave characteristics of gravity. The Poyting vector $\mathbf{S}$ and the density of gravitational energy $u_g$ are well defined quantities given by

$$\mathbf{S} = \frac{\lambda}{4\pi G}\big(\mathbf{R} \times \mathbf{W}\big), \qquad \text{and} \qquad u_g = \frac{1}{8\pi G}\Big(|\mathbf{R}|^2 + |\mathbf{W}|^2\Big), \tag{9}$$

satisfying the continuity equation

$$\nabla \cdot \mathbf{S} + \frac{\partial u_g}{\partial t} = 0. \tag{10}$$

This implies that the gravitational energy velocity is

$$\mathbf{v}_g = \frac{\mathbf{S}}{u_g} = 2\lambda\frac{\mathbf{R} \times \mathbf{W}}{|\mathbf{R}|^2 + |\mathbf{W}|^2}, \tag{11}$$

and the power emitted by gravitational field int he direction $\hat{\mathbf{n}}$ is

$$P_g = \frac{1}{4\pi G\lambda}\big(\mathbf{R} \times \mathbf{W}\big) \cdot \hat{\mathbf{n}}, \tag{12}$$

where $\hat{\mathbf{n}} = \mathbf{x}/|\mathbf{x}|$ is the vector pointing out of the surface of an sphere centered on the particle position and having a radius $r = |\mathbf{x}|$.

## 3. Newtonian Gravitational Waves

It is known that the particular solution of the equations in (8) are given by the convolution of the inhomogeneity with the fundamental solution of the wave equations [26],

$$\Phi(\mathbf{x}, t) = -4\pi G(\mathcal{E} \star \rho), \qquad \mathbf{A}(\mathbf{x}, t) = -\frac{4\pi G}{\lambda}(\mathcal{E} \star \mathbf{J}), \tag{13}$$

where $\mathcal{E}$ is the fundamental solution,

$$\mathcal{E}(\mathbf{x}, t) = -\frac{1}{4\pi}\frac{\delta(t - |\mathbf{x}|/\lambda)}{|\mathbf{x}|}. \tag{14}$$

This brings about the known retarded potentials

$$\Phi(\mathbf{x}, t) = G\int_{\Omega\times\Re}\frac{\rho(\mathbf{x}', t')\delta(t - t' - |\mathbf{x} - \mathbf{x}'|/\lambda)}{|\mathbf{x} - \mathbf{x}'|}d^3\mathbf{x}'dt' \tag{15}$$

and

$$\mathbf{A}(\mathbf{x}, t) = \frac{G}{\lambda}\int_{\Omega\times\Re}\frac{\mathbf{J}(\mathbf{x}', t')\delta(t - t' - |\mathbf{x} - \mathbf{x}'|/\lambda)}{|\mathbf{x} - \mathbf{x}'|}d^3\mathbf{x}'dt', \tag{16}$$

where $\Omega \subset \Re^3$ is the domain where $\rho$ and $\mathbf{J}$ are defined. For a point object of mass $m$ which is moving arbitrarily, having the position $\mathbf{x}_m(t)$ and velocity $\mathbf{v}_m(t)$, with $\rho(\mathbf{x}', t') = m\delta\big(\mathbf{x}' - \mathbf{x}_m(t')\big)$ and $\mathbf{J}(\mathbf{x}', t') = m\mathbf{v}_m(t')\delta\big(\mathbf{x}' - \mathbf{x}_m(t')\big)$, a Liénard-Wiechert potentials are gotten, and the resulting gravitational fields are of the form

$$\mathbf{R} = \mathbf{R}_\beta + \mathbf{R}_{\dot\beta}, \qquad \mathbf{W} = \hat{\mathbf{r}} \times \mathbf{R}, \tag{17}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{x}'$ is the vector going from the object position $\mathbf{x}'$, to the observer position $\mathbf{x}$ , $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$ is the unitary vector, and the gravitational fields $\mathbf{R}_\beta$ and $\mathbf{R}_{\dot\beta}$ are

$$\mathbf{R}_\beta = \frac{Gm(\hat{\mathbf{r}} - \vec{\beta})(1-\beta^2)}{r^2(1-\hat{\mathbf{r}}\cdot\vec{\beta})^3}\bigg|_{t'=t-r/\lambda} \tag{18a}$$

and

$$\mathbf{R}_{\dot\beta} = \frac{Gm}{\lambda}\frac{\hat{\mathbf{r}} \times \big[(\hat{\mathbf{r}} - \vec{\beta}) \times \dot{\vec{\beta}}\big]}{r(1-\hat{\mathbf{r}}\cdot\vec{\beta})^3}\bigg|_{t'=t-r/\lambda}, \tag{18b}$$

being $\vec{\beta} = \mathbf{v}_m(t)/\lambda$ the normalized velocity of the object, and $t'$ is the retarded time. The gravitational power emitted by the accelerated object per solid angle is (Figure 1)

$$\frac{dP_g}{d\Omega} = (1-\hat{\mathbf{r}}\cdot\vec{\beta})r^2(\mathbf{S}\cdot\hat{\mathbf{r}}), \tag{19}$$

and using (12), (17), and (18b), one gets

$$\frac{dP_g}{d\Omega} = \frac{(Gm)^2}{4\pi\lambda}\frac{\big|\hat{\mathbf{r}} \times \big[(\hat{\mathbf{r}} - \vec{\beta}) \times \dot{\vec{\beta}}\big|^2}{(1-\hat{\mathbf{r}}\cdot\vec{\beta})^5} \tag{20}$$

Thus, any object of mass $m$ which is accelerated will emit gravitational radiation. Of course, the object must have a huge mass in order for this radiation to be observed, and any periodic motion of a body of mass $m$ will emit gravitational periodic waves. This result contrast a lot with the given by General Relativity since in this theory the object must have a quadrupole configuration in order to emits gravitational energy [27–29], and the gravitational waves emitted are of quadrupole type waves.

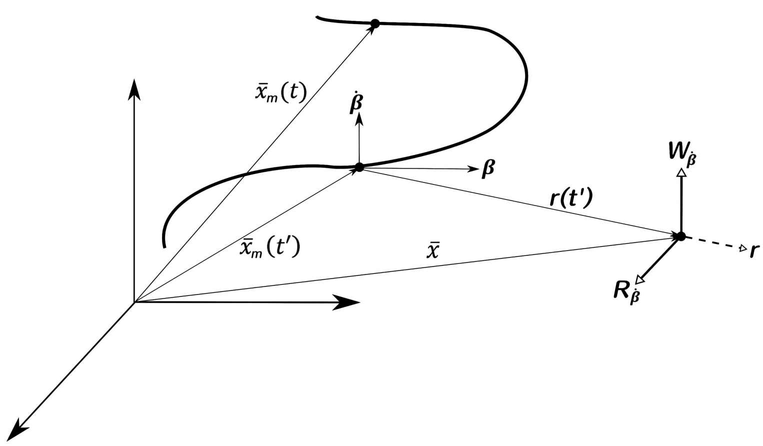


**Figure 1.** Gravitational field of a body of mass "$m$" with an arbitrary motion.

If one has an accelerated charged particle of mass $m$, this particle will emits electromagnetic [30] and gravitational (20) energy such that the ratio of electromagnetic to gravitational energy emitted is

$$\frac{(dP/d\Omega)_{\mathcal{E}}}{(dP_g/d\Omega)} = \left(\frac{q}{4\pi\epsilon_0 Gm}\right)^2. \tag{21}$$

where $\epsilon_0 = 8,854 \times 10^{-12} F/m$ is the constant dielectric of the vacuum. For the electron, $|q_e| = 1.6 \times 10^{-19} C$, this ratio is of the order of $10^{39}$, regardless of its acceleration, that is, the electromagnetic radiation totally dominate the gravitational radiation. In order for a object of mass $m$ and charge $q$ to emits the same gravitational energy as the electromagnetic energy emission, it would require that its mass would be $m \approx q/4\pi\epsilon_0 G$.

## 4. Conclusion and Comments

Within the above approach, it was shown that Maxwell's equations for gravitational field appear without having any relation at all with General Relativity or space-time curved. This is just due to Gauss' theorem and the continuity equation for density of mass and current which allowed to have the existence of a new gravitational field $\mathbf{W}$, with similar properties properties to the magnetic field in Electrodynamics. However, experimental verification of the existence of this new gravitational field is required, although its mathematical existence is out of question. Thus, it is absolutely astonishing the existing closely relation between post Newtonian gravity theory and Electrodynamics theory, indicating that a space-time curved is not needed for the description of linear dipole gravitational waves. Of course, non-linear and some linear (mainly of quadrupole type) gravitational waves are described by general relativity. In addition, one needs to point out that the energy associated to gravitational field or gravitational waves is a well defined concept, and dipole linear gravitational waves appears from this extended Newtonian gravity theory. In summary, gravitational waves radiation appear for any accelerated object of mass $m$, and dipole type of radiation is always expected. Finally, with respect the resent claimed gravitational waves detections [31–33], maybe it is possible to see the dipole linear gravitational waves here. However, this statement would require a deep careful experimental analysis.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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