

Configuration Mixing in Pt Nuclei within Interacting Boson Model-2

Malek A. H. Muhi, Saad N. Abood

Physics Department, College of Science, Al-Nahrain University, Baghdad, Iraq

Email: malek.muhi@gmail.com

How to cite this paper: Muhi, M.A.H. and Abood, S.N. (2020) Configuration Mixing in Pt Nuclei within Interacting Boson Model-2. *Journal of Applied Mathematics and Physics*, 8, 492-506.

<https://doi.org/10.4236/jamp.2020.83039>

Received: January 7, 2020

Accepted: March 10, 2020

Published: March 13, 2020

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Abstract

This article provides to describe a consistent of the even-even $^{182-200}\text{Pt}$ isotopes. This has been achieved using the interacting boson model-2 (IBM-2) and including configuration mixing (IBM-2 CM). Our attention is paid to describe the nuclei shape and to their connecting with shaping coexistence phenomenon. Ten isotopes are studied, ranging from the middle of the neutron shell to very near the doubly closed shell at ^{208}Pb . The same Hamiltonian is used for all the nuclei studied, with parameters which are constant or smoothly varying. In this study, we showed the transition between more axially symmetric deformed features of light Pt isotopes to γ -unstable and vibrational isotopes (near spherical shape) for $^{198-200}\text{Pt}$ isotopes.

Keywords

Configuration Mixing, IBM-2, Pt Isotopes, Coexistence Phenomenon

1. Introduction

Some nuclei near closed shells appear to have both the vibrational structure expected for a near-spherical shape, and rotational structure, which is typical of deformed nuclei [1]. This phenomenon of shape coexistence involves two configurations of the nucleus which have different numbers of active nucleons. In an IBM description, the two configurations have different boson numbers; N_v being the same but N_π different, or vice versa. The most common situation involves a difference in N_π (N_v) of two bosons between the normal configuration and the so-called intruder configuration, corresponding to a pair excitation across a shell or sub-shell gap [1] [2].

Shape coexistence in atomic nuclei has become a very active field of research during the last decades and clear signals of its existence have been obtained at and near proton or neutron closed shells [1] [2] [3], more in particular in light

nuclei with a closed neutron shell at $N = 8, 20, 28,$ and 50 closed shells as well as in heavy nuclei such as the Sn and the Pb nuclei. It seems that, without exception, shape coexistence is associated with the presence of low-lying excited 0^+ states.

In the case of the IBM-2, shape coexistence arises including two-particle two-hole (2p-2h) (or even higher np-nh). Excitations across the closed shells, but they consider them as extra bosons, *i.e.*, pairs of nucleons. This extension is called IBM-2 configuration mixing. In two previous papers [4] [5], we used the IBM-CM to extensively study the Pt nuclei. We carried out a detailed analysis of the energy spectra and absolute $B(E2)$ values for states up to an energy of 1.5 MeV. Garc'ia-Ramos *et al.*, [5], studied shape evolution and shape coexistence in Pt isotopes: Comparing interacting boson model configuration mixing and Gogny mean-field energy surfaces. In this work, the evolution of the total energy surface and the nuclear shape in the isotopic chain $^{172-194}\text{Pt}$ are studied in the framework of the interacting boson model, including configuration mixing. The results are compared with a self-consistent Hartree-Fock-Bogoliubov calculation using the Gogny-D1S interaction and a good agreement between both approaches is found.

In this article, it will be studied to extract the parameters describing the IBM-2 CM Hamiltonian precisely. After showing some theories and laws about IBM-2, it represents the results of the Pt isotopes comparing with some experimental data on energy levels, electromagnetic transition probabilities (B(E2) and B(M1)), monopole transitions, mixing ratios and quadrupole moments.

2. The Interacting Boson Model-2 (IBM-2)

2.1. The Hamiltonian of Interacting Boson Model-2 (IBM-2)

As the IBM-2 model, the protons and neutrons degrees of freedom are taken into account explicitly. Consequently, one can write the Hamiltonian [6] [7] as,

$$H = H_\pi + H_\nu + V_{\pi\nu} \quad (1)$$

$$H = \varepsilon_\pi d_\pi^+ d_\pi^- + \varepsilon_\nu d_\nu^+ d_\nu^- + V_{\pi\pi} + V_{\nu\nu} + \kappa Q_\pi \cdot Q_\nu + M_{\pi\nu} \quad (2)$$

Here ε is the d-boson energy, κ is the strength of the quadrupole interacting between proton and neutron bosons.

The quadrupole moment operator, in the IBM-2 model, has the form:

$$Q_{\rho\rho} = (s^+ d^- + d^+ s^-)_\rho^{(2)} + \kappa_\rho (d^+ d^-)_\rho^{(2)} \quad (3)$$

where $\rho = \pi$ or ν , $Q_{\rho\rho}$ is the quadrupole deformation parameter for protons ($\rho = \pi$) and neutrons ($\rho = \nu$). Where the terms $V_{\nu\nu}$ and $V_{\pi\pi}$ are the proton-proton and neutron-neutron d-boson interacting only and written by:

$$V_{\rho\rho} = \sum_{J=0,2,4} \frac{1}{2} C_{L\rho} (2J+1)^{1/2} \left[(d^+ d^+)_\rho^{(2)} (\tilde{d}\tilde{d})_\rho^{(2)} \right]^{(0)} \quad (4)$$

The Majorana interaction is represented the term $M_{\pi\nu}$, that accounts for the symmetry energy and shifts the states with mixed proton-neutron symmetry state with respect to the fully symmetric ones which affects only the relative location

of the states with mixed symmetry, it is with respect to the totally symmetric states. For the reason that little experimental information is familiar states with mixed symmetry that has the form:

$$M_{\pi\nu} = - \sum_{k=1,3} 2\xi_k (d_{\pi}^+ d_{\pi}^+)^{(k)} (d_{\pi}^- d_{\pi}^-)^{(k)} + \xi_2 (d_{\pi}^+ s_{\nu}^+ - s_{\pi}^- d_{\nu}^-)^{(2)} (d_{\pi}^- s_{\nu}^- - s_{\pi}^+ d_{\nu}^+)^{(2)} \quad (5)$$

2.2. Electromagnetic Transitions and Quadrupole Moments in IBM-2

Generally, the E2 transition operator of one-body in the IBM-2 is

$$T(l) = T_{\pi}(l) + T_{\nu}(l) \quad (6)$$

$$\begin{aligned} T(E2) &= e_{\pi} \left[(s^+ d^- + d^+ s^-)_{\pi}^{(2)} + \chi_{\pi} (d^+ d^-)_{\pi}^{(2)} \right]^{(2)} \\ &+ e_{\nu} \left[(s^+ d^- + d^+ s^-)_{\nu}^{(2)} + \chi_{\nu} (d^+ d^-)_{\nu}^{(2)} \right]^{(2)} \\ T(E2) &= e_{\pi} Q_{\pi} + e_{\nu} Q_{\nu} \end{aligned} \quad (7)$$

which Q_{ρ} is in the form of Equation (3). For simplicity, the χ_{ρ} has the same value as in the Hamiltonian. Also, one suggests it by the single j -shell microscopy. Generally, the E2 transition results are not sensitive to the choice of e_{ν} and e_{π} , whether $e_{\pi} = e_{\nu}$ or not. Thus, the reduced electric quadrupole transition rates between $J_i^+ \rightarrow J_f^+$ states are given by:

$$B(E2; J_i^+ \rightarrow J_f^+) = \frac{1}{2J_i + 1} \left| \langle J_f^+ || T(E2) || J_i^+ \rangle \right|^2 \quad (8)$$

In IBM-2, the electric quadrupole moment is written as:

$$Q_l = \left[\frac{16\pi}{5} \right]^{1/2} \begin{bmatrix} J & 2 & J \\ -J & 0 & J \end{bmatrix} \langle J || T(E2) || J \rangle \quad (9)$$

One can calculate IBM-2 eigen functions and energy eigenvalues are usually achieved numerically by the program code *NPBOS* [8]. Then, the result of eigenvectors can be calculated transition rates and related properties using the program code *NPBTRN* [8]. The relationship is between the parameters of Equation (2).

2.3. Configuration Mixing in Interacting Boson Model-2 (IBM-2 CM)

Configuration mixing can be treated in the IBM-2 using a technique developed by Duval and Barrett [9]. Separate IBM-2 calculations are achieved for the two configurations and the results are then mixed this is done by the interaction configuration mixing can be remedied in the IBM-2 by a technique method developed by Duval and Barrett [9]. Separate IBM-2 calculations are done for the two configurations and the results are then mixed using the interaction

$$V_{mix} = \alpha (s_{\pi}^+ s_{\pi}^+ + s_{\pi}^- s_{\pi}^-)^{(0)} + \beta (d_{\pi}^+ d_{\pi}^+ + d_{\pi}^- d_{\pi}^-)^{(0)} \quad (10)$$

where the intruder configuration is assumed to involve the proton shell. There are three parameters in the mixing calculation, the mixing strengths α and β in Equation (23), and the pair excitation energy, Δ , which gives the relative energies of the two unperturbed configurations.

The total mixing Hamiltonian is then given by

$$H_{mix} = H_1 + H_2 + V_{mix} \quad (11)$$

where H_1 (H_2) is the IBM-2 Hamiltonian for the first (second) configuration, as given by Equation (11), and an amount ϵ_1 has been added to the energies of the second configuration.

The mixed wave functions are used to calculate $B(E2)$ values of observed transitions and quadrupole moments. The $E2$ transition operator is given by [9]:

$$T(E2) = e_2 [(Q_{2\pi} + Q_{2\nu}) + (e_4/e_2)(Q_{4\pi} + Q_{4\nu})] \quad (12)$$

where Q_ρ were defined in Equation (3) and e_j and $e_{\rho j}$ ($j=0,2$) are adjustable parameters. The suffixes 2 and 4 refer to the normal and the intruder configurations respectively.

For simplicity, $e_{2\pi} = e_{2\nu} = e_2$ and $e_{4\pi} = e_{4\nu} = e_4$.

For a mathematical simplicity, the neutron boson and proton boson effective charges are often taken to be equal, and the parameters χ_π and χ_ν (in intruder configuration) are taken to be the same as the Hamiltonian parameters χ_π and χ_ν respectively. The $T(E2)$ transition operator can then be written as:

$$T(E2) = e_2 (Q_\pi + Q_\nu) \quad (13)$$

When two configurations are present, this operator becomes [9]:

$$T(E2) = \sum_{i\rho} e_{i\rho} Q_{i\rho} \quad (14)$$

where $\rho = \pi$ or ν and i denotes the configuration. The effective charges of the two configurations are not the same, in general.

3. Results and Discussion

3.1. Choice of Parameters

The normal configuration for platinum isotopes involves $N_\pi = 2$ (sometimes denoted as 2π , two proton boson holes), counting from the $Z = 82$ closed shell. The neutron configuration for $^{196}_{78}\text{Pt}_{118}$ for example, is $N_\nu = 2$ (four neutron boson holes), counting from the $N = 126$ closed shell. The vibrational spectra can be calculated by diagonalizing the IBM-2 Hamiltonian, (Equation (2)), in the space of two proton and N_ν neutron s and d bosons. In order to describe the rotational states, an alternative configuration must be specified and a separate set of IBM-2 calculations made, based on that configuration. The alternate configuration used for the $^{186-200}\text{Pt}$ isotopes involves a two-particle-four-hole excitation in the shell model proton space [1] [9] [10]. This corresponds to two proton boson particles and two proton boson hole in the IBM-2 space. For simplicity, the proton boson particles and hole are treated equivalently, even though

the underlying fermion pair degrees of freedom originate in different major shells.

The IBM-2 calculations have been done in model spaces with $N_\pi = 2(2\pi)$ and $N_\pi = 4(4\pi)$ to describe the vibrational and rotational states, respectively, the two calculations are combined using Equation (11).

In the phenomenological calculations the parameters appearing in the Hamiltonian ((Equation (2) and Equation (11)) in two configurations may in general depend both on proton (N_π) and neutron (N_ν) boson number. Guided by the microscopic theory as we have assumed that only $\varepsilon^{2\pi}, \varepsilon^{4\pi}$ and $\kappa^{2\pi}, \kappa^{4\pi}$ depend on N_π ($2\pi, 4\pi$) and N_ν *i.e.*, $\varepsilon(N_\pi, N_\nu)$, $\kappa(N_\pi, N_\nu)$, while χ_ν depends only on N_ν and χ_π on N_π , *i.e.*, $\chi_\nu(N_\nu)$, $\chi_\pi(N_\pi)$. Thus a set of isotopes, (constant N_ν) have the same value of χ_ν , while a set of isotones, (constant N_π), have the same value of χ_π . This parameterization allows one to correlate a large number of experimental data. Similarly, when a proton-proton $V_{\pi\pi}$ and neutron-neutron $V_{\nu\nu}$, interaction is added, the coefficients C_L are taken as $C_{L\pi}(N_\pi)$ and $C_{L\nu}(N_\nu)$, *i.e.* the proton boson interaction will only depend on N_π , and the neutron boson on N_ν . Since there is no information on the location of the states with mixed neutron-proton symmetry we kept the coefficients appearing in the Majorana force $M_{\pi\nu}$ (see **Table 1** and **Table 2**).

The values of the parameters used for the present calculations are given in **Table 1**. The value of the parameter (boson energy) ε for the $N_\pi = 4$ configuration, $\varepsilon^{4\pi}$, is constant for all isotopes and the values of ε for the $N_\pi = 2$ configuration, $\varepsilon^{2\pi}$, are nearly constant. The quadrupole-quadrupole interaction strengths κ trends for (2π) both configurations follow the microscopic predictions [9].

The values of the parameter χ_ν used for Pt isotopes are the microscopic predictions from Bijker *et al.*, 1980 [11]. They were reported only for neutron number $102 \leq N \leq 122$. The values of χ_ν for Pt isotopes were determined by extrapolating the microscopic trend to larger neutron number. This was done in

Table 1. IBM-2 Hamiltonian parameters for the Pt isotopes (normal configuration $N_\pi = 2$). All energies are in MeV; $\chi_\pi^{2\pi}$ and $\chi_\nu^{2\pi}$ are dimensionless.

Nuclei	$\varepsilon^{2\pi}$	$\kappa^{2\pi}$	$\chi_\nu^{2\pi}$	$\chi_\pi^{2\pi}$	$C_{0\nu}^{2\pi}$	$C_{2\nu}^{2\pi}$	$C_{4\nu}^{2\nu}$
¹⁸⁶Pt	0.610	-0.148	-0.52	-0.810	-0.250	-0.160	0.00
¹⁸⁸Pt	0.580	-0.161	0.00	-0.810	-0.250	-0.160	0.00
¹⁹⁰Pt	0.580	-0.178	0.46	-0.810	0.00	-0.127	0.00
¹⁹²Pt	0.580	-0.180	0.81	-0.810	0.450	-0.09	0.00
¹⁹⁴Pt	0.580	-0.180	0.95	-0.810	0.500	0.00	0.00
¹⁹⁶Pt	0.580	-0.180	1.05	-0.810	0.560	0.002	0.00
¹⁹⁸Pt	0.580	-0.180	1.07	-0.810	0.750	0.0018	0.00
²⁰⁰Pt	0.590	-0.180	1.08	-0.810	0.800	0.001	0.00

$$\xi_1 = \xi_3 = 0.11, \quad \xi_2 = 0.0, \quad C_{0\pi}^{2\pi} = C_{2\pi}^{2\pi} = C_{4\pi}^{2\pi} = 0.00.$$

Table 2. IBM-2 Hamiltonian parameters for the Pt isotopes (normal configuration $N_\pi = 4$). (2P-2h), All energies are in MeV; $\chi_\pi^{4\pi}$ and $\chi_\nu^{4\pi}$ $\chi_\nu^{2\pi}$ are dimensionless.

Nuclei	$\epsilon^{4\pi}$	$\kappa^{4\pi}$	$\chi_\nu^{4\pi}$	$\chi_\pi^{4\pi}$	$C_{0\nu}^{4\pi}$	$C_{2\nu}^{4\pi}$	$C_{4\nu}^{4\pi}$
¹⁸⁶ Pt	0.550	-0.155	1.080	-1.10	-0.250	-0.160	0.00
¹⁸⁸ Pt	0.550	-0.150	1.070	-1.10	-0.250	-0.160	0.00
¹⁹⁰ Pt	0.550	-0.145	1.050	-1.10	0.00	-0.127	0.00
¹⁹² Pt	0.550	-0.140	0.950	-1.10	0.450	-0.09	0.00
¹⁹⁴ Pt	0.550	-0.135	0.811	-1.10	0.500	0.00	0.00
¹⁹⁶ Pt	0.550	-0.130	0.461	-1.10	0.560	0.002	0.00
¹⁹⁸ Pt	0.550	-0.125	0.00	-1.10	0.750	0.0018	0.00
²⁰⁰ Pt	0.550	-0.120	-0.520	-1.10	0.800	0.001	0.00

$$\xi_1 = \xi_3 = -0.11, \quad \xi_2 = 0.0, \quad C_{0\pi}^{4\pi} = C_{2\pi}^{4\pi} = C_{4\pi}^{4\pi} = 0.00.$$

a manner consistent with the phenomenologically determined χ_ν values which were used for the neighboring platinum and osmium isotones [11]. The same values of χ_ν are used for both configurations.

The emphasis in this work is on describing overall trends with constant or smoothly varying parameters of the Hamiltonian (Equations. (2), (11)), rather than obtaining the best possible fit to the experimental data for each nucleus. This is done in an effort to find a set of IBM-2 Hamiltonian parameters which is appropriate for the entire isotopic chain.

The normal configuration for platinum involves $N_\pi = 2$ (sometimes (2π) denoted as (2π) , two proton boson hole), counting from the $Z = 82$ closed shell. The neutron configuration for ¹⁹⁶Pt for example, is $N_\nu = 4$ (four neutron boson holes), counting from the $N = 126$ closed shell. The vibrational spectra can be calculated by diagonalizing the IBM-2 Hamiltonian, Equation (2), in the space of one proton and $N_\nu = 4$ neutron s and d bosons. In order to describe the rotational states, an alternative configuration must be specified and a separate set of IBM-2 calculations made, based on that configuration. The alternate configuration used for the Pt isotopes involves a four-particle-four-hole excitation in the shell model proton space. This corresponds to two proton boson particles and two proton boson hole in the IBM-2 space. The two configurations are depicted schematically energy levels in **Table 2**. For simplicity, the proton boson particles and hole are treated equivalently, even though the underlying fermion pair degrees of freedom originate in different major shells.

3.2. Energy Spectra

The configuration mixing calculations are done using the computer code NPMIX [10], which calculates the energy eigenvalues and eigenfunctions. The computer codes NPBEMX and BEMIX [10] are subsequently used to calculate matrix elements for transition rates and other properties.

The mixing between the two configurations (2π) and (4π) is apparent in the

experimental data shown in **Figures 1-3**. Consider, for example, the 4_1^+ , 0_2^+ and 4_2^+ , 0_2^+ states in $^{186-196}\text{Pt}$ isotopes. In the latter, the vibrational (2) state is lower in energy than the rotational (4π) 4_1^+ and 4_2^+ states. The states are close in energy and mutually repel, with the vibrational 4^+ state being lower in energy and the rotational 4^+ state higher in energy than would be expected without configuration mixing.

In the second configuration (4π) configuration interaction parameters $C_{2\nu}^{2\pi} = C_{2\nu}^{4\pi}$, $C_{0\nu}^{2\pi} = C_{0\nu}^{4\pi} = 0$ and $C_{4\nu}^{2\pi} = C_{4\nu}^{4\pi} = 0$ for all isotopes. For most nuclei, the spectral structure comes primarily from the pairing and quadrupole-quadrupole terms in the IBM-2 Hamiltonian, Equation (2), and the $V_{\nu\nu}$ term is relatively unimportant. Since there is only two proton boson to interact through $Q.Q$, the residual interaction among the neutron bosons $V_{\nu\nu}$ interaction is comparatively large.

From **Figure 1**, in the normal configuration the ground state band the moment of inertia increases, beta band ($0_2^+, 2_2^+, 4_2^+$) is pushed up, where the state 0_2^+ becomes bandhead (a member of γ -band) of $K^\pi = 0^+$. In $^{194-200}\text{Pt}$ isotopes, only five 0_2^+ states are observed around 1.25 MeV is produced, some of these states do not have a $SU(6)_\pi \otimes SU(6)_\nu$ symmetry.

In the IBM-2-CM approach, the lightest Pt isotopes, are deformed ($^{172-178}\text{Pt}$). In ^{186}Pt isotope a prolate shape and a γ -soft minimum coexist, but a well-deformed prolate minimum quickly develops in ^{180}Pt isotope, becoming the most pronounced prolate minimum at the mid-shell, *i.e.*, in ^{182}Pt isotope with the prolate shape remaining well pronounced up to ^{186}Pt isotope. Moving towards heavier mass Pt isotopes, γ -flat energy surfaces start to develop. For ^{188}Pt isotope, a much extended energy surface develops in the γ -direction (deviation from symmetry axis), becoming completely γ -unstable when reaching $^{190-196}\text{Pt}$ isotopes, the $^{198-200}\text{Pt}$ isotopes tends to spherical shape because approach to magic number.

We show in **Figure 1** the evolution of the energy of the ground state band (2_1^+ , 4_1^+ , 6_1^+ , and 8_1^+) increases toward the middle of the major shell with the number of the valence neutrons and remain almost constant for $180 \leq A \leq 186$ isotopes. Although these tendencies are well reproduced, the rotational properties or rotational features are somewhat enhanced in the calculated levels for $180 \leq A \leq 184$, which are slightly agreement in energy with the experimental data. From both results, the IBM-2 CM and the experimental data, we observed the fingerprints for structural evolution with a jump between ^{186}Pt and ^{188}Pt isotopes, also the shape of the nucleus changed from prolate (^{186}Pt isotope) to oblate (^{188}Pt isotope) deformation, this case is called phase transition. The ^{188}Pt isotope takes the oblate shape because the yrast states gradually departure to neutron closed shell.

One can also find signatures for a shape or phase transition in the systematics of the quasi- β -band levels shown in **Figure 2**. In $^{180-186}\text{Pt}$ isotopes configuration mixing, the 0_2^+ bandhead and the state 2_3^+ nearly constant in both IBM-2 data and experimental data. The two levels are pushed up rather significantly from

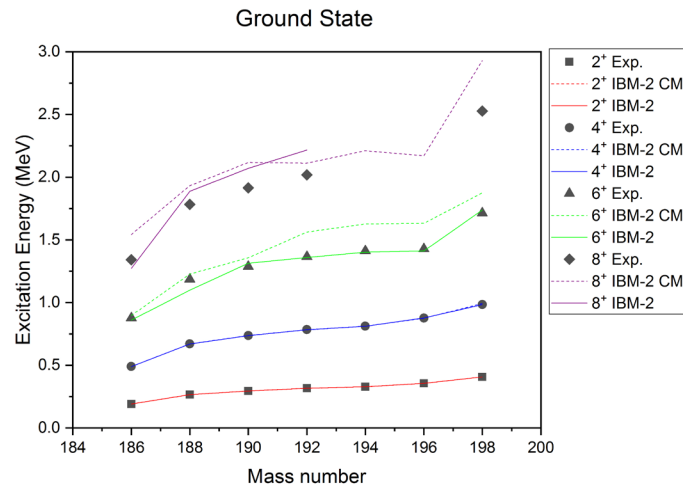


Figure 1. Ground state band in Pt isotopes. Experimental data are taken from Refs. [12]-[21].

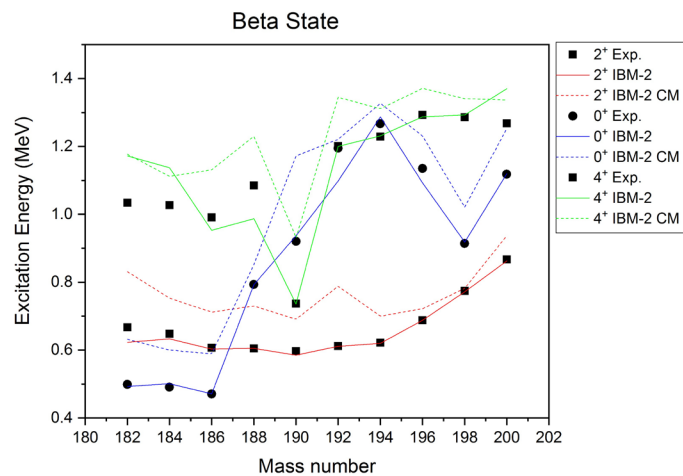


Figure 2. Beta state band in Pt isotopes. Experimental data are taken from Refs. [12]-[21].

^{186}Pt isotope to ^{188}Pt isotope. Consistently with the systematics in the ground-state band. The calculated 0_2^+ and 2_3^+ states are higher than, but still follow, the experimental data. The 3_1^+ state lies close to the 4_2^+ state. However, this calculation predicts that this trend persists even for $^{188-198}\text{Pt}$ isotopes, whereas the energy spacing between the experimental 3_1^+ and 4_2^+ states for these isotopes is larger. Similar deviation occurs for high spin states. This means that our calculations suggest the feature characteristic of the O (6) symmetry (γ -unstable).

The γ -band states shown in **Figure 3**, we can see the good agreement between IBM-2 calculations and experimental data for $180 \leq A \leq 188$, where the 3_1^+ , 4_3^+ and 2_3^+ states are energetically closed.

The high spin states for example., 6_2^+ , 3_2^+ , 4_4^+ , states, we observe some deviation occurs for these states. This means that our calculations suggest the feature characteristic of the O (6) symmetry, where the staggering occurs as high spin beta band states such as 2_3^+ , 4_3^+ , 6_2^+ , ..., etc., However, the experimental

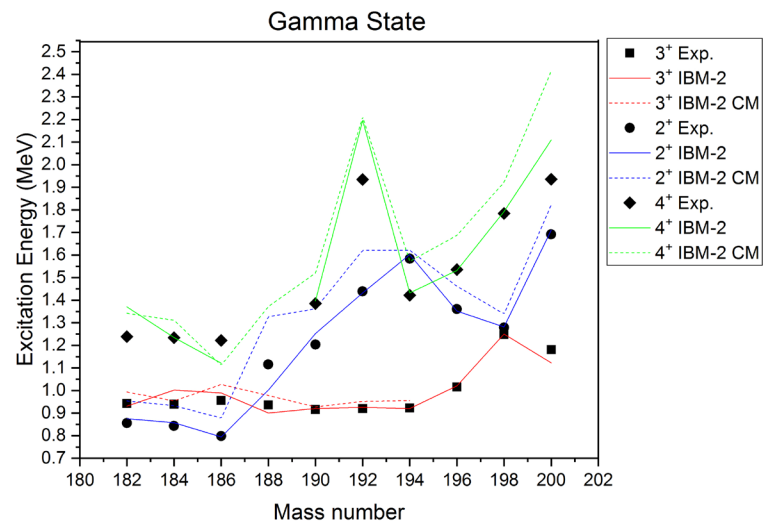


Figure 3. Gamma state band in Pt isotopes. Experimental data are taken from Refs. [12]-[21].

levels are lying more regularly, particularly for $188 \leq A \leq 196$, and thus appear to be in between the $O(6)$ symmetry and triaxial rotor symmetry. The deviation of the γ -band structure seems to be nothing more than a consequence of an algebraic nature of the IBM-2, and indeed has also been found in existing phenomenologically calculations of IBM-2.

A characteristic feature of the γ -unstable limit of the IBM-2 is a bunching of γ -band states according to 2^+ , $(3^+, 4^+)$, $(5^+, 6^+)$, ..., that is, 3^+ and 4^+ are close in energy, etc. This even-even staggering is observed in certain $O(6)$ nuclei but not in all and in some it is, in fact, replaced by the opposite bunching $(2^+, 3^+)$, $(4^+, 5^+)$, ... which is typical of a rigid triaxial rotor [22]. From these qualitative observations it is clear that the even-even γ -band staggering is governed by the γ -degree of freedom (*i.e.*, triaxiality) as it changes character in the transition from a γ -soft vibrator to a rigid triaxial rotor. The energy surface becomes rather flat, evolving towards a spherical minimum at ^{200}Pt and beyond. The possibility of triaxial deformation was not considered (tends to spherical shape).

3.3. Electric Transition Probability $B(E2)$

Calculations of electromagnetic properties give us a good test of the nuclear models prediction. The electromagnetic matrix elements between eigenstates were calculated using the programs *NPBTRN* and *NPBEMX* for IBM-2 and IBM-2 CM model respectively.

From Equations (7), we note that an E2 transition mainly depends on identifying proton and neutron bosons effective charges e_π and e_ν . These isotopes lying in region between SU(3) limit (rotational nuclei) and $O(6)$ limit (γ -soft nuclei), therefore, the relationship between (e_π, e_ν) and the reduced transition probability $B(E2)$ for rotational limit SU(3) and γ -soft limit $O(6)$ is given in the form [23] [24]:

For O(6) limit

$$B(E2; 2_1^+ \rightarrow 0_1^+) = \frac{(N+4)(e_\pi N_\pi + e_\nu N_\nu)}{5N} \quad (16)$$

For SU(3) limit

$$B(E2; 2_1^+ \rightarrow 0_1^+) = \frac{(2N+3)(e_\pi N_\pi + e_\nu N_\nu)}{5N} \quad (17)$$

where $B(E2; 2_1^+ \rightarrow 0_1^+)$ is the experimental reduced transition probability from the first excited states (2_1^+) to the ground state (0_1^+), N is the total number of bosons.

The relations (26) and (27), was used to estimate the effective boson charges for proton and neutron bosons (e_π , e_ν). In these calculations, we use the following criteria to determine the effective charges. $e_\pi = 0.173$ e.b is a constant throughout the whole isotopic chain and the e_ν changes with neutron number. This is true if the neutron (proton) interaction does not depend on the proton (neutron) configurations. The values of e_π and e_ν are determined by fitting to the five $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 2_2^+ \rightarrow 2_1^+)$ in ^{194}Pt and ^{196}Pt isotopes from the first configuration ($N_\pi = 2$). They are given in **Table 3**.

For the configuration mixing, the effective charges for bosons is evaluated in the same manner in normal configuration, For simplicity, $e_{2\pi} = e_{2\nu} = e_2$ and $e_{4\pi} = e_{4\nu} = e_4$. The ratio of the two quadrupole effective charges, e_4/e_2 in Equation (25) is taken to be the same quadrupole interaction strengths, as the ratio of the corresponding $\kappa^{4\pi}/\kappa^{2\pi}$, for each isotope (see Equation (12) and **Table 1**, **Table 2**). This is reasonable, since the effective charge and the strength of the quadrupole interaction are both proportional to the mean square proton radius. Thus, the only new parameter needed to determine the reduced transition rates is e_2 . The value for the Pt isotopes was determined by fitting of $B(E2; 2_1^+ \rightarrow 0_1^+)$ in ^{196}Pt isotope.

It is well known that absolute gamma ray transition probabilities offer the possibility of a very sensitive test of nuclear models and the majority of the information on the nature of the ground state has come from studies of the energy level spacing. The transition probability values of the excited state in the ground state band constitute another source of nuclear information. Yrast levels of even-even nuclei ($J^\pi = 2^+, 4^+, 6^+, \dots$) usually decay by $E2$ transition to the lower lying yrast level with $J_f^+ = J_i^+ - 2$.

In **Table 4** we show the $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 4_1^+ \rightarrow 2_1^+)$ values, which are of the same order of magnitude and display a typical decrease towards the middle of the shell.

As a consequence of possible $M1$ admixture the $B(E2; 3_1^+ \rightarrow 2_1^+)$ quantity is rather difficult to measure. There is no experimental data to compare the values

Table 3. Effective charge used in E2 transition calculations ($e_\pi = 0.173$ e.b).

Isotopes	^{180}Pt	^{182}Pt	^{184}Pt	^{186}Pt	^{188}Pt	^{190}Pt	^{192}Pt	^{194}Pt	^{196}Pt	^{198}Pt	^{200}Pt
e_ν (eb)	0.1243	0.126	0.1286	0.128	0.129	0.1295	0.132	0.138	0.142	0.143	0.144

Table4. Electric transition probability $B(E2; J_i^+ \rightarrow J_f^+)$ in e^2b^2 units.

Nuclei	$J_i^+ \rightarrow J_f^+$	Exp.	IBM-2	IBM-2 CM
Pt- 186	$2_1^+ \rightarrow 0_1^+$	0.59 [14]	0.572	0.583
	$2_2^+ \rightarrow 0_1^+$	-	0.032	0.0431
	$2_2^+ \rightarrow 2_1^+$	-	0.112	0.143
	$4_1^+ \rightarrow 2_1^+$	-	0.140	0.173
Pt- 188	$2_1^+ \rightarrow 0_1^+$	0.520 (94) [25]	0.522	0.522
	$2_2^+ \rightarrow 0_1^+$	-	0.0021	0.027
	$2_2^+ \rightarrow 2_1^+$	-	0.698	0.0963
	$4_1^+ \rightarrow 2_1^+$	-	0.721	0.822
Pt- 190	$2_1^+ \rightarrow 0_1^+$	0.350 (44) [25]	0.350	0.355
	$2_2^+ \rightarrow 0_1^+$	-	0.018	0.021
	$2_2^+ \rightarrow 2_1^+$	-	0.440	0.472
	$4_1^+ \rightarrow 2_1^+$	-	0.634	0.827
Pt- 192	$3_1^+ \rightarrow 2_1^+$	1.800 (13) [26]	1.972	1.873
	$2_1^+ \rightarrow 0_1^+$	0.382 (12)[27]	0.382	0.389
	$2_2^+ \rightarrow 0_1^+$	0.0044 (5) [27]	0.006	0.0072
	$2_2^+ \rightarrow 2_1^+$	0.460 (5) [27]	0.481	0.487
	$4_1^+ \rightarrow 2_1^+$	0.580 (30) [27]	0.597	0.544
	$3_1^+ \rightarrow 2_1^+$	0760 (15) [25]	0.852	0.776
	$2_1^+ \rightarrow 0_1^+$	0.324 (16) [28]	0.324	0.327
	$4_1^+ \rightarrow 2_1^+$	0.470 (3) [28]	0.476	0.467
Pt- 194	$6_1^+ \rightarrow 4_1^+$	0.320 (8) [28]	0.331	0.351
	$8_1^+ \rightarrow 6_1^+$	0.360 (11) [28]	0.421	0.339
	$4_2^+ \rightarrow 4_2^+$	0.280 (12) [28]	0.310	0.263
	$6_2^+ \rightarrow 4_2^+$	0.280 (12) [28]	0.279	0.331
	$2_2^+ \rightarrow 0_1^+$	0.0014 (2) [28]	0.0019	0.0175
	$2_2^+ \rightarrow 2_1^+$	0.423 (15) [29]	0.520	0.631
	$4_2^+ \rightarrow 4_1^+$	0.870 (43) [29]	0.887	0.883
	$4_2^+ \rightarrow 2_1^+$	0.01 (5) [29]	0.027	0.0289
Pt- 196	$2_1^+ \rightarrow 0_1^+$	0.280(8) [25]	0.280	0.281
	$2_2^+ \rightarrow 0_1^+$	0.318 (23) [11]	0.320	0.325
	$4_1^+ \rightarrow 2_1^+$	0.409 (22) [11]	0.412	0.411
	$4_2^+ \rightarrow 2_1^+$	0.003 (1) [30]	0.006	0.0081
	$4_2^+ \rightarrow 4_1^+$	0.139 (97) [11]	0.192	0.187
	$0_2^+ \rightarrow 2_2^+$	0.142 (77) [25]	0.144	0.146
	$0_2^+ \rightarrow 2_1^+$	0.022 (10) [11]	0.023	0.042
	$4_2^+ \rightarrow 2_2^+$	0.177 (25) [30]	0.182	0.207
	$6_1^+ \rightarrow 4_1^+$	0.421 (116) [30]	0.427	0.521
	$2_2^+ \rightarrow 0_1^+$	0.318 (23) [30]	0.337	0.355
	$3_1^+ \rightarrow 2_1^+$	0.130 (11) [30]	0.136	0.147

Continued

Pt- 198	$2_1^+ \rightarrow 0_1^+$	0.212 (10) [25]	0.212	0.200
	$2_2^+ \rightarrow 0_1^+$	0.0003 (1) [30]	0.00076	0.0005
	$2_2^+ \rightarrow 2_1^+$	0.262 (38) [30]	0.289	0.311
	$4_1^+ \rightarrow 2_1^+$	0.2700 (23) [30]	0.297	0.297
Pt- 200	$2_1^+ \rightarrow 0_1^+$	-	0.200	0.202
	$2_2^+ \rightarrow 0_1^+$	-	0.00089	0.0008
	$2_2^+ \rightarrow 2_1^+$	-	0.275	0.245
	$4_1^+ \rightarrow 2_1^+$	-	0.281	0.310

of IBM-2. For ^{190}Pt isotope, we give the agreement with experimental value, from these values seems to decrease for $^{190-196}\text{Pt}$ isotopes.

In **Table 4**, we show $B(E2; 2_2^+ \rightarrow 0_1^+)$ values. Experimentally the results are radically different for the Pt isotopes. In some Pt isotopes the value seems to decrease towards the middle of the shell, whereas in another ^{196}Pt isotope is increased. Our calculations could not reproduce these contradictory features simultaneously. The results for $B(E2; 2_2^+ \rightarrow 0_1^+)$ values are shown in **Table 4**. This quantity is rather small since this transition is forbidden in all three symmetries of the IBM-2.

As a consequence of possible $M1$ admixture the $B(E2; 2_2^+ \rightarrow 2_1^+)$ quantity is rather difficult to measure. For some Pt isotopes, we give the different, conflicting experimental results and we see that no general feature be derived from them, from these values seems to decrease for Pt isotopes.

In ^{194}Pt isotope, there is a good agreement between experimental and calculated $B(E2; 0_2^+ \rightarrow 2_1^+)$ value. This confirms our earlier statement about the nature of the lowest 0_2^+ state in this isotope. Other transitions such as $B(E2; 6_1^+ \rightarrow 4_1^+)$, $B(E2; 8_1^+ \rightarrow 6_1^+)$ and $B(E2; 4_2^+ \rightarrow 2_2^+)$ are small values in sometimes because these transitions between different bands (cross over transitions) and the selection rules which determine these transition.

In **Table 5**, the quadrupole moment, qualitatively, for first excited state $Q(2_1^+)$, second excited states $Q(2_2^+)$ and $Q(4_1^+)$, for the ground state band, the positive $Q(2_1^+)$ and $Q(4_1^+)$ mean a negative intrinsic quadrupole moment (for ground state Q_0). For the beta band, a negative $Q(2_1^+)$ means a negative intrinsic quadrupole moment Q_0 . The negative Q_0 implies that the nucleus has an oblate shape; the positive intrinsic quadrupole moment Q_0 means that the nucleus has a prolate shape. The overall the IBM-2 and IBM-2 CM results is a good agreement with the experimental data.

4. Concluding Remarks

In the present work, we have analyzed the spectroscopic properties for the Pt isotopic chain $^{182-200}\text{Pt}$ in terms of the interacting boson model-2 (IBM-2) and interacting boson model configuration mixing (IBM-2 CM). Calculations of energy levels for even-even $^{182-200}\text{Pt}$ isotopes were performed with the whole

Table 5. Quadrupole moments $Q(J_i^+)$ in eb Units for Pt isotopes.

Nuclei	J_i^+	Exp.	IBM-2	IBM-2 CM
Pt-186	2_1^+	-	0.096	0.091
	2_2^+	-	-0.082	-0.065
	4_1^+	-	0.088	0.079
Pt-188	2_1^+	-	0.081	0.087
	2_2^+	-	-0.072	-0.079
	4_1^+	-	0.182	0.188
Pt-190	2_1^+	-	0.556	0.621
	2_2^+	-	-0.522	-0.529
	4_1^+	-	0.571	0.627
Pt-192	2_1^+	0.55 (21) [17]	0.53	0.580
	2_2^+	-	-0.62	-0.68
	4_1^+	-	0.58	0.61
Pt-194	2_1^+	[18] 0.63 (6)	0.650	0.697
	2_2^+	[18] -0.5 (5)	-0.52	-0.62
	4_1^+	[18] 0.5 (10)	0.503	0.58
Pt-196	2_1^+	[19] 0.62 (8)	0.64	0.71
	2_2^+	[19] -0.39 (16)	-0.33	-0.421
	4_1^+	[19] 1.03 (12)	1.007	1.018
Pt-198	2_1^+	[20] 0.42 (12)	0.44	0.490
	2_2^+	-	-0.25	-0.281
	4_1^+	-	0.38	0.422

Hamiltonian (Equation (2) and Equation (11)) using *NPBOS* computer code. $^{182-200}\text{Pt}$ nuclei ($Z = 78$) have two proton bosons ($N_\pi = 2$ normal configuration) and the neutron boson varied from $N_\nu = 13$ for ^{182}Pt isotope to $N_\nu = 4$ for ^{200}Pt isotope.

The energy levels in Pt isotopes especially (ground state band) are a good agreement with the experimental data. In this study, we see that the excitation energy of the state 3_1^+ in $^{186-200}\text{Pt}$ isotopes (heavier Pt isotopes) is predicted sometimes too high in comparison with experimental data.

In the IBM-2-CM approach, the lightest Pt isotopes are deformed ($^{172-178}\text{Pt}$). In ^{186}Pt isotope a prolate shape and a γ -soft minimum coexist, but a well-deformed prolate minimum quickly develops in ^{180}Pt isotope, becoming the most pronounced prolate minimum at the mid-shell, *i.e.*, in ^{182}Pt isotope with the prolate shape remaining well pronounced up to ^{186}Pt isotope. Moving towards heavier mass Pt isotopes, γ -flat energy surfaces start to develop. The $^{198-200}\text{Pt}$ isotopes tend to spherical shape because of the approach to magic number.

Concerning electric transition probability $B(E2)$ values, we find that in all calculations the overall trend is reproduced reasonably very well in some transitions, but notice some discrepancies present case of the decay of the 0_2^+ states

in heavier Pt isotopes, in general, better than the values calculated by Bijker *et al.*, [11].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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