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Type-2 Fuzzy Point

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Abstract

The important role of the concept of type-2 fuzzy point in the formation of type-2 fuzzy open sets such as type-2 fuzzy $\tilde{\tilde{\delta}}$-closed set this important role make the main objective of this paper is to introduce the concept type-2 fuzzy point of type-2 fuzzy set an important definitions in the composition of this concept as $\tilde{\tilde{\alpha}}$-plane and the support of type-2 fuzzy set after preliminaries we present the definition of type-1 fuzzy set (fuzzy set) and fuzzy point and the special concepts that helped to configure them as support.

Keywords

Type-2 Fuzzy Set, $\tilde{\tilde{\alpha}}$-Plane, Fuzzy Point, Type-1 Fuzzy Set, Type-2 Fuzzy Point

1. Introduction

The main concept of a type-1 fuzzy set (fuzzy set) is introduced by Zadeh in 1965 [1]. In [2], he introduced the concept, namely, type-2 fuzzy set in 1973 to classify a fuzzy set of type-1 fuzzy set when dealing with type-2 fuzzy set. Many of the scientists then introduced the concepts of type-2 fuzzy set, in 1976 introducing Mizumoto and Tanaka some properties of type-2 fuzzy set [3] and how to find the operations of type-2 fuzzy sets using the extension principle, it is the same rule that it has established Zadeh in 1973. Scientists then introduced several important concepts of the type-2 fuzzy set that correspond to the main concepts of the type-1 fuzzy set. In [4], Liu introduced the $\tilde{\tilde{\alpha}}$-plane concept of type-2 fuzzy set corresponding to the concept $\tilde{\alpha}$-level of the type-1 fuzzy set and the concept of support type-2 fuzzy set was developed by Mendel [5] corresponding to the concept of support type-1 fuzzy set. Mendel then introduced a very important concept interval type-2 fuzzy sets [5]. The study of type-2 fuzzy sets was expanded to include the topology study of type-1 fuzzy set with the introduction of fuzzy topology by Chang in 1968 [6]. Zhang in 2013 offered the interval type-2 fuzzy topological space [7] and then introduced Mohammad and Munir General type-2 fuzzy topological space [8]. The type-2 fuzzy open sets such as type-2 fuzzy $\tilde{\tilde{\delta}}$-preopen set and type-2 fuzzy $\tilde{\tilde{e}}$-open set, we cannot configure its own definition only after introduce type-2 fuzzy point concept that belongs to the general fuzzy type-2 topological space.

2. Preliminaries

This section paves the way to introducing the concept of type-2 fuzzy point by providing definitions of special important concept type-1 fuzzy set and type-2 fuzzy set after submitting the special definition to the clusters.

2.1. Definition [1]

If X is a collection of objects with generic element χ, then a fuzzy subset $\tilde{A}$ in X is characterized by a membership function; $\tilde{\aleph}_{\tilde{A}}: X \to I$, where I is the closed unit interval [0, 1], then we write a fuzzy set $\tilde{A}$ by the set of points:
$\tilde{A} = \left\{\left(\chi, \tilde{\aleph}_{\tilde{A}}(\chi)\right) \mid \chi \in X, 0 \le \tilde{\aleph}_{\tilde{A}}(\chi) \le 1\right\}$.

2.2. Definition [9]

The support of a type-1 fuzzy set $\tilde{A}$ (denoted by $S\left(\tilde{A}\right)$) which is the crisp set of all $\chi \in X$, $\left\{\chi : \tilde{\aleph}_{\tilde{A}}(\chi) > 0\right\}$.

In order to represent an element of a type-1fuzzy set $\tilde{A}$, we provide a concept in type-1 fuzzy set theory that is a special case of $\tilde{\alpha}$-level sets, which is called fuzzy points used for inclusion of elements to fuzzy sets.

2.3. Definition [10]

A type-1 fuzzy point $\tilde{\wp}$ in a set X is also a type-1 fuzzy set with membership function:

$$\tilde{\aleph}_{\tilde{\wp}}(\chi) = \begin{cases} \kappa & \text{for } \chi = \gamma \\ 0 & \text{for } \chi \ne \gamma \end{cases},$$

where $\chi \in X$ and $0 < \kappa < 1$, γ is called the support of $\tilde{\wp}$ and κ the value of $\tilde{\wp}$. We denote this type-1 fuzzy point by $\tilde{\wp}$. Two fuzzy points $\tilde{\wp}_{\chi_\kappa}$ and $\tilde{\wp}_{\gamma_\kappa}$ are said to be distinct if and only if $\chi \ne \gamma$. A type-1 fuzzy point $\tilde{\wp}$ is said to belong to a type-1 fuzzy subset $\tilde{A}$ in X, denoted by $\tilde{\wp} \in \tilde{A}$ if and only if $\kappa \le \tilde{\aleph}_{\tilde{A}}(\chi)$.

2.4. Definition [2] [5]

Let X be a finite and non-empty set , which is referred to as the universe a type-2 fuzzy set, denoted by $\tilde{\tilde{A}}$ is characterized by a type-2 memberships function $\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi, \tilde{u})$, as

$$\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}: X \times I \to I^{J_\chi}\left(J_\chi \subseteq I\right), I = [0,1],$$

where $\chi \in X$ and $\tilde{u} \in J_\chi$, that is

$\tilde{\tilde{A}} = \left\{\left((\chi,\tilde{u}),\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u})\right): \text{where } \chi \in X \text{ and } \tilde{u} \in J_{\chi} \subseteq I, \text{where } 0 \le \tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u}) \le 1\right\}$.

We can give a new wording to $\tilde{\tilde{A}}$

$$\tilde{\tilde{A}} = \sum_{\chi\in X}\sum_{\tilde{u}\in\upsilon_{\chi}} \frac{\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u})}{(\chi,\tilde{u})},$$

$$\tilde{\tilde{A}} = \sum_{\chi\in X}\sum_{\tilde{u}\in\upsilon_{\chi}} \frac{\frac{\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u})}{\tilde{u}}}{\chi},\ \upsilon_{\chi} \subseteq [0,1]$$

where $\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u}) = \tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u})$ and $\Sigma\Sigma$ denote the union in discrete sets and Σ is replaced by $\int$ is continuous universes are set. The class of all type-2 fuzzy set of $\chi \in X \ne \varnothing$ denoted by $\tilde{\tilde{\mathbb{F}}}_{T-2}(X)$.

A type-2 fuzzy set universes set [3], denoted by, such that

$$\tilde{\tilde{X}} = \sum_{\chi\in X}\sum_{\tilde{u}\in[1,1]} \frac{\frac{1}{\tilde{u}}}{\chi}.$$

A type-2 fuzzy empty set [3], denoted by

$$\tilde{\tilde{\varnothing}} = \sum_{\chi\in X}\sum_{\tilde{u}\in[0,0]} \frac{\frac{1}{\tilde{u}}}{\chi}.$$

Interval type-2 fuzzy set [5], when all the $\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u}) = 1$, for all $\chi \in X$.

The operations of type-2 fuzzy set [3], consider $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are two type-2 fuzzy sets and the membership grades of $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ respectively, we can represented by

$$\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi) = \sum_{\tilde{u}\in\upsilon_{\chi}^{\tilde{u}}} \frac{\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u})}{\tilde{u}} \text{ and } \tilde{\tilde{\aleph}}_{\tilde{\tilde{B}}}(\chi) = \sum_{\tilde{w}\in\upsilon_{\chi}^{\tilde{w}}} \frac{\tilde{\tilde{\mathcal{R}}}_{\chi}(\tilde{w})}{\tilde{w}},$$

where $\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u}), \tilde{\tilde{\mathcal{R}}}_{\chi}(\tilde{w}) \in \mathfrak{T} = [0,1]$ and.

The union of two type-2 fuzzy sets is defined as

$$\tilde{\tilde{A}} \cup \tilde{\tilde{B}} \Leftrightarrow \tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}\cup\tilde{\tilde{B}}}(\chi) = \sum_{\tilde{u}\in\upsilon_{\chi}^{\tilde{u}}}\sum_{\tilde{w}\in\upsilon_{\chi}^{\tilde{w}}} \frac{\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u}) \tilde{\wedge} \tilde{\tilde{\mathcal{R}}}_{\chi}(\tilde{w})}{\tilde{u} \tilde{\vee} \tilde{w}}.$$

The intersection of two type-2 fuzzy sets is defined as

$$\tilde{\tilde{A}} \cap \tilde{\tilde{B}} \Leftrightarrow \tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}\cap\tilde{\tilde{B}}}(\chi) = \sum_{\tilde{u}\in\upsilon_{\chi}^{\tilde{u}}}\sum_{\tilde{w}\in\upsilon_{\chi}^{\tilde{w}}} \frac{\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u}) \tilde{\wedge} \tilde{\tilde{\mathcal{R}}}_{\chi}(\tilde{w})}{\tilde{u} \tilde{\wedge} \tilde{w}}.$$

The containment type-2 fuzzy sets are defined as $\tilde{\tilde{A}} \subseteq \tilde{\tilde{B}} \Leftrightarrow \tilde{\tilde{A}} \cap \tilde{\tilde{B}} = \tilde{\tilde{A}}$.

The complement of type-2 fuzzy set defined as $\left(\neg\tilde{\tilde{A}}\right) = \sum_{\tilde{u}\in\upsilon_{\chi}^{\tilde{u}}} \frac{\tilde{\tilde{\mathcal{F}}}_{\chi}(\tilde{u})}{1-\tilde{u}}$.

A normal type-1 fuzzy set $\tilde{A}$ is one for height equals 1, otherwise it is called subnormal [9].

2.5. Definition [3]

A normal type-2 fuzzy set $\tilde{\tilde{A}}$ is one for which $\max_{\chi\in X} \tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u}) = 1$.

2.6. Definition [5]

The support of a type-2 fuzzy set $\tilde{\tilde{A}}$ denoted $S\left(\tilde{\tilde{A}}\right)$ comprises all $(\chi,\tilde{u}) \in X \times I$ such that $\tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u}) > 0$.

2.7. Definition [4]

The two damnation $\tilde{\tilde{\alpha}}$ -plane , denoted $\tilde{\tilde{A}}_{\tilde{\tilde{\alpha}}}$ is the union of all primary membership whose secondary grades are greater than or equal special value $\tilde{\tilde{\alpha}}$ that is: $\tilde{\tilde{A}}_{\tilde{\tilde{\alpha}}} = \bigcup_{\chi \in X} \left\{ (\chi,\tilde{u}) : \tilde{\tilde{\aleph}}_{\tilde{\tilde{A}}}(\chi,\tilde{u}) \ge \tilde{\tilde{\alpha}}, \tilde{\tilde{\alpha}} \in [0,1] \right\}$.

2.8. Definition [8]

Let $\tilde{\tilde{T}}$ be the collection of type-2 fuzzy sets over $X \neq \varnothing$ then $\tilde{\tilde{T}}$ is called to be general type-2 fuzzy topology on $X \neq \varnothing$.

If

1) $\tilde{\tilde{X}}, \tilde{\tilde{\varnothing}} \in \tilde{\tilde{T}}$;
2) $\tilde{\tilde{A}} \cap \tilde{\tilde{B}} \in \tilde{\tilde{T}}$;
3) $\bigcup_{i \in N} \tilde{\tilde{A}} \in \tilde{\tilde{T}}$ for $\tilde{\tilde{A}}_i \in \tilde{\tilde{T}}$.

The pair $\left(\tilde{\tilde{X}}, \tilde{\tilde{T}}\right)$ is said to general type-2 fuzzy topological space over X and the member of $\tilde{\tilde{T}}$ is said to be type-2 fuzzy $\tilde{\tilde{T}} \sim$ open sets in X and type-2 fuzzy sets are said type-2 fuzzy $\tilde{\tilde{T}} \sim$ closed sets in X, if its complement $\neg\tilde{\tilde{A}} \in \tilde{\tilde{T}}$.

We must note that all the type-2 fuzzy sets are normal type-2 fuzzy sets so as to complete the topological construction and especially check identity law ($\tilde{\tilde{A}} \cap \tilde{\tilde{\varnothing}} = \tilde{\tilde{\varnothing}}$).

3. Type-2 Fuzzy Point

The introduction of this section is the end of which the paper developed by the special case of $\tilde{\tilde{\alpha}}$ -plane, which claims type-2 fuzzy point.

3.1. Definition

A type-2 fuzzy point $\tilde{\tilde{\wp}}$ in a set X is also a type-2 fuzzy set with secondary membership function:

$$\tilde{\tilde{\aleph}}_{\tilde{\tilde{\wp}}}(\chi,\tilde{u}) = \begin{cases} \kappa & \text{for } \chi = \gamma \\ 0 & \text{for } \chi \neq \gamma \end{cases},$$

where $\chi \in X$ and $0 < \kappa < 1$, γ is called the support of $\tilde{\tilde{\wp}}$ and κ the value of $\tilde{\tilde{\wp}}$.

3.2. Example

Let $X = \{a,b\}$, $\tilde{\tilde{A}} = \{((a,0.5),0.4),((b,0.8),0)\}$ is a type-2 fuzzy point

$$S\left(\tilde{\tilde{A}}\right) = \{(a,0.5)\} \text{ and we have } \tilde{\tilde{\aleph}}_{\tilde{\tilde{\wp}}}(\chi,\tilde{u}) = \begin{cases} \kappa = 0.4 & \chi = \gamma \\ 0 & \chi \neq \gamma \end{cases}$$

$$\Rightarrow \text{that is } \tilde{\tilde{\aleph}}_{\tilde{\tilde{\wp}}}(\chi,\tilde{u}) = \tilde{\tilde{\aleph}}_{\tilde{\tilde{\wp}}}(a,0.5) = \kappa = 0.4$$
$$\tilde{\tilde{\aleph}}_{\tilde{\tilde{\wp}}}(\chi,\tilde{u}) = \tilde{\tilde{\aleph}}_{\tilde{\tilde{\wp}}}(b,0.8) = 0 .$$

We denote this a type-2 fuzzy point by $\tilde{\tilde{\wp}}$. Two a type-2 points $\tilde{\tilde{\wp}}_{\chi_\kappa}$ and $\tilde{\tilde{\wp}}_{\gamma_\lambda}$ are said to be distinct if and only if $\chi \neq \gamma$.

3.3. Serious Results

The means of determining belonging set or fuzzy point in the subject of the fuzzy set is to compare with the membership function, but in the second type-2 fuzzy set we use the containment property given by Mizumotoand Tanaka $\tilde{\tilde{A}} \subseteq \tilde{\tilde{B}} \Leftrightarrow \tilde{\tilde{A}} \cap \tilde{\tilde{B}} = \tilde{\tilde{A}}$ [3].

3.4. Example

Let

$$X = \{\chi_1, \chi_2, \chi_3\}$$
$$\tilde{\tilde{A}} = \{((\chi_1,0.7),0.8),((\chi_1,0.6),1),((\chi_2,0.4),1),((\chi_3,0.8),0.9),((\chi_3,0.5),1)\}$$
$$\tilde{\tilde{\wp}} = \{((\chi_1,0.6),0.8),((\chi_2,0.3),0.8),((\chi_3,0.5),0)\}$$
$$\tilde{\tilde{\wp}} \cap \tilde{\tilde{A}}(\chi_1) = \frac{0.8 \wedge 0.8}{0.7 \wedge 0.6} + \frac{1 \wedge 0.8}{0.6 \wedge 0.6}$$
$$\tilde{\tilde{\wp}} \cap \tilde{\tilde{A}}(\chi_1) = (0.6, \max(0.8,0.8)) = ((\chi_1,0.6),0.8)$$
$$\tilde{\tilde{\wp}} \cap \tilde{\tilde{A}}(\chi_2) = \frac{1 \wedge 0.8}{0.4 \wedge 0.3} \Rightarrow \frac{0.8}{0.3} = ((\chi_2,0.3),0.8)$$
$$\tilde{\tilde{\wp}} \cap \tilde{\tilde{A}}(\chi_3) = \frac{0.9 \wedge 0}{0.8 \wedge 0.5} + \frac{1 \wedge 0}{0.5 \wedge 0.5}$$
$$\tilde{\tilde{\wp}} \cap \tilde{\tilde{A}}(\chi_3) = (0.5, \max(0,0)) = ((\chi_3,0.5),0)$$
$$\Rightarrow \text{therefore } \tilde{\tilde{\wp}} \cap \tilde{\tilde{A}} = \tilde{\tilde{\wp}} \Leftrightarrow \tilde{\tilde{\wp}} \subseteq \tilde{\tilde{A}} .$$

4. Conclusion

The important role played by the type-2 fuzzy point in configuring continuous function after building new concepts from open type-2 fuzzy set made us offer this new concept. This concept allows for future dealings with the concept of neighborhood in general type-2 fuzzy topological space. In order for us to be able to configure such as type-2 fuzzy $\tilde{\tilde{\delta}}$-preopenset and type-2 fuzzy $\tilde{\tilde{e}}$-open set only and to examine the relationships among them.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Zadeh, L.A. (1965) Fuzzy Sets. *Information and Control*, **8**, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Zadeh, L.A. (1975) The Concept of a Linguistic Variable and Its Application to Approximate Reasoning—1. *Information Sciences*, **8**, 199-249. https://doi.org/10.1016/0020-0255(75)90036-5

[3] Mizumoto, M. and Tanaka, K. (1976) Some Properties of Fuzzy Sets of Type-2. *Information and Control*, **31**, 312-340. https://doi.org/10.1016/S0019-9958(76)80011-3

[4] Liu, F.L. (2008) An Efficient Centroid Type-Reduction Strategy for General Type-2 Fuzzy Logic System. *Information Sciences*, **178**, 2224-2236. https://doi.org/10.1016/j.ins.2007.11.014

[5] Mendel, J.M. (2001) Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. Prentice-Hall, New Jersey.

[6] Chang, C.L. (1968) Fuzzy Topological Spaces. *Journal of Mathematical Analysis and Applications*, **24**, 182-190. https://doi.org/10.1016/0022-247X(68)90057-7

[7] Zhang, Z.M. (2013) On Characterization of Generalized Interval Type-2 Fuzzy Rough Sets. *Information Sciences*, **219**, 124-150. https://doi.org/10.1016/j.ins.2012.07.013

[8] Hussan, M.S.M. and AL-Khafaji, M.A.K. (2018) General Type-2 Fuzzy Topological Spaces, *Advances in Pure Mathematics*, **8**, 771-781. https://doi.org/10.4236/apm.2018.89047

[9] Klir, G.J. and Yuan, B. (1995) Fuzzy Sets and Fuzzy Logic Theory and Application. Prentice Hall, New Jersey.

[10] Wong, C.K. (1974) Fuzzy Points and Local Properties of Fuzzy Topology. *Journal of Mathematical Analysis and Applications*, **46**, 316-328. https://doi.org/10.1016/0022-247X(74)90242-X