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Ambient Noise Tomography, Green's Function and Earthquakes

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Abstract

Green's function is well-known, among others, in the application of ambient noise tomography methodologies that may demonstrate the potential of hydrocarbon entrapment in the study area. Here it is also shown to be of key importance in identifying the fractal dimension in the unified scaling law for earthquakes as well as in studying an explicit relationship of a future strong earthquake epicenter to the average earthquake potential score. Such studies are now in progress.

Keywords

Surface Wave Tomography, Shear Velocity Model, Inversion, Green's Function, Ambient Noise, Earthquakes

1. Introduction

Seismic waves from earthquakes and/or artificial sources travel through the Earth carrying information about the Earth's subsurface structure and properties. Aki [1] and Claerbout [2] contributed a lot to the development of the technique for seismic applications. A signal at location A can be cross correlated with a signal at location B to reproduce a virtual source-receiver pair. It has been demonstrated (e.g., [3]) that this cross correlation can reproduce the surface waves of the Earth's impulse response, or the Green's function, as if triggered by a point source. By studying the dispersion relation of these surface waves between multiple pairs of stations, surface wave tomography is possible [4].

It is the scope of this work to reveal the decisive importance of Green's function in the study of earthquakes and in particular along the following two directions that are described in detail below, in Sections 2 and 3: First, the determination of the fractal dimension of the earthquake epicenters projected onto the Earth's surface, e.g. Fig. 7d of the work by Lei and Kusunose [5]. Second, the interrelation between the epicenter of a future strong earthquake and the average earthquakes potential score (EPS) maps.

Before proceeding, we emphasize that scale invariance methods such as fractal dimension, correlation dimension and multifractal spectrum have potential applications in emerging fields of science, engineering, and seismology. Numerous studies have been undergoing to study the applications in seismology, in particular, see references [6]-[24].

2. Additional Usefulness of Green's Function

2.1. Fractal Dimension of the Earthquake Epicenters Projected onto the Earth's Surface

We have recently [25] shown that an estimate of the epicenter location of a future strong earthquake can be obtained by combining a new analysis of seismicity termed natural time analysis [26] [27] [28] [29] with earthquake networks based on similar activity patterns [30] and earthquake nowcasting [31]-[38]. This is based on the construction of average EPS maps.

A simple model was developed to understand the process of self-consistently averaging EPS e.g., see Fig. 5 of Reference [25]. To keep this model simple, we assume that all EQs occur practically in the center O of circular region of radius R. After a long enough period without a large EQ, the EPS takes the value of unity for all points inside and, since we assumed that no other EQ takes place, the EPS will be zero outside it. Hence, the value of ⟨EPS⟩ at a point P lying at a distance d away from O simplifies to:

$$\langle \text{EPS} \rangle = \frac{A(d,R)}{\pi R^2} \tag{1}$$

where $A(d,R)$ is the overlapping area of the two intersecting circles, equaled:

$$A(d,R) = 2R^2 \arccos\left(\frac{d}{2R}\right) - d\sqrt{R^2 - \left(\frac{d}{2}\right)^2} \tag{2}$$

Hence, the value of "EPS" in polar coordinates (ρ, θ) with center at O equaled to:

$$f(\rho) = \frac{2}{\pi}\arccos\left(\frac{\rho}{2R}\right) - \frac{\rho}{\pi R}\sqrt{1-\left(\frac{\rho}{2R}\right)^2} \tag{3}$$

for $\rho < 2R$ and zero otherwise. If we now calculate the mean value of "EPS":

$$m(R,R') = \frac{\int_0^{R'}\int_0^{2\pi} f(\rho)\rho \mathrm{d}\theta \mathrm{d}\rho}{\pi R^2} = \frac{2\int_0^{R'} f(\rho)\rho \mathrm{d}\rho}{R^2} \tag{4}$$

in a circular region of radius $R' > 2R$, we obtain that:

$$m(R,R') = \left(\frac{R}{R'}\right)^2 \tag{5}$$

by virtue of the integrals:

$$\int_0^{2R} \arccos\left(\frac{\rho}{2R}\right)\rho \mathrm{d}\rho = \pi R^2 \tag{6}$$

$$\int_0^{2R} \rho^2 \mathrm{d}\rho \sqrt{1-\left(\frac{\rho}{2R}\right)^2} = \frac{\pi R^3}{2} \tag{7}$$

The validity of the latter Equation is verified by assuming that EQs occur according to the aforementioned simple model and, using the computer programs used for the calculation of the actual ⟨EPS⟩ maps shown in Fig. 4 of Reference [25], the numerically found $m_n(R,R')$ has the form:

$$m_n\left(R,R'\right) = \left(\frac{R}{R'}\right)^{d_f} \tag{8}$$

When the quantity $m_n(R,R')$ was studied for the ⟨EPS⟩ maps of Fig. 4 of Reference [25] we find, $d_f = 1.32$. This value of d_f differed only slightly from the value $d_f \approx 1.2$, which Bak *et al.* [39] found to describe the fractal dimension of the location of epicenters projected onto the surface of the Earth in a unified scaling law obeyed by the distribution of waiting times between EQs occurring in California and ranging from tens of seconds to tens of years.

2.2. On the Relation between Average EPS Maps and the Epicenter of a Future Strong Earthquake

We now proceed to another usefulness of Green's function. A self-consistent method of producing average EPS maps, also written ⟨EPS⟩ maps, using a radius R has been suggested and applied to the Eastern Mediterranean area in [25]. To construct such a map, one first estimates EPS for disks of radius R at the points $\left(x_{ij}, y_{ij}\right)$ of a lattice to obtain EPS$_{ij}$ and then averages for each point $\left(x_{i_0 j_0}, y_{i_0 j_0}\right)$ the estimated EPS values within the same radius R, *i.e.*,

$$\langle \mathrm{EPS} \rangle \left(x_{i_0 j_0}, y_{i_0 j_0}\right) = \frac{1}{N} \sum_{i,j}^{d\left(x_{i_0 j_0}, y_{i_0 j_0}; x_{ij}, y_{ij}\right) \le R} \mathrm{EPS}_{ij} \tag{9}$$

where the summation is restricted to the lattice points whose distance $d\left(x_{i_0 j_0}, y_{i_0 j_0}; x_{ij}, y_{ij}\right)$ from the observation point is smaller than or equal to R, and N stands for the number of lattice points included in the sum.

It has been shown that the study of ⟨EPS⟩ close to the epicenters of forthcoming strong EQs exhibits a logarithmic dependence on R, reminiscent of the Green's function of the Poisson equation in two dimensions, while the mean value $\overline{\langle \mathrm{EPS} \rangle}$ of ⟨EPS⟩ over all the lattice points scales with R as a power law with an exponent d_f, *i.e.*, $\overline{\langle \mathrm{EPS} \rangle} \propto R^{d_f}$, see Section 3 and Equation (12) of Reference [25].

A clear relation between such made ⟨EPS⟩ maps and the epicenter of an impending strong EQ has been observed in the respective regional studies of Reference [25].

3. Conclusions

Green's function, beyond its usefulness in the application of ambient noise tomography methodologies that may demonstrate the potential of hydrocarbon entrapment in the study area, is shown to be of usefulness in the study of earthquakes along the following two directions:

1) In the identification of the fractal dimension of the earthquake epicenters projected onto the Earth's surface in the unified scaling law for earthquakes obtained by Bak *et al.* [39], and

2) The two-dimensional Green's function has recently been recovered when investigating the relationship of a future strong earthquake epicenter to the average earthquake potential score.

Such additional studies are now in progress in various areas due to their importance.

This importance is further strengthened by the fact that very recent aspects (see for example Reference [40]) are focused on the spatio-temporal variations of the correlation fractal dimension for earthquakes with magnitude M equal or greater than 2.5 in southern and Baja California to ascertain the incidence of seismic precursors before strong earthquakes [40].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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