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Scaling Behavior for the Susceptibility of the Vacuum

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Abstract

Using the two-component superfluid model of Winterberg for space, two models for the susceptibility of the cosmic vacuum as a function of the cosmic scale parameter, a, are presented. We also consider the possibility that Newton's constant can scale, *i.e.*, $G^{-1} = G^{-1}(a)$, to form the most general scaling laws for polarization of the vacuum. The positive and negative values for the Planckion mass, which form the basis of the Winterberg model, are inextricably linked to the value of G, and as such, both G and Planck mass are intrinsic properties of the vacuum. Scaling laws for the non-local, smeared, cosmic susceptibility, $\bar{\chi}(a)$, the cosmic polarization, $\bar{P}(a)$, the cosmic macroscopic gravitational field, $\bar{g}(a)$, and the cosmic gravitational field mass density, $\overline{\rho_{gg}}(a)$, are worked out, with specific examples. At the end of recombination, *i.e.*, the era of last scattering, using the polarization to explain dark matter, and the gravitational field mass density to explain dark energy, we find that, $(\Omega_{rad,1}, \Omega_{b,1}, \Omega_{c,1}, \Omega_{\Lambda,1}) = (0.37, 0.19, 0, 0.44)$. While this is an unconventional assignment, differing from the ΛCDM model, we believe this is correct, as localized dark matter (LDM) contributions can be much higher in this epoch than cosmic smeared values for susceptibility. All density parameter assignments in Friedmanns' equation are cosmic averages, valid for distance scales in excess of 100 Mpc in the current epoch. We also evaluate the transition from ordinary matter dominance, to dark matter dominance, for the cosmos as a whole. We obtain for the transition points, $z = 1.66$, for susceptibility model I, and, $z = 2.53$, for susceptibility model II.

Keywords

Extended Model of Gravity, Dark Matter, Dark Energy, Cosmic Evolution of Density Parameters, Gravitational Susceptibility of the Vacuum, Vacuum

Energy, Polarization of Space

1. Introduction

Cosmic susceptibility, and cosmic polarization are natural consequences of a Hajdukovic/Winterberg model for space. Hajdukovic [1] [2] [3] [4] was the first to entertain polarization of the vacuum as a model for dark matter. The particles which are polarized were virtual, positively and negatively charged, pion pairs. Their vacuum expectation value, he argued, leads to extra energy/mass, which he proffered as a possible explanation for the rotation curves of stars within galaxies, the halo effect around galaxies, the motion of galaxies within superclusters, and gravitational lensing. Later, he included dark energy as a consequence of this polarization as well. Due to the anti-screening feature of the gravity, the universe not only has additional effective bound mass, but an additional energy of the sort, that would tend to pull empty space apart.

Winterberg considered a different version of the quantum vacuum [5]-[11]. According to him, the vacuum is comprised not only of blackbody radiation, but also literally of real, positive and *negative mass* particles, which he called Planckions. They have ±Planck mass, and form a two-component superfluid, where each mass maintains a fixed distance of separation from the other masses of the same species, due to fluid forces. Because of the mass compensating effect, already at the sub-microscopic level ($<10^{-18}$ m), the vacuum appears massively neutral, and has zero net gravitational energy, zero net gravitational pressure, and zero net entropy in the undisturbed state. This vast assembly, or sea, of positive and negative mass Planckions populate the vacuum, and thereby create an ether-like medium, which is seemingly not there, due to their mass compensating effect. Winterberg did not consider the polarization of the vacuum, per se, but considered many other aspects relating to quantum mechanics and the general theory of relativity, which he considers to be asymptotic limits within his more encompassing theory.

The two models of Hajdukovic and Winterberg were combined in a recent work by this author [12] to form a new model for space. Using the idea of polarization due to Hajdukovic, and the notion of positive and negative mass Planckions due to Winterberg, the author developed a theory of gravi statics, which can be used to explain the present day density parameters in Friedmanns' equation. He found that, as a consequence, the cosmic susceptibility in the present epoch amounts to, $\overline{\chi_0} = 0.842$. The cosmic polarization, on a grand scale, equals, $\overline{P_0} = 2.396\ \mathrm{kg/m^2}$. Both are smeared values holding for distance scales in excess of 100 Mpc, and hence the bar over these cosmic averages. These values also hold in the present epoch only, which we denote by the subscript, "0". What needs to be determined is how these cosmic quantities scale upon expansion of the universe. This paper deals with that question in detail.

That the cosmic susceptibility, and cosmic polarization, of the vacuum should change from epoch to epoch is not in question. Cosmic susceptibility, and cosmic polarization, are both dependent on the CMB temperature, and because the CMB temperature of the blackbody photons changes, we would expect changes in the amount of cosmic smeared polarization, and degree of cosmic susceptibility. The question is whether, G, Newton's constant also changes. We believe that it might, and have given our reasons in references, [13] [14] [15]. This we include as a possibility. If G changes, then the Planck mass defined by, $M_{PL} \equiv (\hbar c/G)^{1/2}$, must also evolve with cosmological time. This would affect the value for the cosmic polarization, and susceptibility, as well. Hence we include this as a possibility. If G is truly a constant of nature, however, then we can easily accommodate this state of affairs in all formulas given in this paper, by taking an appropriate limit.

A net macroscopic polarization of space can be induced by the gravitational fields produced by ordinary source matter. Thus, in the surrounding regions of space, we can have a net polarization of space, if the conditions are right. The source gravitational fields would have to be strong enough, the gravitational dipole moments large enough, and the ambient temperatures low enough. It is really a tug of war situation, where the gravitational field promotes order, and the temperature frustrates all such attempts. Locally, a polarization cloud will form, about the free or source mass distribution, $\rho_F(\vec{x})$, which is matter built up from ordinary matter, *i.e.*, matter made up of quarks and leptons. This source mass distribution, $\rho_F(\vec{x})$, produces an applied field in the surrounding space, $\overrightarrow{g^{(0)}}(\vec{x})$, which takes on the same symmetry as $\rho_F(\vec{x})$. The, $\overrightarrow{g^{(0)}}(\vec{x})$, in turn, induces a polarized gravitational field, $\overrightarrow{g^{(1)}}(\vec{x})$, within the vacuum, the gravitic, which is our gravitational version of a dielectric. The total macroscopic gravitational field, $\vec{g}(\vec{x}) = \overrightarrow{g^{(0)}}(\vec{x}) + \overrightarrow{g^{(1)}}(\vec{x})$, is greater than the original field, $\overrightarrow{g^{(0)}}(\vec{x})$, and hence, we have anti-screening, where the induced field adds to the source field. This is in contrast to electrostatics, where the induced field, $\overrightarrow{E^{(1)}}(\vec{x})$, takes away from the original field, $\overrightarrow{E^{(0)}}(\vec{x})$. All gravitational fields reflect the symmetry of the original source distribution, $\rho_F(\vec{x})$. A simple example of such a symmetry would be spherical symmetry.

The *net polarization* of the vacuum, cosmically, when averaged over the entire universe, was found to equal, $\overline{P_0} = \varepsilon_0 \overline{\chi_0}\,\overline{g_0} = 2.396\ \text{kg/m}^2$, in the present era, as shown in reference [12]. In this equation, $\overline{\chi_0}$, is the cosmic susceptibility, a smeared quantity. The cosmic net macroscopic gravitational field equals, $\overline{g_0} = 2.387\text{E}{-9}\ \text{m/s}^2$. This is another smeared quantity, obtained from Gauss's law, which holds point for point in the universe, but only if huge distance scales are considered, greater than 100 Mpc in the current era. This value takes into account both source and bound matter within the universe. The gravitational permittivity, ε_0, is defined, by analogy to electrostatics, as, $\varepsilon_0 \equiv 1/(4\pi G_0) = 1.192\text{E}9\ (\text{MKS})$, where, G_0, is Newton's constant. The above values for, $\overline{\chi_0}$, and, $\overline{g_0}$, above, were imposed upon us in order to make sense of the present-day density parameters in Friedmanns' equation, within the ΛCDM model. It is important to realize that for polarization to exist a-priori, either locally or cosmically, the positive and negative mass Planckions must be spatially anchored or locked in position, somehow. The specific mechanism of anchoring was discussed in some detail in section, II, of reference, [12].

In this work, we wish to build upon our model, as presented in reference, [12]. We want to look into the specifics of gravitational polarization formation, and introduce two scaling laws for the cosmic susceptibility, $\overline{\chi} = \overline{\chi}(a)$, where, a, is the cosmic scale parameter dependent on CMB temperature. In fact, we wish to take this further and develop scaling laws for all the macroscopic variables of interest, which were introduced in our previous work. This is our primary objective.

A second goal is to consider a cosmologically varying gravitational constant. We wish to build in this additional feature, as it ties in directly with the positive and negative mass value for the Planckions, as well as other considerations presented elsewhere [13] [14] [15]. The Planck mass is only a constant, if Newtons' constant, G, is a true constant of nature. If G varies cosmologically, very slowly in the current era so as not to upset, too much, the accepted and very successful ΛCDM model, then we have Planckion masses which change with cosmological time. For the broadest possible scaling laws, and to provide an intimate connection in later work between electrostatics, and Planckion theory, we will also include this possibility.

The outline of this paper is as follows. In section II, we include the possibility that, $G = G(a)$. There are many reasons for assuming this, which were discussed elsewhere. We will make use of two specific models for, $G = G(a)$, which were called models, A, and, B, in references [13] [14] [15]. In our view, $\overline{\chi}(a)$, and, $G(a)$, are both intrinsic properties of the vacuum, although the latter does not require any source mass, nor the gravitational field produced by such.

We know that ordinary matter is made up of elementary particles, *i.e.*, quarks and leptons, which only came into existence, at temperatures well below, 10E16 Kelvin (about 1 TeV) [16] [17] [18] [19]. If Newton's constant varies at all, it was calculated that, $G^{-1}(a)$, surfaces, or forms, at an inception CBR temperature of about, 10E22 Kelvin. In fact, both models, A, and, B, lead to very similar inception temperatures even though they are modeled quite differently. For the polarization of space and susceptibility, in general, we need ordinary matter, and that source mass, which is made up of quarks and leptons, didn't even begin to freeze out until well below, 10E16 Kelvin. So, the susceptibility of space would seem to have little to do with the cosmic development of, G^{-1}, as both have very different inception temperatures. There is, however, an intimate connection. The mass of the constituent positive and negative mass Planckions is directly determined by the value of, G. The positive and negative mass value will also determine polarization, because their masses will allow us to define an intrinsic gravitational dipole moment. If we wish to see how the vacuum evolves, it would be a mistake not to include the possibility that the masses of the Planckions can vary, as they comprise and populate the vacuum, as well as determine the polarization of space.

In section III, we give two models for cosmic susceptibility, $\bar{\chi} = \bar{\chi}(a)$. We first differentiate between ionic and orientation polarization, and show that both lead to essentially the same results qualitatively. Quantitatively there is a difference. Model I, is based on ionic polarization, whereas model II, has orientation polarizability as its basis. Orientation polarization, model II, assumes pre-existing dipoles before any applied field is introduced. When a source field is applied, these dipole moments will orient, or align, themselves in the sense of the applied field, if conditions are right, in order to minimize their gravitational potential energy. Those pre-existing dipoles could be due to blackbody photon bombardment, as these photons would cause non-vanishing root mean square amplitudes for the oscillating positive and negative mass Planckion pairs, which make up the vacuum.

In section IV, macroscopic quantities, important in a discussion of polarization for the cosmos as a whole, will be considered. We will derive the scaling laws for these quantities as the universe expands. We will also focus on one or two epochs of special interest, such as the era of last photon scattering, 380,000 years after the big bang. With our two models we will see that when the CMB temperature was about, 3000 Kelvin, the cosmic $\bar{\chi}(a)$ values are rather small. However the localized values for, $\chi(\vec{x})$, can still be quite large. The coolest regions in the universe will have the greatest amount of local susceptibility, and thus those coolest pockets will have the greatest amount of dark matter. This can be important in interpreting the acoustic peaks in the power spectrum correctly. We also consider the cosmological point where dark matter starts to dominate over ordinary matter. Cosmically, this happened rather recently, when the universe as a whole is considered. Local deviations will follow their own rules, independent of the universe as a whole with respect to scaling. Finally, in section V, we present our summary and conclusions.

2. G = G(a) Models

We are interested in the scaling behavior of macroscopic quantities relevant to our polarization model. One of these quantities is Newtons' constant, G. There are many reasons why G could vary with cosmological time [13] [14] [15], and we include this possibility here. We keep in mind, however, that our formulas are easily modified, should G turn out to be a true constant of nature. All results in reference, [13], revert to the standard ΛCDM model, in the limit where the quintessence parameter, w, equals negative one. We assumed, namely, in reference [13], that, $w = -0.98$, a slight deviation from the ΛCDM assumed value of negative unity. The value, $w = -0.98$, is what is actually observed, although, in fairness, $w = -1$, is easily accommodated within observational error. Choosing, $w = -0.98$, allowed us to derive two specific functions for $G(a)$, which we designated as models, A, and, B. Except in the very early universe, the deviations from the predictions of the ΛCDM model, were slight.

A cosmologically varying G has a long and interesting history, starting with the work of Dirac and his large number hypothesis [20] [21] [22], already formulated in 1936. He was among the first to recognize that G is unusual because of its very weak value when compared to the other coupling constants found in nature, and its inherent canonical dimension. Soon afterwards, Jordan [23] [24] [25] [26] related a cosmologically time varying G to Hubble's constant. Since then, there have been many attempts to observe such a variation, with limited success. Some of that history is presented in reference, [13], and will not be repeated here. It is extensive. We mention it here only to give some context.

Model, in reference [13], assumes a G^{-1} scaling behavior as follows,

$$G^{-1}(T) = G_{\infty}^{-1}\left(1 - e^{b/T}\right) \qquad \text{(model A)} \qquad (2\text{-}1)$$

In Equation (2-1), T stands for the CMB temperature, and G_{∞}^{-1} is a saturation value, achieved in the limit where, $T \to 0$. The constant, "b", was determined to equal, $b = 11.663\ \text{Kelvin}$, by fixing the quintessence parameter to equal, $w = -0.98$. In model A, the, $G_{\infty}^{-1} = 1.014 G_0^{-1}$, where G_0 is Newtons' constant.

Another way to write Equation (2-1), is to make use of the cosmic scale parameter, a, defined by, $a \equiv T_0/T = R/R_0 = (1+z)^{-1}$. All subscripts, "0", denote the current era, and we are using the convention where, $a_0 = 1$. The, R, stands for the Hubble radius, the, T, denotes CMB temperature, and the, z, equals the redshift. In the present epoch, $T_0 = 2.725\ \text{Kelvin}$. When re-expressed in terms of the cosmic scale parameter, Equation (2-1), reads,

$$G^{-1}(T) = G_{\infty}^{-1}\left(1 - e^{-4.28a}\right) = 1.014 G_0^{-1}\left(1 - e^{-4.28a}\right) \qquad \text{(model A)} \qquad (2\text{-}2)$$

This equation came into being at a temperature estimated to be approximately, 6.20E21 Kelvin. We are close to full saturation in the present epoch since, $G_0 = 1.014 G_{\infty}$. Saturation will occur at roughly, $a \cong 10$, *i.e.*, when the observable universe is roughly ten times its current radius in this model. Equation (2-2), was modeled as a charging capacitor. What is charging up as a function of cosmological time, is the mass squared of the planckions, as will be seen shortly.

Model B assumes an entirely different scaling law. Here,

$$G^{-1}(T) = G_{\infty}^{-1}\left[\coth(b/T) - T/b\right] \qquad \text{(model B)} \qquad (2\text{-}3)$$

Again, G_{∞}^{-1} is the saturated value, applicable in the limit where the CMB temperature, $T \to 0$. The constant, "b", was determined to equal, $b = 48.15\ \text{Kelvin}$, in order to guarantee that the quintessence parameter, $w = -0.98$. Here, in model B, it turns out that, $G_{\infty}^{-1} = 1.054 G_0^{-1}$.

A second way to rewrite Equation (2-3), is to make use of the identity, $a = T_0/T = 2.725/T$. Substituting this into Equation (2-3), and making use of the numerical value for "b", we find,

$$\begin{aligned} G^{-1}(a) &= G_{\infty}^{-1}\left[\coth(17.67a) - 1/(17.67a)\right] \\ &= 1.054 G_0^{-1}\left[\coth(17.67a) - 1/(17.67a)\right] \end{aligned} \qquad \text{(model B)} \qquad (2\text{-}4)$$

This order parameter, $G^{-1}(a)$, surfaced at a Curie temperature of roughly, 7.01E21 Kelvin, which is very close to the value above, in model A. This is remarkable because both functions, indicated by Equations (2-1) and (2-3), are quite distinct from one another. In model B, Newtons' constant, G_0, is also close to the final saturation value since it is found that, $G_0 = 1.054 G_\infty$. Effective saturation in model B, is achieved when the cosmic CMB temperature drops to one-half current value, or when the Hubble radius is twice the current radius. Model B, is modeled much like magnetization, and we call this model the magnetization model for *G*. Both, G^{-1}, and magnetization, have the same inherent canonical dimension. It should be noted that both Equations (2-3) and (2-4), involve the Langevin function, $L(x) = \coth x - 1/x$, where in this instance, the variable, $x = b/T = 17.67a$. The Langevin function is often used to model paramagnetism. We can think of space as somehow consisting of polarized gravitational domains, which can be ordered, much like magnetic domains.

The inverse Newtonian "constant", G^{-1}, in both models A, and, B, are one-parameter, non-linear functions, which have specific inception temperatures, and rise dramatically at very high temperatures. In fact, both models give a $G^{-1}(a)$ value, which is inversely proportional to temperature at very high temperatures. More correctly, if T_C equals the inception temperature, then G^{-1} is proportional to, $1/(T - T_C)$, which is typical order parameter behavior. As the universe expands, and the CMB temperature cools, the $G^{-1}(a)$ functions will start to level off and flatten. Close to saturation, the G^{-1} approaches a constant value, G_∞^{-1}. In the current era, we are close to full saturation since, $G_0 \cong G_\infty$. When plotted as a function of cosmic scale parameter, *a*, both Equations (2-2) and (2-4), look very similar.

The inverse Newtonian gravitational constant, $G^{-1}(a)$, is directly related to ±Planckion mass. To see this, we start with the formal definition of the Planck mass,

$$M_{Pl} \equiv (\hbar c/G)^{1/2} \tag{2-5}$$

We square this result, and rewrite the mass as a field,

$$M_{Pl}^2 = \hbar c G^{-1} = \langle 0 | \varphi^2 | 0 \rangle \tag{2-6}$$

Here, the M_{Pl}^2 is no longer a constant, but the vacuum expectation value (VEV) of a scalar field, φ, squared. As the scalar field squared, φ^2, freezes out of the vacuum, the G^{-1} will change its value, a process lasting eons. In our scenario, G^{-1} is no longer a constant, and neither is the Planck mass. We identify the scalar field in Equation (2-6), with the scalar field of Jordan, first introduced already in the year, 1937 [23].

It should be noted that M_{Pl}^2 has the same canonical dimensions as magnetization in condensed matter physics, or $M_{W_\pm}^2$ in particle physics. Thus, it could very well be an order parameter based on inherent dimension alone. In the theory of weak interactions, it is well known that $M_{W_\pm}^2$ is essentially the inverse Fermi constant, G_F^{-1}, which effectively fades at high energies, and is only constant below approximately 100 GeV. Above, 100 GeV, the momentum squared term starts to take over, and dominate over the mass squared term, in the propagator. The, $M_{W_\pm}$, is the mass of the $W_\pm$ boson. Newton's constant, and the Fermi constant, are the *only two* known coupling constants in physics, which have an inherent canonical dimension, and that canonical dimension is the same for both. It can be expressed as inverse mass, or inverse momentum, squared. We are modeling the gravitational constant much like the Fermi constant in the electro-weak interaction.

The current value for G, is, of course, $G_0 = 6.674\text{E}{-11}(\text{MKS units})$. If we insert this into Equation (2-5), then we obtain the familiar Planck mass, $M_{Pl} = 2.176\text{E}{-8}\text{ kg}$. Using this value, we can write in place of Equation (2-6), the following expression.

$$M_{Pl}^2 = \hbar c G^{-1} = (G_0/G)(2.176\text{E}{-8}\text{ kg})^2 \quad (2\text{-}7)$$

Since G^{-1} will increase, with an increase in cosmological time, so too will, M_{Pl}^2. The Planck mass will start out from a zero value in our models, and increase in accordance with Equation (2-7).

As a specific example for our formulas, Equations (2-2) and (2-4), we consider the era of last scattering, where the CMB temperature was, $T_1 = 3000\text{ Kelvin}$. This specifies a particular epoch, where, $a_1 = T_0/T_1 = 2.725/3000 = 1100^{-1}$. We substitute this value into both Equations (2-2) and (2-4), and find that,

$$G_1/G_0 = 254 \quad \text{(model A)} \quad (2\text{-}8a)$$

$$G_1/G_0 = 177 \quad \text{(model B)} \quad (2\text{-}8b)$$

Both functions give a larger G value for this cosmological time, when the universe was 1/1100 its present Hubble radius. By Equation (2-7), both the positive and negative Planckion mass, are reduced in magnitude, by a factor of, $1/\sqrt{254} = 0.063$, and $1/\sqrt{177} = 0.075$, respectively. The ±Planckions were less massive in that previous epoch.

3. Two Models for Cosmic Susceptibility, χ(a)

Two types of polarization will be considered, ionization polarization, and orientation polarization. For each, we will present a specific function, $\chi(a)$. Ionic polarization, designated as model I, involves induced gravitational dipole moments. Consider a source gravitational field, $\overrightarrow{g^{(0)}}$, pointing from right to left. The positive mass Planckion will get displaced from its equilibrium position, and move slightly to the left, being attracted to the source mass. Call that displacement, $\overrightarrow{d_+}$. The negative mass planckion will also get shifted, but to the right, being repelled by the source field, $\overrightarrow{g^{(0)}}$. Seeing that the source field is uniform (a small enough region of space is considered), we can expect the displacement of the negative mass to equal in magnitude the positive mass displacement. However, the sense of direction is opposite, *i.e.*, $\overrightarrow{d_-} = -\overrightarrow{d_+}$. The induced dipole moment is thus, $\overrightarrow{p_d} = M_{Pl}\vec{d} = M_{Pl}(2d_+)(-\hat{i})$, where, $(\hat{i})$, is a unit vector pointing from left to right. This is the simplest kind of polarization possible, where, d_+, will depend on the amount of the applied field, $\overrightarrow{g^{(0)}}$. The full macroscopic field, $\vec{g}$, is the vector sum of the source field, $\overrightarrow{g^{(0)}}$, and the induced field, $\overrightarrow{g^{(1)}}$, both pointing in the same direction, $(-\hat{i})$. By definition, $\overrightarrow{p_d}$, will always point from the negative mass to the positive mass, just like for charges in electrostatics.

The gravitational potential energy here for dipole ordering is, $U = -\overrightarrow{p_d} \cdot \overrightarrow{g^{(2)}} = -M_{Pl}\left(2\overrightarrow{d_+}\right) \cdot \overrightarrow{g^{(2)}}$, where, $\vec{g}$, gets replaced by the localized field, $\overrightarrow{g^{(2)}}$. The localized field is sometimes called the local Lorentz field, or the "molecular field" in electrostatics, and it takes into account the other neighboring dipoles in the vicinity. This is the field that a particular dipole directly experiences within the lattice. If there is no displacement of positive and negative mass, then no dipole is formed. In this situation, the symmetry between the positive and negative masses within the undisturbed vacuum prevents any particular direction in space being singled out. The gravitational potential energy also averages out to zero. The factor of two is necessary because both positive and negative masses undergo displacement in an applied field.

A second type of polarization is orientation polarization, which we call model II. Here we have permanent or inherent dipoles within the medium (vacuum). These will try to self-organize and align in a particular direction in an applied $\overrightarrow{g^{(0)}}$ field against the disruptive effects of temperature. The gravitational potential energy here equals, $U = -\overrightarrow{p_d} \cdot \overrightarrow{g^{(2)}} = -\left|\overrightarrow{p_d}\right|\left|\overrightarrow{g^{(2)}}\right|\cos(\theta)$, where, $\overrightarrow{g^{(2)}}$, is, again, the local Lorentz field, or molecular field, described above, which also takes into account the gravitational field produced by the neighboring dipoles. The permanent dipoles will orient themselves three-dimensionally in a $\overrightarrow{g^{(2)}}$ field, in order to achieve the lowest possible potential energy, against a backdrop of CMB temperature, which will attempt to disrupt and frustrate any such attempts. The permanent dipoles can be due to inherent and constant collisions with CMB blackbody photons. This will cause oscillations about the center of mass for the dipole Planckion pairs, and a root-mean-square amplitude for simple harmonic motion results. In a $\overrightarrow{g^{(2)}}$ field, the axis of vibration or oscillation would want to align itself with the gravitational field, with the positive mass facing the source.

If the vector sum of the individual gravitational dipoles can overcome disruption due to temperature, then we can have partial, or even full, alignment. In either case, we then have polarization in the amount

$$\vec{P} = n_{MAX}\left\langle \overrightarrow{p_d} \right\rangle \tag{3-1}$$

where, n_{MAX}, is the maximum gravitational dipole density, $n_{MAX} = n_{MAX}(\vec{x})$, and, $\left\langle \overrightarrow{p_d} \right\rangle$, is some average taking into account thermal disturbances. It turns out that, in the case of orientation polarization, we can set

$$\left\langle \overrightarrow{p_d} \right\rangle = \overrightarrow{p_d}\left\langle \cos(\theta) \right\rangle = \overrightarrow{p_d} L(x) = \overrightarrow{p_d}\left[\coth(x) - 1/x\right] \tag{3-2}$$

In Equation (3-2), $L(x)$ is the Langevin function, defined as, $L(x) \equiv \left[\coth(x) - 1/x\right]$. This Langevin function can be viewed as a probability or percentage of total dipole alignment. The Langevin function depends on ambient temperature, T, dipole moment, $\overrightarrow{p_d}$, and, $\overrightarrow{g^{(2)}}$ field. The argument of the Langevin function, x, in Equation (3-2), is namely defined as,

$$x \equiv -U/(k_B T) = \left(\left|\overrightarrow{p_d}\right|\left|\overrightarrow{g^{(2)}}\right|\right) \Big/ (k_B T) \tag{3-3}$$

The, k_B, refers to Boltzmann's constant.

If dealing with an expanding universe, and space on a grand scale, all quantities in the definition of variable, above, are smeared quantities. Then, we would write in place of Equation (3-3),

$$\overline{x} = a\overline{x_0} \equiv a\left(\overline{p_d}\,\overline{g^{(2)}}\right) \Big/ (k_B T_0) \tag{3-4}$$

In this equation, T_0, represents the present CMB temperature, $T_0 = 2.725\ \text{Kelvin}$, and, a, is the cosmic scale parameter, $a = T_0/T = (1+z)^{-1}$. The redshift is specified by the variable, z. Equations (3-3) and (3-4), look similar, but one is local, Equation (3-3), and the other Equation (3-4), is cosmic, where all variables are smeared cosmic averages, which hold only when huge distance scales are considered. A local equation is one where all variables depend on position, $\vec{x}$. For, $\chi(a)$, we choose, $\chi(\overline{x}) = L(\overline{x})$, where, $\overline{x}$, is specified by Equation (3-4). This is our model II, for cosmic susceptibility. One will note that the maximum value for cosmic susceptibility, $\chi(\overline{x}) = L(\overline{x})$, is unity, which indicates 100% alignment.

In the case of ionic polarization, model I, we will use a different function for, $\chi = \chi(a)$, not the Langevin function. Instead of choosing, $\chi(\overline{x}) = L(\overline{x})$, as specified in Equation (3-2), we will use, instead,

$$\chi(x) = \left[1 - \mathrm{e}^{-x}\right] \qquad \text{(model I)} \tag{3-5}$$

The variable, x, is defined as in Equation (3-4). We are looking at cosmic susceptibility, $\chi(x) = \chi(a)$, which holds only when the universe is taken as a whole. The maximum value for Equation (3-5), is also unity. It is achieved in the limit where, $x \to \infty$, or equivalently, when $T \to 0$. The Langevin function has those same limits.

The physical motivation for Equation (3-5), is somewhat different than that of Equation, (3-2). Equation (3-2), treats the cosmic susceptibility as a kind of magnetization. Localized domains in space, create an average or smeared cosmic value, and, $L(x) = L(a) = \chi(a)$, is the result. Equation (3-5), on the other hand, looks more like a charging capacitor model where bound mass for the universe is being "charged up" within the gravitic, which is what we call the vacuum. The gravitic is a gravitational version of a dielectric. From previous work [12], bound mass, or polarized mass, is identified as dark matter, M_B. The M_B is related to source mass, M_F, by means of the equation,

$$M_B = (\chi/K) M_F = \left[\chi/(1-\chi)\right] M_F \tag{3-6}$$

This is a non-local smeared equation. As the universe expands, and the CMB temperature decreases, cosmic susceptibility, $\chi(a)$, will increase. As a consequence, bound mass will build up as a function of cosmological time, but not linearly. As χ gets larger according to Equation (3-5), the bound mass will increase even more dramatically because of the denominator decreasing at the same time. The M_F value stays the same. In the current epoch, it was determined that the cosmic value for χ equals, $\chi_0 = \chi(a_0 = 1) = 0.842$. The high degree of susceptibility is due to the very dilute mass density value of the universe in the present epoch.

The counterpart to Equation (3-5), will hold for orientation polarization. This we called model II, our magnetization model for the cosmic susceptibility. Once more, this can be written in terms of a Langevin function as,

$$L(x) = \chi(x) = \left[\coth(x) - 1/x\right] \qquad \text{(model II)} \qquad (3\text{-}7)$$

Equations (3-5) and (3-7), are two quite distinct functions. Yet, when plotted, they look remarkably similar. They are both one parameter, nonlinear functions, and both mimic order parameter behavior. The one parameter that has to be fixed in both models is, $x_0 \equiv \left(\overline{p_d}\,\overline{g^{(2)}}\right)\Big/\left(k_B T_0\right)$. See Equation (3-4). What we are really determining is dipole gravitational potential energy, *U*, for both model I, and model II, since $T_0 = 2.725\ \text{Kelvin}$. See Equation (3-3). This dipole energy, *U*, determines dipole ordering, or alignment, in a $\overline{g^{(2)}}$ field. Once x_0 is determined, we are in a position to find the cosmic susceptibility for both our models, I, and, II, using Equations (3-5) and (3-7), respectively. We keep in mind that, $= ax_0$, where, *a*, is cosmic scale parameter.

To find the parameter, x_0, in either model, we use the present epoch value for cosmic susceptibility. This has been found [12] to equal, $\chi_0 = \overline{\chi}(a_0 = 1) = 0.842$. Inserting this value in Equation (3-5), and solving gives,

$$x_0 = 1.845, \quad x = ax_0 \qquad \text{(model I)} \qquad (3\text{-}8)$$

For Equation (3-7), we proceed likewise. Set the right hand side equal to 0.842, keeping in mind that this holds for, $a = 1$, and solve for x_0. The result is,

$$x_0 = 6.338, \quad x = ax_0 \qquad \text{(model II)} \qquad (3\text{-}9)$$

With these values for x_0, we can easily find, $x = ax_0$, for any given cosmological epoch. We just have to specify the cosmic scale parameter, *a*, or, equivalently, the redshift. Substituting the$x$ value in the appropriate Equations (3-5) or (3-7), will give us our cosmic susceptibility.

One will have noticed that Equations (3-5), and (3-7), have the same form as Equations (2-2), and (2-4), in section II. This is no accident. A charging capacitor model, or a magnetization model, seem to us very good parametrizations, for both, $\overline{\chi} = \overline{\chi}(a)$, and, $G^{-1} = G^{-1}(a)$. We emphasize however, that they both model entirely physical processes. The inverse Newtonian "constant", $G^{-1}(a)$, has an inception temperature of about, 10E22 Kelvin, and effectively models the development of Planck mass squared. See Equations (2-6) or (2-7). The $\overline{\chi}(a)$, on the other hand, models cosmic susceptibility, or polarization of space, when the cosmos is treated as a whole. This is a smeared value. The inception temperature for $\overline{\chi}(a)$ is much less than that for $G^{-1}(a)$. $\overline{\chi}(a)$ came into being much later cosmically speaking, after BBM (Big Bang Nucleosynthesis), or when, $T \le 10\text{E}9\ \text{Kelvin}$. At a CMB temperature of 3000 Kelvin, we will see that the $\bar{\chi}(a)$ value is already about a thousand times smaller than what it is today. This we show next.

As a numerical example of Equations (3-5) and (3-7), let us evaluate both $\bar{\chi}(a)$ values at a CMB temperature of 3000 Kelvin, the era of last photon scattering. The appropriate scale parameter value here is, $a_1 = T_0/T_1 = 2.725/3000 = 1100^{-1}$. In model I, we substitute this a_1 value, together with the x_0 value specified in Equation (3-8), into Equation (3-5). We find that

$$\chi_1 = \chi\left(a_1 = 1100^{-1}\right) = 1.675\text{E}{-3} \qquad \text{(model I)} \qquad (3\text{-}10)$$

This is much less than the current cosmic value of, $\chi_0 = 0.842$. In fact, it is about 0.002 as large. For model II, we proceed likewise. We substitute the a_1 value above, and the x_0 value as indicated by Equation (3-9), into Equation (3-7). Doing this, and evaluating the result gives,

$$\chi_1 = \chi\left(a_1 = 1100^{-1}\right) = 1.920\text{E}{-3} \qquad \text{(model II)} \qquad (3\text{-}11)$$

This result is also much less than the current value for cosmic susceptibility. It is only about. 0023 times as large. We notice that at this CMB temperature, both models give much reduced values for cosmic susceptibility, and they are approximately equal.

For what is needed later, let us also evaluate the corresponding cosmicgravitic constant, or relative gravitational permittivity, defined as, $K_1 \equiv 1 - \chi_1$, for the above two models. We find that,

$$K_1 = K\left(a_1 = 1100^{-1}\right) = 0.9983 \qquad \text{(model I)} \qquad (3\text{-}12a)$$

$$K_1 = K\left(a_1 = 1100^{-1}\right) = 0.9981 \qquad \text{(model II)} \qquad (3\text{-}12b)$$

There is virtually no cosmic polarization, and hence, the relative gravitational permittivity is close to unity. Finally we evaluate the ratio, χ_1/K_1, at the end of recombination. Using the results of Equations (3-10), (3-11), and, (3-12a, b), we obtain,

$$\chi_1/K_1 = 1.678\text{E}{-3} \qquad \text{(model I)} \qquad (3\text{-}13a)$$

$$\chi_1/K_1 = 1.924\text{E}{-3} \qquad \text{(model II)} \qquad (3\text{-}13b)$$

These ratios are very small. In the present epoch, by contrast, we have, $\chi_0/K_0 = 0.842/0.158 = 5.327$. The values indicated above for, χ_1, K_1, and, χ_1/K_1, are cosmic averages, or smeared quantities, which do not hold locally.

Locally, $\chi = \chi(\vec{x})$, and we cannot use the, x_0, values listed above, in Equations (3-8) or (3-9). The gravitational field is totally different locally, and not a smeared value. Also, we have different values for the gravitational dipole moments, and ambient temperature. To make a long story short, the potential energy is different, and we can no longer use the cosmic values determined in Equations (3-3) and (3-4). Local values for, $\chi = \chi(\vec{x})$, can be quite large in the era of last photon scattering, and exist, even at much, much higher temperatures than 3000 Kelvin. Remember that the CMB temperature, $T_1 = 3000\text{ Kelvin}$, is a thermal average holding for the universe as a whole, in that era. What counts for local susceptibility, is the local dipole moment, the local gravitational field, and the local ambient temperature, all of which have to be specified before we can use a variation of our susceptibility models, I, and, II. In principle, however, it should be possible to model local situations, as well, if these inputs can be determined.

We have seen that Equation (3-1), is one way to specify polarization, Another way is to use a macroscopic formulation [12], $\vec{P} = \varepsilon\chi\vec{g}$, a result familiar from electrostatics, but now applied to situations in gravistatics. Equating both equations gives,

$$\vec{P} = \varepsilon\chi\vec{g} = n_{MAX}\left\langle\overrightarrow{p_d}\right\rangle = n_{MAX}\,\chi\,\overrightarrow{p_d} = n\,\overrightarrow{p_d} \tag{3-14}$$

In this equation, $\vec{g}$, is the macroscopic gravitational field taking into account an induced field due to dipole ordering. The, n_{MAX}, stands for the maximum gravitational dipole density, and, n, equals the effective dipole density, which takes susceptibility into account. Only a percentage of the maximum available dipole moments will self-organize, or align macroscopically. For gravitational polarization, the gravitational permittivity, ε, is defined by, $\varepsilon \equiv 1/(4\pi G)$, where, G, is Newton's constant. In the present epoch, $\varepsilon = \varepsilon_0 = 1.192\text{E}9(\text{MKS})$. As mentioned, we leave open the possibility that, G, can vary.

Equation (3-14), can be thought of as a cause and effect relation. A gravitational field, the cause, will produce a net polarization, but only if there is a net susceptibility. In other words, an induced field, $\overrightarrow{g^{(1)}} \equiv \chi\vec{g}$, must exist. If the susceptibility is unequal to zero, then we will have anet effective macroscopicdipole alignment, or ordering, which is the effect, in the amount, $n = \chi n_{MAX}$. The equation can be interpreted both, locally, or cosmically, like so many of our equations. If treated as a cosmic equation, then the $\overline{g}(a)$, the $\overline{\chi}(a)$, and, the $\overline{p_d}(a)$, are all smeared values, holding for the universe as a whole. We would also have an effective smeared dipole number density, $\overline{n}(a)$, as well as a maximum smeared dipole number density, $\overline{n_{MAX}}(a)$.

4. The Scaling Behavior of Cosmic Gravitational Fields, Dark Matter, and Dark Energy

We next consider the scaling laws for the macroscopic quantities introduced in reference [12]. Upon expansion of the universe, we wish to determine how the cosmic gravitational fields, the cosmic polarization, the net bound mass density (dark matter), and net gravitational field mass density (dark energy), change as a function of cosmic scale parameter. First a quick review.

Dark matter was identified [12] as the mass produced within the vacuum, due to dipole alignment, or ordering. This is what we referred to as bound mass. We had four mass density terms in Friedmann's equation,

$$H^2 = (8\pi G/3)\left(\rho_{Rad} + \rho_F + \rho_B + \rho_{gg}\right) \tag{4-1}$$

The first, ρ_{Rad}, is the mass density associated with radiation. Although this is a negligible contribution in the current epoch, it becomes the dominant term in the early universe. It is well known that blackbody radiation due to photons and neutrinos scale as,

$$\rho_{Rad}/\rho_{Rad,0} = a^{-4} \tag{4-2}$$

All subscripts, "0", on variables refer to the present epoch. Variables without a subscript refer to other cosmological epochs. The, *a*, is the cosmic scale parameter. In the present epoch, the radiative component has the value, $\rho_{Rad,0} = \Omega_{Rad,0}\rho_0 = (8.3\text{E}-5)(8.624\text{E}-27\ \text{kg/m}^3)$. This contribution is negligible when compared to the other contributions on the right hand side of Equation (4-1), in the present epoch. All values for mass densities are taken from the latest WMAP/Planck cosmological data collaboration [27] [28] [29].

The second contribution to total mass density on the right hand side of Equation (4-1), is, ρ_F. This is due to ordinary mass found in the universe, made up of quarks and leptons. We sum up the individual masses of all the gases, molecules, atoms, stars, planets, galaxies, etc. to come up with a total mass, and then divide by the Hubble volume, to arrive at this, ρ_F, value. Its current value is estimated to equal, $\rho_{F,0} = \Omega_{F,0}\rho_0 \cong (0.0486)(8.624\text{E}-27\ \text{kg/m}^3)$. This is also well known to scale as,

$$\rho_F/\rho_{F,0} = a^{-3} \tag{4-3}$$

The mass densities in Friedmanns' equation are smeared values holding for distance scales in excess of, 100 Mpc. Only then is the universe fairly homogeneous and isotropic. Technically we should have bars over all such quantities, which indicate a cosmic average. We will dispense with this in this section for ease of writing.

The third term on the right hand side of Equation (4-1), is, ρ_B, which we identify as dark matter. As mentioned, this is bound mass, which is produced within the vacuum, and surrounds ordinary matter. This contribution, in our model, is due to the positive and negative mass Planckions forming dipoles within the vacuumgravitic. Fornet macroscopic ordering, or alignment, of such dipoles, within that space, we need a non-vanishing susceptibility. In the current epoch, the estimate for dark matter amounts to, $\rho_{B,0} = \Omega_{B,0}\rho_0 = (0.2589)(8.624\text{E}-27\ \text{kg/m}^3)$. This will not scale like ordinary matter in our model. Counter to the ΛCDM standard model, we will propose a different scaling law for, ρ_B. Our scaling law for, ρ_B, is,

$$\rho_B/\rho_{B,0} = [(\chi/K)/(\chi_0/K_0)](\rho_F/\rho_{F,0}) = [(\chi/K)/(\chi_0/K_0)]a^{-3} \tag{4-4}$$

This follows since, $\overline{\rho_B}$, is related to, $\overline{\rho_F}$, via the relation [1],

$$\overline{\rho_B} = (\chi/K)\overline{\rho_F} \tag{4-5}$$

Equation, (4-5), also follows from Equation (3-6). We know the value of the ratio, χ_0/K_0, in the present epoch. This equals, $\chi_0/K_0 = (0.842/0.158) = 5.329$. For the, χ/K, value in another epoch, we need to specify the scale parameter, *a*, and then use either Equations (3-5) with (3-8), for model, I, or, Equations (3-7) with (3-9), for model, II. In both models we know that, $= ax_0$.

As a particular example, we can consider the era of last photon scattering. There, the CMB temperature was, $T_1 = 3000\ \text{Kelvin}$. The, χ_1/K_1, values have already been worked out for models I, and II, and are indicated in Equations (3-13a, b). Substituting these values into Equation (4-4), we find that

$$\rho_{B,1}/\rho_{B,0} = 4.191\text{E}5 \qquad \text{(model I)} \qquad (4\text{-}6a)$$

$$\rho_{B,1}/\rho_{B,0} = 4.805\text{E}5 \qquad \text{(model II)} \qquad (4\text{-}6b)$$

Both of these ratios are far less than those assumed in the ΛCDM standard model. In the standard model, one expects that the dark matter mass density scales as source matter density. Therefore, in place of the right hand sides of equations, (4-6a, b), we would have instead, $a^{-3} = 1100^3 = 1.331\text{E}9$. Clearly, equations, (4-6a, b), indicate far less or values. Dark matter, in our models, is virtually non-existent as a cosmic average, at the end of recombination. We keep in mind, however, that localized values for dark matter can still be quite large. We postpone further discussion on this point until later.

The fourth term on the right hand side of Equation (4-1), is, ρ_{gg}. This we interpreted as dark energy [12]. This contribution is really made up of two separate components, a part which does not depend on polarization, and another part which does. Dark energy is interpreted as the energy density associated with gravitational fields, due to both source matter, and bound matter. Following electrostatics, we claimed that [12],

$$\begin{aligned}\rho_{gg} &= 1/\left(2c^2\right)K\varepsilon g^2 = \Omega_{gg}\rho = 1/\left(2c^2\right)\varepsilon g g^{(0)} = 1/\left(2c^2\right)\varepsilon g^{(0)}\left(g^{(0)} + g^{(1)}\right) \\ &= 1/\left(2c^2\right)\varepsilon g^{(0)}g^{(0)} + 1/\left(2c^2\right)\varepsilon g^{(0)}g^{(1)} = \rho_{AA} + \rho_{BB}\end{aligned} \qquad (4\text{-}7)$$

In this equation,

$$\rho_{AA} \equiv 1/\left(2c^2\right)\varepsilon g^{(0)}g^{(0)} \qquad (4\text{-}8a)$$

$$\rho_{AB} \equiv 1/\left(2c^2\right)\varepsilon g^{(0)}g^{(1)} = \left(\chi/K\right)\rho_{AA} \qquad (4\text{-}8b)$$

Equations (4-8a, b), are formal definitions. The gravitational field mass density associated with just ordinary matter, or source matter, is ρ_{AA}. This is proportional to, $g^{(0)}$ squared, as indicated by Equation (4-8a). The gravitational field mass density associated with, $g^{(0)}$, coupled to the gravitational field associated with bound matter, $g^{(1)}$, is ρ_{AB}. By Equation (4-8b), this involves both the source gravitational field, $g^{(0)}$, and the polarized gravitational field, $g^{(1)}$. In the limit where the cosmic susceptibility vanishes, this contribution, ρ_{AB}, also approaches zero.

From Equation (4-5), which is a non-local equation, we saw how bound mass density, or dark matter density, is related to free mass density, sometimes referred to as source mass density. Dark matter is formed in the space surrounding ordinary matter, and for dark matter a non-vanishing susceptibility is needed. It should come as no surprise then, that in the second line of Equation (4-8b), we have a similar relation, but now relating the gravitational field mass densities. For ρ_{AB} to exist, a non-vanishing cosmic susceptibility, χ, is needed. If cosmic susceptibility vanishes, then we only have the following contributions to mass density, ρ_{Rad}, ρ_F, and ρ_{AA}, in Friedmanns' equation. The ρ_B, and the ρ_{AB}, necessarily vanish.

We next consider the scaling behavior for dark energy, ρ_{gg}, As we have seen, this is made up of two components, ρ_{AA}, and, ρ_{AB}. We first focus on the ρ_{AA} component, defined by Equation (4-8a). This equation is really a smeared equation, and, as such, should have bars placed over both the quantities, ρ_{AA}, and, $g^{(0)}$. Averages for the universe as a whole are epoch dependent. Cosmic averages also cannot be used to determine scaling. To see this, we argue as follows.

Let us imagine the universe as a three dimensional sphere, the Hubble bubble, within which we place dots representing significant mass sources such as galaxies. Around each dot, draw dashed concentric bubbles, some smaller in radius, and some larger, depending on how much source mass is present. These dashed bubbles represent the localized susceptibility field, *i.e.*, the extent to which, $\overrightarrow{g^{(0)}}(\vec{x})$, reaches, and polarizes the surrounding vacuum. In some instances, there will be no dashed bubble, because there is no localized susceptibility due to the ambient temperature being too high, or the source gravitational field too weak. In those instances where susceptibility prevails, the *dashed bubbles are gravitationally bound to the source mass* distribution. As such, the ρ_{AA} must scale like ordinary matter. We obtain,

$$\rho_{AA}/\rho_{AA,0} = \rho_F/\rho_{F,0} = a^{-3} \tag{4-9}$$

What expands is the space between concentric bubbles, and not the bubbles themselves.

Equation (4-9), also makes sense from a conservation of energy point of view. The ratio of, ρ_{AA} to ρ_F, must stay, more or less, constant as the universe expands. Therefore, $\rho_{AA}/\rho_F = \rho_{AA,0}/\rho_{F,0}$, and Equation (4-9), follows. We have used Equation (4-3). The, ρ_{AB}, on the other hand, involves a coupling of $g^{(0)}$ with $g^{(1)}$, where $g^{(1)}$ is induced in the surrounding vacuum. This can, and will, involve a different scaling law than that for pure source matter.

Let us use the second line in Equation (4-8b), to determine this scaling law. From this equation, it should be apparent that,

$$\rho_{AB}/\rho_{AB,0} = \left[(\chi/K)/(\chi_0/K_0)\right]\left(\rho_{AA}/\rho_{AA,0}\right) = \left[(\chi/K)/(\chi_0/K_0)\right]a^{-3} \tag{4-10}$$

The, $\chi_0/K_0 = (0.842/0.158) = 5.329$, in the present epoch. See the discussion following Equation (4-5). We also can make use of the models from the previous section to determine the ratio, χ/K. We use either Equations (3-5) with (3-8), for model I, or, Equations (3-7) with (3-9), for model II. In both models, $x \equiv ax_0$. All we need to do is specify the cosmic parameter, *a*, or redshift, *z*, and we can evaluate the cosmic ratio, χ/K, in any given epoch.

We'll work out one numerical example. Let us consider the end of recombination, our familiar example, where the CMB temperature equals, $T_1 = 3000\text{ Kelvin}$. The, χ_1/K_1, values have been evaluated. These are specified by equations, (3-13a, b). We substitute these values into Equation (4-10), and find,

$$\rho_{AB,1}/\rho_{AB,0} = (3.149\text{E}-4)(1100)^3 \qquad \text{(model I)} \qquad (4\text{-}11a)$$

$$\rho_{AB,1}/\rho_{AB,0} = (5.734\text{E}-4)(1100)^3 \qquad \text{(model II)} \qquad (4\text{-}11b)$$

These are very small contributions when compared to the, $\rho_{AA,1}/\rho_{AA,0} = \rho_{F,1}/\rho_{F,0} = (1100)^3$, evaluated in Equation (4-9).

The total dark energy mass density, ρ_{gg}, is the sum of ρ_{AA}, and, ρ_{AB}. This can be written as,

$$\rho_{gg} = (1+\chi/K)\rho_{AA} = \rho_{AA}/K \qquad (4\text{-}12)$$

We have utilized the second line in Equation (4-8b). We also remember that, in gravistatics, the identity, $(K+\chi)\equiv 1$, holds. Thus the second equality follows in Equation (4-12). From Equation (4-12), we obtain,

$$\rho_{gg}/\rho_{gg,0} = (K_0/K)(\rho_{AA}/\rho_{AA,0}) = (K_0/K)a^{-3} \qquad (4\text{-}13)$$

The relative gravitational permittivity in the present epoch equals, $K_0 = 0.158$. At the end of recombination where, $T_1 = 3000\text{ Kelvin}$, we find using equations, (3-12a, b), that

$$\rho_{gg,1}/\rho_{gg,0} = (0.1583)(1100)^3 \qquad \text{(model I)} \qquad (4\text{-}14a)$$

$$\rho_{gg,1}/\rho_{gg,0} = (0.1583)(1100)^3 \qquad \text{(model II)} \qquad (4\text{-}14b)$$

There is no difference between the two models. Dark energy scales according to Equations (4-14a, b). It will be noticed that none of the mass density scaling laws in Friedmanns' equation involve, *G*, Newton's constant. This will be different when we look at the smeared, or cosmic, gravitational field strengths.

Before we consider the individual cosmic gravitational field scaling laws, let us evaluate the various contributions to mass density in the era of last scattering. We have all the relations needed. We start with radiation mass density, ρ_{Rad}. From Equation (4-2), we find that,

$$\rho_{Rad} = a_1^{-4}\rho_{Rad,0} = (1100)^4(8.3\text{E}-5)\rho_0 = 1.2152\text{E}8\rho_0 \qquad (4\text{-}15)$$

The total mass density in the present epoch, ρ_0, equals, $\rho_0 = 8.624\text{E}-27\text{ kg/m}^3$, corresponding to a present rate of expansion of, $H_0 = 67.74\text{ km}/(\text{s}\cdot\text{Mpc})$. For ordinary matter, we use Equation (4-3). At the CMB temperature, $T_1 = 3000\text{ Kelvin}$, we obtain,

$$\rho_F = a_1^{-3}\rho_{F,0} = (1100)^3(0.0486)\rho_0 = 0.6469\text{E}8\rho_0 \qquad (4\text{-}16)$$

Dark matter comes next. For this we use either Equation (4-6a), or Equation (4-6b), as our scaling law. We find,

$$\rho_{B,1} = (4.191\text{E}5)(0.2589)\rho_0 = 1.085\text{E}5\rho_0 \qquad \text{(model I)} \qquad (4\text{-}17a)$$

$$\rho_{B,1} = (4.805\text{E}5)(0.2589)\rho_0 = 1.244\text{E}5\rho_0 \qquad \text{(model II)} \qquad (4\text{-}17b)$$

And, finally we have dark energy. This scaling law is determined by using either one of Equations, (4-14a, b). Using these equations, we can claim that, in the era of last photon scattering,

$$\rho_{gg,1} = (0.1583)(1100)^3(0.6911)\rho_0 = 1.456\text{E}8\rho_0 \qquad \text{(model I)} \qquad (4\text{-}18a)$$

$$\rho_{gg,1} = (0.1583)(1100)^3(0.6911)\rho_0 = 1.456\text{E}8\rho_0 \qquad \text{(model II)} \qquad (4\text{-}18b)$$

We sum over all the various contributions to mass density in Friedmanns' Equation (4-1). At the end of recombination, we find that, $\rho_1 = \rho_{Rad,1} + \rho_{F,1} + \rho_{B,1} + \rho_{gg,1} = 3.3192\text{E}8\rho_0$. This holds for susceptibility model I. For model II, the sum is, $\rho_1 = 3.3193\text{E}8\rho_0$, which is almost in distinguihable from that of model I. We define the density parameters at the end of recombination by the equation, $\Omega_{i,1} \equiv \rho_{i,1}/\rho_1$. Thus, at a CMB temperature of, $T_1 = 3000\ \text{Kelvin}$, we find that,

$$\left(\Omega_{Rad,1}, \Omega_{F,1}, \Omega_{B,1}, \Omega_{gg,1}\right) = (0.366, 0.195, 0, 0.439) \quad \text{(models I \& II)} \qquad (4\text{-}19)$$

This result holds for both susceptibility models, I and, II, when taken to three significant figures. It will be noticed that cosmic dark matter does not exist at this CMB temperature, given our models for cosmic susceptibility.

The values indicated in Equation (4-19), are very different from those commonly assumed in the standard model. In the ΛCDM model, dark matter scales like ordinary matter, and there is no dark energy in this epoch. In the standard cosmological model, the result to be expected is, $\left(\Omega_{Rad,1}, \Omega_{F,1}, \Omega_{B,1}, \Omega_{gg,1}\right) = (0.229, 0.122, 0.649, 0)$. When we compare this with the above, we see noticeable differences. In the ΛCDM model, and even in the extended models where we have quintessence, dark energy barely scales. Therefore, as a consequence, dark energy is virtually non-existent at the end of recombination in the standard scenario.

This brings us to an interesting dilemma. Dark matter is thought to be needed at recombination in order to aggregate ordinary matter in gravitational potential wells, without which, the present structure of the universe would be difficult to explain. Also, when looking at the CMB power spectrum obtained from WMAP/Planck satellite data, the height of the third acoustic peak stands in a certain proportion and relation to the height of the first peak. The third peak is identified with dark matter, whereas the first peak denotes ordinary matter. At the end of recombination, one could expect that, $\Omega_{B,1}/\Omega_{F,1} = 0.2589/0.0486$, just as is the case in the present epoch.

We will still maintain, however, that Equation (4-19), is correct. There are several important points which must be considered. The first is that the localized dark matter contributions are much different than the smeared or cosmic average, dark matter contributions. Even though cosmic dark matter effectively disappears at this high CMB temperature, localized dark matter does not. In fact, localized dark matter (LDM) must be much higher in value near the somewhat cooler source matter, since in the cosmic voids, where there is little to no source matter, there must also be little to no dark matter. If the average cosmic value for dark matter is weak, and if, in the voids, there is negligible dark matter, then near the source masses we must have localized values for dark matter which are particularly strong to compensate for the close to zero values in the voids.

Second, as indicated in Equations (2-8a, b), Newton's gravitational constant has a much higher value. This would help aggregate ordinary matter into gravitational potential energy wells, perhaps even without the need for localized dark matter. Third, as we shall see shortly, the gravitational fields have enhanced values due to an increase in G value. Those stronger gravitational fields would also enhance clumping of ordinary matter. Finally, *dark energy*, itself, may even play a role in the aggregation of ordinary matter. Dark energy is a mass density associated with gravitational fields. This mass density can exert an added pressure on ordinary matter, causing the ordinary matter to clump up. In summary, there are many reasons which can explain source mass clumping in the era of photon decoupling. As to explaining the height of the third acoustic peak in relation to the first, we leave that for another paper. This is a technical point, and an area for further research and study.

As another example of the cosmic susceptibility scaling laws, we consider, specifically, the cosmic era where dark matter starts to dominate over ordinary matter for the universe. This happened fairly recently, in cosmological time. To find this point we set,

$$\overline{\rho_B} = \overline{\rho_F}$$
$$\left(\overline{\chi_2}/\overline{K_2}\right)\overline{\rho_F} = \overline{\rho_F}$$
$$\overline{\chi_2} = 1 - \overline{\chi_2}$$
$$\overline{\chi_2} = 1/2 \qquad (4\text{-}20)$$

This equation can be solved for both models, I and II. For cosmic susceptibility model I, we use Equations (3-5) with (3-8), in order to fix the value for the cosmic scale parameter, $a_2 = (1+z_2)^{-1}$. Setting Equation (3-5), equal to 0.5, as indicated by Equation (4-20), we find that,

$$\left[1 - \mathrm{e}^{-(1.845 a_2)}\right] = 0.5 \qquad (4\text{-}21)$$

If we consider the second cosmic susceptibility model, model, we would have to use Equations (3-7) with (3-9), instead, in order to fix this parameter, a_2. Demanding that Equation (3-8) equal the right hand side of Equation (4-20), we have the condition that,

$$\left[\coth(6.338 a_2) - 1/(6.338 a_2)\right] = 0.5 \qquad (4\text{-}22)$$

Both Equations (4-21) and (4-22), are easily solved. The solutions are,

$$a_2 = 0.376\,, \quad z_2 = 1.66 \qquad \text{(model I)} \qquad (4\text{-}23a)$$
$$a_2 = 0.284\,, \quad z_2 = 2.53 \qquad \text{(model II)} \qquad (4\text{-}23b)$$

The two models give different predictions, with model II indicating an earlier epoch for dark matter dominance.

What about the gravitational field scaling laws? How do these change as the universe expands? The cosmic gravitational fields are all cosmic average quantities, holding when distance scales in excess of 100 Mpc are considered in the present epoch. At such distance scales, the universe is spherically symmetric, and homogeneous. Because the gravitational fields are smeared values, they will not apply locally.

We start with Equation (4-9), and use our definition, (4-8a). This allows us to re-express Equation (4-9), as,

$$\varepsilon \overline{g^{(0)}}^{2} \Big/ \left(\varepsilon_0 \overline{g_0^{(0)}}^{2} \right) = a^{-3} \tag{4-24}$$

We next bring the gravitational permittivity terms over to the right hand side, and keep in mind that, $\varepsilon \equiv 1/(4\pi G)$. This leads to,

$$\overline{g^{(0)}}^{2} \Big/ \overline{g_0^{(0)}}^{2} = (G/G_0)\, a^{-3} \tag{4-25}$$

Finally, taking the square root of both sides of this equation gives us the cosmic gravitational field scaling law, which is due to source mass in the universe. We find that

$$\overline{g^{(0)}} \Big/ \overline{g_0^{(0)}} = (G/G_0)^{1/2}\, a^{-3/2} \tag{4-26}$$

We notice that this scaling law *does involve* the Newtonian constant. If G does not scale, then, obviously, $= G_0$, and the right hand side above simplifies to, $a^{-3/2}$.

As a concrete example, we consider the era of last scattering, where, $a_1 = 1100^{-1}$. For this epoch, the G values have been calculated, and they are given by Equations (2-8a, b). Substituting these values into Equation (4-46), results in,

$$\overline{g_1^{(0)}} \Big/ \overline{g_0^{(0)}} = 5.77\text{E}5 \qquad \text{(model A)} \tag{4-27a}$$

$$\overline{g_1^{(0)}} \Big/ \overline{g_0^{(0)}} = 4.85\text{E}5 \qquad \text{(model B)} \tag{4-27b}$$

The cosmic gravitational field due to source mass is enhanced quite dramatically in this earlier epoch.

Another cosmic gravitational field is that due to both source mass, and bound (polarized) mass. This gravitational field was designated as, $\overline{g}$. We know, however, that, $\overline{g^{(0)}} = K\overline{g}$. Using this relation, we can claim that for, $\overline{g}/\overline{g_0}$, the following scaling behavior applies.

$$\overline{g} \Big/ \overline{g_0} = (K_0/K)(G/G_0)^{1/2}\, a^{-3/2} \tag{4-28}$$

For this result, we have made use of Equation (4-26). This scaling law also involves the Newtonian constant, but in addition, the susceptibility scaling laws, because of the factor, (K_0/K), on the right hand side.

As a numerical example, we focus on, $a_1 = 1100^{-1}$, the end of recombination. The G values are again given by equations, (2-8a, b). We also have the appropriate K values, for our two susceptibility models, I, and, II. These are found in equations, (3-12a, b). Substituting all these values into Equation (4-28), renders,

$$\overline{g_1} \Big/ \overline{g_0} = 9.13\text{E}4 \qquad \text{(model A, I)} \tag{4-29a}$$

$$\overline{g_1} \Big/ \overline{g_0} = 9.13\text{E}4 \qquad \text{(model A, II)} \tag{4-29b}$$

$$\overline{g_1} \Big/ \overline{g_0} = 7.68\text{E}4 \qquad \text{(model B, I)} \tag{4-29c}$$

$$\overline{g_1}/\overline{g_0} = 7.68\text{E}4 \qquad \text{(model B, II)} \qquad (4\text{-}29\text{d})$$

We see that there is no difference between cosmic susceptibility models. Any variation is due to the G model chosen, A or, B.

Finally, as far as cosmic gravitational fields are concerned, we still have, $\overline{g^{(1)}}$, which is the contribution due to just bound, or polarized, mass in the universe. This gravitational field is dude to dipole ordering. Here we will make use of the fundamental relation, $\overline{g^{(1)}} = \chi\overline{g}$. We start with Equation (4-28), and multiply this equation through by the factor, χ/χ_0. This allows us to write,

$$\overline{g^{(1)}}/\overline{g_0^{(1)}} = (\chi/\chi_0)(K_0/K)(G/G_0)^{1/2}a^{-3/2} \qquad (4\text{-}30)$$

Again, both the gravitational constant, and the susceptibility model come into play. The scaling behavior is complicated, even in the limit where the Newtonian constant stays the same when switching between epochs.

We will work out these values when, $a_1 = 1100^{-1}$. The scaling laws for G are indicated by Equations (2-8a, b). For the ratio, (χ_1/K_1), use Equations (3-13a, b). We also keep in mind that, $\chi_0/K_0 = 0.842/0.158$. Inserting all of this into Equation (4-30), gives us the following scaling behavior,

$$\overline{g_1^{(1)}}/\overline{g_0^{(1)}} = 1.82\text{E}2 \qquad \text{(model A, I)} \qquad (4\text{-}31\text{a})$$

$$\overline{g_1^{(1)}}/\overline{g_0^{(1)}} = 2.09\text{E}2 \qquad \text{(model A, II)} \qquad (4\text{-}31\text{b})$$

$$\overline{g_1^{(1)}}/\overline{g_0^{(1)}} = 1.53\text{E}2 \qquad \text{(model B, I)} \qquad (4\text{-}31\text{c})$$

$$\overline{g_1^{(1)}}/\overline{g_0^{(1)}} = 1.76\text{E}2 \qquad \text{(model B, II)} \qquad (4\text{-}31\text{d})$$

These cosmic or smeared gravitational fields do not magnify by nearly as much as the other cosmic or smeared gravitational fields. But then, this is field associated with dipole moments, which, in and of themselves, should fall off quite rapidly with increasing temperature.

In summary, the cosmic gravitational fields seem to scale fairly similarly, irrespective of the model combination chosen. All these scaling laws involve a variation in G value, if G does, in fact, scale. If G is a true constant, then we set, $G = G_0$, in all of the above equations, resulting in less complicated scaling behavior. The factor, $(G/G_0)^{1/2}$, is common to all scaling laws for every type of cosmic gravitational field. This factor, at the end of recombination, equals, $\sqrt{254} = 15.9$, for model, A, and, $\sqrt{177} = 13.3$, for model, B. The increased values for gravitational fields due to these factors can contribute to the aggregation of ordinary matter in this epoch.

To close this section, we present one final scaling law, and that is for cosmic polarization, $\overline{P}$. It is known that the cosmic polarization is given by the equation, $\overline{P} = \varepsilon\chi\overline{g} = \varepsilon\overline{\chi}\overline{g} = \varepsilon\overline{g^{(1)}}$. See Equation (3-14). From this equation, it should be apparent that,

$$\begin{aligned}\overline{P}/\overline{P_0} &= (\varepsilon/\varepsilon_0)\left(\overline{g^{(1)}}/\overline{g_0^{(1)}}\right) \\ &= (G_0/G)(\chi/\chi_0)(K_0/K)(G/G_0)^{1/2}a^{-3/2} \\ &= (\chi/\chi_0)(K_0/K)(G_0/G)^{1/2}a^{-3/2}\end{aligned} \qquad (4\text{-}32)$$

Use of Equation (4-30), has been made. This scaling law depends on both G scaling and χ scaling. We have two models for each quantity which can scale, and thus four permutations.

To work out a specific example, let us look at our familiar example, $a_1 = 1100^{-1}$. For the G variation, Equations (2-8a, b), can be used. For the ratio, χ_1/K_1, use Equations (3-13a, b). If we insert all these values into our cosmic polarization scaling law, Equation (4-32), we find that,

$$\overline{P_1}/\overline{P_0} = 0.72 \quad \text{(model A, I)} \tag{4-33a}$$

$$\overline{P_1}/\overline{P_0} = 0.83 \quad \text{(model A, II)} \tag{4-33b}$$

$$\overline{P_1}/\overline{P_0} = 0.87 \quad \text{(model B, I)} \tag{4-33c}$$

$$\overline{P_1}/\overline{P_0} = 0.99 \quad \text{(model B, II)} \tag{4-33d}$$

Surprisingly, the net cosmic polarization, in the era of last photon scattering, is about the same as the net cosmic polarization in the current epoch. In the present epoch we found that, $\overline{P_0} = 2.396\ \text{kg}/\text{m}^2$. We are within, 72% to 99%, of this current value at the end of recombination.

5. Summary and Conclusions

We have considered the gravitational susceptibility of space assuming that space is made up of a vast assembly (sea) of positive and negative mass particles, called Planckions. These particles, first put forward by Winterberg, form a very stiff, two-component superfluid, interact with particles within their species, and offer possible explanations for the vacuum energy, quantum mechanical indeterminacy (the Heisenberg relation), the Schroedinger equation, and, now, dark matter with dark energy. It is specifically the polarization of space and bound mass, which leads to dark matter. For dark energy, we are led to an identification with gravitational field mass density, due to both source, as well as bound, mass, within the universe. For the polarization of space, gravitational dipoles are needed, which can be ordered or aligned in some sense. These are formed from the positive and negative mass Planckions themselves in our model, which are assumed to be real, versus virtual, particles. We presented two specific models for cosmic susceptibility, Equations (3-5) with (3-8), which we call model, I, and Equations (3-7) with (3-9), which is referred to as model, II. These susceptibilities do not hold locally, but cosmically as smeared quantities. Thus they hold for the universe as a whole, when the universe is considered homogeneous. Cosmic susceptibility is thought to be epoch dependent, and can be expressed in terms of the cosmic scale parameter, a as, $\chi = \overline{\chi} = \overline{\chi}(a)$.

With the help of our two models for, $\overline{\chi}(a)$, we can predict how space, *i.e.*, the vacuum, will polarize as a function of cosmological time. We worked out several numerical examples. We can also have a localized version of susceptibility, $\chi = \chi(\vec{x})$, where no specific models are given. For that we need a comprehensive microscopic theory, which is being worked on. The ratio of applied gravitational field, $\overrightarrow{g^{(0)}}$, which promotes order to ambient temperature, which promotes disorder, is crucial. The applied field will lead through a series of steps to the molecular field, $\overline{g^{(2)}}$, which is what the individual dipole experiences within the superfluid. This takes into account the gravitational field set up by the neighboring dipoles. What is also important in determining, $\chi(\vec{x})$, are the gravitational dipole moments themselves. These dipole moments, $\overrightarrow{p_d} = M_{Pl}\vec{d}$, are formed from the positive and negative mass Planckions comprising the vacuum. The theory is involved, and will be left for another paper. We keep in mind that even though the cosmic susceptibility may be quite low in certain epochs, such as in the era of last scattering, the localized values for susceptibility within the same epoch can be quite high.

This result is significant because we have worked out the cosmic susceptibility at the end of recombination, the era of last photon scattering, 380,000 years after the big bang. The results are given in Equations (3-10) and (3-11). For comparison, the present epoch value for cosmic susceptibility is, $\chi_0 = 0.842$. In the era of last scattering, the cosmic susceptibility is very small, leading to virtually no cosmic dark matter in this epoch. Localized pockets of dark matter, however, can exist at this CMB temperature of 3000 Kelvin (a cosmic average), and even, at much higher temperatures. The bullet cluster has considerable dark matter, and it is known that the temperature in the surrounding space is very, very high. This tells us that the gravitational fields, and the dipole moments associated with these gravitational fields, are substantial enough to overcome the disruptive effects of ambient temperature, if our thinking is correct. Localized dark matter (LDM) is probably needed for the aggregation of ordinary matter at the end of recombination. Other mechanisms, however, can also contribute to the clumping of ordinary matter into gravitational wells in this epoch.

The polarization of the vacuum will also depend on the value of Newtons' constant. Newtons' constant determines the mass of the positive and negative mass Planckions in any given era. See Equation (2-6), where this is made explicit. We believe that Newton's constant may vary cosmologically with time, and we include that possibility in this paper. The reasons for this are presented elsewhere. In a follow up paper, there is also compelling evidence for this conjecture. For the most general scaling laws for the cosmic polarization of space, we have also considered a time varying gravitational constant. Two models for, $G^{-1} = G^{-1}(a)$, were included. Model, A, has Equation (2-2), as its basis. Model, B, uses a different function to model, $G^{-1}(a)$, namely Equation (2-4). All the equations for the scaling of specific quantities in this paper, can accommodate both scenarios, a varying *G*, or a non-varying *G*. For a constant *G* value, simply let, $= G_0$, in all equations. With the help of two sets of scaling laws, one set for, $\overline{\chi}(a)$, and another set for, $G(a)$, we can predict how the polarization of space on a cosmic level will evolve, as the universe expands.

Dark matter, and to some extent, dark energy, are thought to depend on the susceptibility of the vacuum. We also have cosmic polarization, $\overline{P} = \varepsilon\chi\overline{g}$ $= \varepsilon\overline{\chi}\overline{g} = \varepsilon\overline{g^{(1)}}$, where the cosmic susceptibility and cosmic gravitational field, are smeared quantities holding for the universe as a whole. The gravitational permittivity is defined by, $\varepsilon \equiv 1/(4\pi G)$. Dark matter is given by Equation, (4-5). The scaling law is relation, (4-4). This scaling law involves, $\chi = \bar{\chi} = \bar{\chi}(a)$. Dark energy is identified as Equations (4-7) with (4-8a, b). The scaling laws, here, are Equations (4-9) and (4-10). At the end of recombination, dark matter scales numerically by the amount indicated in Equations (4-6a, b). For dark energy, in the era of last scattering, we have the specific increases over present value, specified by Equations (4-14a, b). Also, the density parameters at the end of recombination have been worked out. Their relative weightings are indicated by Equation (4-19). Although this is a somewhat unconventional prediction, we believe that it could be correct. We gave several reasons for how this could be reconciled with power spectrum data, not the least being that localized pockets of dark energy (LDM)can survive at this temperature, and, in fact, at much, much higher temperatures. There is a fundamental difference between cosmic dark matter (CDM), and, localized dark matter (LDM). Localized dark matter follows its own rules, as it is not a smeared quantity.

The transition from ordinary matter dominance to dark matter dominance in the cosmos can also be determined using our cosmic susceptibility models. We obtained either Equations (4-23a) or (4-23b), depending on the model. These values are for the universe as a whole. We also found the gravitational field scaling laws as one changes epochs. These will depend on Newtons' constant, and whether a cosmological evolution for this quantity exists. We have Equations (4-26), (4-28) and (4-30). Particular numerical values have been worked out, which hold at the end of recombination. Those results are presented in equations (4-27a, b), (4-29a, b, c, d), and (4-31a, b, c, d). These assume that G varies according to either Equations (2-8a) or (2-8b). If G does not vary, minor modifications have to be made in those equations. Finally the cosmic polarization scaling law has been ascertained. We believe that Equation (4-32), could be valid. At, $a_1 = 1100^{-1}$, the era of last photon scattering, we obtain equations, (4-33a, b, c, d). The results are surprising because there is virtually little change in cosmic polarization in that era versus the current era.

We are currently working on a detailed microscopic theory of space as it relates to positive and negative mass Planckions. Other work is in progress.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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