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Calibration of Four Nonlinear Failure Envelopes from Triaxial Test Data and Influence of Nonlinearity on Geotechnical Computations

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Abstract

It is now recognized that many geomaterials have nonlinear failure envelopes. This non-linearity is most marked at lower stress levels, the failure envelope being of quasi-parabolic shape. It is not easy to calibrate these nonlinear failure envelopes from triaxial test data. Currently only the power-type failure envelope has been studied with an established formal procedure for its determination from triaxial test data. In this paper, a simplified procedure is evolved for the development of four different types of nonlinear envelopes. These are of invaluable assistance in the evaluation of true factors of safety in problems of slope stability and correct computation of lateral earth pressure and bearing capacity. The use of the Mohr-Coulomb failure envelopes leads to an overestimation of the factors of safety and other geotechnical quantities.

Keywords

Calibration of Nonlinear Failure Envelope, Triaxial Test Data, Modified Maksimovic Envelope, Power-Type Envelope, Polynomial-Type Envelope, Hoek-Brown Envelope, Standard Error of Estimate

1. Introduction

Comprehensive literature reviews on nonlinear envelopes can be found in [1] [2] and a repetition here will be superfluous. A nonlinear or curved failure envelope is shown in Figure 1. The friction angle ϕ is a continuously varying quantity. The non-linearity of failure envelopes is most marked at lower stress levels, the failure envelope being of quasi-parabolic shape [3]. The most commonly used nonlinear

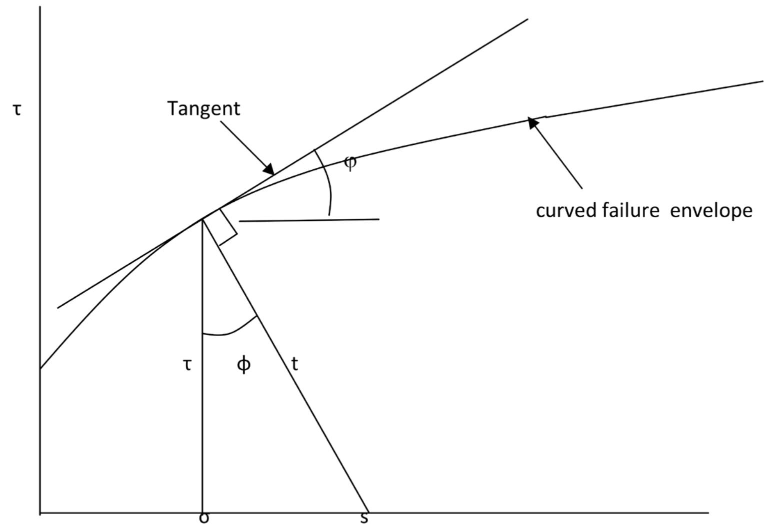

Figure 1. Curved failure envelope.

envelope is the power-type envelope $\tau = \tau_o \left(1 + \frac{\sigma'}{\sigma'_t}\right)^{1/m}$ or $\tau = \left(a + b\sigma'\right)^n$ where τ = shear stress on failure arc; σ' = effective normal stress on failure arc = $\sigma - u$; σ = total normal stress; u = pore water pressure; σ_t = tensile strength of geomaterial; τ_o = cohesion of geomaterial; m is a constant. In the alternative representation a, b, and n are constants with the case of $a = 0$ being common and denotes a purely granular geomaterial. This had been used in several studies on slope stability [4] [5] [6]. Investigations carried out by Anyaegbunam [2] revealed that the power-type equation is a valid envelope for soil for all $n > 0$ (except $n = 0.5$) if

$$a > \left[b^2 n(1-2n)\right]^{\frac{1}{2(1-n)}} \tag{1}$$

prior to this discovery the power-type equation was affirmed to be a valid envelope for $n > 0.5$ only [2] [5].

Additionally, it was discovered that the quadratic equation

$$\tau = \left(a + b\sigma'\right)^{0.5} \tag{2}$$

can only be a legitimate failure envelope if

$$a \geq \frac{b^2}{4} \tag{3}$$

a and b are parameters in the equation of quadratic failure envelope.

The aims of this manuscript are to

1) Calibrate the modified Maksimovic nonlinear failure envelope from triaxial test data;

2) Develop a simplified procedure for calibrating the polynomial type failure envelope from triaxial test data;

3) Develop a simple methodology for calibrating the power-type failure envelope from triaxial test data;

4) Produce a simpler procedure for calibrating the power-type failure envelope equivalent to the Hoek-Brown failure envelope using triaxial test data;

5) Compute the lateral earth pressure for a material with modified Maksimovic failure criterion;

6) Determine the factor of safety of a slope made of material with modified Maksimovic criterion;

7) Determine the factor of safety of a slope made of material with Hoek-Brown criterion.

This work has never been presented in the literature before. As had been mentioned real soil envelopes are nonlinear and the calibration of these nonlinear envelopes will help in determining the true response of soils. The accurate calibration of the Maksimovic failure envelope from triaxial test data has never been attempted before. The influences of nonlinearity on lateral earth pressure and factor of safety of slope with material made of modified Maksimovic law have never been attempted before and are quite difficult to implement.

2. Methodology

The calibration of a nonlinear failure envelope from triaxial test data is a problem of considerable difficulty [2] [7] [8]. To ease the calibration the formulas derived first by Balmer [9] come in handy.

They are:

$$\sigma' = \sigma_3' + \frac{\sigma_1' - \sigma_3'}{1 + \frac{\partial \sigma_1'}{\partial \sigma_3'}} \tag{4}$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{1 + \frac{\partial \sigma_1'}{\partial \sigma_3'}} \sqrt{\frac{\partial \sigma_1'}{\partial \sigma_3'}} \tag{5}$$

where σ' and τ have their previous meanings, σ_1' = effective major principal stress at failure, σ_3' = effective minor principal stress at failure.

Given a set of experimental determined (σ_3', σ_1') values then the normal stress and shear stress on the failure plane can be calculated from Equations (4) and (5) respectively.

It could be shown that

$$\frac{\partial \sigma_1'}{\partial \sigma_3'} = N_\varphi = \frac{1 + \sin \varphi'}{1 - \sin \varphi'} \tag{6}$$

where φ' = the effective instantaneous friction angle.

These authors deduced that the effective instantaneous cohesion c' is given by

$$c' = \frac{1}{2}\left[\sigma_1' \sqrt{\frac{\partial \sigma_3'}{\partial \sigma_1'}} - \sigma_3' \sqrt{\frac{\partial \sigma_1'}{\partial \sigma_3'}}\right] \tag{7}$$

Equation (7) does not seem to exist in the literature. Fu and Liao (2010) seems to have derived the effective instantaneous cohesion for Hoek-Brown criterion that requires iteration to obtain.

2.1. Calibration of Modified Maksumovic Failure Envelope

Maksimovic's [10] failure criterion has been determined to be excellent for rock-fill.

According to Srbulov [11] this failure envelope provides the best fit to experimental data over a very wide stress range unlike the power-type envelopes that give low angle of friction at large normal stresses.

Maksimovic [10] proposed a hyperbolic failure law for rock-fills that can be expressed as

$$\tau = \sigma' \tan\left(a_1 + \frac{1}{a_2 + a_3\sigma'}\right) \tag{8}$$

which has three parameters that needs to be determined. a_1, a_2 and a_3 are related to the Maksimovic's parameters by $\varphi_B = a_1$, $\Delta\varphi = 1/a_2$, $P_N = a_2/a_3$. The determination of a_1, a_2 and a_3 is not straight forward at all. The parameters were determined for the experimental data presented in test No. O of Holtz and Gibbs [12] for a sample containing 20% gravel at a relative density of 50% shown in Table 1. It was discovered that the original Maksimovic [10] law Equation (8) does not provide an excellent match to the data.

Therefore a modified Makumovic hyperbolic law, that provided an improved match, was proposed namely

$$\tau = \sigma' \tan\left(a_1 + \frac{\sigma'}{a_2 + a_3\sigma'}\right) \tag{9}$$

For a drained test on a granular material $\sigma' = \sigma$. By using Equations (4) and (5) values of σ and τ are computed in Table 1. Simple finite difference approximation has been used to compute values of $\partial\sigma_1/\partial\sigma_3$. These seem to be crude approximations but in practice have been found to give excellent results.

By substituting for the largest values of σ and τ in Table 1, a relationship is derived between the constants, namely.

$$a_1 + \frac{\sigma_m}{a_2 + a_3\sigma_m} = \arctan\left(\frac{\tau_m}{\sigma_m}\right) \tag{10}$$

with $\sigma_m = 1105.69$ and $\tau_m = 814.85$.

Table 1. First two columns: Test data O from page 20 of Holtz and Gibbs (1952).

σ_3 KNm^{-2}	σ_1 KNm^{-2}	$\partial\sigma_1/\partial\sigma_3$	σ KNm^{-2}	τ KNm^{-2}
24.2	114.8	4.37	41.02	35.16
44.9	204.90	5.14	70.94	59.07
89.0	447.8	3.76	164.35	146.14
174.6	692.8	3.63	286.49	213.21
345.7	1380	3.90	556.77	416.84
690.0	2703	3.84	1105.69	814.85

To obtain the best values of a_2 and a_3 the method of least squares is used. To implement the least squares method Equation (9) should be expressed as

$$\frac{\sigma}{\arctan\left(\frac{\tau}{\sigma}\right)-a_1}=y=a_2+a_3\sigma \quad (11)$$

Applying the method of least squares it is obtained that

$$a_2N+a_3\sum\sigma_i=\sum y_i \quad (12a)$$

$$a_2\sum\sigma_i+a_3\sum\sigma_i^2=\sum\sigma_i y_i \quad (12b)$$

where subscript i will run from 1 to N = number of data points which is 6 in this case. Equation (12) can be solved to obtain

$$a_3=\frac{N\sum\sigma y-\sum\sigma\sum y}{N\sum\sigma^2-\left(\sum\sigma\right)^2}, \quad (13a)$$

$$a_2=\frac{\sum y-a_3\sum\sigma}{N} \quad (13b)$$

The best fit values of a_2 and a_3 should be obtained as follows: A certain value of a_1 near one is chosen and then values of a_2 and a_3 are calculated as shown above. Equation (10) is used to calculate another value of a_1. if the chosen and the calculated a_1 are equal then the correct solutions have been obtained. If they are different, then, another iteration should be done.

When the above routine is implemented it is obtained that the parameters for the modified Maksimovic failure law for the triaxial data given in Table 1 are

$$a_1=1.0,\ \ a_2=-64.35,\ \ a_3=-2.6837.$$

The Mohr-Coulomb approximation to Table 1 is

$$\tau=11.42+0.7269\sigma \quad (14a)$$

with a SEE of 28.73. SEE = the standard error of estimate calculated from

$$\text{SEE}=\sqrt{\frac{1}{N}\sum\left(\sigma_{1pred}-\sigma_1\right)^2}$$

The modified Maksumovic approximation to Table 1 is

$$\tau=\sigma\tan\left(1.0-\frac{\sigma}{64.35+2.6837\sigma}\right) \quad (14b)$$

with a slightly lower SEE of 28.04 and the envelope passing through $\tau = 0$ as required. The modified Maksimovic failure envelope would give a better factor of safety when the stability of geomaterial of shallow depth is considered. In Figure 2 the M-C and M-M failure envelopes are compared. In Table 2 is shown the data for test W of Holtz and Gibbs [12] for a sample containing 20% gravel at a relative density of 70%. The Mohr-Coulomb approximation to Table 2 is

$$\tau=31.95+0.7542\sigma \quad (15a)$$

with a SEE of 31.82.

The modified Maksimovic approximation to Table 2 is

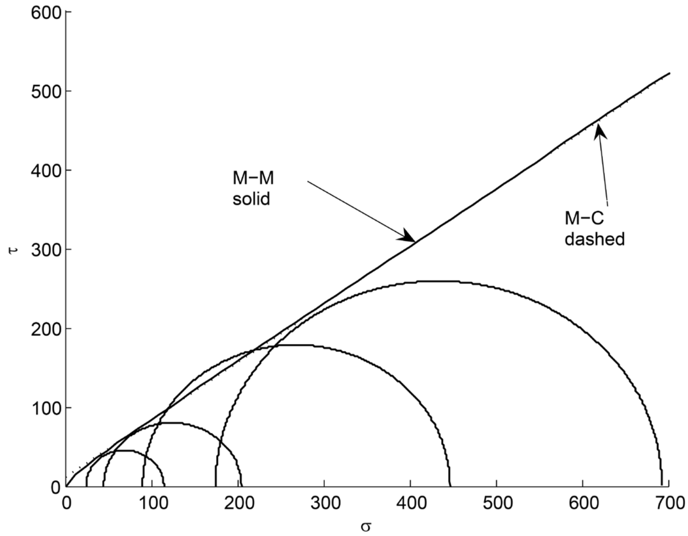

Figure 2. The M-C and M-M failure envelopes are compared wrt Mohr circles of Test data O.

Table 2. Test data W from page 21 of Holtz and Gibbs (1952).

σ_3 KNm^{-2}	σ_1 KNm^{-2}
22.8	192.5
45.5	280.1
88.3	523.7
173.9	825.9
347.0	1572
691.0	2886

$$\tau = \sigma \tan\left(1.0 - \frac{\sigma}{254.55 + 2.7241\sigma}\right) \quad (15b)$$

with a lower SEE of 23.78. This time around the difference is clear. The two envelopes are compared in **Figure 3**.

2.2. Calibration of Polynomial Failure Envelope

Yuanming *et al.* [13] derived a cubic polynomial $\sigma - \tau$ failure envelope for frozen sandy clay from $\sigma_3 - \sigma_1$ triaxial test data shown in **Table 3**. In this case effective normal stress = total normal stress. They calibrated the coefficients of the polynomial via a not really straightforward procedure. It is the purpose of this section to illustrate a much simpler procedure for doing the calibration.

Yuanming *et al.* [13]'s procedure was proceeded by a derivation by regression of a $\sigma_1 - \sigma_3$ relation (Equation (16)) that enables an analytical determination of $\partial\sigma_1/\partial\sigma_3$ but these authors used a finite difference approximation for $\partial\sigma_1/\partial\sigma_3$.

$$\sigma_1 = \left(K_o\right)^{\sigma_3/P_{atm}} \sigma_c \left(1 + \frac{\sigma_3}{\sigma_T}\right)^{b_o} \quad (16)$$

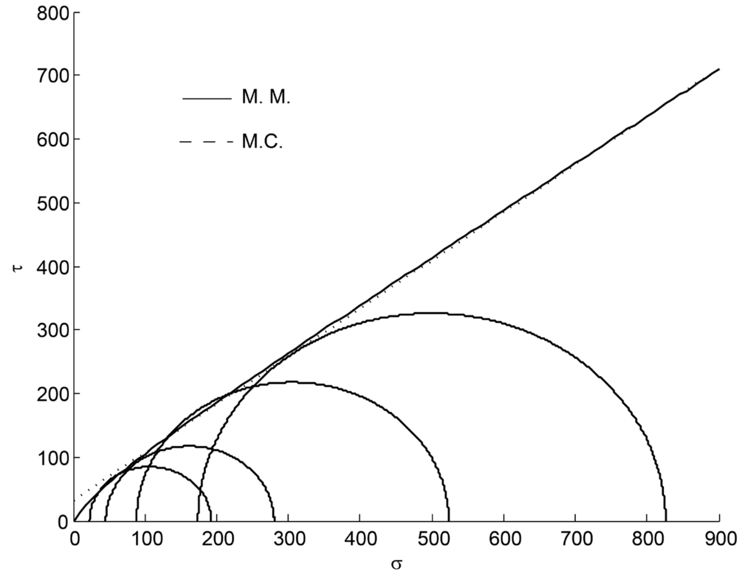

Figure 3. The M-C and M-M failure envelopes are compared wrt Mohr circles of Test data W.

Table 3. Test data for frozen sany clay at −6˚C from Yuanming *et al.* (2010) page 51.

σ_3 MNm^{-2}	σ_1 MNm^{-2}	σ_3 MNm^{-2}	σ_1 MNm^{-2}
0.0	2.285	5.0	11.877
0.3	3.289	6.0	12.924
0.6	4.155	8.0	14.924
0.8	4.726	10.0	17.161
1.0	5.308	12.0	19.381
2.0	7.392	14.0	20.795
3.0	9.541	16.0	22.571
4.0	11.140	18.0	24.953

where σ_c and σ_T are the uni-axial compressive and tensile strengths of the specimen respectively.

K_o and b_o are experimental parameters. Equation (16) is not needed in this study.

Using Equations (4) and (5) on Table 3 a set of values of σ and τ are obtained.

Let the cubic polynomial failure envelope be

$$\tau = b_1 + b_2\sigma + b_3\sigma^2 + b_4\sigma^3 \quad (17)$$

Applying the method of least squares regression directly to this the following 4 equations are obtained

$$\sum\tau = Nb_1 + b_2\sum\sigma + b_3\sum\sigma^2 + b_4\sum\sigma^3 \quad (18a)$$

$$\sum\sigma\tau = b_1\sum\sigma + b_2\sum\sigma^2 + b_3\sum\sigma^3 + b_4\sum\sigma^4 \quad (18b)$$

$$\sum\sigma^2\tau = b_1\sum\sigma^2 + b_2\sum\sigma^3 + b_3\sum\sigma^4 + b_4\sum\sigma^5 \quad (18c)$$

$$\sum \sigma^3 \tau = b_1 \sum \sigma^3 + b_2 \sum \sigma^4 + b_3 \sum \sigma^5 + b_4 \sum \sigma^6 \quad (18d)$$

where N is number of data points and the subscripts have been deleted without loss of meaning.

When Equations (18a) to (18d) are solved for the constants b_1 to b_4 it is obtained that

$$b_1 = 0.6549, b_2 = 0.6690, b_3 = -0.04815, b_4 = 1.0773 \times 10^{-3}$$

Yuanming *et al.* [13] obtained $b_1 = 0.6667$, $b_2 = 0.6290$, $b_3 = -0.0423$, $b_4 = 0.90 \times 10^{-3}$

The two sets of constants can be seen to be approximately the same.

According to this paper the polynomial envelope is

$\tau = 0.6549 + 0.6690\sigma - 0.04815\sigma^2 + 1.0773 \times 10^{-3}\sigma^3$

The standard error of estimate was determined to be SEE = 0.3727. The Mohr-Coulomb envelope is determined to be $\tau = 2.036 + 0.0969\sigma$ with an SEE = 1.1870. Thus, the Polynomial envelope provides a better fit to the experimental data. It is believed that the procedure used to do the calibration here is much more simpler and readily understood than the method of Yuanming *et al.*'s. A Q basic computer program [14] was used for doing all the relevant calculations and this program is available from the corresponding author on request. **Figure 4** shows the Mohr circles of the experimental data of **Table 3** and the associated M-C and polynomial failure envelopes.

2.3. Calibration of the Power-Type Failure Envelope

The power-type failure envelope takes the form $\tau = (a + b\sigma)^n$. A somehow complicated procedure for calibrating the power-type failure envelope has been given in Baker [1] and Anyaegbunam [2]. In **Table 4** is shown experimental data lifted from Anyaegbunam [2].

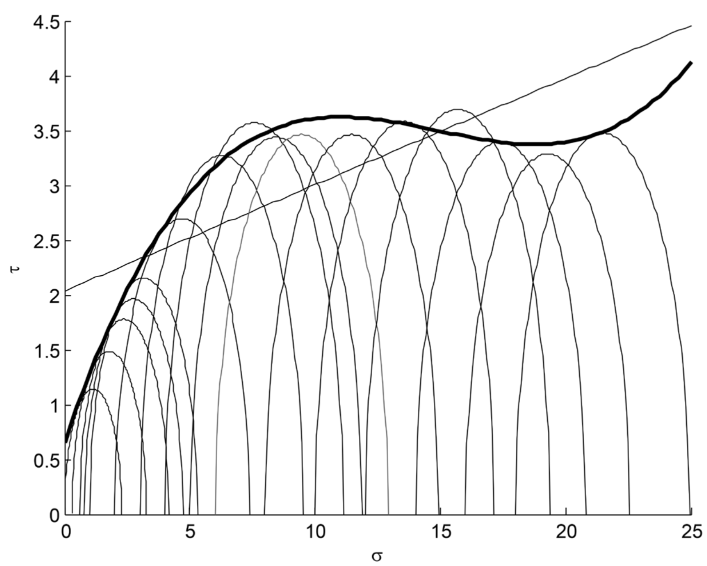

Figure 4. Mohr circles of the experimental data of **Table 3** and the associated M-C and polynomial failure envelopes.

Equations (4) and (5) are used to obtain the corresponding σ and τ values at failure. The power-type equation is then expressed as

$$a + b\sigma = \tau^{1/n} \tag{19}$$

By assuming several values of n Equation (19) is subjected to least squares regression for each value of n. The value of n that yields the minimum SEE is chosen as the correct failure envelope. This has been programmed in the afore mentioned QBASIC 4.5 program for the automatic determination of n, a and b. For the data of Table 4 it is obtained that $\tau = (0.389 + 2.61\sigma)^{0.748}$ with an SEE of 0.25 that is almost exact match to the experimental data. This agrees almost exactly with Anyaegbunam [2] that gave $\tau = (0.439 + 2.612\sigma)^{0.748}$ using a more complicated procedure. The Mohr-Coulomb envelope for the data of Table 4 is $\tau = 29.44 + 0.3812\sigma$ that corresponds to $\varphi = \arctan(0.3812) = 20.9^{\circ}$. Figure 5 shows the Mohr circles of the experimental data of Table 4 and the associated linear M-C and power-type failure envelopes.

2.4. Calibration of the Power-Type Failure Envelope for the Hoek-Brown Criterion

Hoek and Brown [15] presented a useful and practical equation for the insitu

Table 4. Consolidated undrained test data for a sample of laterite from Table 2 of Anyaegbunam (2015).

σ_3 KNm^{-2}	σ_1 KNm^{-2}
69.0	225.8
138	383.7
276	664.4

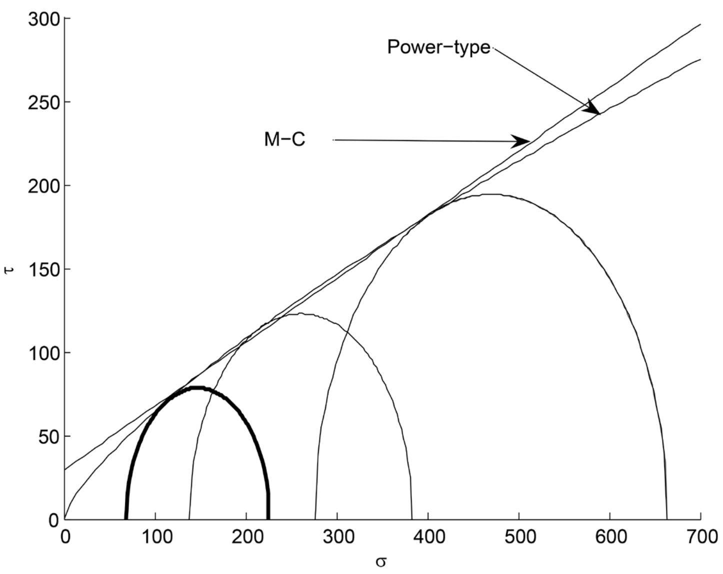

Figure 5. Mohr circles of the experimental data of Table 4 and the associated linear M-C and power-type failure envelopes.

strength of rock mass. The 2002 version of this equation [16] (the modified Hoek-Brown criterion) is:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(\frac{m_b\sigma_3}{\sigma_{ci}} + s_b\right)^{\eta} \tag{20}$$

where $m_b = m_i \exp\left(\frac{\mathrm{GSI}-100}{28-14D}\right)$, $s_b = \exp\left(\frac{\mathrm{GSI}-100}{9-3D}\right)$

$$\eta = \frac{1}{2} + \frac{1}{6}\left[\exp\left(-\mathrm{GSI}/15\right) - \exp\left(-20/3\right)\right]$$

where GSI is the Geological Strength Index, D is the disturbance factor, σ_{ci} = the compressive strength of the intact rock, m_i is a material constant of the intact rock.

The Hoek-Brown criterion can be expressed simply as

$$\sigma_1 = \sigma_3 + \left(\kappa\sigma_3 + \beta\right)^{\eta} \tag{21}$$

where $\kappa = m_b \sigma_{ci}^{\frac{1}{\eta}-1}$ and $\beta = s_b \sigma_{ci}^{\frac{1}{\eta}}$

Letting $F = \kappa\sigma_3 + \beta$ and $den = \kappa\eta + 2F^{1-\eta}$ (22a, b)

then it can be shown that

$$\sigma = \sigma_3 + \frac{F}{den} \quad \text{and} \quad \tau = \frac{F^{\frac{1+\eta}{2}}}{den}\sqrt{den - F^{1-\eta}} \tag{23a, b}$$

Equations (23a, b) are the exact parametric (implicit) equations of the Mohr envelope of the H-B criterion. These equations with σ_3 as the paramaeter are not very much useful in practice because the value of τ are not easily calculated from that of σ, vice versa. Hence, it is necessary to determine a single equation connecting τ and σ.

The H-B criterion utilized herein have the following constants σ_{ci} = 40 MN/m^2, m_i = 10, GSI = 45, D = 0.9 with the rest calculated to be m_b = 0.281, η = 0.508, s_b = 1.616 × 10^{-4}.

The Hoek-Brown equation cannot be directly used in slope stability analysis because it is defined in terms of principal stresses. Considerable difficulty is encountered when Hoek-Brown equation is used directly in strength-reduction finite element type of slope stability analysis [17] [18] hence it will useful to obtain its Mohr envelope for use in limit equilibrium analysis. Baker [1] shows that the power-type failure envelope $\tau = \left(a + b\sigma\right)^{n}$ provides an excellent fit to the Hoek-Brown criterion in the Mohr (σ-τ) plane. Therefore the Mohr envelope obtained in this section is of the power-type. Equation (21) has been used to derive **Table 5** and Equations (23a and b) was used in the program to obtain values of σ and τ.

Figure 6 shows the Mohr circles of the H-B criterion and the associated Power-type envelope and M-C envelope. The derived power-type envelope has the equation $\tau = \left(353.3 + 22.343\sigma\right)^{0.684}$ with a SEE of 6.63. The Mohr-Coulomb envelope has the equation $\tau = 159.86 + 0.8014\sigma$ or $\tau = 159.86 + \sigma\tan\left(38.71^{\circ}\right)$ with

Table 5. Data for Hoek-Brown criterion specified in the text.

σ_3 KNm^{-2}	σ_1 KNm^{-2}
0.0	474.2
20.0	671.6
40.0	831.1
80.0	1095.5
160.0	1519.8
300.0	2114.8
440.0	2619.1
614.0	3176.5

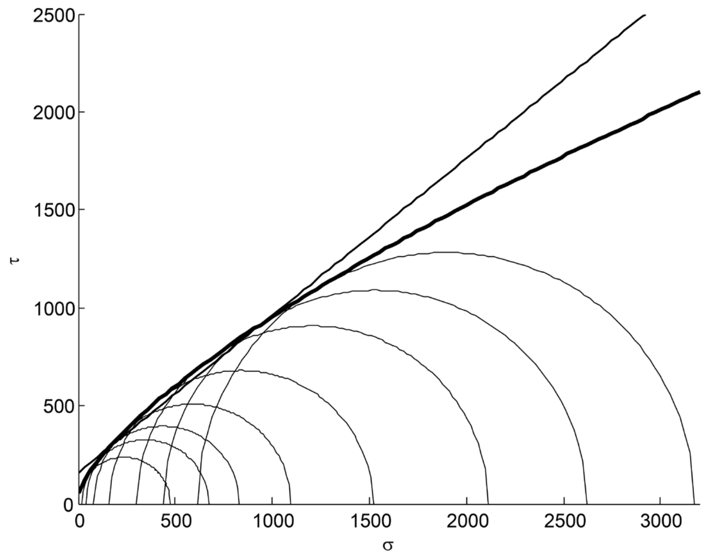

Figure 6. Mohr circles of the H-B criterion and comparison with linear M-C and nonlinear power-type failure envelopes.

a SEE = 123.94. The envelopes derived in Anyaegbunam [2] namely $\tau = (357.302 + 22.337\sigma)^{0.684}$ with a SEE = 5.77 and the M-C fit of c = 157.0 KN/m^2 and φ = 38.7˚ with a SEE = 131.77 can be seen to be in excellent agreement. The method used in this paper can be seen to be much simpler and readily understood than Anyaegbunam [2]. Deng *et al.* [17] presented an approximate limit equilibrium technique for slope stability analysis using the Hoek-Brown criterion.

3. Influence of Nonlinearity of Failure Envelope on Geotechnical Computations

3.1. Passive Pressure on a Smooth Wall Due to Soil with Modified Maksimovic Law Derived from Table 2

This shall be illustrated for a 2.0 m smooth high wall that is a portion of a wall embedded in homogeneous soil of unit weight γ = 18 KN/m^3. The vertical stress, which is the minimum principal stress, is given by $\sigma_3 = \gamma z(i)$ where $z(i)$ = vertical distance from the ground surface. Assume that the wall height is divided into a number of divisions such that σ_3 can be denoted by $\sigma_3(i)$ and that the number of divisions is num. On the basis of a Mohr-Coulomb law the passive pressure at a given point is given by $\sigma_{1MC}(i)=\sigma_3(i)N_\varphi+2C\sqrt{N_\varphi}$. where C and φ are the M-C shear strength parameters and $N_\varphi=\tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$. C = 31.95 KN/m^2, tanφ = 0.7542, a_1 = 1.0, a_2 = −254.55, a_3 = −2.7241.

The pseudo code for evaluating the passive pressure according to the modified Maksimovic law will be as follows:

Dimension the variables

set i = 1: eps = 0.001

450 compute $\sigma_3(i)$: set $\sigma_3=\sigma_3(i)$' 450 is line numbering in the code

$$\text{compute } \sigma_{1MC}(i)\text{: set } \sigma_{1MC}=\sigma_{1MC}(i) \tag{24}$$

Let σ_n = normal stress on the failure arc. Estimate σ_n as σ_{n1}.

$$\sigma_{n1}=0.45\left[(\sigma_{1MC}+\sigma_3)-(\sigma_{1MC}-\sigma_3)\sin\varphi_{MC}\right]$$

where φ_{MC} = M-C friction angle φ

$$500 \quad \tau=\sigma_{n1}\tan\left(a_1+\frac{\sigma_{n1}}{a_2+a_3\sigma_{n1}}\right)$$

Let $der=\frac{d\tau}{d\sigma}$

$$der=\frac{\tau}{\sigma_{n1}}+\frac{a_2\sigma_{n1}}{(a_2+a_3\sigma_{n1})^2}\left[1+\frac{\tau^2}{\sigma_{n1}^2}\right]$$

$$t=\tau\sqrt{1+der^2}$$

From Figure 1 it could be shown that

$$\sigma_{n2}=\sigma_n=\sigma_3+t-\sqrt{t^2-\tau^2}$$

If i = num + 1 then END

If $|\sigma_{n2}-\sigma_{n1}|>eps$ then $\sigma_{n1}=\sigma_{n2}$: goto 500

If $|\sigma_{n2}-\sigma_{n1}|\le eps$ then $\sigma_1(i)=\sigma_3+2t$: $i=i+1$: goto 450

In Figure 7 are shown the Passive pressures calculated for M-C and M. M. material having test data W.

The M-C soil has a passive force and moment of 401.4 KN/m and 353.1 KNm/m at the base of the wall. The M.M. soil has a passive force and moment of 284.3 KN and 203.5 KNm/m at the base of the wall which are 29.2% and 42.4% less than the M-C values respectively. This is usually the case for short walls that are less than 6.0 m in height.

3.2. Factor of Safety of a Slope with Material Governed by a Modified Maksimovic Law and Comparison with M-C Equivalent

It is proposed to determine the factor of safety of a 45˚ homogeneous slope of

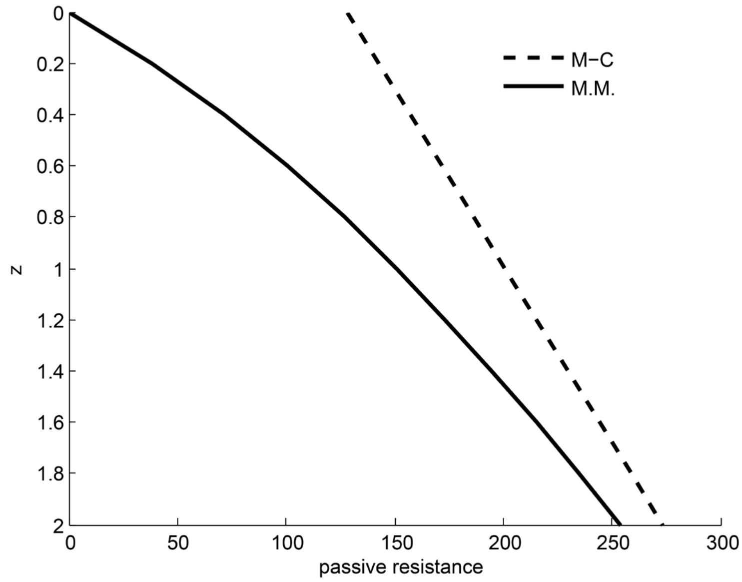

Figure 7. Passive Pressures of soil with M-C and M.M. envelopes for test data W.

height 10 m via a Maksimovic material model and M-C material model using the Bishop simplified technique [19].

The material models were those derived from the triaxial data of **Table 2** and are follows: For the M-C model: $C = 31.95\ \text{KN/m}^2$, $\phi = 37.02^\circ$ and for the M. M. model: $a_1 = 1.0$, $a_2 = -254.55$, $a_3 = -2.7241$.

After the soil mass is divided into vertical slices the Bishop method for M-C material is given by

$$F_s = \frac{1}{\sum \gamma h_i \sin \alpha_i} \sum \frac{c_i' + (\gamma h_i - u_i)\tan \varphi'}{\cos \alpha_i \left(1 + \frac{\tan \alpha_i \tan \varphi'}{F_s}\right)} \tag{25}$$

where F_s = factor of safety, h_i = midheight of slice i, α_i = inclination of base of slice i, u = pore pressure on base of slice i.

The Bishop method for modified Maksimovic (M.M.) material is given by

$$F_s = \frac{1}{\sum \gamma h_i \sin \alpha_i} \sum \frac{\sigma_i'}{\cos \alpha_i} \tan\left[a_1 + \frac{\sigma_i'}{a_2 + a_3 \sigma_i'}\right] \tag{26}$$

where σ_i' is obtained for each slice from the equation

$$\sigma_i' + \tan \alpha_i \frac{\sigma_i'}{F_s} \tan\left[a_1 + \frac{\sigma_i'}{a_2 + a_3 \sigma_i'}\right] - (\gamma h_i - u_i) = 0 \tag{27}$$

The factor of safety F_s is determined by assuming an initial value of F_s and calculating σ_i' from Equation (27). Thereafter σ_i' are substituted into Equation (26) to re-calculate F_s. The calculation of F_s using the Bishop method on a M.M. material model does not exist in the literature.

Using the M-C material model and 10 slices, the following results are obtained: $F_s = 2.40$ with the critical toe circle of radius 14.75 m centered at (−0.64, 14.74) with a central angle of 68.8˚. Using the M.M. material model the $F_s = 1.64$

Table 6. Comparison of F_s via Deng *et al.*'s H-B LEM and this paper's H-B power-type LEM Bishop method.

H	β	Deng's method	This paper's method
15	30˚	2.570	2.586
15	45˚	1.905	1.903
15	60˚	1.482	1.453

with the critical toe circle of radius 16.87 m centered at (−4.46, 16.27) with a central angle of 52.8˚. The use of the M.M. material model is seen to have a strong influence on the calculated factor of safety which is seen to be smaller.

Unfortunately the M.M. law has not been used for stability calculations by other authors and comparison of the results herein cannot be compared with previous results.

3.3. Factor of Safety of Slope Using Deng *et al.*'s Approximate LEM with Hoek-Brown Parameters Compared with This Paper's Rigorous Hoek-Brown Method

Deng *et al.* [17] developed an approximate limit equilibrium method (LEM) with Hoek–Brown parameters for determining the factor of safety of a rock slope.

The H-B parameters of the rock slope are $D = 0$, $GSI = 100$, $m_i = 10$, $\sigma_{ci} = 140$ KNm^{-2}, $\alpha = \eta = 0.5$. m_b and s_b are calculated from Equation (20). The rock unit weight is $\gamma = 23$ KN/m^3.

$$\text{Also,}\quad \sigma_{cm} = \sigma_{ci}\frac{\left[m_b + 4s_b - \alpha\left(m_b - 8s_b\right)\right]\left[\frac{m_b}{4} + s_b\right]^{\alpha-1}}{2\left(1+\alpha\right)\left(2+\alpha\right)} \tag{28}$$

$$\sigma_{3\max} = 0.72\sigma_{cm}\left(\frac{\sigma_{cm}}{\gamma H}\right)^{-0.91} \tag{29}$$

σ_3 values are listed from values of 0 to $\sigma_{3\max}$ and the H-B relation is used to calculate corresponding values of σ_1. From these the power-type equivalent to this Hoek-Brown law is deduced by section (3.4) to be $\tau = \left(134.53 + 14.431\sigma\right)^{0.643}$. The derived SEE = 1.26. In **Table 6** is shown the results of factors of safety of slope stability obtained via the two methods. The F.S. for a slope using the power-type envelope was obtained using a modification of the method of Charles and Soares [20]. The F.S. of a slope using Deng *et al.*'s method was obtained using Equation (18) of their paper. The calculated factors of safety can be seen to be close except when $\beta > 60˚$ when they differ appreciably as Deng *et al.* discovered.

4. Conclusion

Four Mohr failure envelopes have been presented and the methods of for deriving them from experimental triaxial test data have been explained. The method for computing the passive resistance for granular soil with M.M. (modified Maksimovic) envelope is presented. In addition, the influence of a M.M. envelope on the factor of safety of slope is presented and it is shown that the use of a M.M. material model results in a significant reduction in the calculated factor of safety. Also, it is shown that the factors of safety of rock slope obtained by this paper's power-type approximation and LEM are close to Deng *et al.*'s factors of safety using H-B approximation and LEM. A computer program written in QBASIC version 4.5 for doing all the calibrations has been developed by the authors and is available on request from the corresponding author. This program is a useful contribution to geotechnical engineering practice.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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