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Mathematical Modeling of the Bulk Density Property of Knitted Fabrics

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Abstract

The paper focuses on the prediction of the volumetric density property of knitting fabrics using mathematical modeling. Based on the graphical analysis of the obtained mathematical models, the results of the raw material properties for the futer knitted fabric on the values of alternative indicators are presented.

1. Introduction

In the development of Uzbekistan, the demand for the production of knitting goods is growing and improving day by day. This, of course, in turn requires caution to increase the range of new types of knitwear, to obtain new textures, to produce competitive products.

The main purpose of theoretical and experimental research is to compare the results of the experiment with the ideas put forward by the working hypothesis. Different criteria are used to compare theoretical (according to the working hypothesis) and experimental data [1] [2] [3] [4]. For example, the minimum, average, and maximum deviations obtained from experimental data given due to

calculations based on theoretical dependencies. However, the most reliable are the same (appropriate) criteria related to the experimental theory [1].

Comparing the working hypothesis with the experimental data, the following cases can be observed:

- The working hypothesis is confirmed in full or almost complete experiment. In this case, the working hypothesis is a theoretical rule, proven by theory;
- The working hypothesis is partially confirmed in the experiment; in other cases it contradicts it. In this case, the working hypothesis is modified for work that is fully or almost completely adapted to the experimental result. A corrective experiment is performed to confirm the change in the working hypothesis. The hypothesis then becomes a theory, as in the first case [2] [3].
- The working hypothesis is not confirmed experimentally. In this case, the previously accepted hypothesis is fully considered, *i.e.* a new one is developed. Negative scientific results allow to narrow the scope of the search for a new hypothesis.

Once the hypothesis is recognized as a theoretical rule, the conclusions (or) suggestions are expressed, that is, the new essence obtained as a result of the research is put forward. The amount of the main conclusions should not exceed 5 - 10. In addition to the basic conclusions, other conclusions can be drawn in some cases. All conclusions are divided into two groups: scientific and production. Scientific findings show the contribution of novelty, which will be introduced into science as a result of the research done [4] [5] [6]. The conclusions of production are related to profits, which are provided by experiments conducted in the field of economics.

It is advisable to make effective use of multifactor mathematical planning methods in the study of the parameters that determine the structure of the futer knitted fabric. In the production of foot-knitted fabrics, it is observed that the efficiency of work is influenced by factors that are not related to each other at different levels at the same time. Futer has shown that in the production of fabrics, mainly the linear density of the back yarn, the type of weaving and the raw material, *i.e.* the fiber content of the yarn, have a predominant effect [7] [8] [9] [10].

Futer knitted fabrics used in enterprises have some fiber components (polyester, lycra) mainly due to imports from abroad, and as a raw material, it consists almost entirely of polyester and lycra fiber yarn, which leads to an increase in body price. The research was conducted to improve the quality of knitted fabrics using local raw cotton fibers (partly polyester and lycra) in order to reduce costs and improve quality. The study examines the factors influencing the quality of futer fabric, such as air permeability, moisture absorption, thickness, tensile strength, abrasion resistance, deformation and volume densities, and studies to find the optimal values of these factors are the amount of polyester and lycra in the knitted fabric.

The variation of the surface density of futer fabrics depends on the linear den-

sity of the yarns, the type of weaving, the coefficient, the density on the body and the back. If the surface density is at the required level of binding and filling coefficient in the direction of the body and back system yarns, the penetration of futer knitted fabrics after washing will be different. Therefore, the surface density property of the futer knitted fabric is the most important indicator.

2. Research Planning Matrix and Processing of Results

The front length of the loop strip of knitted fabric.

$$X_1 = 1.5 \div 3.5 \text{ mm}$$

Knitted fabric loop step.

$$X_2 = 0.4 \div 0.6 \text{ mm}$$

Changes in the fiber content of fabric yarns (Table 1).

$$X_3 = 5 \div 25\%$$

Output parameter:

Y_1-Dimensional density, mg/cm^3.

On a general basis, we move from the natural values of the factors to the coded values. From the TOT results, it became clear that the process under study was represented by a higher-level equation. Therefore, in order to obtain a secondary regression mathematical model, the central non-composite experience Central Failure Positional Experiment (CNCE) was selected and implemented, which is much simpler and more convenient than other methods, as well as widely used in the study of technological processes in the textile industry.

The CNCE working matrix and the results of the experiments are given in the following Table 2.

Based on the experimental results, we look for a secondary regression multivariate mathematical model. As a result of this experiment we can obtain the following general regression model:

$$Y_R = b_0 + \sum_{i=1}^{M} b_i x_i + \sum_{\substack{i=j=1 \\ j \neq 1}}^{n} b_{ij} x_i x_j + \sum_{i=1}^{M} b_{ii} x_i^2 \text{ ;}$$

Table 1. The factors under study are the selection of levels and intervals of change.

Name and designation of factors	Change levels			Change interval
	−1	0	+1	
The front length of the loop strip of knitted fabric	1.5	2.5	3.5	1
Knitted fabric loop step	0.4	0.5	0.6	0.1
Changes in the fiber content of fabric yarns	95% cotton, 5% laycra	75% cotton, 23% poly, 2% laycra	75% cotton, 25% poly	-

Table 2. Central non-composite experimental matrix.

N°	Factors			x_1x_2	x_1x_3	x_2x_3	x_1^2	x_2^2	x_3^2	Y_1	$S_u^2(Y_1)$
	x_1	x_2	x_3								
1	+	+	0	+	0	0	+	+	0	202	11.0
2	+	-	0	-	0	0	+	+	0	180	9.0
3	-	+	0	-	0	0	+	+	0	175	9.0
4	-	-	0	+	0	0	+	+	0	160	12.0
5	+	0	+	0	+	0	+	0	+	245	8.0
6	+	0	-	0	-	0	+	0	+	211	12.0
7	-	0	+	0	-	0	+	0	+	189	11.0
8	-	0	-	0	+	0	+	0	+	176	8.0
9	0	+	+	0	0	+	0	+	+	209	9.0
10	0	+	-	0	0	-	0	+	+	211	11.0
11	0	-	+	0	0	-	0	+	+	196	8.0
12	0	-	-	0	0	+	0	+	+	169	9.0
13	0	0	0	0	0	0	0	0	0	206	7.0
14	0	0	0	0	0	0	0	0	0	214	5.0
15	0	0	0	0	0	0	0	0	0	211	15.0

The values (+), (-) and 0 should be used in the matrix

Or, because of the three factors are involved in our experience, the above expression takes the following form:

$$Y_R = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2$$

In the equation:

$b_0, b_1, \cdots$: Regression coefficients,

x_1, x_2, x_3: Intended values of factors.

Determine the regression coefficients:

$$b_0 = \frac{1}{N_s}\sum_{u=1}^{N_s} \overline{Y_u} = \frac{1}{3}(206 + 214 + 211) = 212;$$

$$b_i = g_3 \sum_{u=1}^{N} x_{iu}\overline{Y_u}$$

To calculate the regression coefficients, we use the values from literature A. G. Sevostyanov "Methods and means of research of mechanical technological processes of textile and light industry" [11].

$$g_1 = 0.2; \quad g_2 = 0.166;$$

$$g_3 = 0.125; \quad g_4 = 0.25;$$

$$g_5 = 0.125; \quad g_6 = 0.0625;$$

$$g_7 = 0.3125;$$

$$b_1 = 0.125(200 + 180 - 175 - 160 + 245 + 211 - 189 - 176) = 17.25;$$

$$b_2 = 0.125(200 - 180 + 175 - 160 + 209 + 211 - 196 - 169) = 11.5;$$

$$b_3 = 0.125(245 - 211 + 189 - 176 + 209 - 211 + 196 - 169) = 18;$$

$$b_{ij} = g_4 \sum_{u=1}^{N} x_{iu} x_{ju} \overline{Y}_u;$$

$$b_{12} = 0.25(202 - 180 - 175 + 160) = 1.75;$$

$$b_{13} = 0.25(245 - 211 - 189 + 176) = 5.26;$$

$$b_{23} = 0.25(209 - 211 - 196 + 169) = -3.625;$$

$$b_{ii} = g_5 \sum_{u=1}^{N} x_{iu}^2 \overline{Y}_u + g_6 \sum_{i=1}^{M} \sum_{u=1}^{N} x_{iu}^2 \overline{Y}_u - g_2 \sum_{u=1}^{N} \overline{Y}_u;$$

$$\sum x_1^2 \overline{Y}_u = 202 + 180 + 175 + 160 + 245 + 211 + 189 + 176 = 1538;$$

$$\sum x_2^2 \overline{Y}_u = 202 + 180 + 175 + 160 + 209 + 211 + 196 + 169 = 1502;$$

$$\sum x_3^2 \overline{Y}_u = 245 + 211 + 189 + 176 + 209 + 211 + 196 + 169 = 1606;$$

$$\sum \overline{Y}_u = 202 + 180 + 175 + 160 + 245 + 211 + 189 + 176 + 209$$
$$+ 211 + 196 + 169 + 206 + 214 + 211 = 2959;$$

$$\sum_{i=1}^{M} x_i^2 \overline{Y}_u = 1538 + 1502 + 1606 = 4646;$$

$$b_{11} = 0.125 \times 1538 + 0.0625 \times 4646 - 0.166 \times 2959 = -8.57;$$

$$b_{22} = 0.125 \times 1502 + 0.0625 \times 4646 - 0.166 \times 2959 = -13.07;$$

$$b_{33} = 0.125 \times 1606 + 0.0625 \times 4646 - 0.166 \times 2959 = -0.07.$$

Taking into account the determined regression coefficients, we write the equation:

$$Y_R = 212 + 17.25x_1 + 11.5x_2 + 18x_3 + 1.75x_1x_2 + 5.26x_1x_3$$
$$- 3.625x_2x_3 - 8.57x_1^2 - 13.07x_2^2 - 0.07x_3^2.$$

We determine the significance of the regression coefficients.

For this, we determine the variance of the output parameters.

$$S^2\{Y\} = S_m^2\{Y\} = \frac{1}{N_s - 1} \sum_{u=1}^{N_s} S^2\{\overline{Y}\};$$

$$S^2\{\overline{Y}\} = \frac{1}{3-1} \times 27 = 13.5;$$

and on this basis we calculate the variance in determining the regression coefficients:

$$S^2\{b_0\} = g_1 S^2\{\overline{Y}\} = 0.2 \times 13.5 = 2.7;$$

$$S^2\{b_i\} = g_3 S^2\{\overline{Y}\} = 0.125 \times 13.5 = 1.69;$$

$$S^2\{b_{ij}\} = g_4 S^2\{\overline{Y}\} = 0.25 \times 13.5 = 3.38;$$

$$S^2 \{b_{ii}\} = g_7 S^2 \{\bar{Y}\} = 0.3125 \times 13.5 = 4.22.$$

We find the standard deviation in determining the regression coefficients:

$$S \{b_0\} = 1.64;$$

$$\{b_i\} = 1.3;$$

$$S \{b_{ij}\} = 1.84;$$

$$S \{b_{ii}\} = 2.05.$$

We then determine the calculated value of the Student's Criterion using the following equation:

$$t_R \{b_i\} = \frac{|b_i|}{S \{b_i\}};$$

$$t_R \{b_0\} = \frac{|212|}{1.64} = 129; \quad t_R \{b_{12}\} = \frac{|1.75|}{1.84} = 0.95;$$

$$t_R \{b_1\} = \frac{|17.25|}{1.3} = 13.3; \quad t_R \{b_{13}\} = \frac{|5.25|}{1.84} = 8.86;$$

$$t_R \{b_2\} = \frac{|11.5|}{1.3} = 8.85; \quad t_R \{b_{23}\} = \frac{|-3.625|}{1.84} = 1.97;$$

$$t_R \{b_3\} = \frac{|18|}{1.3} = 13.9; \quad t_R \{b_{11}\} = \frac{|-8.57|}{2.05} = 4.17;$$

$$t_R \{b_{22}\} = \frac{|-2.97|}{2.05} = 3.505; \quad t_R \{b_{33}\} = \frac{|-0.07|}{2.05} = 0.03.$$

We get table value of the Student criterion from Appendix 3 [11]:

$$t_j \left[P_D = 0.95; f \{S_s^2\} = 3 - 1 = 2 \right] = 2.77.$$

It is known that if the calculated value of the criterion is less than the table value, that coefficient is not significant and we subtract it from equation [11]. In the study, it was found that the coefficients b_{12}, b_{23}, b_{33} were insignificant for the parameters under study:

We rewrite the equation with significant coefficients:

$$Y_R = 212 + 17.25x_1 + 11.5x_2 + 18x_3 + 5.26x_1 x_3 - 8.57x_1^2 - 13.07x_2^2$$

In order to check whether the above regression mathematical model is adequate or not, we determine it using the calculated value of the Fisher criterion:

$$F_R = \frac{S_{nad}^2 \{Y\}}{S^2 \{\bar{Y}\}};$$

here:

$$S^2 \{\bar{Y}\} = \frac{\sum_{u=1}^{N} S^2 \{Y\}}{N_s - 1} = \frac{27}{3 - 1} = 13.5;$$

$$S_{nad}^2 \{Y\} = \frac{\sum_{u=1}^{N-N_s+1} \left(Y_{Ru} - \bar{Y}_u\right)^2}{N - N_{k.en} - \left(N_s - 1\right)^2};$$

$$N - N_{k.en} - \left(N_s - 1\right)^2 = 15 - 7 - \left(3 - 1\right)^2 = 4;$$

$$N - N_s + 1 = 15 - 3 + 1 = 13;$$

$$\sum_{u=1}^{N-N_s+1} \left(Y_{Ru} - \overline{Y}_u\right)^2 = 1859;$$

$$S_{nad}^2 \{Y\} = \frac{144}{4} = 36.$$

It is known that, if the calculated value of the criterion is less than the value of **Table 3**, that coefficient is adequate and proves that the calculations were performed correctly [12].

$$F_R = \frac{S_{nad}^2 \{Y\}}{S^2 \{\overline{Y}\}} = \frac{36}{13.5} = 2.67;$$

$$F_j \left[P_D = 0.95; f \left\{ S_{nad}^2 \{Y\} \right\} = 15 - 6 - \left(3 - 1\right) = 5; f \left\{ S_u^2 \right\} = 3 - 1 = 2 \right] = 4.74;$$

$$F_R = 2.67 < 4.74 = F_j.$$

Hence, the obtained regression mathematical model represents the studied process with sufficient accuracy:

$$Y_R = 212 + 17.25x_1 + 11.5x_2 + 18x_3 + 5.26x_1x_3 - 8.57x_1^2 - 13.07x_2^2.$$

Since the equation constructed to determine the volumetric density as the outgoing parameter for the study is three-dimensional, one of the input factors in the analysis is the two-dimensional graph of the field of change for two factors of value $X_i = 0$, we build.

Table 3. Simplifying the calculations.

N°	$\overline{Y}_u$	Y_{Ru}	$\left(Y_{Ru} - \overline{Y}_u\right)$	$\left(Y_{Ru} - \overline{Y}_u\right)^2$
1	202	219.11	17.11	292.82
2	180	196.11	16.11	259.60
3	175	184.61	9.61	92.39
4	160	161.61	1.61	2.60
5	245	238.68	−6.32	39.9
6	211	202.68	−8.32	69.2
7	189	204.18	15.18	230.5
8	176	168.18	−7.82	61.1
9	209	228.43	19.43	377.6
10	211	192.43	−18.57	344.8
11	196	205.43	9.43	88.9
12	169	169.43	0.43	0.2

The third factor for the analysis of the mathematical model is the appearance of our mathematical model, taking the average ($x_3 = 0$, 75% cotton, 23% poly, 2% laycra) state of the values of change in the fiber content of the yarns of knitted fabric.

$$Y_R = 212 + 17.25x_1 + 11.5x_2 - 8.57x_1^2 - 13.07x_2^2$$

As can be seen from the graph above (**Figure 1**), when the unwanted first (x_1) and second (x_2) factors change from the accepted minimum (−1), to the maximum (1), and the third factor ($x_3 = 0$, 75% cotton, 23% poly, 2% laycra) is averaged using (Y_1) volumetric density values are described. Using the drawing (X_1) knit fabric loop length front length in the range of 0.25 0.5 0.75, *i.e.* at a value of 0.525 ÷ 0.575 mm, and (X_2) factor knitting ring pitch 0.25 - 0.5 at intermediate values (Y_R) reach the highest values of bulk density.

1) The second factor is the appearance of our mathematical model, taking the average ($x_2 = 0$, 0.5 mm) state of the surface density of knitted fabric as follows.

$$Y_R = 212 + 17.25x_1 + 18x_3 + 5.26x_1x_3 - 8.57x_1^2$$

As can be seen from the diagram (**Figure 2**), the front length of the loop yarn (X_1) is in the range of 0.25, 0.5, 0.75, *i.e.* 0.525 ÷ 0.575 mm, and the factor (X_3) is the change in the fibrous composition of the yarn in the fabric −1 interval. In other words, 75% cotton and 25% poly fiber have the highest bulk density (Y_1).

The first factor is the appearance of our mathematical model, taking the state of the average position ($x_1 = 0$, 2.5 mm) of the amount of the front length of the loop strip of knitted fabric.

2) $Y_R = 212 + 11.5x_2 + 18x_3 - 13.07x_2^2$

As can be seen from the diagram (**Figure 3**), the front length of the loop thread of knitted fabric (X_2) is in the range of 1.0 *i.e.* 0.6 mm, and the change in the fiber content of (X_3) knitted yarn is in the range of −1 at 75% cotton, 25% poly fiber (Y_1).
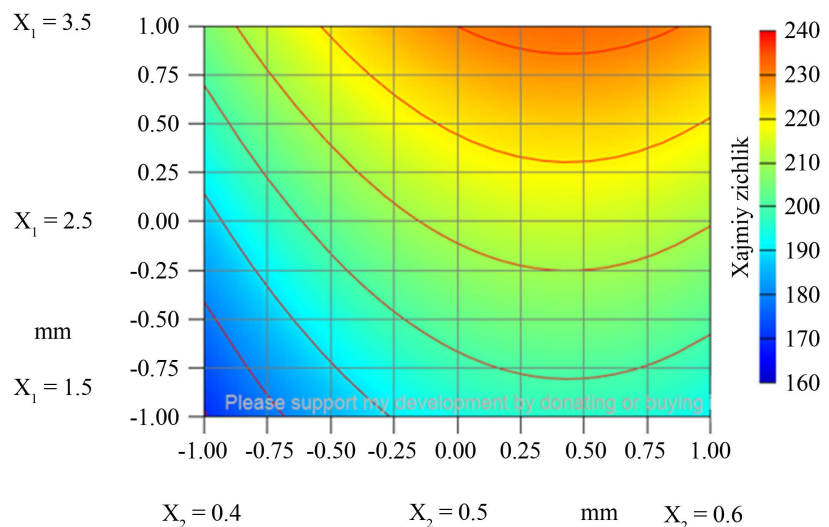

Figure 1. Values of the bulk density indicator when $x_3 = 0$ using a mathematical model.
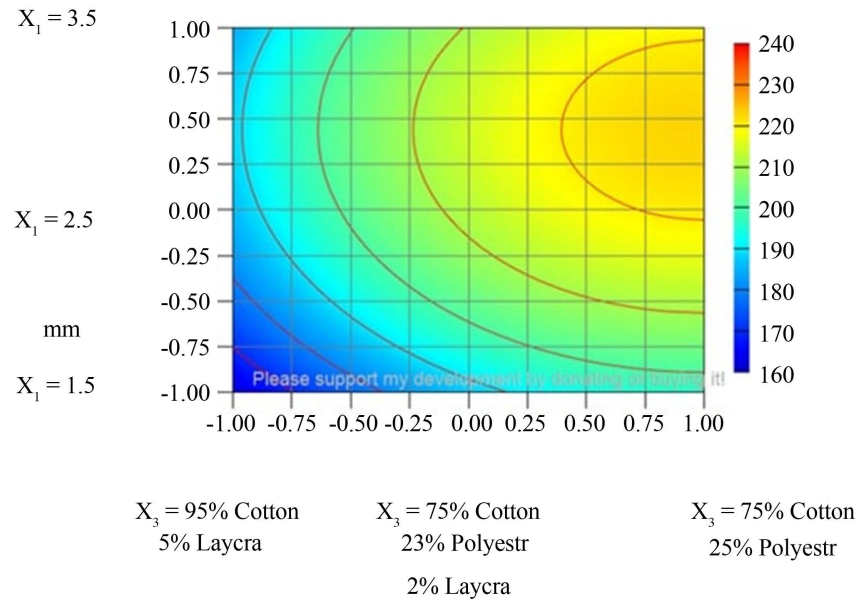
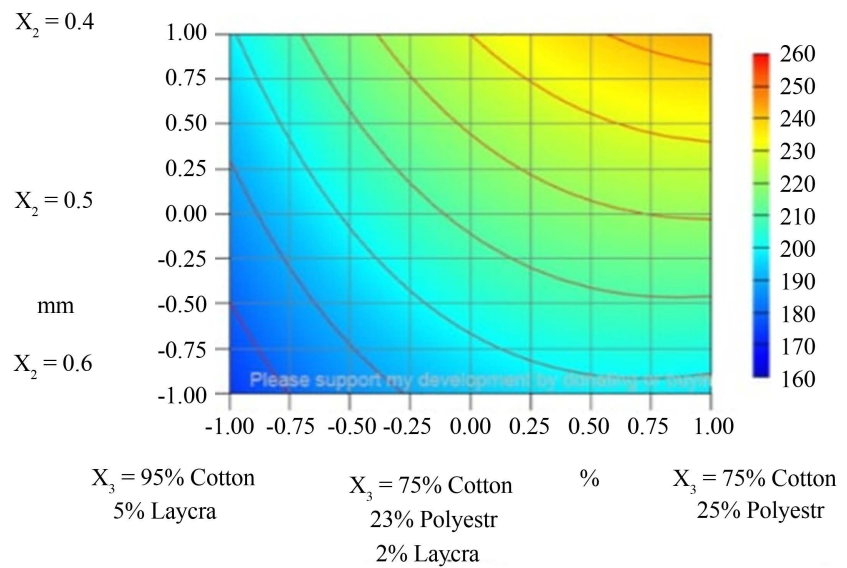**Figure 2.** The value of the bulk density indicators when $x_2 = 0$ using a mathematical model.

Figure 3. Values of the bulk density indicator on $x_1 = 0$ using a mathematical model.

In this research work, the results of the study are summarized in 3 cases (X_1) knitted fabric in the range of 0.25. 0.5. 0.75 *i.e.* 0.525 ÷ 0.575 mm, (X_2) knitted fabric loop yarn front length in the range of 1.0 *i.e.* 0.6 mm, and (X_3) knitted fabric yarn fiber content change in the range −1, *i.e.* 75% cotton, 25% poly fiber (Y_1) achieves the highest values of bulk density.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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