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The Adomian Decomposition Method for Solving Nonlinear Partial Differential Equation Using Maple

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Abstract

The nonlinear partial differential equation is solved using the Adomian decomposition method (ADM) in this article. A number of examples have been provided to illustrate the numerical results, which is the comparison of the exact and numerical solutions, and it has been discovered through the tables that the amount of error between the exact and numerical solutions is very small and almost non-existent, and the graph also shows how the exact solution of absolutely applies to the numerical solution. This demonstrates the precision of the Adomian decomposition method (ADM) for solving the nonlinear partial differential equation with Maple18. And that in terms of obtaining numerical results, this approach is characterized by ease, speed, and high accuracy.

Keywords

Nonlinear Partial Differential Equation, Adomian Decomposition Method, Maple18

1. Introduction

The aim of this study is to use Maple18 to solve the Volterra-Fredholm integral equation with the Adomian decomposition process. Integral equations are fundamental sciences in our everyday lives; they describe physical, chemical, engineering, and medical phenomena, and they also help us find analytical and numerical solutions to these problems.

A reliable Modification of Adomain Decomposition Method [1]. A new Modification of the Adomain Decomposition Method for Linear and Nonlinear Operators [2]. Lorenz equations are solved using a decomposition method [3]. Using Adomian's decomposition method to solve the Riccati differential equation [4].

Numeric-analytic integration of strongly nonlinear and chaotic oscillators using Adomian decomposition [5]. For fourth-order boundary value problems, the extended Adomian decomposition method [6]. The Adomian decomposition approach has been used to solve multipoint boundary value problems [7]. A new algorithm for evaluating Adomian polynomials has been developed [8]. An efficient algorithm for the multivariable Adomian polynomials [9]. Convenient analytic recurrence algorithms for the Adomian polynomials [10]. A review of the Adomian decomposition method and its applications to fractional differential equations are discussed in this paper [11]. A bibliography of the Adomian decomposition method's theory and applications [12]. Nonlinear integral equation solutions are more difficult to solve than linear integral equation solutions [13], and there are several analytical and computational methods for solving both linear and nonlinear integral equations [14]-[19].

MATLAB and Maple were used to implement the Adomian decomposition method for the Fredholm integral equation of the second kind. To solve the Fredholm integral equation of the second kind [20], the Adomian decomposition method was employed. In addition, using Maple, a Modified research approach for solving the Volterra integral equation of the second kind [21]. The Adomian Decomposition Method for Solving Volterra-Fredholm Integral Equation Using Maple [22].

In this article, we used the Maple algorithm to apply the Adomian decomposition method to various cases, such as finding the approximate solution, comparing it to the exact solution, and determining the amount of error between the approximate solution and the exact solution.

2. Adomian Decomposition Method

Consider the nonlinear partial differential equation given in an operator form

$$L_x u(x,y) + L_y u(x,y) + R(u(x,y)) + F(u(x,y)) = g(x,y) \quad (1)$$

where L_x is the highest order differential in x, L_y is the highest order differential in y, R contains the remaining linear terms of lower derivatives, $F(u(x,y))$ is an analytic nonlinear term, and $g(x,y)$ is an inhomogeneous or forcing term.

Assuming that the operator L_x meets the two bases of selection, therefore we set

$$L_x u(x,y) = g(x,y) - L_y u(x,y) - R(u(x,y)) - F(u(x,y)) \quad (2)$$

Applying L_x^{-1} to both sides of (1) gives

$$u(x,y) = \varnothing_0 - L_x^{-1} g(x,y) - L_x^{-1} L_y u(x,y) - L_x^{-1} R(u(x,y)) - L_x^{-1} R(u(x,y)), \quad (3)$$

$$\varnothing_0 = \begin{cases} u(0,y) & \text{for } L = \dfrac{\partial}{\partial x} \\ u(0,y) + xu_x(0,y) & \text{for } L = \dfrac{\partial^2}{\partial x^2} \\ u(0,y) + xu_x(0,y) + \dfrac{1}{2!}x^2 u_{xx}(0,y) & \text{for } L = \dfrac{\partial^3}{\partial x^3} \\ u(0,y) + xu_x(0,y) + \dfrac{1}{2!}x^2 u_{xx}(0,y) + \dfrac{1}{3!}x^3 u_{xxx}(0,y) & \text{for } L = \dfrac{\partial^4}{\partial x^4} \end{cases}$$

We proceed in exactly the same manner by calculating the solution $u(x,y)$. In a series form

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y), \tag{4}$$

And the nonlinear term

$$F(u(x,y)) = \sum_{n=0}^{\infty} A_n,$$

where A_n are Adomian polynomials that can be generated for all forms of nonlinearity. Based on these assumptions, Equation (2) becomes

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x,y) = \varnothing_0 - L_x^{-1} g(x,y) - L_x^{-1} L_y \left(\sum_{n=0}^{\infty} u_n(x,y)\right) \\ - L_x^{-1} R\left(\sum_{n=0}^{\infty} u_n(x,y)\right) - L_x^{-1}\left(\sum_{n=0}^{\infty} A_n\right), \end{aligned} \tag{5}$$

The components, $u_n(x,y), n \geq 0$ of the solution $u(x,y)$ can be recursively determined by using the relation

$$u_0(x,y) = \varnothing_0 - L_x^{-1} g(x,y),$$

$$u_{k+1}(x,y) = -L_x^{-1} L_y u_k(x,y) - L_x^{-1} R(u_k(x,y)) - L_x^{-1} A_k, \quad k \geq 0$$

Using the algorithms described before for calculating A_n for the nonlinear term $F(u)$.

The first few components can be identified by

$$u_0(x,y) = \varnothing_0 - L_x^{-1} g(x,y),$$

$$u_1(x,y) = -L_x^{-1} L_y u_0(x,y) - L_x^{-1} R(u_0(x,y)) - L_x^{-1} A_0,$$

$$u_2(x,y) = -L_x^{-1} L_y u_1(x,y) - L_x^{-1} R(u_1(x,y)) - L_x^{-1} A_1,$$

$$u_3(x,y) = -L_x^{-1} L_y u_2(x,y) - L_x^{-1} R(u_2(x,y)) - L_x^{-1} A_2,$$

$$u_4(x,y) = -L_x^{-1} L_y u_3(x,y) - L_x^{-1} R(u_3(x,y)) - L_x^{-1} A_3,$$

where each components can be determined by using the preceding component. Having calculated the components $u_n(x,y)$.

3. Numerical Examples

In this section, we solve some examples, and we can compare the numerical results with the exact solution.

Example 1. Consider the nonlinear partial differential equation

$$u_t + uu_x = 0,\ u(x,0) = x,\ t > 0, \tag{6}$$

the exact Solution $u(x) = \frac{x}{1+t}, |t| < 1$.

Applying the Adomian decomposition method using Maple18.

Example 2. Consider the nonlinear partial differential equation.

$$u_t + uu_x = 1 + t \cdot \cos x + \sin x \cos x,\ u(x,0) = \sin x \tag{7}$$

the exact Solution $u(x) = t + \sin x$.

Applying the Adomian Decomposition Method using Maple18.

Example 3. Consider the nonlinear partial differential equation

$$u_t = x^2 + \frac{1}{4}u_x^2,\ u(x,0) = 0 \tag{8}$$

the exact Solution $u(x) = x^2 \tan t$.

Applying the Adomian Decomposition Method using Maple18.

Example 4. Consider the nonlinear partial differential equation

$$u_t + \frac{1}{36}xu_{xx}^2 = x^3,\ u(x,0) = 0, \tag{9}$$

the exact Solution $u(x,t) = x^3 \tanh t$.

Applying the Adomian Decomposition Method using Maple.

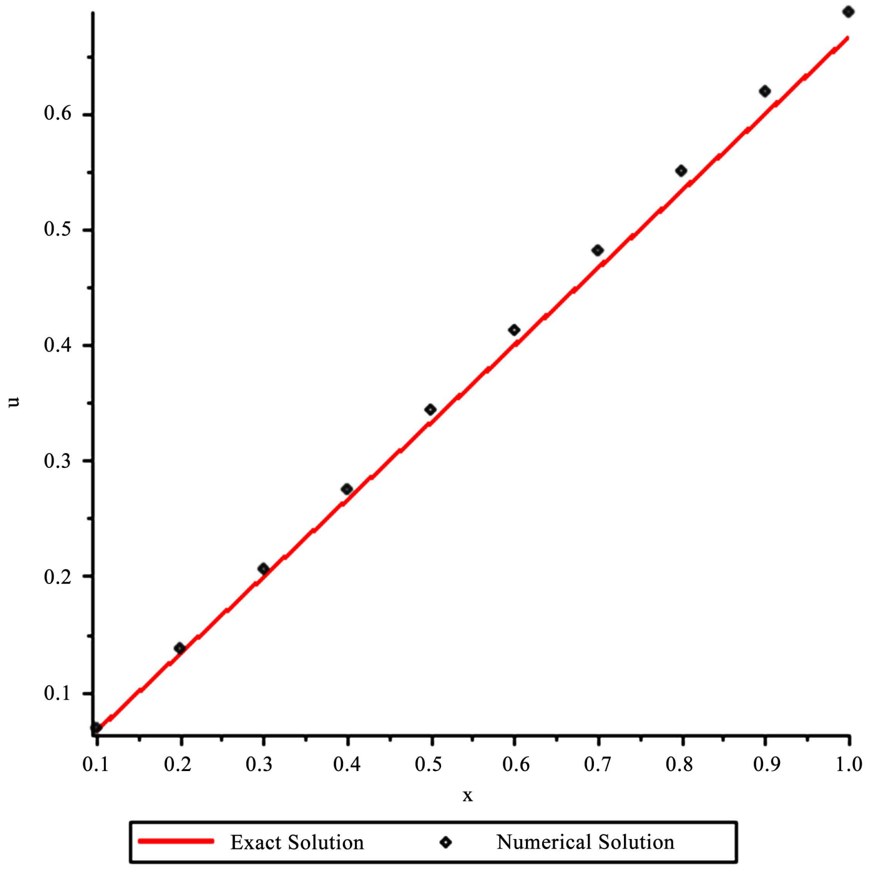

Figure 1. Plot of the solutions of nonlinear partial differential equation for example 1.

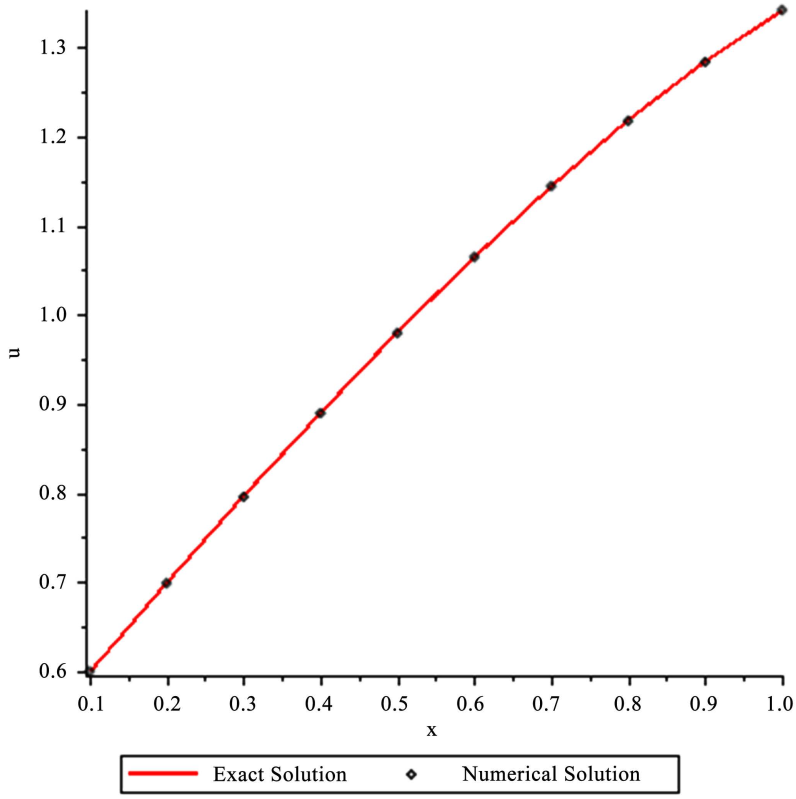

Figure 2. Plot of the solutions of nonlinear partial differential equation for example 2.

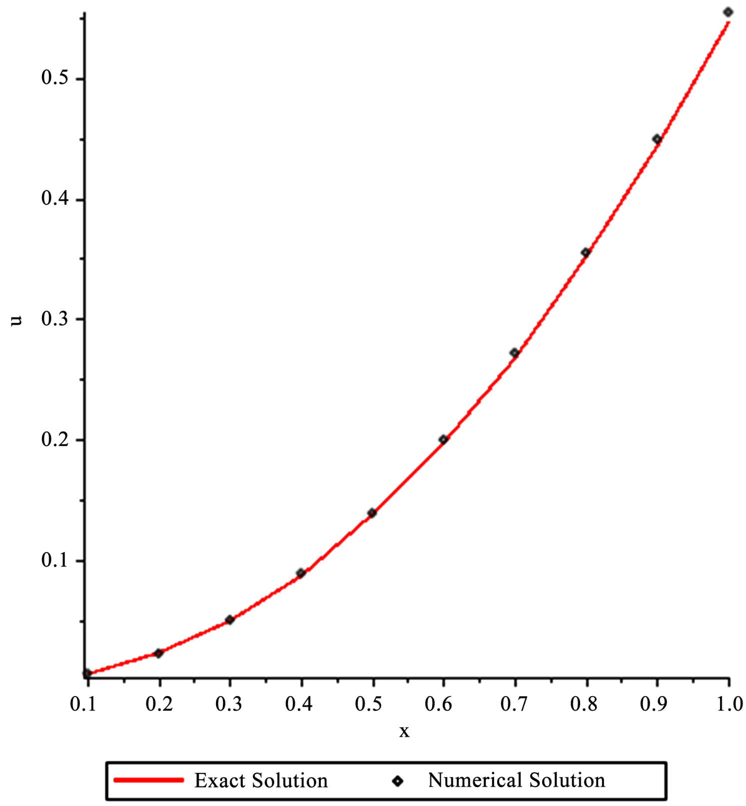

Figure 3. Plot of the solutions of nonlinear partial differential equation for example 3.

Table 1. Approximation solution and exact solution of nonlinear partial differential equation for example 1.

x	u	$Exact2 = \frac{x}{1+t}$	$Error = \lvert Exact1 - u \rvert$
0.10000	0.0666667	0.0687500	0.0020833
0.20000	0.1333333	0.1375000	0.0041667
0.30000	0.2000000	0.2062500	0.0062500
0.40000	0.2666667	0.2750000	0.0083333
0.50000	0.3333333	0.3437500	0.0104167
0.60000	0.4000000	0.4125000	0.0125000
0.70000	0.4666667	0.4812500	0.0145833
0.80000	0.5333333	0.5500000	0.0166667
0.90000	0.6000000	0.6187500	0.0187500
1.00000	0.6666667	0.6875000	0.0208333

Table 2. Approximation solution and exact solution of nonlinear partial differential equation for example 2.

x	u	$Exact2 = \sin(x)$	$Error = \lvert Exact2 - u \rvert$
0.10000	0.5998334	0.5998334	0.0000000
0.20000	0.6986693	0.6986693	0.0000000
0.30000	0.7955202	0.7955202	0.0000000
0.40000	0.8894183	0.8894187	0.0000003
0.50000	0.9794255	0.9794271	0.0000015
0.60000	1.0646425	1.0646480	0.0000055
0.70000	1.1442177	1.1442339	0.0000162
0.80000	1.2173561	1.2173973	0.0000412
0.90000	1.2833269	1.2834208	0.0000938
1.00000	1.3414710	1.3416667	0.0001957

Table 3. Approximation solution and exact solution of Volterra Fredholm integral equations for example 3.

x	u	$Exact3 = x^2 \tan t$	$Error = \lvert Exact3 - u \rvert$
0.10000	0.0054630	0.0055459	0.0000829
0.20000	0.0218521	0.0221835	0.0003314
0.30000	0.0491672	0.0499129	0.0007457
0.40000	0.0874084	0.0887341	0.0013257
0.50000	0.1365756	0.1386471	0.0020715
0.60000	0.1966689	0.1996518	0.0029829
0.70000	0.2676882	0.2717483	0.0040600
0.80000	0.3496336	0.3549365	0.0053029
0.90000	0.4425050	0.4492165	0.0067115
1.00000	0.5463025	0.5545883	0.0082858

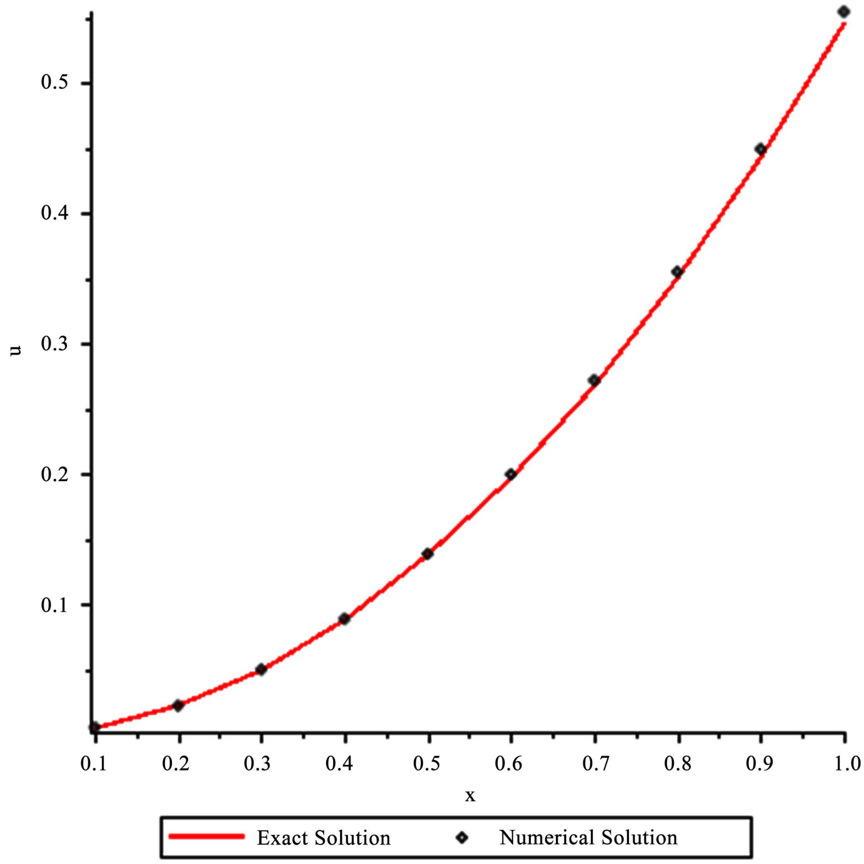

Figure 4. Plot of the solutions of of nonlinear partial differential equation for example 4.

Table 4. Approximation solution and exact solution of nonlinear partial differential equation for example 4.

x	u	$Exact4 = x^3 \tanh t$	$Error = \|Exact4 - u\|$
0.10000	0.0004621	0.0004704	0.0000083
0.20000	0.0036969	0.0037633	0.0000664
0.30000	0.0124772	0.0127011	0.0002240
0.40000	0.0295755	0.0301063	0.0005309
0.50000	0.0577646	0.0588015	0.0010368
0.60000	0.0998173	0.1016089	0.0017916
0.70000	0.1585062	0.1613512	0.0028450
0.80000	0.2366040	0.2408508	0.0042468
0.90000	0.3368834	0.3429301	0.0060467
1.00000	0.4621172	0.4704117	0.0082945

4. Conclusion

In this paper, the Adomian decomposition method is applied to solve the nonlinear partial differential equation using Maple18 software. Results were obtained by tables and drawing with figures. The exact solution and numerical solution are shown in **Tables 1-4** and **Figures 1-4**. By comparing the numerical results, we find that the numerical solution is widely applied to the precise solution, which proves the efficiency of the method used and the ability to obtain the numerical solution corresponding to the precise solution easily and conveniently using Maple 18 software. Moreover, high accuracy of the results obtained.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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