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Symmetrical Distribution of Primes and Their Gaps

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Abstract

Primes are of great importance and interest in mathematics partially due to their hard-to-predict distribution. A corollary of the Goldbach Conjecture is that two primes are equally distanced from a mid-point integer. Here the authors demonstrate that most primes are bilateral symmetrically distributed on the both sides of the halves of super products (or their integer multiples) of primes. This pattern suggests that greater primes may be obtained more efficiently by subtracting smaller ones from constants equal to super products (or their integer multiples) of primes.

Keywords

Prime Number, Distribution, Bilateral Symmetry, Super Product, Pairwise

1. Introduction

Primes appear to distribute randomly and they catch much attention from mathematicians for long time [1]-[11]. The Goldbach Conjecture states that every even number greater than 4 is a sum of two primes. A corollary of the conjecture is that every single integer greater than 3 is equally distanced from two primes, implying that at least two primes are paired on both sides of an integer. This pattern has been documented in previous publications [10] [12] [13]. However, how many prime pairs there are on both sides of an integer remains to be an unanswered question. Trying to answer this question, we found that more primes tend to be paired on both sides of the halves of super products (or their integer multiples) of primes. Many mathematicians have noted the existence of prime gaps as well as twin primes (which have a difference of 2 in between) [5] [14], but whether there is any regularity about the occurrence of such gaps remains an open question. Here we demonstrate that the pairwise (bilateral symmetrical) distributions of primes and their gaps near the super products (or its integer multiples), hoping it will trigger more interesting investigations.

2. Methods

Initially, the target of our investigation was to figure out how many prime pairs have the same sum. The statistics indicated pair number peaks at super products of primes (or their integer multiples). We analyzed and proved the rationality underlying these peaks, and proved the existence of gaps around super products of primes (or their integer multiples) and validated a routine generating primes.

3. Results

After we manually obtained number of prime pairs for every even number under 220 (**Table 1**) and did statistics (**Figure 1**), it became obvious that there are local peaks at 30, 60, 90, 120 …, namely, the number peaks periodically.

4. Theoretical Analysis and Proof

The nth prime is denoted as P_n. The product of the first $n - 1$ primes is designated as Super Product of P_n, and denoted as X_n [9].

Theorem 1. There are at most two primes, namely, $X_n - 1$ and $X_n + 1$, in $(X_n - P_n, X_n + P_n)$, and these two, if both valid, constitute twin primes.

Proof.

1) By definition, X_n is a composite.

2) Since $a \mid X_n$ and $a \nmid 1$ for all $a \in \{P_i \mid 1 \le i < n\}$, therefore $a \nmid (X_n - 1)$ and $a \nmid (X_n + 1)$. As $(X_n + 1) = (X_n - 1) + 2$, so if both $(X_n - 1)$ and $(X_n + 1)$ are primes, they constitute twin primes.

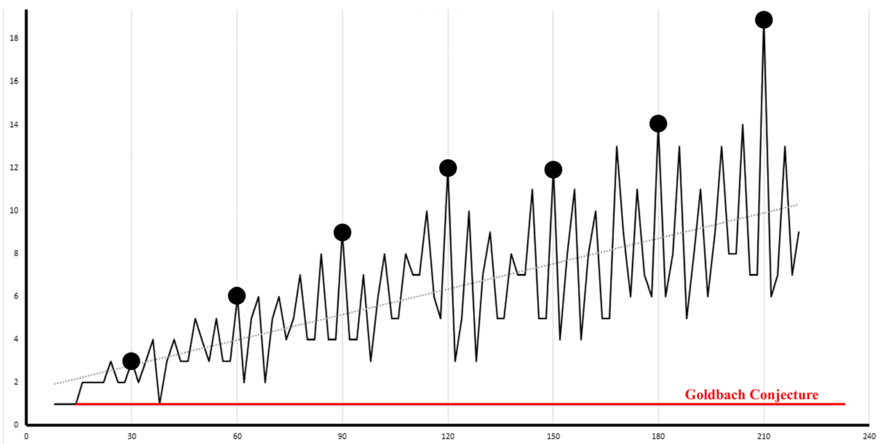

Figure 1. Number of prime pairs for each even number under 220 demonstrates a general increasing trend, and it peaks periodically at the multiples of 30. Note that all the numbers are equal to or greater than 1, the number required by the Goldbach Conjecture.

Table 1. The first 107 even numbers and the paired primes.

Sums	Paired primes (#1 + #2)	#pairs
220	23 + 197, 29 + 191, 41 + 179, 47 + 173, 53 + 167, 71 + 149, 83 + 137, 89 + 131, 107 + 113	9
218	7 + 211, 19 + 199, 37 + 181, 61 + 157, 67 + 151, 79 + 139	6
216	5 + 211, 17 + 199, 19 + 197, 23 + 193, 37 + 179, 43 + 173, 53 + 163, 59 + 157, 67 + 149, 79 + 137, 89 + 127, 103 + 113, 107 + 109	13
214	3 + 211, 17 + 197, 23 + 191, 41 + 173, 47 + 167, 83 + 131, 101 + 113	7
212	13 + 199, 19 + 193, 31 + 181, 61 + 151, 73 + 139, 103 + 109	6
210	11 + 199, 13 + 197, 17 + 193, 19 + 191, 29 + 181, 31 + 179, 37 + 173, 43 + 167, 47 + 163, 53 + 157, 59 + 151, 61 + 149, 71 + 139, 73 + 137, 79 + 131, 83 + 127, 97 + 113, 101 + 109, 103 + 107	19
208	11 + 197, 17 + 191, 29 + 179, 41 + 167, 59 + 149, 71 + 137, 101 + 107	7
206	7 + 199, 13 + 193, 43 + 163, 67 + 139, 69 + 137, 79 + 127, 97 + 109	7
204	5 + 199, 7 + 197, 11 + 193, 13 + 191, 23 + 181, 31 + 173, 37 + 167, 41 + 163, 47 + 157, 53 + 151, 67 + 137, 73 + 131, 97 + 107, 101 + 103	14
202	3 + 199, 5 + 197, 11 + 191, 23 + 179, 29 + 173, 53 + 149, 71 + 131, 89 + 113	8
200	3 + 197, 7 + 193, 19 + 181, 37 + 163, 43 + 157, 61 + 139, 73 + 127, 97 + 103	8
198	5 + 193, 7 + 191, 17 + 181, 19 + 179, 31 + 167, 41 + 157, 47 + 151, 59 + 139, 61 + 137, 67 + 131, 71 + 127, 89 + 109, 97 + 101	13
196	3 + 193, 5 + 191, 17 + 179, 23 + 173, 29 + 167, 47 + 149, 59 + 137, 83 + 113, 89 + 107	9
194	3 + 191, 13 + 181, 31 + 163, 37 + 157, 43 + 151, 67 + 127	6
192	11 + 181, 13 + 179, 19 + 173, 29 + 163, 41 + 151, 43 + 149, 53 + 139, 61 + 131, 79 + 113, 83 + 109, 89 + 103,	11
190	11 + 179, 17 + 173, 23 + 167, 41 + 149, 53 + 137, 59 + 131, 83 + 107, 89 + 101	8
188	7 + 181, 31 + 157, 37 + 151, 61 + 127, 79 + 109	5
186	5 + 181, 7 + 179, 13 + 173, 19 + 167, 23 + 163, 29 + 157, 37 + 149, 47 + 139, 59 + 127, 73 + 113, 79 + 107, 83 + 103, 89 + 97	13
184	3 + 181, 5 + 179, 11 + 173, 17 + 167, 47 + 137, 53 + 131, 71 + 113, 83 + 101	8
182	3 + 179, 19 + 163, 31 + 151, 43 + 139, 73 + 109, 79 + 103	6
180	7 + 173, 13 + 167, 17 + 163, 23 + 157, 29 + 151, 31 + 149, 41 + 139, 43 + 137, 53 + 127, 67 + 113, 71 + 109, 73 + 107, 79 + 101, 83 + 97	14
178	5 + 173, 11 + 167, 29 + 149, 41 + 137, 47 + 131, 71 + 107	6
176	3 + 173, 13 + 163, 19 + 157, 37 + 139, 67 + 109, 73 + 103, 79 + 97	7
174	7 + 167, 11 + 163, 17 + 157, 23 + 151, 37 + 137, 43 + 131, 47 + 127, 61 + 113, 67 + 107, 71 + 103, 73 + 101	11
172	5 + 167, 23 + 149, 41 + 131, 59 + 113, 71 + 101, 83 + 89	6
170	3 + 167, 7 + 163, 13 + 157, 19 + 151, 31 + 139, 43 + 127, 61 + 109, 67 + 103, 73 + 97	9
168	5 + 163, 11 + 157, 17 + 151, 19 + 149, 29 + 139, 31 + 137, 37 + 131, 41 + 127, 59 + 109, 61 + 107, 67 + 101, 71 + 97, 79 + 89	13
166	3 + 163, 17 + 149, 29 + 137, 53 + 113, 59 + 107	5
164	7 + 157, 13 + 151, 37 + 127, 61 + 103, 67 + 97	5
162	5 + 157, 11 + 151, 13 + 149, 23 + 139, 31 + 131, 53 + 109, 59 + 103, 61 + 101, 73 + 89, 79 + 83	10
160	3 + 157, 11 + 149, 23 + 137, 29 + 131, 47 + 113, 53 + 107, 59 + 101, 71 + 89	8
158	7 + 151, 19 + 139, 31 + 127, 61 + 97	4
156	5 + 151, 7 + 149, 17 + 139, 19 + 137, 29 + 127, 43 + 113, 47 + 109, 53 + 103, 59 + 97, 67 + 89, 73 + 83	11
154	3 + 151, 5 + 149, 17 + 137, 23 + 131, 41 + 113, 47 + 107, 53 + 101, 71 + 83	8

Continued

152	3 + 149, 13 + 139, 43 + 109, 73 + 79	4
150	11 + 139, 13 + 137, 19 + 131, 23 + 127, 37 + 113, 41 + 109, 43 + 107, 47 + 103, 53 + 97, 61 + 89, 67 + 83, 71 + 79	12
148	11 + 137, 17 + 131, 41 + 107, 47 + 101, 59 + 89	5
146	7 + 139, 19 + 127, 37 + 109, 43 + 103, 67 + 79	5
144	5 + 139, 7 + 137, 13 + 131, 17 + 127, 31 + 113, 37 + 107, 41 + 103, 43 + 101, 47 + 97, 61 + 83, 71 + 73	11
142	3 + 139, 5 + 137, 11 + 131, 29 + 113, 41 + 101, 53 + 89, 59 + 83	7
140	3 + 137, 13 + 127, 31 + 109, 37 + 103, 43 + 97, 61 + 79, 67 + 73	7
138	7 + 131, 11 + 127, 29 + 109, 31 + 107, 37 + 101, 41 + 97, 59 + 79, 67 + 71	8
136	5 + 131, 23 + 113, 29 + 107, 47 + 89, 53 + 83	5
134	3 + 131, 7 + 127, 31 + 103, 37 + 97, 61 + 73	5
132	5 + 127, 19 + 113, 23 + 109, 29 + 103, 31 + 101, 43 + 89, 53 + 79, 59 + 73, 61 + 71	9
130	3 + 127, 17 + 113, 23 + 107, 29 + 101, 41 + 89, 47 + 83, 59 + 71	7
128	19 + 109, 31 + 97, 61 + 67	3
126	13 + 113, 17 + 109, 19 + 107, 23 + 103, 29 + 97, 37 + 89, 43 + 83, 47 + 79, 53 + 73, 59 + 67	10
124	11 + 113, 17 + 107, 23 + 101, 41 + 83, 53 + 71	5
122	13 + 109, 19 + 103, 43 + 79	3
120	7 + 113, 11 + 109, 13 + 107, 17 + 103, 19 + 101, 23 + 97, 31 + 89, 37 + 83, 41 + 79, 47 + 73, 53 + 67, 59 + 61	12
118	5 + 113, 11 + 107, 17 + 101, 29 + 89, 47 + 71	5
116	3 + 113, 7 + 109, 13 + 103, 19 + 97, 37 + 79, 43 + 73	6
114	5 + 109, 7 + 107, 11 + 103, 13 + 101, 17 + 97, 31 + 83, 41 + 73, 43 + 71, 47 + 67, 53 + 61	10
112	3 + 109, 5 + 107, 11 + 101, 23 + 89, 29 + 83, 41 + 71, 53 + 59	7
110	3 + 107, 7 + 103, 13 + 97, 31 + 79, 37 + 73, 43 + 67, 47 + 63	7
108	5 + 103, 7 + 101, 11 + 97, 19 + 89, 29 + 79, 37 + 71, 41 + 67, 47 + 61	8
106	3 + 103, 5 + 101, 17 + 89, 23 + 83, 47 + 59	5
104	3 + 101, 7 + 97, 31 + 73, 37 + 67, 43 + 61	5
102	5 + 97, 13 + 89, 19 + 83, 23 + 79, 29 + 73, 31 + 71, 41 + 61, 43 + 59	8
100	3 + 97, 11 + 89, 17 + 83, 29 + 71, 41 + 59, 47 + 53	6
98	19 + 79, 31 + 67, 37 + 61	3
96	7 + 89, 13 + 83, 17 + 79, 23 + 73, 29 + 67, 37 + 59, 43 + 53	7
94	5 + 89, 11 + 83, 23 + 71, 41 + 53	4
92	3 + 89, 13 + 79, 19 + 73, 31 + 61	4
90	7 + 83, 11 + 79, 17 + 73, 19 + 71, 23 + 67, 29 + 61, 31 + 59, 37 + 53, 43 + 47	9
88	5 + 83, 17 + 71, 29 + 59, 41 + 47	4
86	3 + 83, 7 + 79, 13 + 73, 19 + 67	4
84	5 + 79, 11 + 73, 13 + 71, 17 + 67, 23 + 61, 31 + 53, 37 + 47, 41 + 43	8
82	3 + 79, 11 + 71, 23 + 59, 29 + 53	4
80	7 + 73, 13 + 67, 19 + 61, 37 + 43	4

Continued

78	5 + 73, 7 + 71, 11 + 67, 17 + 61, 19 + 59, 31 + 47, 37 + 41	7
76	3 + 73, 5 + 71, 17 + 59, 23 + 53, 29 + 47	5
74	3 + 71, 7 + 67, 13 + 61, 31 + 43	4
72	5 + 67, 11 + 61, 13 + 59, 19 + 53, 29 + 43, 31 + 41	6
70	3 + 67, 11 + 59, 17 + 53, 23 + 47, 29 + 41	5
68	7 + 61, 31 + 37	2
66	5 + 61, 7 + 59, 13 + 53, 19 + 47, 23 + 43, 29 + 37	6
64	3 + 61, 5 + 59, 11 + 53, 17 + 47, 23 + 41	5
62	3 + 59, 19 + 43	2
60	7 + 53, 13 + 47, 17 + 43, 19 + 41, 23 + 37, 29 + 31	6
58	5 + 53, 11 + 47, 17 + 41	3
56	3 + 53, 13 + 43, 19 + 37	3
54	7 + 47, 11 + 43, 13 + 41, 17 + 37, 23 + 31	5
52	5 + 47, 11 + 41, 23 + 29	3
50	3 + 47, 7 + 43, 13 + 37, 19 + 31	4
48	5 + 43, 7 + 41, 11 + 37, 17 + 31, 19 + 29	5
46	3 + 43, 5 + 41, 17 + 29	3
44	3 + 41, 7 + 37, 13 + 31	3
42	5 + 37, 11 + 31, 13 + 29, 19 + 23	4
40	3 + 37, 11 + 29, 17 + 23	3
38	7 + 31	1
36	5 + 31, 7 + 29, 13 + 23, 17 + 19	4
34	3 + 31, 5 + 29, 11 + 23	3
32	3 + 29, 13 + 19	2
30	7 + 23, 11 + 19, 13 + 17	3
28	5 + 23, 11 + 17	2
26	3 + 23, 7 + 19	2
24	5 + 19, 7 + 17, 11 + 13	3
22	3 + 19, 5 + 17	2
20	3 + 17, 7 + 13	2
18	5 + 13, 7 + 11	2
16	3 + 13, 5 + 11	2
14	3 + 11	1
12	5 + 7	1
10	3 + 7	1
8	3 + 5	1

3) Since $a \mid X_n$ and $a \mid a$ for all $a \in \{P_i \mid 1 \le i < n\}$, therefore $a \mid (X_n - a)$ and $a \mid (X_n + a)$, namely, all $X_n - a$ and $X_n + a$ are composites for all

$a \in \{P_i \mid 1 \le i < n\}$.

4) If a is a composite smaller than P_n, a must have all of its prime factors $b \in \{P_i \mid 1 \le i < n\}$. Since $b \mid X_n$ and $b \mid a$, therefore $b \mid (X_n - a)$ and $b \mid (X_n + a)$, therefore all $X_n - a$ and $X_n + a$ are composites.

In summary, $X_n - 1$ and $X_n + 1$ are the only numbers in $(X_n - P_n, X_n + P_n)$ that can be primes and with a difference of 2 in between.

This completes the proof.

Note. $X_n - 1$ and $X_n + 1$ does not necessarily be a prime, either of them may be divided exactly by a prime equal to or greater than P_n.

In case none of $X_n - 1$ and $X_n + 1$ is a prime, $(X_n - P_n, X_n + P_n)$ is a $2P_n$ long prime gap. An interesting inference is "As P_n approaches to the infinite, length of the gap also approaches the infinite". This answers the Question vii asked by Dr. Hua on page 90 of his book [15]. Considering the known greatest prime is more than 24 million digits long [6], it is amazing to conceive that there exists such a long gap of primes: at most only two primes are immersed in zillions and zillions of composites! This provides one solution for the problem of prime gap (A8) and raw material for hypotheses on difference between consecutive primes mentioned in [4] [11].

Assuming $X_n + 1$ and $X_n + P_n$ both are primes, let's try to search for the next prime after X_n. Starting from X_n, the next number is $X_n + 1$ which is a prime. We finish the search with only one test, with 100% success rate. Starting from $X_n + 2$, we cannot succeed until reaching $X_n + P_n$. The success rate is $2/(P_n - 2)$ if we only test all the odds in the range. This rate decreases as the value of P_n grows greater. Knowing the above knowledge about prime gap, the test can be restricted to all odds in $[X_n + P_{n-1} + 1, X_n + P_n]$, the success rate is $2/(P_n - P_{n-1})$ (Figure 2).

Theorem 2. For $a \in \{P_i \mid i \ge n\}$ and $b \in \{P_i \mid 1 \le i < n\}$, b cannot divide $X_n - a$ exactly.

Proof.

Since $b \mid X_n$ and $b \nmid a$, therefore $b \nmid (X_n - a)$.

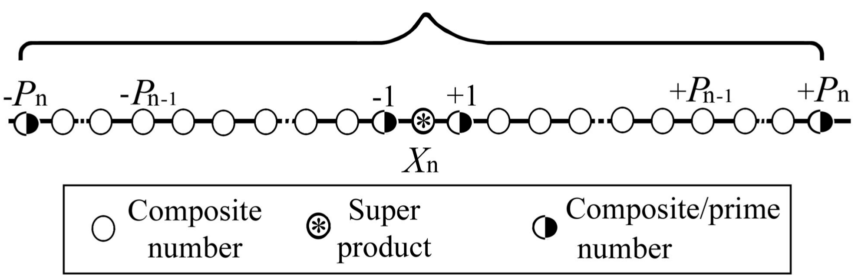

Figure 2. Prime gap $(X_n - P_n, X_n + P_n)$ centered around X_n. Note the primality of $X_n - 1$ and $X_n + 1$ determines length of the gap: if both are composites, the gap is $2P_n$ long; if both are primes, they are the only primes in $(X_n - P_n, X_n + P_n)$ and they are twin primes, and the lengths of the gap are reduced to $P_n - 1$; if one of them is a prime, then the other is the only prime in $(X_n - P_n, X_n + P_n)$, and the lengths of the gap are reduced to $P_n - 1$ and $P_n + 1$, respectively. A few examples of such gaps are (23, 37), (53, 67), (83, 97), (113, 127), (143, 157), (173, 187), (199, 221) with 29 and 31, 59 and 61, 89, none, 149 and 151, 179 and 181, 211, respectively, as prime exceptions.

This completes the proof.

Note. This does not necessarily mean that $X_n - a$ is a prime, as it may be divided exactly by a prime equal or greater than P_n. This is the shortcoming of this paper, namely, we cannot eliminate the all influence of numbers greater than P_n. However, it does imply that $X_n - a$ is very likely a prime. This constitutes the rationality underlying the bilateral symmetrical distribution of primes shown in Figures 3-5.

5. Implications on Relationship between Primes

As implied by Theorem 2, primes can be paired under certain condition. Now we designate S as a constant equal to X_n (or its integer multiples), then $S/2$ can be a mid-point integer between of prime pairs on its both sides. The relationship between such prime pairs is termed **complementary** here since the sums of such pairs are always equal to a constant S, as shown in Figures 3-5.

Note 1. Although 1 is not a prime, its complementary may be a prime.

Note 2. Some of the numbers in ($S/2$, S) obtained by subtracting smaller primes may be composites. These exceptions can be eliminated case-by-case by calculating all combination products of all primes in [P_n, S/P_n]: if any of their products falls in (P_n, $S/2$), add the complementary of the product into the list; if any of their products falls in (S, $S/2$), delete the product from the generated list.

Note 3. As shown in Figure 3, when $S = 30$, $P_n = 7$, $S/P_n = 4$, the range [P_n, S/P_n] becomes [7,4], which is an empty range. This explains the lack of exceptions in Figure 3 although such exceptions occur in Figure 4 and Figure 5.

6. Algorithm Generating Primes

Taking advantage of the above described pairing relationship between primes, routine generating greater primes includes the following steps (using Figure 3 as an example).

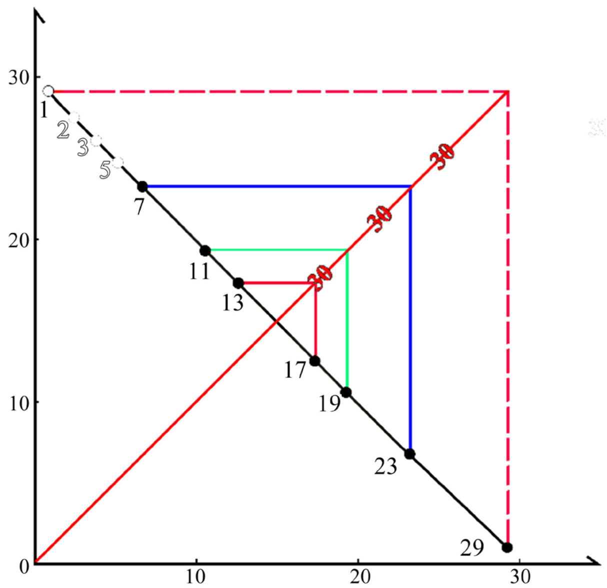

Figure 3. Prime pairs with sums equal to 30, with no exception.

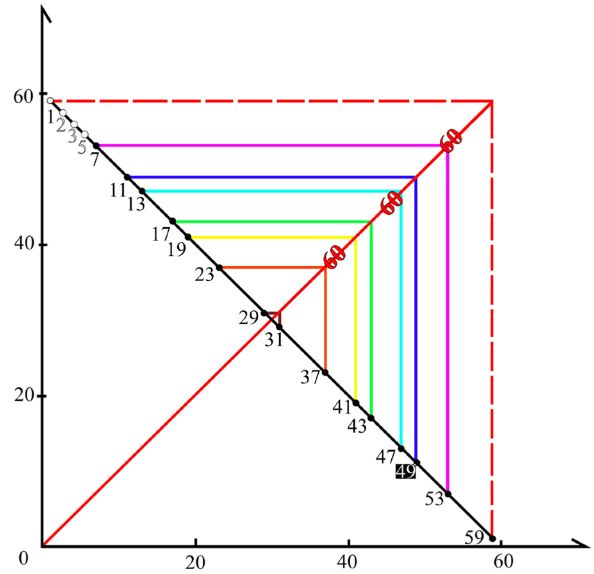

Figure 4. Prime pairs with sums equal to 60, with one exception of 49 (=7 × 7).

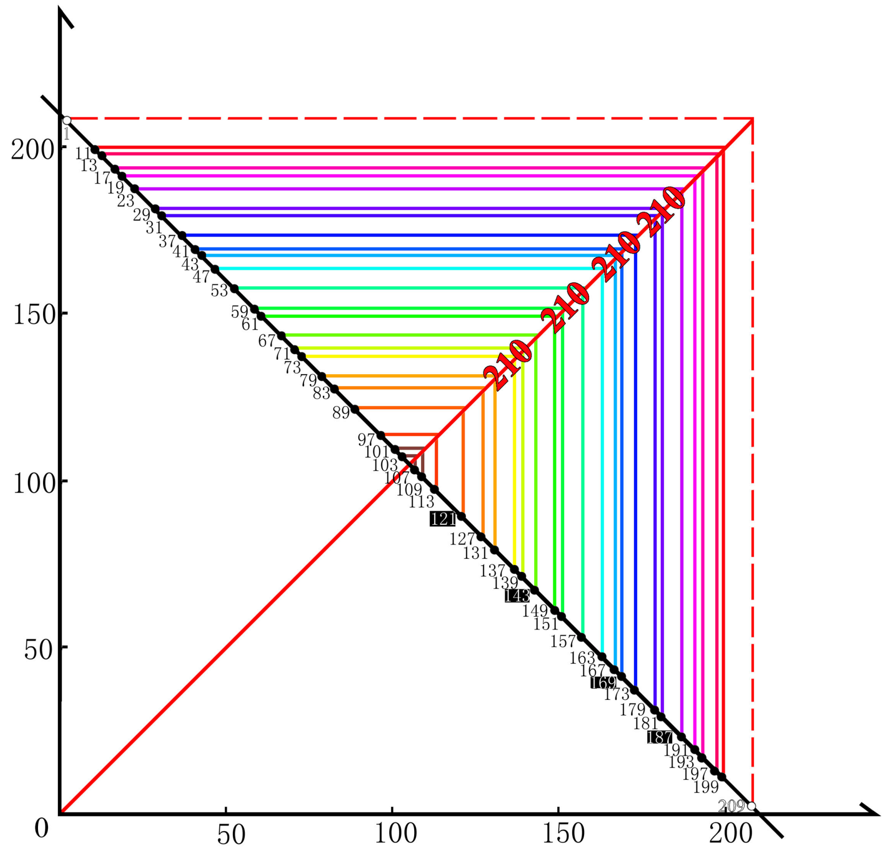

Figure 5. Prime pairs with sums equal to 210, with four exceptions of 121 (=11 × 11), 143 (=11 × 13), 169 (=13 × 13), and 187 (=11 × 17).

Step 1. Calculate the value of S (X_n or its integer multiples).

Step 2. Subtract 1 and all primes in $[P_n, S/2)$ from S, save the result in an ascending order as a list.

Step 3. Calculate all combination products in (P_n, S) of all primes in [P_n, S/P_n]. If such a product is within the list obtained in Step 2, delete it from the list.

Step 4. Save the above generated list. Finish.

7. Discussions

Compared to the existing routines generating primes, the present one has the following advantages:

1) The calculation involved is computationally cheap. The candidate list of greater primes can be obtained by subtracting smaller ones from a constant.

2) The result is dense, namely, all primes within scope are covered.

3) Although the applicable range of each run is limited, the applicable range of the routine can be extended exponentially into the infinite, as it is hinged with super products of primes.

8. Conclusion

Primes tend to be pairwise distributed. Such pairing relationship implies that greater primes can be obtained in a computationally cheap way. There is either one continuous $2P_n$ long prime gap or two at least $P_n - 1$ long prime gaps around X_n. One or two of $X_n - 1$ and $X_n + 1$ may be the only primes within ($X_n - P_n$, $X_n + P_n$).

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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