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The Genesis of Prime Numbers—Revealing the Underlying Periodicity of Prime Numbers

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Abstract

Prime numbers are the integers that cannot be divided exactly by another integer other than one and itself. Prime numbers are notoriously disobedient to rules: they seem to be randomly distributed among natural numbers with no laws except that of chance. Questions about prime numbers have been perplexing mathematicians over centuries. How to efficiently predict greater prime numbers has been a great challenge for many. Most of the previous studies focus on how many prime numbers there are in certain ranges or patterns of the first or last digits of prime numbers. Honestly, although these patterns are true, they help little with accurately predicting new prime numbers, as a deviation at any digit is enough to annihilate the primality of a number. The author demonstrates the periodicity and inter-relationship underlying all prime numbers that makes the occurrence of all prime numbers predictable. This knowledge helps to fish all prime numbers within one net and will help to speed up the related research.

Keywords

Prime Number, Genesis, Periodicity, Rule, Prediction, Number Theory, Evolution

1. Introduction

Prime numbers appear random and irregularly distributed, especially when viewed from a linear perspective. Many mathematicians have been studying the density of prime numbers such as function $\pi(x)$, which has intrigued numerous publications [1], and focus on how many prime numbers there are in certain ranges. However, even if this function is well understood, it remains elusive as to exactly which one among numerous numbers is a prime number. There are many publications about the pattern of the first or last digit of prime numbers [2] [3]. These studies lack the power of prediction as the unique feature, primality, of prime numbers requires no error at any digit. Twin prime numbers are of interest for many, and mathematicians are eager to learn more about their distribution [4] [5] [6]. Although the currently known greatest prime number is already 24,862,048 digit-long [7], greater one remains a further target for many [1]. To help solve these problems, it is necessary to find patterns underlying prime numbers. Here I tried to demonstrate a novel periodic pattern underlying all prime numbers, and hoped to fuel the related research.

2. Promising Primality

Definition 1. The nth prime number is denoted as P_n. For example, $P_1 = 2$, $P_4 = 7$.

Definition 2. The product of the first n-1 prime numbers is designated as **Super Product of P_n**, denoted as X_n. For example, $X_1 = 1$, $X_4 = 2 \times 3 \times 5 = 30$.

Theorem 1. All prime numbers smaller than P_n cannot divide exactly the sum of an integer multiple of X_n and a prime number greater than or equal to P_n.

Proof. Suppose a prime number $q \geq P_n$, another prime number $d < P_n$, and an integer $S = a \times X_n + q$ ($a \in N$).

Since $d | X_n$, then $d | (a \times X_n)$, and $d \nmid q$, therefore $d \nmid (a \times X_n + q)$, namely, $d \nmid S$. This completes the proof.

Theorem 2. All prime numbers smaller than P_n cannot divide exactly the sum of an integer multiple of X_n and the product of prime numbers greater than or equal to P_n.

The proof of this theorem is similar to that of Theorem 1.

Note. Both theorems imply 1) that a prime number smaller than P_n cannot divide exactly the sum specified in the theorems; 2) that, however, a prime number greater than or equal to P_n may divide exactly the sum specified in the theorems. The latter implication is the reason for exceptions in **Figures 3-5**. It is also the shortcoming of this paper, namely, I cannot eliminate all composite number exceptions *a priori*.

3. The Underlying Periodicity

The above theorems suggest that X_n is the step length between neighboring prime numbers on the same radius (in the same series) in Stage N, suggestive of periodicity underlying prime numbers. Following I will demonstrate the truthful existence of such periodicity of prime numbers stage by stage (**Figures 1-5**).

4. Inferences

More further stages are not allowed to be shown given the limited space of this paper. It is enough to say that more stages of prime numbers following similar regularity are there to show. Each stage takes all elements (except the first one) of the preceding stage as its initial ancestor circle (innermost). Every next outer derived circle (descendant circle) is generated by adding the step length of present stage to the elements of the preceding circle. This process is repeated multiple times until the total number of circles reaches P_n. Therefore, although the periodicity in a stage is restricted to its own stage, as there is no upper limit for the number of stages that is hinged with number of prime numbers (which is infinite, as proven), apparently, similar (although slightly different) regularity can be extended to the infinite (Figure 6).

It is interesting to extract the following generalizations.

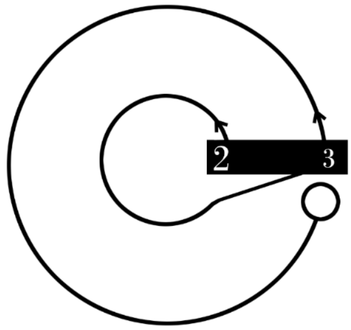

Figure 1. Stage I: 1 series starting with 2, step length 1 (the unit in integers), including 2 elements (2, 3).

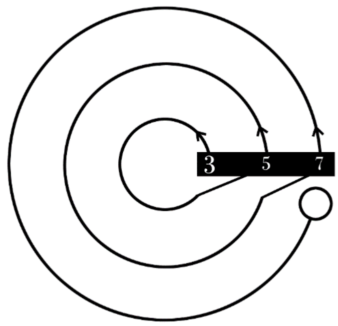

Figure 2. Stage II: 1 series, starting with 3, step length 2, including 3 elements (3, 5, 7).

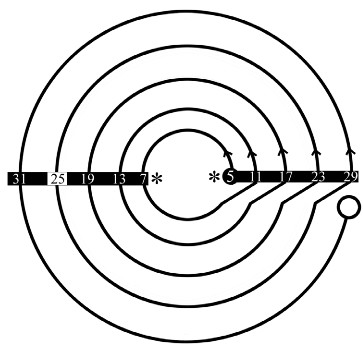

Figure 3. Stage III: 2 series (starting with 5 and 7, respectively), step length 6, including 10 elements (5, 11, 17, 23, 29; 7, 13, 19, 25, 31), with 1 twin prime number series and 1 exception (25).

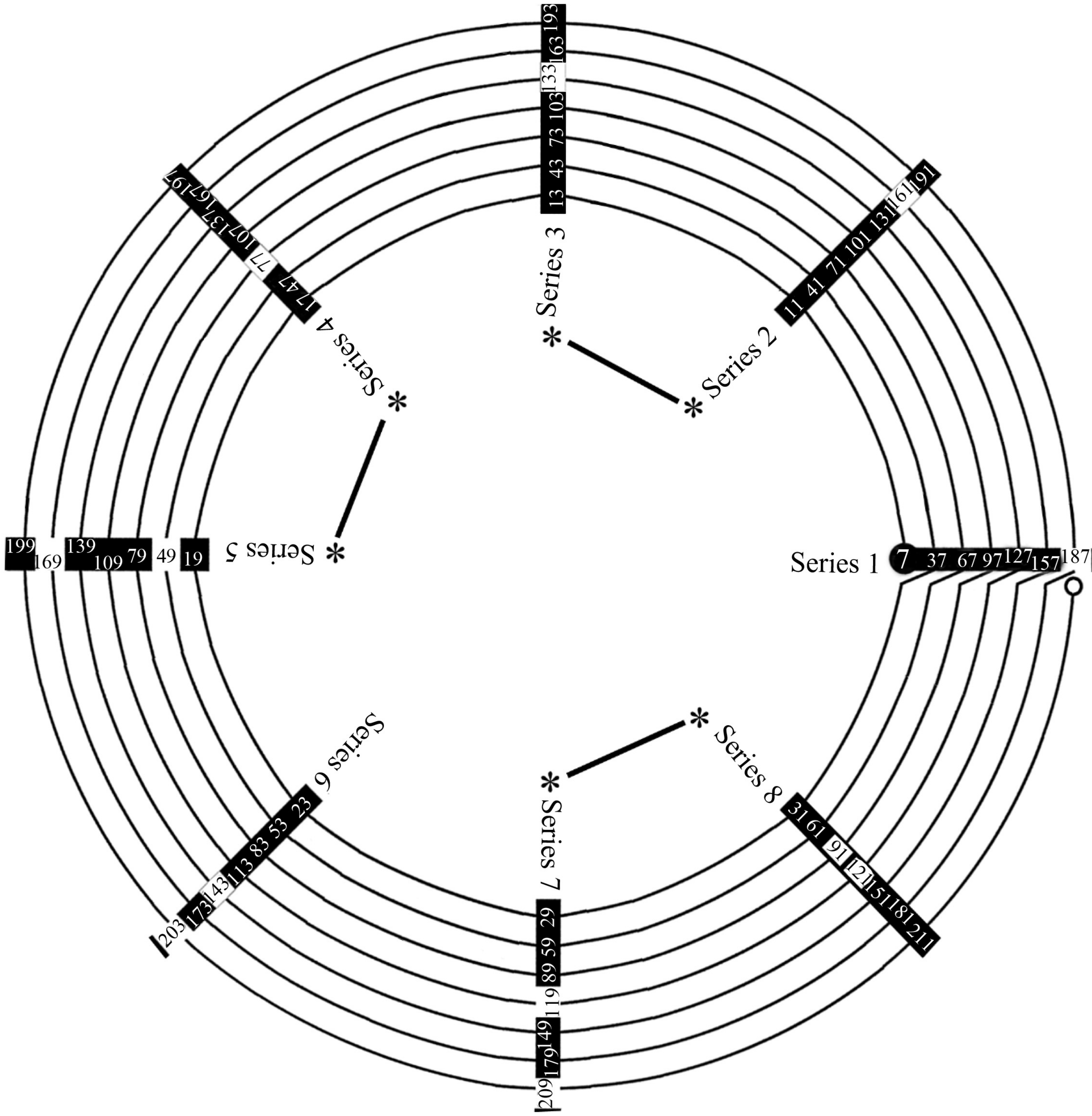

Figure 4. Stage IV: 8 series (starting with 7, 11, 13, 17, 19, 23, 29, 31, respectively), step length 30, including 56 elements (7, 37, 67, 97, 127, 157, 187; 11, 41, 71, 101, 131, 161, 191; 13, 43, 73, 103, 133, 163, 193; 17, 47, 77, 107, 137, 167, 197; 19, 49, 79, 109, 139, 169, 199; 23, 53, 83, 113, 143, 173, 203; 29, 59, 89, 119, 149, 179, 209; 31, 61, 91, 121, 151, 181, 211), with 3 twin prime number series (starting with 11 and 13, 17 and 19, 29 and 31, respectively) and 12 exceptions (49, 77, 91, 119, 121, 133, 143, 161, 169, 187, 203, 209).

1) Stage N starts with P_n.

2) Prime numbers deployed on the same radii of the circles have fixed difference (step length in Stage N = X_n) with the neighboring ones.

3) All prime numbers are included within the generated series.

4) A prime number greater than 7 can be a sum of a composite number and a prime/composite number, except twin prime numbers with a difference 2 in

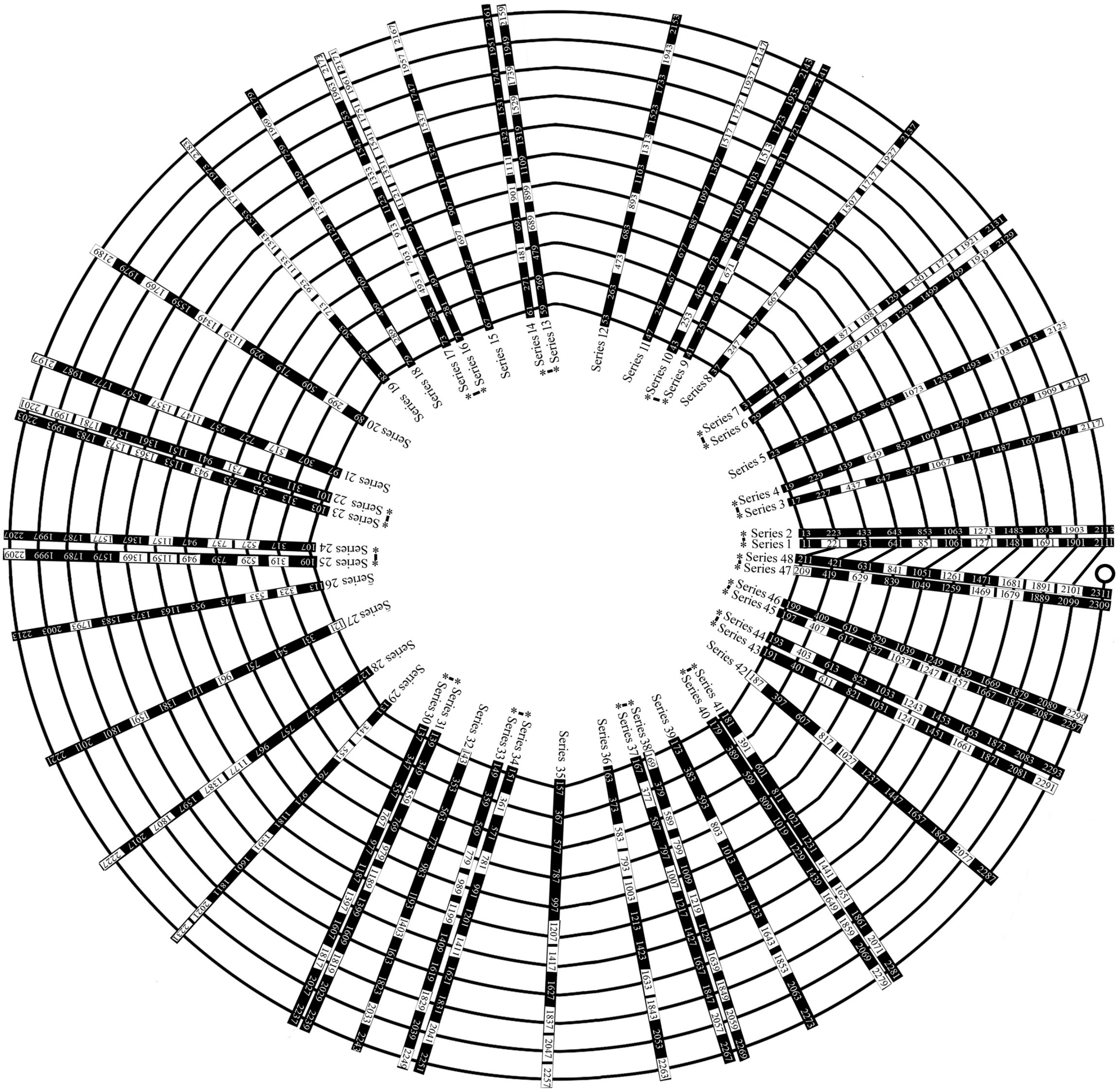

Figure 5. Stage V: 48 series starting with 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 121, 127, 131, 137, 139, 143, 149, 151, 157, 163, 167, 169, 173, 179,181, 187, 191, 193, 197, 199, 209, 211, respectively, step length 210, including 528 elements, with 15 twin prime number series (starting with 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73, 101 and 103, 107 and 109, 137 and 139, 149 and 151, 167 and 169, 179 and 181, 191 and 193, 197 and 199, 209 and 211, respectively) and 188 exceptions.

between.

5) The existence of certain composite numbers exceptional to the regularity are necessary for the increasingly sparse density of prime numbers in later generations and for the genesis of some prime numbers.

6) Twin prime numbers give birth to new twin prime numbers.

7) It is intriguing to test whether there is a comparability between the genesis of prime numbers and that of other entities, such as organisms.

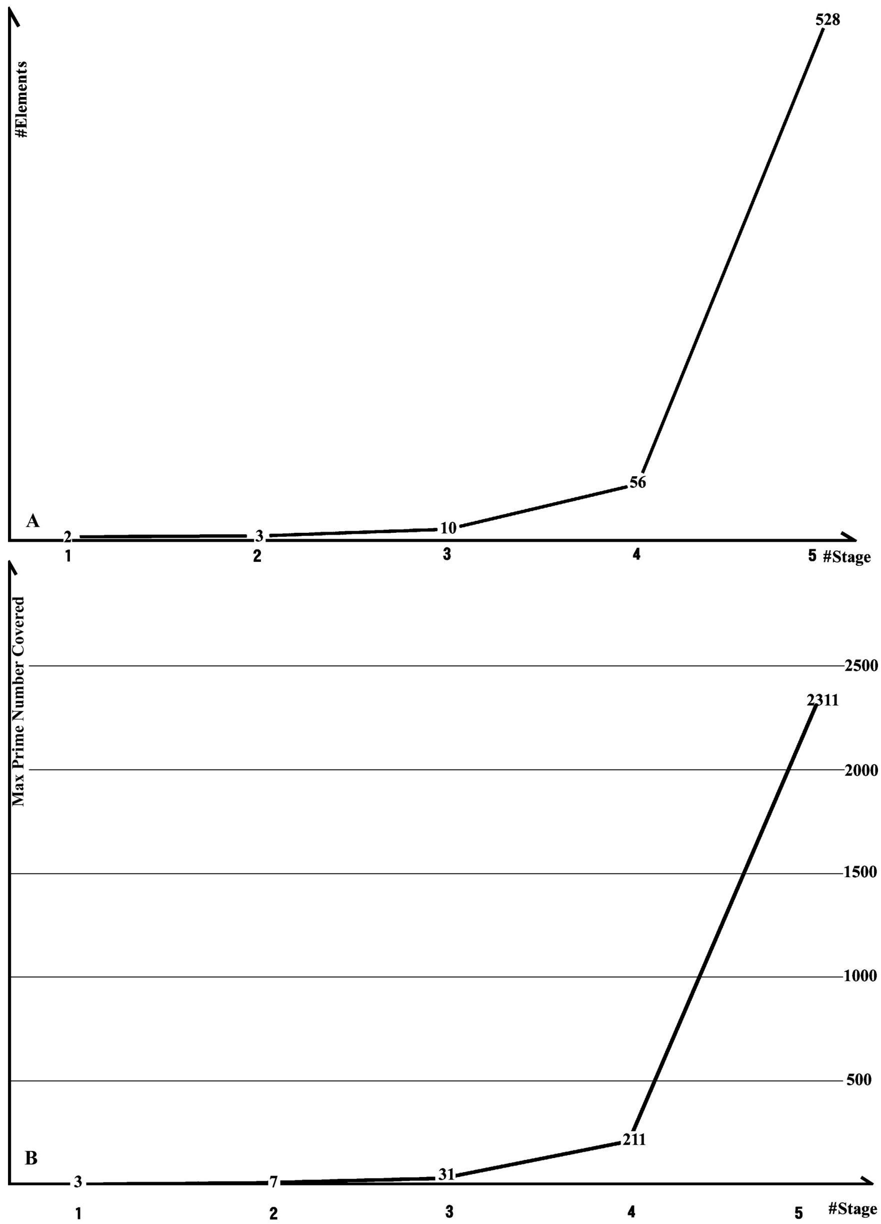

Figure 6. The number of elements (A) and maximal prime number (B) in the stages increase exponentially with the number of stages.

5. Conclusion

If the numbers on the innermost circle in each stage are taken as ancestor prime numbers that give birth to more prime numbers, the numbers on the outer circles can be taken as descendants of these ancestors. The differences between neighboring descendants are fixed within a stage. The periodicity underlying prime numbers elaborated here makes prime numbers tame and predictable.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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