Advances in Pure Mathematics, 2020, 10, 540-544
https://www.scirp.org/journal/apm
ISSN Online: 2160-0384
ISSN Print: 2160-0368

Solutions of Indefinite Equations

Zengyong Liang

MCHH of Guangxi, Nanning, China
Email: lzy8ok@163.com

How to cite this paper: Liang, Z.Y. (2020) Solutions of Indefinite Equations. *Advances in Pure Mathematics*, **10**, 540-544.
https://doi.org/10.4236/apm.2020.109033

Received: August 31, 2020
Accepted: September 18, 2020
Published: September 21, 2020

Abstract

Indefinite equation is an unsolved problem in number theory. Through exploration, the author has been able to use a simple elementary algebraic method to solve the solutions of all three variable indefinite equations. In this paper, we will introduce and prove the solutions of Pythagorean equation, Fermat's theorem, Bill equation and so on.

Keywords

Indefinite Equation, Fermat's Last Theorem, Algebraic Transformation, L-Algorithm

1. Definition of Indefinite Equation

We also know indefinite equation (also called Diophantine equation) that all unknowns and known numbers are positive integers [1] [2]. The indefinite equation, especially the higher-order indefinite equation, is a difficult problem which has not been solved thoroughly in number theory. In this paper, we will introduce their new solutions one by one.

2. First Order Indefinite Equation Some Theorems

Theorem 1. *Here is the three variable indefinite equation as*

$$A+B=C \tag{1}$$

If one of the terms is an arbitrary positive integer, the Equation (1) *must have a solution.*

Proof. Suppose C is any positive integer, A and B are two positive integers, then the sum of A and B must be positive integers. We can use the number axis to verify that C is also a positive integer. □

3. Pythagorean Equation (Pythagorean Theorem)

Pythagorean theorem is the most familiar mathematical formula that has been proved by many kinds of proofs [3]. Namely Pythagorean theorem is

$$A^2 + B^2 = C^2 \tag{2}$$

However, its simple way of solutions has never been seen. Next we will find and prove the solutions.

By Equation (2), we get

$$A^2 = C^2 - B^2,$$

$$A^2 = (C - B)(C + B).$$

Analysis:

By $C - B = d$, $d = 1, 2, 3, \cdots$, we will derive many solutions to Equation (2), as Table 1.

Very regular! It is not difficult to sum it up that general solutions are: $A = 3k$, $B = 4k$, $C = 5k$, $k = 1, 2, 3, \cdots$.

Theorem 2. *If positive integers a, b, c are a series of positive integer solutions of following equation*

$$a^2 + b^2 = c^2, \tag{3}$$

when $c - b = 1$.

Then as $A = ka$, $B = kb$, $C = kc$ *also are solutions of equation as* $A^2 + B^2 = C^2$.

Proof. Suppose Equation (3) is tenable. Multiplying k^2 on both sides of the Equation (3), we obtain

$$(ka)^2 + (kb)^2 = (kc)^2. \tag{4}$$

So the solutions of $A^2 + B^2 = C^2$ are $A = ka$, $B = kb$, $C = kc$. □

In addition, we also proof when $(C - B) = 1$, have array solutions as $A = 2m + 1$, $B = 4m(m + 1)$, $C = B + 1$.

They are called original numbers of shares [4]. Obviously, (3) if there is a solution of array $(c - b) = 1$, then (2) also has countless series of positive integer solutions. In this way, we can find the solution of the infinite Pythagorean equations.

4. Higher Degree Indefinite Equation

Definition of higher degree indefinite equation is an indefinite equation in which the exponents of all powers are greater than or equal to 3, as

$$A^n + B^n = C^n, \tag{5}$$

Table 1. A table of A, B, C, $C - B$.

A	B	C	$C - B$
3	4	5	1
6	8	10	2
9	12	15	3
	…		

or

$$A^x + B^y = C^z . \tag{6}$$

where $n \geq 3$, $x \geq 3$, $y \geq 3$, $z \geq 3$.

4.1. Fermat's Last Theorem

We have known that Fermat's last theorem has been proved by British Mathematician Andew. Wiles using the properties of elliptic curves (1993) [5]. Now we present a proof using elementary algebra as following.

Theorem 3 (Fermat's Last Theorem). *If* A, B, C *are tree positive integers*, $n \geq 3$, *than equation*

$$A^n + B^n = C^n \tag{7}$$

is no integer solutions.

Proof. Suppose Equation (7) is true. Let $A = C - a$, $B = C - b$, then

$$A^n = (C-a)^n = C^n - nC^{n-1}a + \cdots + nCa^{n-1} + a^n , \tag{8}$$

$$B^n = (C-b)^n = C^n - nC^{n-1}b + \cdots + nCb^{n-1} + b^n . \tag{9}$$

(8) + (9):

$$A^n + B^n = \left(C^n - nC^{n-1}a + \cdots + nCa^{n-1} + a^n\right) + \left(C^n - nC^{n-1}b + \cdots + nCb^{n-1} + b^n\right),$$

$$A^n + B^n = 2C^n - nC^{n-1}(a+b) + \cdots + nC\left(a^{n-1} + b^{n-1}\right) + a^n + b^n . \tag{10}$$

By Equations (7) and (10), we have

$$C^n - nC^{n-1}(a+b) + \cdots + nC\left(a^{n-1} + b^{n-1}\right) + \left(a^n + b^n\right) = 0 . \tag{11}$$

If

$$C^n - nC^{n-1}q + \cdots + nCq^{n-1} + q^n = 0 . \tag{12}$$

Comparisons Equations (11) and (12), we have:

$$a + b \to q ,$$

$$a^2 + b^2 \to q^2 ,$$

$$\cdots$$

$$a^n + b^n \to q^n .$$

So, by theorem 3, if Equation (12) is true, also need $q = C$, and we obtain that if Equation (12) is true, require the following are true:

$$a + b = C ,$$

$$a^2 + b^2 = C^2 ,$$

$$\cdots$$

$$a^n + b^n = C^n .$$

Obviously, these equations are contradictory by $n \geq 3$, it is impossible. So, the hypothesis is not valid, the Equation (11) is not true, and the Equation (7) also is not true. □

Theorem 3. *If the following indefinite equation is holds, need* $C = q$, *if* $n \ge 3$:

$$C^n - nC^{n-1}q + \cdots + nCq^{n-1} + q^n = 0. \tag{13}$$

Proof. Obviously, when $n \ge 3$, only have $C = q$ to get polynomial

$$(C-q)^n = C^n - nC^{n-1}q + \cdots + nCq^{n-1} + q^n = 0. \square$$

Therefore, the equation as (7) is no integer solutions.

4.2. Beal Equation

4.2.1. Beal's Conjecture

Beal's conjecture: if the indefinite equation

$$A^x + B^y = C^z \tag{14}$$

is true, there $x \ne y, y \ne z, x \ne z$ and $x \ge 3, y \ge 3, z \ge 3$; than A, B, C must have a common factor.

We have proved that Beal's conjecture is true in the paper of "Proof of Beal Conjecture" [6].

4.2.2. Solutions by L-Algorithm

The equations as Equation (14) are called Beal equations. Let's find their solutions by L-algorithm.

By Theorem 1, there is always $a^x + b^y = c$ (it is called original equation of the Equation (14)) than multiplying the least common factor c^s on both sides of original equation

$$a^x c^s + b^y c^s = cc^s.$$

Lest $s = xy$, than

$$a^x c^{xy} + b^y c^{xy} = cc^{xy},$$

also is

$$\left(ac^y\right)^x + \left(bc^x\right)^y = c^{xy+1} \tag{15}$$

So, if $z|(xy+1)$, or $xy + 1 = kz$, then $A = ac^y$, $B = bc^x$, $C = c^k$ are solutions for the Equation (14).

This is solution for the Beal equations. Therefore, we can use the above method to solve other indefinite equations.

Example 1. Solve the following indefinite equation

$$A^5 + B^7 = C^3. \tag{16}$$

Firstly, a, b, d are selected to satisfy $a^x + b^y = c$, as

$$5^5 + 3^7 = 5312 \tag{17}$$

Then, choose c^s, let $s = xy = 35$, according to (15), multiplying 5312^{35} on the two sides of (17):

$$5^5 \times 5312^{35} + 5^7 \times 5312^{35} = 5312 \times 5312^{35},$$

we obtain

$$\left(5\times5312^{7}\right)^{5}+\left(5\times5312^{5}\right)^{7}=\left(5312^{12}\right)^{3},$$

So, that solutions of Equation (16) are $A = 5 \times 5312^{7}$, $B = 5 \times 5312^{5}$, $C = 5312^{12}$. □

Example 2. Solve the following indefinite equation

$$A^{5}+B^{7}+C^{3}=D^{4}. \tag{18}$$

By Example 1, we know the original of Equation (18) is

$$5^{5}+3^{7}+1^{3}=5313,$$

$$\left(5^{5}+3^{7}+1^{3}\right)5313^{315}=5313^{315+1}. \tag{19}$$

$$\left(5\times5313^{63}\right)^{5}+\left(3\times5313^{45}\right)^{7}+\left(5313^{105}\right)^{3}=\left(5313^{79}\right)^{4}. \tag{20}$$

Thus, by Equation (20), $A = 5 \times 5313^{63}$, $B = 3 \times 5313^{45}$, $C = 5313^{105}$, $D = 5313^{79}$. □

Therefore, we have found that the solutions the indefinite equations as (14) and (18) or more.

5. Conclusion

Through the above introduction, we understand the new solution of the indefinite equation, which shows that the indefinite equation can be solved by the method of elementary algebra. According to this method flexibly, we will solve more higher degree indefinite equations. It adds a new way to solve the higher order indefinite equation for number theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Harly, G.H. and Wright, E.M. (2007) An Introduction to the Theorem of Number. People's Post and Telecommunications Press, Beijing, 205-215.

[2] Baiks, Indefinite Equation. https://baike.so.com/doc/4001025-4197595.html

[3] Euclid (2011) Geometric Origin. Translated by Weng Shi, Tianjing People's Press, Tianjing, 39-40.

[4] Rosen, K.H. (2005) Elementary Number Theory and Its Application. Machinery Industry Press, Beijing.

[5] Aczel, A.D. (2016) Fermat's Last Theorem. Shanghai Scientific & Technological Literature Press, Shanghai.

[6] Liang, Z.Y. (2019) Proof of Beal Conjecture. Journal of *Advances in Pure Mathematics*, **9**. http://www.scirp.org/jourrnal.jamp