Connectivity Analysis of Vehicles in Highway with One Entry/Exit and One RSU

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Abstract

In this paper, we consider a unidirectional highway road with one entry/exit uniformly distributed on the road interval. One road side unit (RSU) is located on the position of the entry/exit. We build an analytical model to study the network connectivity problem. In building the analytical model, we take into account several parameters, such as vehicle arrival rate, vehicle moving speed, vehicle communication radius, RSU communication radius, highway road length and the probability of vehicles driving through the entry/exit. The analytical model is verified by using simulation tools.

Keywords

Connectivity, Entry/Exit, RSU, Vehicular Network

1. Introduction

Network connectivity is one of the most fundamental issues in Vehicular Ad Hoc Networks (VANETs). A higher connectivity in VANETs can ensure the timeliness for various kinds of data transmission. The previous studies indicated that the entries/exits in highway scenario can affect the network connectivity [1] [2] and deploying RSUs on the entries/exits is an effective method to improve the network connectivity [3] [4]. As so far, there is no study that focuses on the network connectivity modeling of such scenario. Thus, it is necessary to build an analytical model to study the network connectivity of highway scenario with entries/exits and RSUs, and it is also interesting to analyze the impacts of several parameters on the network connectivity in such scenario.

In this paper, we build an analytical model to study the network connectivity of highway scenario with one entry/exit and one RSU. We consider a unidirectional highway road with one entry/exit uniformly distributed on the road in-

terval. One RSU is located on the position of the entry/exit. In building the analytical model, we divide the highway road into several road intervals, and analyze both the connectivity of each road interval and the connectivity of neighbor road interval. Moreover, we take into account several parameters, such as vehicle arrival rate, vehicle moving speed, vehicle communication radius, RSU communication radius, highway road length and the probability of vehicles driving through the entry/exit. The analytical model is verified by using simulation tools, and the analytical model can be used to analyze the impacts of several parameters on the network connectivity.

The remainder of the paper is organized as follows. Section II reviews related work in the literature. Section III derives the connectivity analytical model. Section IV presents simulation results to verify the accuracy of the analytical model. Section V concludes this paper.

2. Related Work

Connectivity analysis for a highway scenario in a VANET has been studied in the literature [5] [6] [7]. Most of the present work studies the connectivity of highway scenario without considering entries/exits [5] [6] [7]. In [5], Neelakantan *et al.* focused on the connectivity analysis of a VANET from a physical perspective and investigated the minimum transit power used by vehicles to guarantee network connectivity. In [6], Shao *et al.* investigated the connectivity probabilities for V2V and V2I communication scenarios in one-way and two-way platoon-based VANETs and designed a connectivity-aware MAC protocol. In [7], Kwon *et al.* modeled the 1-D network as geometric elements of lines and points and analyze the connectivity of the network using geometric probability. However, in all of the above work, the network connectivity was studied for a highway scenario without considering entries/exit. In [1], Khabazian *et al.* presented a connectivity analytical model for a highway scenario with multiple entries/exits, but without considering the deployment of RSUs. This work is on the basis of our previous work [2] which considers the two-way connectivity in highway scenario with one entry/exit.

3. Connectivity Analysis

In this section, we build an analytical model to calculate the network connectivity probability of highway scenario with one entry/exit and one RSU.

3.1. Network Scenario

Consider a unidirectional highway road scenario, as shown in **Figure 1**. The vehicles are moving on the highway road. One entry/exit is distributed on the highway road. One RSU is deployed on the position of the entry/exit. The communication radius of vehicles is denoted as R_v and the communication radius of RSU is denoted as R_u, here assumed that $R_u \geq R_v$. The highway road interval is denoted as $[0, L]$, where L represents the length of the highway road and
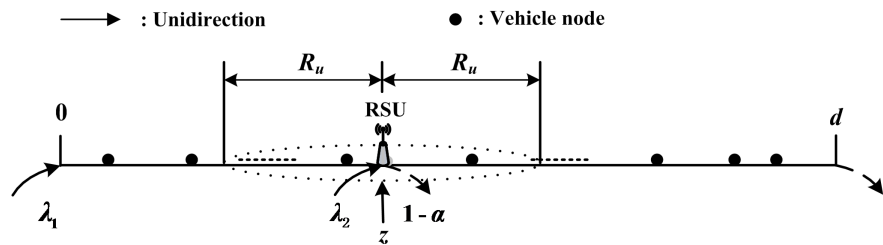
Figure 1. Basic network scenario.

$L \geq 2R_u$. The entry/exit is denoted as z. Assume that z follows uniform distribution on road interval $[R_u, L - R_u]$. The entry/exit divides the highway road interval into two sub-intervals which are represented as $[0, z]$ and $[z, L]$. The vehicle arrival process on $[0, z]$ is Poisson process with mean rate λ_1. When the vehicles on $[0, z]$ arriving at the position of the entry/exit, the vehicles can keep moving onto the next road interval $[z, L]$ with probability $\alpha (0 \leq \alpha \leq 1)$, or depart from the highway road at the entry/exit with probability $1 - \alpha$. The vehicles moving on the road interval $[z, L]$ contains two parts: one is the vehicles from the road interval $[0, z]$ and the other is the new arriving vehicles from the entry/exit. The new arriving vehicles on the road interval $[z, L]$ are also Poisson process with mean rate λ_2. The vehicle speed is denoted as random variant f_v which follows uniform distribution on interval $[v_{min}, v_{max}]$, where v_{min} and v_{max} respectively represents the minimum and the maximum value of the allowed moving speed for vehicles. The average value of the vehicle speed is expressed by $v = E[f_v]$, where $E[\cdot]$ represents mathematical expectation. According to the property of Poisson process, the vehicle arrival rates of $[0, z]$ and $[z, L]$ can be expressed by γ_1 and γ_2, where $\gamma_1 = \lambda_1/v$ and $\gamma_2 = (\alpha \cdot \lambda_1 + \lambda_2)/v$. Therefore, the network connectivity in this paper is building an analytical model to calculate the connectivity probability of the vehicles in the network.

3.2. Network Connectivity Probability

Similar to Ref. [2], consider the two-way connectivity which requires that the vehicle and RSU can communicate with each other only and only if the distance between them is not larger than the vehicle communication radius. For ease of analysis, as shown in **Figure 2**, divide the interval $[z - R_u, z + R_u]$ into four sub-intervals which are denoted as a_1, a_2, a_3 and a_4, respectively. And the notations which are used after are listed as follows:

H_a: the event that there are vehicles on road segment a.

$\bar{H}_a$: the event that there is no vehicle on road segment a.

C_a: the event that road segment a is connected.

$\bar{C}_a$: the event that road segment a is disconnected.

$\Pr\{V\}$: the probability that event V occurs.

From **Figure 2**, it is obvious that the length of a_1 is not more than the communication radius of vehicle. If there are vehicles in a_1, the vehicles in a_1 is
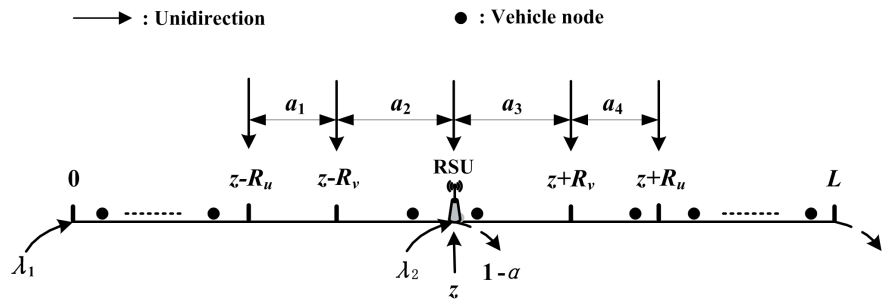
Figure 2. Analysis diagram.

connected. The connectivity of a_2, a_3 and a_4 is similar to a_1. Due to the influence of the entry/exit to the network connectivity, the connectivity of a_2 and a_3 is needed to be firstly considered. According to the case that whether there are vehicles on a_2 and a_3, we perform the analysis into following four sub-cases: 1) There are vehicles both on a_2 and a_3; 2) There are vehicles on a_2, but there is no vehicle on a_3; 3) There are vehicles on a_3, but there is no vehicle on a_2; 4) There is no vehicle on a_2 and a_3.

The probabilities that there are vehicles on a_2 and a_3 are respectively given by

$$\Pr\{H_{a_2}\} = 1 - e^{-\gamma_1 \cdot R_v}, \quad \Pr\{H_{a_3}\} = 1 - e^{-\gamma_2 \cdot R_v}.$$

The probabilities that there is no vehicle on a_2 and a_3 are respectively calculated by

$$\Pr\{\bar{H}_{a_2}\} = 1 - \Pr\{H_{a_2}\} = e^{-\gamma_1 \cdot R_v}, \quad \Pr\{\bar{H}_{a_3}\} = 1 - \Pr\{H_{a_3}\} = e^{-\gamma_2 \cdot R_v}$$

1) There are vehicles both on a_2 and a_3.

It is seen from **Figure 2**, if there are vehicles on a_2, a_2 is connected for the case that the length of a_2 is not larger than the communication radius of vehicle and the distance between any two vehicles on a_2 is not larger than the communication radius of vehicle. The connectivity of a_3 is similar to that of a_2. If there are vehicles both on a_2 and a_3, the vehicles separately from a_2 and a_3 can two-way connect to each other by the RSU. For further analysis of the network connectivity, we divide a into four sub-cases.

a) There are vehicles both on a_1 and a_4.

In this case, we can calculate the probabilities that there are vehicles both on a_1 and a_4 as follows

$$\Pr\{H_{a_1}\} = 1 - e^{-\gamma_1 \cdot (R_u - R_v)}, \quad \Pr\{H_{a_4}\} = 1 - e^{-\gamma_2 \cdot (R_u - R_v)}.$$

Then we analyze the connectivity of sub-interval $[0, z]$ and $[z, L]$.

When there are vehicles both on $[0, z - R_u]$ and $[z + R_u, L]$, if the vehicles on sub-interval $[0, z]$ are connected and the vehicles on sub-interval $[z, L]$ are connected, the road interval $[0, L]$ is connected. The probabilities that there are vehicles both on $[0, z - R_u]$ and $[z + R_u, L]$ are calculated as follows

$$\Pr\{H_{[0, z-R_u]}\} = 1 - e^{-\gamma_1 \cdot (z - R_u)}, \quad \Pr\{H_{[z+R_u, L]}\} = 1 - e^{-\gamma_2 \cdot (L - z - R_u)}.$$

In this case, there is no entry/exit in sub-interval $[0, z]$ and sub-interval $[z, L]$. The connectivity probabilities of sub-interval $[0, z]$ and sub-interval $[z, L]$ are obtained according to Ref. [8]. In Ref. [8], for a highway road without entries and exits, if vehicle arrivals follow a Poisson process, the connectivity probability of the road is given by

$$\Pr\{\lambda_v, R_v, L'\} = e^{-\lambda_v \cdot L'} \sum_{j=0}^{\lfloor L'/R_v \rfloor} \frac{(-1)^j}{j!} \left[\lambda_v (L' - jR_v)\right]^{j-1}$$
$$\times \left[j + \lambda_v (L' - jR_v)\right] e^{\lambda_v \cdot (L' - jR_v)}$$

where L' is the road length, R_v is a vehicle's transmission radius, λ_v is the vehicle arrival rate, and $\lfloor x \rfloor$ is the largest integer that is not larger than x. Thus, the connectivity probabilities of sub-interval $[0, z]$ and sub-interval $[z, L]$ are

$$\Pr\{C_{[0,z]}\} = \Pr\{\gamma_1, R_v, z\}, \quad \Pr\{C_{[z,L]}\} = \Pr\{\gamma_2, R_v, L - z\}.$$

The connectivity probability of road interval $[0, L]$ is

$$p_{1.1.1} = \Pr\{H_{[0,z-R_u]}\} \cdot \Pr\{H_{[z+R_u,L]}\} \cdot \Pr\{C_{[0,z]}\} \cdot \Pr\{C_{[z,L]}\}. \tag{1}$$

When there are vehicles on $[0, z - R_u]$, but there is no vehicle on $[z + R_u, L]$, if the vehicles on sub-interval $[0, z]$ are connected and the vehicles on sub-interval $[z, z + R_u]$ are connected, the road interval $[0, L]$ is connected. The probability that there is no vehicle on $[z + R_u, L]$ is calculated as follows

$$\Pr\{\bar{H}_{[z+R_u,L]}\} = 1 - \Pr\{H_{[z+R_u,L]}\} = e^{-\gamma_2 \cdot (L - z - R_u)}.$$

The connectivity probability for the vehicles on $[z, z + R_u]$ is

$$\Pr\{C_{[z,z+R_u]}\} = \Pr\{\gamma_2, R_v, R_u\}.$$

In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.1.2} = \Pr\{H_{[0,z-R_u]}\} \cdot \Pr\{\bar{H}_{[z+R_u,L]}\} \cdot \Pr\{C_{[0,z]}\} \cdot \Pr\{C_{[z,z+R_u]}\}. \tag{2}$$

When there are vehicles on $[z + R_u, L]$, but there is no vehicle on $[0, z - R_u]$, if the vehicles on sub-interval $[z - R_u, z]$ are connected and the vehicles on sub-interval $[z, L]$ are connected, the road interval $[0, L]$ is connected. The probability that there is no vehicle on $[0, z - R_u]$ is

$$\Pr\{\bar{H}_{[0,z-R_u]}\} = 1 - \Pr\{H_{[0,z-R_u]}\} = e^{-\gamma_1 \cdot (z - R_u)}.$$

The connectivity probability for the vehicles on $[z - R_u, z]$ is

$$[z - R_u, z] \Pr\{C_{[z-R_u,z]}\} = \Pr\{\gamma_1, R_v, R_u\}.$$

In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.1.3} = \Pr\{\bar{H}_{[0,z-R_u]}\} \cdot \Pr\{H_{[z+R_u,L]}\} \cdot \Pr\{C_{[z-R_u,z]}\} \cdot \Pr\{C_{[z,L]}\}. \tag{3}$$

When there is no vehicle on $[0, z - R_u]$ and $[z + R_u, L]$, if the vehicles on $[z - R_u, z]$ are connected and the vehicles on $[z, z + R_u]$ are connected, the

road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.1.4} = \Pr\left\{\bar{H}_{[0, z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u, L]}\right\} \cdot \Pr\left\{C_{[z-R_u, z]}\right\} \cdot \Pr\left\{C_{[z, z+R_u]}\right\}. \tag{4}$$

Thus, the connectivity probability of road interval $[0, L]$ in a-1) is

$$p_{1.1} = \Pr\left\{H_{a_1}\right\} \cdot \Pr\left\{H_{a_4}\right\} \cdot \left(p_{1.1.1} + p_{1.1.2} + p_{1.1.3} + p_{1.1.4}\right). \tag{5}$$

b) There are vehicles on a_1, but there is no vehicle on a_4.

In b), the probability that there is no vehicle on a_4 is

$$\Pr\left\{\bar{H}_{a_4}\right\} = 1 - \Pr\left\{H_{a_4}\right\} = e^{-\gamma_2 \cdot (R_u - R_v)}.$$

When there are vehicles both on $[0, z-R_u]$ and $[z+R_u, L]$, if the vehicles on sub-interval $[0, z]$ are connected, the vehicles on sub-interval $[z+R_u, L]$ are connected, and the vehicles separately from a_3 and $[z+R_u, L]$ are connected, the road interval $[0, L]$ is connected. Since there is no entry/exit in $[z+R_u, L]$, the connectivity probability of $[z+R_u, L]$ is

$$\Pr\left\{C_{[z+R_u, L]}\right\} = \Pr\left\{\gamma_2, R_v, L - z - R_u\right\}.$$

Next, we analyze the connectivity of vehicles separately from a_3 and $[z+R_u, L]$. According to the definition of two-way connectivity and the assumption that $R_u \geq R_v$. From **Figure 2**, we can easily obtain that in the case $R_u > 2R_v$, the distance between two vehicles which separately from a_3 and $[z+R_u, L]$ is larger than the communication radius of vehicles. In this case, the vehicles which separately from a_3 and $[z+R_u, L]$ have no chance to connect to each other, the road interval $[0, L]$ is disconnected. Otherwise, if $R_u \leq 2R_v$, the vehicles which separately from a_3 and $[z+R_u, L]$ have chance to connect to each other. Thus, in the following analysis, we further assume that $R_u \leq 2R_v$. When there are vehicles on $[z+R_u, z+2R_v]$, if the vehicles separately from a_3 and $[z+R_u, z+2R_v]$ are connected, a_3 and $[z+R_u, L]$ is connected. Otherwise, when there is no vehicle on $[z+R_u, z+2R_v]$, the road interval $[0, L]$ is disconnected. The probability that there are vehicles on $[z+R_u, z+2R_v]$ is

$$\Pr\left\{H_{[z+R_u, z+2R_v]}\right\} = 1 - e^{-\gamma_2 \cdot (2R_v - R_u)}.$$

According to Ref. [1], we can obtain that the connectivity probability of vehicles separately from a_3 and $[z+R_u, z+2R_v]$ is

$$\Pr\left\{C_{a_3, [z+R_u, z+2R_v]}\right\} = \frac{1}{2}.$$

The connectivity probability of vehicles separately from a_3 and $[z+R_u, L]$ is

$$\Pr\left\{C_{\{a_3, [z+R_u, L]\}}\right\} = \Pr\left\{H_{[z+R_u, z+2R_v]}\right\} \cdot \Pr\left\{C_{\{a_3, [z+R_u, z+2R_v]\}}\right\}.$$

In this case, when $R_u > 2R_v$, the connectivity probability of road interval $[0, L]$ is 0; when $R_u \leq 2R_v$, the connectivity probability of road interval $[0, L]$ is calculated as follows

$$p_{1.2.1} = \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[0,z]}\right\}$$
$$\cdot \Pr\left\{C_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{\{a_3,[z+R_u,L]\}}\right\}. \tag{6}$$

When there are vehicles on $[0, z - R_u]$, but there is no vehicle on $[z + R_u, L]$, if vehicles on $[0, z]$ is connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.2.2} = \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[0,z]}\right\}. \tag{7}$$

When there are vehicles on $[z + R_u, L]$, but there is no vehicle on $[0, z - R_u]$, if the vehicles on $[z - R_u, z]$ are connected, the vehicles on $[z + R_u, L]$ are connected, and the vehicles separately from a_3 and $[z + R_u, L]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.2.3} = \Pr\left\{\bar{H}_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z-R_u,z]}\right\}$$
$$\cdot \Pr\left\{C_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{\{a_3,[z+R_u,L]\}}\right\}. \tag{8}$$

When there is no vehicle both in $[0, z - R_u]$ and $[z + R_u, L]$, if vehicles on $[z - R_u, z]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.2.4} = \Pr\left\{\bar{H}_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z-R_u,z]}\right\}. \tag{9}$$

Thus, the connectivity probability of road interval $[0, L]$ in a-2) is

$$p_{1.2} = \Pr\left\{H_{a_1}\right\} \cdot \Pr\left\{\bar{H}_{a_4}\right\} \cdot \left(p_{1.2.1} + p_{1.2.2} + p_{1.2.3} + p_{1.2.4}\right). \tag{10}$$

c) There are vehicles on a_4, but there is no vehicle on a_1.

In c), the probability that there is no vehicle on a_1 is

$$\Pr\left\{\bar{H}_{a_1}\right\} = 1 - \Pr\left\{H_{a_1}\right\} = e^{-\gamma_1 \cdot (R_u - R_v)}.$$

When there are vehicles both on $[0, z - R_u]$ and $[z + R_u, L]$, if vehicles on $[z, L]$ are connected, vehicles on $[0, z - R_u]$ are connected, and vehicles separately from a_2 and $[0, z - R_u]$ are connected, the road interval $[0, L]$ is connected. The probability that there are vehicles on $[0, z - R_u]$ is

$$\Pr\left\{C_{[0,z-R_u]}\right\} = \Pr\left\{\gamma_1, R_v, z - R_u\right\}.$$

The analysis for the connectivity probability of vehicles separately from a_2 and $[0, z - R_u]$ is similar to that of vehicles separately from a_3 and $[z + R_u, L]$. Therefore, when $R_u > 2R_v$, the connectivity of road interval $[0, L]$ is 0; when $R_u \leq 2R_v$, the connectivity of road interval $[0, L]$ is calculated as follows

$$p_{1.3.1} = \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z,L]}\right\}$$
$$\cdot \Pr\left\{C_{[0,z-R_u]}\right\} \cdot \Pr\left\{C_{\{a_2,[0,z-R_u]\}}\right\} \tag{11}$$

where

$$\Pr\left\{C_{\{a_2,[0,z-R_u]\}}\right\} = \Pr\left\{H_{[z-2R_v,z-R_u]}\right\} \cdot \Pr\left\{C_{\{a_2,[z-2R_v,z-R_u]\}}\right\},$$

$$\Pr\left\{H_{[z-2R_v,z-R_u]}\right\} = 1 - e^{-\gamma_1 \cdot (2R_v - R_u)}.$$

$$\Pr\left\{C_{\{a_2,[z-2R_v,z-R_u]\}}\right\} = \frac{1}{2}.$$

When there are vehicles on $[0, z-R_u]$, but there is no vehicle on $[z+R_u, L]$, if vehicles on $[z, z+R_u]$ are connected, vehicles on $[0, z-R_u]$ are connected, and vehicles separately from a_2 and $[0, z-R_u]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$\begin{aligned}
p_{1.3.2} &= \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z,z+R_u]}\right\} \\
&\quad \cdot \Pr\left\{C_{[0,z-R_u]}\right\} \cdot \Pr\left\{C_{\{a_2,[0,z-R_u]\}}\right\}.
\end{aligned} \tag{12}$$

When there are vehicles on $[z+R_u, L]$, but there is no vehicle on $[0, z-R_u]$, if vehicles on $[z, L]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.3.3} = \Pr\left\{\bar{H}_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z,L]}\right\}. \tag{13}$$

When there is no vehicle both on $[0, z-R_u]$ and $[z+R_u, L]$, if vehicles on $[z, z+R_u]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.3.4} = \Pr\left\{\bar{H}_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z,z+R_u]}\right\}. \tag{14}$$

Thus, the connectivity probability of road interval $[0, L]$ in a-3) is

$$p_{1.3} = \Pr\left\{\bar{H}_{a_1}\right\} \cdot \Pr\left\{H_{a_4}\right\} \cdot \left(p_{1.3.1} + p_{1.3.2} + p_{1.3.3} + p_{1.3.4}\right). \tag{15}$$

d) There is no vehicle on a_1 and a_4.

In d), for analyzing the connectivity of road interval $[0, L]$, we only need to consider the case that whether there are vehicles on $[0, z-R_u]$ and $[z+R_u, L]$.

When there are vehicles both on $[0, z-R_u]$ and $[z+R_u, L]$, if vehicles on $[0, z-R_u]$ are connected, vehicles on $[z+R_u, L]$ are connected, vehicles separately from a_2 and $[0, z-R_u]$ are connected, and vehicles separately from a_3 and $[z+R_u, L]$ are connected, the road interval $[0, L]$ is connected. In this case, when $R_u > 2R_v$, the connectivity probability of vehicles separately from a_2 and $[0, z-R_u]$ is 0, the connectivity probability of vehicles separately from a_3 and $[z+R_u, L]$ is 0, and the connectivity probability of road interval $[0, L]$ is 0. When $R_u \le 2R_v$, the connectivity probability of road interval $[0, L]$ is

$$\begin{aligned}
p_{1.4.1} &= \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z+R_u,L]}\right\} \\
&\quad \cdot \Pr\left\{C_{[0,z-R_u]}\right\} \cdot \Pr\left\{C_{\{a_2,[0,z-R_u]\}}\right\} \cdot \Pr\left\{C_{\{a_3,[z+R_u,L]\}}\right\}.
\end{aligned} \tag{16}$$

When there are vehicles on $[0, z-R_u]$, but there is no vehicle on $[z+R_u, L]$,

if vehicles on $[0, z-R_u]$ are connected, and vehicles separately from a_2 and $[0, z-R_u]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.4.2} = \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[0,z-R_u]}\right\} \cdot \Pr\left\{C_{\{a_2,[0,z-R_u]\}}\right\}. \quad (17)$$

When there are vehicles on $[z+R_u, L]$, but there is no vehicle on $[0, z-R_u]$, if vehicles on $[z+R_u, L]$ are connected, and vehicles separately from a_3 and $[z+R_u, L]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.4.3} = \Pr\left\{\bar{H}_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{[z+R_u,L]}\right\} \cdot \Pr\left\{C_{\{a_3,[z+R_u,L]\}}\right\}. \quad (18)$$

When there is no vehicle both on $[0, z-R_u]$ and $[z+R_u, L]$, the connectivity probability of the road interval $[0, L]$ is 1. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{1.4.4} = \Pr\left\{\bar{H}_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{[z+R_u,L]}\right\}. \quad (19)$$

Thus, the connectivity probability of road interval $[0, L]$ in a-4) is

$$p_{1.4} = \Pr\left\{\bar{H}_{a_1}\right\} \cdot \Pr\left\{\bar{H}_{a_4}\right\} \cdot \left(p_{1.4.1} + p_{1.4.2} + p_{1.4.3} + p_{1.4.4}\right). \quad (20)$$

Considering all of above, the connectivity probability of road interval $[0, L]$ in *a)* is

$$p_1 = \Pr\left\{H_{a_2}\right\} \cdot \Pr\left\{H_{a_3}\right\} \cdot \left(p_{1.1} + p_{1.2} + p_{1.3} + p_{1.4}\right). \quad (21)$$

2) There are vehicles on a_2, but there is no vehicle on a_3.

In 2), if there are vehicles on $[z+R_v, L]$, the distance between any two vehicles separately from $[z+R_v, L]$ and a_2 is larger than the communication radius of vehicles, the road interval $[0, L]$ is disconnected. Therefore, we only need to consider the connectivity of the interval $[0, z]$ in the case that there is no vehicle on $[z+R_v, L]$. The probability that there is no vehicle on $[z+R_v, L]$ is

$$\Pr\left\{\bar{H}_{[z+R_v,L]}\right\} = e^{-\gamma_2 \cdot (L-z-R_v)}.$$

When there are vehicles both on $[0, z-R_u]$ and a_1, if the vehicles on $[0, z]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of the road interval $[0, L]$ is

$$p_{2.1} = \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{H_{a_1}\right\} \cdot \Pr\left\{C_{[0,z]}\right\}. \quad (22)$$

When there are vehicles on $[0, z-R_u]$, but there is no vehicle on a_1, if vehicles on $[0, z-R_u]$ are connected, and the vehicles separately from a_2 and $[0, z-R_u]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of the road interval $[0, L]$ is

$$p_{2.2} = \Pr\left\{H_{[0,z-R_u]}\right\} \cdot \Pr\left\{\bar{H}_{a_1}\right\} \cdot \Pr\left\{C_{[0,z-R_u]}\right\} \cdot \Pr\left\{C_{\{a_2,[0,z-R_u]\}}\right\}. \quad (23)$$

When there are vehicles on a_1, but there is no vehicle on $[0, z-R_u]$, if ve-

hicles on $[z - R_u, z]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of the road interval $[0, L]$ is

$$p_{2.3} = \Pr\left\{\bar{H}_{[0, z - R_u]}\right\} \cdot \Pr\left\{H_{a_1}\right\} \cdot \Pr\left\{C_{[z - R_u, z]}\right\}. \tag{24}$$

When there is no vehicle both on $[0, z - R_u]$ and a_1, the connectivity of road interval $[0, L]$ is 1. In this case, the connectivity probability of the road interval $[0, L]$ is

$$p_{2.4} = \Pr\left\{\bar{H}_{[0, z - R_u]}\right\} \cdot \Pr\left\{\bar{H}_{a_1}\right\}. \tag{25}$$

Considering all of above, the connectivity probability of road interval $[0, L]$ in b) is

$$p_2 = \Pr\left\{H_{a_2}\right\} \cdot \Pr\left\{\bar{H}_{a_3}\right\} \cdot \Pr\left\{\bar{H}_{[z + R_v, L]}\right\} \cdot \left(p_{2.1} + p_{2.2} + p_{2.3} + p_{2.4}\right). \tag{26}$$

3) There are vehicles on a_3, but there is no vehicle on a_2.

In 3), if there are vehicles on $[0, z - R_v]$, and the distance between any two vehicles separately from $[0, z - R_v]$ and a_3 is larger than the communication radius of vehicles, the road interval $[0, L]$ is disconnected. Therefore, we only need to consider the connectivity of $[z + R_v, L]$ in the case that there is no vehicle on $[0, z - R_v]$. The probability that there is no vehicle on $[0, z - R_v]$ is

$$\Pr\left\{\bar{H}_{[0, z - R_v]}\right\} = \mathrm{e}^{-\gamma_1 \cdot (z - R_v)}.$$

When there are vehicles both on $[z + R_u, L]$ and a_4, if vehicles on $[z, L]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{3.1} = \Pr\left\{H_{[z + R_u, L]}\right\} \cdot \Pr\left\{H_{a_4}\right\} \cdot \Pr\left\{C_{[z, L]}\right\}. \tag{27}$$

When there are vehicles on $[z + R_u, L]$, but there is no vehicle on a_4, if vehicles on $[z + R_u, L]$ are connected, and the vehicles separately from a_3 and $[z + R_u, L]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{3.2} = \Pr\left\{H_{[z + R_u, L]}\right\} \cdot \Pr\left\{\bar{H}_{a_4}\right\} \cdot \Pr\left\{C_{[z + R_u, L]}\right\} \cdot \Pr\left\{C_{\{a_3, [z + R_u, L]\}}\right\}. \tag{28}$$

When there are vehicles on a_4, but there is no vehicle on $[z + R_u, L]$, if vehicles on $[z, z + R_u]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{3.3} = \Pr\left\{\bar{H}_{[z + R_u, L]}\right\} \cdot \Pr\left\{H_{a_4}\right\} \cdot \Pr\left\{C_{[z, z + R_u]}\right\}. \tag{29}$$

When there is no vehicle both on $[z + R_u, L]$ and a_4, the connectivity probability of the road interval $[0, L]$ is 1. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{3.4} = \Pr\left\{\bar{H}_{[z + R_u, L]}\right\} \cdot \Pr\left\{\bar{H}_{a_4}\right\}. \tag{30}$$

Considering all of above, the connectivity probability of road interval $[0, L]$

in c) is

$$p_3 = \Pr\{\bar{H}_{a_2}\} \cdot \Pr\{H_{a_3}\} \cdot \Pr\{\bar{H}_{[0,z-R_v]}\} \cdot (p_{3.1} + p_{3.2} + p_{3.3} + p_{3.4}). \tag{31}$$

4) There is no vehicle on a_2 and a_3.

In 4), if there are vehicles both on $[0, z - R_v]$ and $[z + R_v, L]$, the distance between any two vehicles separately from $[0, z - R_v]$ and $[z + R_v, L]$ is larger than the communication radius of vehicles, the road interval $[0, L]$ is disconnected. We only need to analyze the connectivity of the case that at most one of the interval $[0, z - R_v]$ and $[z + R_v, L]$ has vehicles.

When there are vehicles on $[0, z - R_v]$, but there is no vehicle on $[z + R_v, L]$, if vehicles on $[0, z - R_v]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{4.1} = \Pr\{H_{[0,z-R_v]}\} \cdot \Pr\{\bar{H}_{[z+R_v,L]}\} \cdot \Pr\{C_{[0,z-R_v]}\}, \tag{32}$$

where

$$\Pr\{H_{[0,z-R_v]}\} = 1 - e^{-\gamma_1 \cdot (z-R_v)}, \quad \Pr\{C_{[0,z-R_v]}\} = \Pr\{\gamma_1, R_v, z-R_v\}.$$

When there are vehicles on $[z + R_v, L]$, but there is no vehicle on $[0, z - R_v]$, if vehicles on $[z + R_v, L]$ are connected, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{4.2} = \Pr\{\bar{H}_{[0,z-R_v]}\} \cdot \Pr\{H_{[z+R_v,L]}\} \cdot \Pr\{C_{[z+R_v,L]}\}, \tag{33}$$

where

$$\Pr\{H_{[z+R_v,L]}\} = 1 - e^{-\gamma_2 \cdot (L-z-R_v)}, \quad \Pr\{C_{[z+R_v,L]}\} = \Pr\{\gamma_2, R_v, L-z-R_v\}.$$

When there is no vehicle both on $[0, z - R_v]$ and $[z + R_v, L]$, the road interval $[0, L]$ is connected. In this case, the connectivity probability of road interval $[0, L]$ is

$$p_{4.3} = \Pr\{\bar{H}_{[0,z-R_v]}\} \cdot \Pr\{\bar{H}_{[z+R_v,L]}\}. \tag{34}$$

Considering all of above, the connectivity probability of road interval $[0, L]$ in d) is

$$p_4 = \Pr\{\bar{H}_{a_2}\} \cdot \Pr\{\bar{H}_{a_3}\} \cdot (p_{4.1} + p_{4.2} + p_{4.3}). \tag{35}$$

According to analysis of 1)-4), we can obtain the connectivity probability of road interval $[0, L]$ as follows

$$p = \frac{1}{L - 2R_u} \int_{R_u}^{L-R_u} (p_1 + p_2 + p_3 + p_4) \, \mathrm{d}z. \tag{36}$$

4. Numerical Results

In this section, we perform simulation using event-driven and Monte Carlo methods by Matlab tool to verify the proposed analysis model. The parameter values are decided according to the assumption vehicular environment of statistics

observations. Then according to the numerical results, we analyze the impacts of several parameters on the connectivity probability. Each simulation result is an average over 10,000 times. It is seen from **Figures 3-5** the analytical results are very close to the simulation results.

Figure 3 shows the impacts of λ_1, λ_2 and α on the network connectivity probability.

With the increasing of λ_1, λ_2 and α, the connectivity probability also increases. This is because larger values of λ_1, λ_2 and α make the number of vehicles in unit road length becoming larger which reduces the distance of neighbor vehicles, and resulting in a larger connectivity probability of neighbor vehicles.
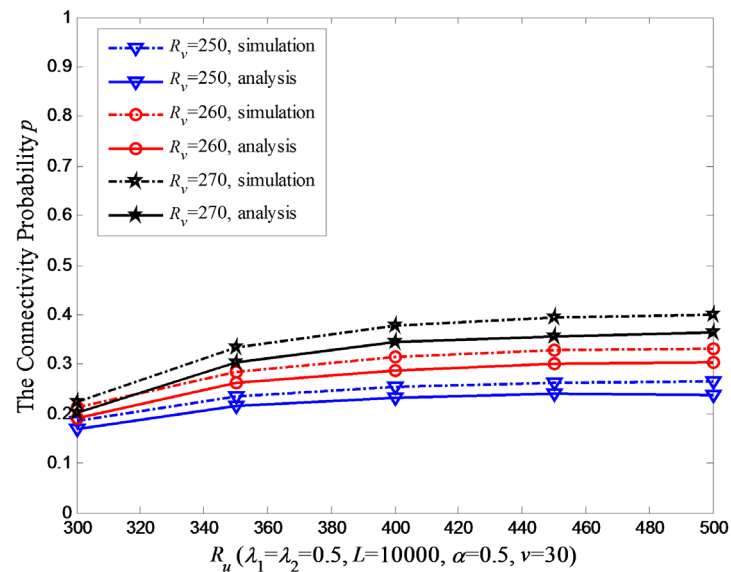
Figure 4 shows the impacts of L and v on the network connectivity probability. With the increasing of L and v, the connectivity probability decreases. This is because with the increase of L, if the arrival rates do not change, there are fewer
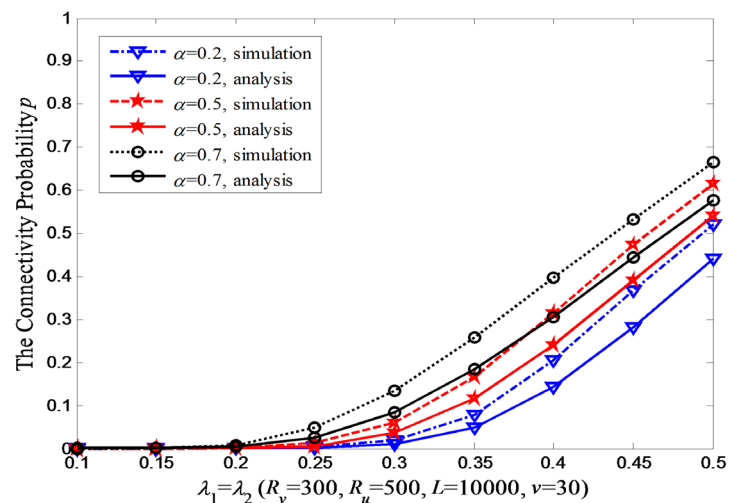
Figure 3. Impacts of λ_1, λ_2 and α on the connectivity probability.
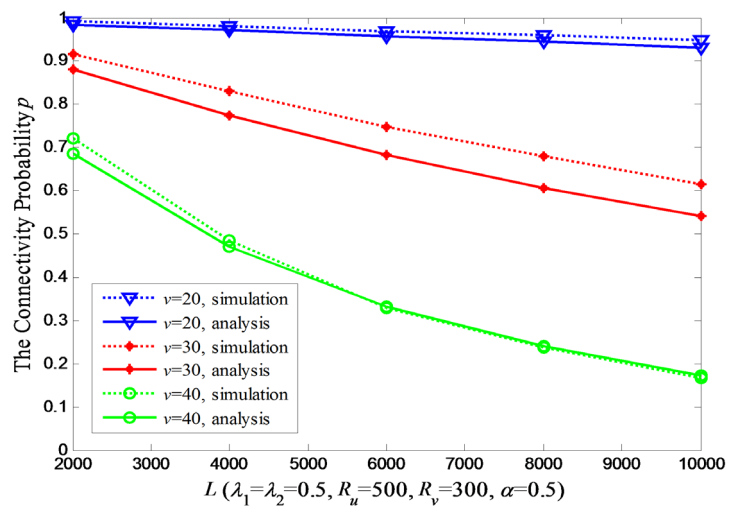
Figure 4. Impacts of L and v on the connectivity probability.

Figure 5. Impacts of R_u and R_v on the connectivity probability.

vehicles on the unit length of the highway road and the distance between two neighbor vehicles becomes larger. As a result, the connectivity probability decreases. Similarly, with the increase of v, if the arrival rates do not change, there are fewer vehicles on the unit length of the highway road and the distance between two neighbor vehicles becomes larger which leads to the decreasing of the network connectivity probability.

Figure 5 shows the impacts of R_u and R_v on the connectivity probability. With the increasing of R_u and R_v, the connectivity probability also increases. This is because a larger value of R_u and R_v makes two neighbor vehicles have a larger chance to communicate with each other.

5. Conclusion

In this paper, we build an analytical model to study the network connectivity of highway scenario with one entry/exit and one RSU. In building the analytical model, we divide the highway road into several road intervals, and analyze both the connectivity of each road interval and the connectivity of neighbor road interval. The analytical model is verified by using simulation tools, and the analytical model can be used to analyze the impacts of several parameters on the network connectivity. The built network connectivity analytical model can be used to provide a reference for the highway scenario with multiple entries/exits and multiple RSUs which are our future work.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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