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New Asymptotic Results on Fermat-Wiles Theorem

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Abstract

We analyse the Diophantine equation of Fermat $x^p + y^p = z^p$ with $p > 2$ a prime, x, y, z positive nonzero integers. We consider the hypothetical solution (a, b, c) of previous equation. We use Fermat main divisors, Diophantine remainders of (a, b, c), an asymptotic approach based on Balzano Weierstrass Analysis Theorem as tools. We construct convergent infinite sequences and establish asymptotic results including the following surprising one. If $z - y = 1$ then there exists a tight bound N such that, for all prime exponents $p > N$, we have $x^p + y^p \neq z^p$.

Keywords

Fermat's Last Theorem, Fermat-Wiles Theorem, Kimou's Divisors, Diophantine Quotient, Diophantine Remainders, Balzano Weierstrass Analysis Theorem

1. Introduction and Main Results

In 1637, Fermat wrote in Latin the following statement translated into modern language: there does not exist a non-zero natural number x, y, z and $n > 2$ such that,

$$x^n + y^n = z^n \tag{1}$$

He claimed to hold a marvellous proof of it that the margin could not contain. In 1670, Samuel de Fermat reissued Bachet's arithmetica augmented with his father's annotations. Then, Fermat's words were made public and popular. Subsequently, they proved to be a powerful stimulus in number theory research and contributed to the development of modern mathematics. We note the following significant facts attached to the problem posed by Fermat.

- In 1670, the proof of Fermat theorem for regular triangle established by using descent infinity method was published. The case $n = 4$ was an immediate consequence of this theorem.
- In 1770, Euler used the infinite descent method and the arithmetic of numbers $a + b\sqrt{-3}$ with *a*, *b* are integers to prove the case $n = 3$ of the problem ([1], pp. 24, 33).
- In 1825, Dirichlet proposed a proof of the cases $n = 5$. His proof will be published in 1928 ([2], pp. 49, 55).
- In 1839, Lamé proved the case $n = 7$. He made use of a polynomial identity of degree 7 ([1], p. 220). In 1840, Lebesgue found a simpler proof ([1], pp. 57, 62).
- There have been numerous papers devoted to the proof of Fermat's theorem for special exponents, other than 7. The methods used were specific to the exponent in question and in most instances not susceptible of generalization ([1], pp. 63, 64).
- Between 1840 and 1847, Lamé and Cauchy produced erroneous evidence for a general solution to FLT ([2], p. 6).
- In 1823, Sophie Germain established that the first case of FLT is true for any prime $p < 100$. A corollary to Sophie Germain's theorem asserted that the first case of FLT for all odd exponents p such that $2p + 1$ is also prime is true. In the same year, Legendre extended this result for exponent p such that $kp + 1$ were $k \in \{4, 8, 10, 16\}$ is also prime ([1], pp. 109, 112).
- In 1846, Kummer introduced the theory of cyclotomic field and proved that the first case of FLT is true for regular prime exponents. He also proved that FLT is true for all exponent $p < 100$ [2].
- In 1909, Wieferich proved that, if FLT fails in the first case, then $2^{p-1} - 1 \equiv 0\left[p^2\right]$ ([1], p. 266).
- In 1977, Terjanian used the modular arithmetic, the residues theorem, and the law of quadratic reciprocity and establish the first case of the exponent $2p$ with p is an odd prime ([1], p. 203).
- In 1952 Harry Vandiver used a SWAC computer to prove FLT for all exponents below 2000. Later, Wagstaff pushed this limit to 125000 ([2], p. 102).
- In 1983, the Falting theorem is proved. One consequence of this result was that Fermat's equation has only a finite number of solutions [3].
- In 1985 Adleman, Heath-Brown, and Fouvry used methods from Sieve's theory, and they established that the first case of FLT is true for an infinity of prime numbers. The tools mentioned go beyond the framework of elementary arithmetic ([1], p. 363). Other authors have investigated the density of the prime exponent for which FLT is true. Their results were published in [4] [5].
- In 1994 André Wiles proposed an indirect and complex proof based on elliptic curves, Galois representations and modular forms. But his proof contained an error. A year later, it was repaired, and FLT was finally demonstrated in 1995. The tools mentioned go beyond the framework of elemen-

tary arithmetic ([1], pp. 366-374, [6]).

Since that time, the hope of finding a direct or elementary proof at FLT disappeared to give way to indirect and sophisticated methods. After a period of euphoria maintained by the proof of Wiles, the idea of a direct and simple proof is reborn but in isolation [7] [8].

It's well known that the proof of FLT would be shortened if the *abc* conjecture were proven. By application of the *abc* conjecture, there exists a number n_0 such that if $n > n_0$ then FLT is true ([1], pp. 364-365; [9], p. 1436). But this evidence is indirect, and today inadmissible because the *abc* conjecture has still not been fully demonstrated.

In 2023 Kimou Prosper and Tanoe François introduced Fermat divisors, quotients and Diophantine remainders and demonstrated results attached to them. These efficient tools have given new possibilities to indeed compute and analyse the Fermat equation and other Diophantine type equations [10] [11] [12]. Our tools have enabled us to find new properties of Fermat's equation or its hypothetical solution that were previously unsuspected. In this paper, we use an original asymptotic approach and Fermat divisors, Diophantine quotients and Diophantine remainders to prove the following main results.

Theorem 1. Let $p > 2$ a prime and let (a_p, b_p, c_p) a non-trivial primitive triple of positive integers solution of Equation (1) with exponent *p* such that $(a_p < b_p < c_p)$. Consider the Diophantine Quotient and Remainders $(q_{1,p}, q_{2,p}, r_{1,p}, r_{2,p})$ of (a_p, b_p, c_p) and a positive real *M*. Then

$$\begin{cases} \lim_{p\to\infty} \frac{b_p}{a_p} = \lim_{p\to\infty} \frac{c_p}{a_p} = q_\infty + l \ \textit{if} \ q_{2,p} \ \textit{is bound} \\ \lim_{p\to\infty} \frac{b_p}{a_p} = \lim_{p\to\infty} \frac{b_p}{a_p} = +\infty \ \textit{if} \ q_{2,p} \ \textit{is'nt bound} \\ \lim_{p\to\infty} \frac{c_p}{b_p} = 1 \end{cases}$$

where $b_p = a_p q_{1,p} + r_{1,p}, c_p = a_p q_{2,p} + r_{2,p}, r_{1,p}, r_{2,p} < a_p$ and $q_\infty = \lim_{p\to\infty} q_{1,p}$.

The 4-uple $(q_{1,p}, q_{2,p}, r_{1,p}, r_{2,p})$ is define as the Diophantine Quotients and Remainders of (a_p, b_p, c_p).

Theorem 2. Let $p > 2$ a prime and let (a_p, b_p, c_p) a non-trivial primitive triple of positive integers solution of Equation (1) with exponent *p* such that $(a_p < b_p < c_p)$. Consider the quotient of the Euclidean division of c_p by a_p. Then

$$q_{2,p} > 1 \Rightarrow \begin{cases} \lim_{p\to\infty} \frac{e_p}{f_p} = 1 \\ \lim_{p\to\infty} \frac{\beta_p}{\gamma_p} = 1 \end{cases}$$

where $e_p = \gcd(b_p, c_p - a_p), f_p = \gcd(c_p, a_p + b_p), b_p = e_p \beta_p, c_p = f_p \gamma_p$

Theorem 3. Let $p>2$ be a prime and let (a,b,c) be a triple of non-trivial primitive solution of Equation (1) with degree *p* such that *a* < *b* < *c*. Consider $c=q_2a+r_2, b=q_1a+r_1$ with $r_2, r_1<a$. Then,

$$q_2=1, \frac{r_1}{a}<\frac{1}{p^2} \Rightarrow \frac{r_2}{a} \underset{p\to\infty}{\sim} \frac{\ln 2}{p}.$$

Theorem 4. Let $p>2$ be a prime and let (a,b,c) be a non-trivial primitive triple solution of Equation (1) with exponent *p* such that $(a<b<c)$. Consider (e,f) the Main Kimou's Divisors of (b,c). Then

$$b\equiv 0[p], c=b+1, f>e \Rightarrow \exists N>2, \forall p>N, a^p+b^p\neq c^p$$

We give definitions, lemmas, and theorems with their proofs in the following secund section. In the third section, we give complementary results. In the section 4, we prove the main results. We conclude and give some perspectives at the end.

2. Preliminary

In this section, we give definitions, lemmas and theorems known in the literature and useful to establish our main results.

Theorem 2.1. (Bolzano-Weierstrass). Every bounded sequence has a convergent sub-sequence.

Definition 2.1. Let $(u_n)_{n\geq 1}$ and $(v_n)_{n\geq 1}$ two positive real sequences. We say that $(u_n)_{n\geq 1}$ and $(v_n)_{n\geq 1}$ are equivalent near infinity and we note $u_n \underset{n\to\infty}{\sim} v_n$ if only if:

$$\lim_{n\to\infty}\frac{u_n}{v_n}=1.$$

Lemma 2.1. Let $(u_n)_{n\geq 1}$ and $(v_n)_{n\geq 1}$ two positive real sequences such that $\lim_{n\to\infty}(u_n+v_n)=0$ Then

$$(u_n)_{n\geq 1}\ \textit{is bound} \Rightarrow \exists s(n), \lim_{n\to\infty}u_{s(n)}=\lim_{n\to\infty}v_{s(n)}=0$$

Proof.

If $(u_n)_{n\geq 1}$ is bound then,

$$\begin{aligned}
\lim_{n\to\infty}u_{s(n)}>0 &\Rightarrow \exists s(n)\, and\, l>0, \lim_{n\to\infty}u_{s(n)}=l\,[Theorem\ 2.1.]\\
&\Rightarrow \lim_{n\to\infty}\left(u_{s(n)}+v_{s(n)}\right)=0\\
&\Rightarrow \lim_{n\to\infty}u_{s(n)}+\lim_{n\to\infty}v_{s(n)}=0\\
&\Rightarrow \lim_{n\to\infty}v_{s(n)}=-l\\
&\Rightarrow Absurd\ because\ v_{s(n)}>0\,\forall n>1.
\end{aligned}$$

Notation 2.1.

We denote by F_p the set of non-trivial hypothetical primitive solutions of Equation (1) with degree p:

$$F_p=\left\{(a,b,c)\in\mathbb{N}^{*3}: a^p+b^p=c^p, \gcd(a,b,c)=1\right\}.$$

Definition 2.2. Let $p>2$ a prime and $(a,b,c)\in F_p$. We recall that the Kimou's Divisors (or Fermat Divisors) is the sextuple $(d,e,f,\alpha,\beta,\gamma)$ defined that:

$$d=\gcd(a,c-b), e=\gcd(b,c-a),$$
$$f=\gcd(c,a+b) \text{ and } a=d\alpha, b=e\beta, c=f\gamma,$$

where (d,e,f) is the Main Kimou's Divisors (MKD) and (α,β,γ) the Auxiliary Kimou's divisors (AKD).

Remark 2.1. The elements of the 6-uplet $(d,e,f,\alpha,\beta,\gamma)$ were originally called Fermat's divisors. We now call them Kimou's Divisors after the author who first introduced them in 2023 [10]. This new name will distinguish them from Fermat's divisors, which are all the integers that divide the polynomial x^n-x.

Theorem 2.2. Let $p>2$ be a prime and let $(a,b,c)\in F_p$. Consider (d,e,f) the Main Kimou's Divisors of (a,b,c). Then

$$abc\not\equiv 0[p]\Rightarrow\begin{cases}2a=d^p-e^p+f^p\\2b=-d^p+e^p+f^p\\2c=d^p+e^p+f^p\end{cases}, c\equiv 0[p]\Rightarrow\begin{cases}2a=d^p-e^p+\dfrac{f^p}{p}\\2b=-d^p+e^p+\dfrac{f^p}{p}\\2c=d^p+e^p+\dfrac{f^p}{p}\end{cases}.$$

Proof. See [10], Theorem 1.

Theorem 2.3. Let $p>2$ be a prime and let $(a,b,c)\in F_p$. Consider (d,e,f) the Main Kimou's Divisors of (a,b,c). Then

$$a\equiv 0[p]\Rightarrow\begin{cases}2a=\dfrac{d^p}{p}-e^p+f^p\\2b=-\dfrac{d^p}{p}+e^p+f^p\\2c=\dfrac{d^p}{p}+e^p+f^p\end{cases}, b\equiv 0[p]\Rightarrow\begin{cases}2a=d^p-\dfrac{e^p}{p}+f^p\\2b=-d^p+\dfrac{e^p}{p}+f^p\\2c=d^p+\dfrac{e^p}{p}+f^p\end{cases}.$$

Proof. See [10], Theorem 1.

Lemma 2.2. Let $p>2$ be a prime and let $(a,b,c)\in F_p$. Consider (d,e,f) the Main Kimou's Divisors of (a,b,c). We have:

$$abc\not\equiv 0[p]\Rightarrow\begin{cases}c-b=d^p\\c-a=e^p\\a+b=f^p\end{cases}, c\equiv 0[p]\Rightarrow\begin{cases}c-b=d^p\\c-a=e^p\\a+b=\dfrac{f^p}{p}\end{cases}.$$

Proof. See [10], Theorem 1.

Lemma 2.3. Let $p>2$ be a prime and let $(a,b,c)\in F_p$. Consider (d,e,f) the Main Kimou's Divisors of (a,b,c). We have:

$$a \equiv 0[p] \Rightarrow \begin{cases} c-b = \dfrac{d^p}{p} \\ c-a = e^p \\ a+b = f^p \end{cases}, b \equiv 0[p] \Rightarrow \begin{cases} c-b = d^p \\ c-a = \dfrac{e^p}{p} \\ a+b = f^p \end{cases}.$$

Proof. See [10], Theorem 1.

Remark 2.2. We also have,

$$\begin{aligned}(a,b,c) \in F_p &\Rightarrow e = \gcd(b, c-a) \\ &\Rightarrow \begin{cases} e = \gcd\left(\beta e, \dfrac{e^p}{p}\right) \text{if} b \equiv 0[p] \\ e = \gcd(\beta e, e^p) \text{otherwise} \end{cases} \\ &\Rightarrow \begin{cases} 1 = \gcd\left(\beta, \dfrac{e^{p-1}}{p}\right) \text{if} b \equiv 0[p] \\ 1 = \gcd(\beta, e^{p-1}) \text{otherwise} \end{cases} \\ &\Rightarrow \gcd(\beta, e) = 1\end{aligned}$$

Lemma 2.4. *Let* $p > 2$ a prime. If $(a,b,c) \in F_p$ such that $a < b < c$ then, there exist (q_1, r_1) and (q_2, r_2) such that $b = aq_1 + r_1$ and $c = aq_2 + r_2$ with $r_1, r_2 < a$.

Proof. Immediate consequence of the Euclidean Division Theorem.

Theorem 2.5. Let $p > 2$ a prime and let a, b and c a pairwise relativity prime such that $a < b < c$. Consider (q_1, r_1) and (q_2, r_2) the respective quotient and the remainder of Euclidean divisibility of b and c by a. Then

$$(a,b,c) \in F_p \Rightarrow q_2 = q_1 \text{ or } q_2 = q_1 + 1.$$

Proof. See [11] (p. 201).

Theorem 2.6. Let $p > 2$ a prime and let $(a,b,c) \in F_p$ such that $a < b < c$. Consider (q_1, r_1) and (q_2, r_2) the respective quotient and the remainder of Euclidean divisibility of b and c by a. Then

$$c - b = 1 \Rightarrow q_2 = q_1.$$

Proof. See [11] (p. 201).

3. New Complementary Results

In this section, we establish news results necessary to prove our main results. A little further, we use the asymptotic approach whose principle is as follow. Let $p > 2$ be a prime parameter. We assume that for all $p > 2$ it is possible to construct primitive solutions (a_p, b_p, c_p) or $(a_{s(n)}, b_{s(n)}, c_{s(n)})$ of Equation (1) where $(s(n))_{n \geq 1}$ is a sub-sequence of prime numbers. We consider the set $\mathcal{P}$ of prime numbers and define the sequence s of primes number as follow:

$$\begin{aligned} s : \mathbb{N}^* &\to \mathcal{P} \\ n &\mapsto p_n. \end{aligned}$$

If $s(n)$ is a finite sequence, Fermat Last Theorem is asymptotically proved. Otherwise, we consider that $s(n)$ is an infinite sequence. Then $a_{s(n)}, b_{s(n)}$ and $c_{s(n)}$ form three infinite sequences of positive integers. Consider the Diophantine quotients and remainders (q_1, q_2, r_1, r_2) of $(a_{s(n)}, b_{s(n)}, c_{s(n)})$. We construct two other sequences $\left(\frac{r_1}{a}\right)_{n\geq 1}$ and $\left(\frac{r_2}{a}\right)_{n\geq 1}$ then we prove their convergence by using. the Bolzano-Weierstrass analysis theorem.

Let note that when we study FLT with arbitrary exponent, we implicitly work with sequences.

Lemma 3.2. Let $p > 2$ be a prime and let $(a,b,c) \in F_p$ such that $a < b < c$. Consider the Principal Kimou's Divisors (d,e,f) of (a,b,c) and the Diophantine quotient and remainder (q_2, r_2) of *b*. We have:

$$q_2 > 1 \Rightarrow \begin{cases} \dfrac{e^p}{pa} > 1 \text{ if } b \equiv 0\,[p] \\ \dfrac{e^p}{a} > 1 \text{ otherwise} \end{cases}.$$

Proof. With the hypothesis of Lemma 2.7. We have:

$$q_2 > 1 \Rightarrow c - a = (q_2 - 1)a + r_2 \text{ because } c = aq_2 + r_2$$

$$\Rightarrow \begin{cases} \dfrac{e^p}{p} = (q_2 - 1)a + r_2 \text{ if } b \equiv 0[p] \\ e^p = (q_2 - 1)a + r_2 \text{ otherwise} \end{cases} [\text{Lemmas } 2.1, 2.2]$$

$$\Rightarrow \begin{cases} \dfrac{e^p}{p} > (q_2 - 1)a \text{ if } b \equiv 0[p] \\ e^p > (q_2 - 1)a \text{ otherwise} \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{e^p}{p} > a \text{ if } b \equiv 0[p] \\ e^p > a \text{ otherwise} \end{cases} \text{because } q_2 \geq 1.$$

Lemma 3.3. Let $p > 2$ be a prime and let (a,b,c) be a triple of positive integers such that $a < b < c$. Then,

$$(a,b,c) \in F_p \Rightarrow \begin{cases} d < e < f \text{ if } abc \not\equiv 0[p] \\ \dfrac{d}{\sqrt{p}} < e < f \text{ if } a \equiv 0[p] \\ d < \dfrac{e}{\sqrt{p}} < f \text{ if } b \equiv 0[p] \\ d < e < \dfrac{f}{\sqrt{p}} \text{ if } c \equiv 0[p] \end{cases}.$$

where (d,e,f) is the Main Kimou's Divisors of (a,b,c).

Proof.

$$(a,b,c) \in F_p \Rightarrow c - b < c - a < a + b$$

According to lemmas 2.1., 2.2. and taking account each case, we have:

$$(a,b,c)\in F_p \Rightarrow \begin{cases} d^p < e^p < f^p \ if \ abc \not\equiv 0[p] \\ \dfrac{d^p}{p} < e^p < f^p \ if \ a \equiv 0[p] \\ d^p < \dfrac{e^p}{p} < f^p \ if \ b \equiv 0[p] \\ d^p < e^p < \dfrac{f^p}{p} \ if \ c \equiv 0[p] \end{cases}$$

$$\Rightarrow \begin{cases} d < e < f \ if \ abc \not\equiv 0[p] \\ \dfrac{d}{\sqrt{p}} < e < f \ if \ a \equiv 0[p] \\ d < \dfrac{e}{\sqrt{p}} < f \ if \ b \equiv 0[p] \\ d < e < \dfrac{f}{\sqrt{p}} \ if \ c \equiv 0[p] \end{cases}$$

Lemma 3.4. Let $p > 2$ be a prime and let (a,b,c) a triple of positive integers such that $a < b < c$. Then,

$$(a,b,c)\in F_p \Rightarrow \begin{cases} f < \sqrt[p]{3pe} \ if \ c \equiv 0[p] \\ f < \sqrt[p]{\dfrac{3}{p}}e \ if \ b \equiv 0[p] \\ f < \sqrt[p]{3}e \ otherwise \end{cases}$$

where (d,e,f) is the Main Kimou's Divisors of (a,b,c).

Proof.

In the one hand,

$$c \equiv 0[p], q_2 > 1 \Rightarrow \frac{e^p}{a} > 1, \frac{e^p}{a} = \frac{2e^p}{2a} [Lemma \ 3.2]$$

$$\Rightarrow \frac{e^p}{a} > 1, \frac{e^p}{a} = \frac{2e^p}{d^p - e^p + \dfrac{f^p}{p}} [Theorem \ 2.2]$$

$$\Rightarrow \frac{e^p}{a} > 1, \frac{e^p}{a} = \frac{2}{\dfrac{d^p}{e^p} - 1 + \dfrac{f^p}{pe^p}}$$

$$\Rightarrow 0 < \frac{d^p}{e^p} - 1 + \frac{f^p}{pe^p} < 2$$

$$\Rightarrow \frac{d^p}{e^p} + \frac{f^p}{pe^p} < 3$$

$$\Rightarrow \frac{f^p}{pe^p} < 3$$

$$\Rightarrow f < \sqrt[p]{3pe}.$$

In the other hand,

$$b \equiv 0[p], q_2 > 1 \Rightarrow \frac{e^p}{pa} > 1 and \frac{e^p}{pa} = \frac{2e^p}{2pa} [Lemma \ 3.2, Theorem \ 2.2]$$

$$\Rightarrow \frac{e^p}{pa} > 1, \frac{e^p}{pa} = \frac{2}{p\frac{d^p}{e^p} - 1 + p\frac{f^p}{e^p}} [Theorem\ 2.2]$$

$$\Rightarrow 0 < p\frac{d^p}{e^p} - 1 + p\frac{f^p}{e^p} < 2$$

$$\Rightarrow p\frac{d^p}{e^p} + p\frac{f^p}{e^p} < 3$$

$$\Rightarrow p\frac{f^p}{e^p} < 3$$

$$\Rightarrow f < \sqrt[p]{\frac{3}{p}}e.$$

For the other cases ($abc \not\equiv 0[p]$ *and* $a \equiv 0[p]$) we proceed in the same way and obtain

$$abc \not\equiv 0[p], a \equiv 0[p] \Rightarrow f < \sqrt[p]{3}e.$$

Remark 3.1. We also have,

$$b \equiv 0[p], q_2 > 1 \Rightarrow f < \sqrt[p]{\frac{3}{p}}e \Rightarrow f < e.$$

Lemma 3.5. Let $p > 2$ a prime. We have,

$$(a,b,c) \in F_p \Rightarrow p(c-b)\left(\frac{b}{a}\right)^{p-1} < a < p(c-b)\left(\frac{c}{a}\right)^{p-1}.$$

Proof. Let $p > 2$ a prime.

$$\begin{aligned}(a,b,c) \in F_p &\Rightarrow a^p + b^p = c^p \\ &\Rightarrow a^p = c^p - b^p \\ &\Rightarrow a^p = (c-b)\sum_{k=0}^{p-1} c^k b^{p-1-k} \\ &\Rightarrow (c-b)\sum_{k=0}^{p-1} b^k b^{p-1-k} < a^p < (c-b)\sum_{k=0}^{p-1} c^k c^{p-1-k} \\ &\Rightarrow (c-b)\sum_{k=0}^{p-1} b^{p-1} < a^p < (c-b)\sum_{k=0}^{p-1} c^k c^{p-1-k} \\ &\Rightarrow p(c-b)b^{p-1} < a^p < p(c-b)c^{p-1} \\ &\Rightarrow p(c-b)\left(\frac{b}{a}\right)^{p-1} < a < (c-b)\left(\frac{c}{a}\right)^{p-1}.\end{aligned}$$

Lemma 3.6. Let $p > 2$ a prime, $(a,b,c) \in F_p$ and (q_1, q_2) the quotient of (b,c) by a. Then

$$pq_1^{p-1}(c-b) < a < p(1+q_2)^{p-1}(c-b).$$

Proof. Let $(a,b,c) \in F_p$ such that $a \not\equiv 0[p]$, we have:

$$(a,b,c) \in F_p \Rightarrow p(c-b)\left(\frac{b}{a}\right)^{p-1} < a < (c-b)\left(\frac{c}{a}\right)^{p-1} [Lemma\ 3.5]$$

$$\Rightarrow p(c-b)\left(q_1 + \frac{r_1}{a}\right)^{p-1} < a < (c-b)\left(q_2 + \frac{r_2}{a}\right)^{p-1} [Lemma\ 2.3.]$$

$$\Rightarrow p(c-b)q_1^{p-1} < a < (c-b)(q_2+1)^{p-1} \text{ because } \frac{r_1}{a}, \frac{r_2}{a} < 1.$$

Remark 3.2. According to Lemmas 2.1, 2.2 and 3.6. we deduce that,

$$(a,b,c) \in F_p \Rightarrow \begin{cases} d^p q_1^{p-1} < a < d^p (q_2+1)^{p-1} \; if \; a \equiv 0[p] \\ pd^p q_1^{p-1} < a < pd^p (q_2+1)^{p-1} \; otherwise \end{cases}.$$

Lemma 3.7. Let $p > 2$ be a prime and let (a,b,c) be a triple of integers such that $a < b < c$. Then

$$(a,b,c) \in F_p \Rightarrow \frac{c-b}{a} < \frac{1}{p}.$$

Proof. Let $(a,b,c) \in F_p$ and q_1 the quotient of Euclidean division of b by a. We have

$$\begin{aligned} a \equiv 0[p] &\Rightarrow a > q_1^{p-1}(c-b)[lemma\ 3.6] \\ &\Rightarrow a > p(c-b) \\ &\Rightarrow a > p(c-b) \\ &\Rightarrow \frac{c-b}{a} < \frac{1}{p}. \end{aligned}$$

Lemma 3.8. Let $p > 2$ be a prime and let (a,b,c) a triple of positive integers such that $a < b < c$. Then,

$$(a,b,c) \in F_p \Rightarrow \left(\frac{c-b}{a}\right)_\infty = 0.$$

Proof.

$$\begin{aligned} (a,b,c) \in F_p &\Rightarrow 0 < \frac{c-b}{a} < \frac{1}{p}[Lemma\ 3.4] \\ &\Rightarrow 0 \le \left(\frac{c-b}{a}\right)_\infty \le 0 \\ &\Rightarrow \left(\frac{c-b}{a}\right)_\infty = 0. \end{aligned}$$

Lemma 3.9. Let $p > 2$ be a prime and let (a,b,c) be a triple of positive integers such that $a < b < c$. Consider the Main Kimou's Divisors (d,e,f) of (a,b,c). Then,

$$(a,b,c) \in F_p \Rightarrow \begin{cases} (q_2 - q_1)_\infty = 0 \\ \left(\frac{r_2 - r_1}{a}\right)_\infty = 0 \end{cases}.$$

Proof.

$$\begin{aligned} (a,b,c) \in F_p &\Rightarrow \frac{c-b}{a} = q_2 - q_1 + \frac{r_2 - r_1}{a} \\ &\Rightarrow 0 < q_2 - q_1 + \frac{r_2 - r_1}{a} < \frac{c-b}{a} \\ &\Rightarrow \left(q_2 - q_1 + \frac{r_2 - r_1}{a}\right)_\infty = 0 \end{aligned}$$

$$\Rightarrow (q_2 - q_1)_\infty = 0 \text{ and } \left(\frac{r_2 - r_1}{a}\right)_\infty = 0 \text{ because } \frac{r_2 - r_1}{a} < 1.$$

Theorem 3.10. Let $p > 2$ be a prime and let (a,b,c) be a triple of positive integers such that $a < b < c$. Consider the quotients q_1 and q_2 of the Euclidean division of *b* and *c* by *a*. Then,

$$(a,b,c) \in F_p \Rightarrow \exists N, p > N, q_2 = q_1.$$

Proof.

$$\begin{aligned}(a,b,c) \in F_p, q_2 = q_1 + 1 &\Rightarrow q_2 - q_1 = 1 \\ &\Rightarrow (q_2 - q_1)_\infty = 1 \\ &\Rightarrow 0 = 1 [\textit{Lemma } 3.9] \\ &\Rightarrow \text{Absurd}\end{aligned}$$

Hence there exist a rank *N* such that $q_2 = q_1$ for any $p > N$.

Lemma 3.11. Let $p > 2$ be a prime and let $(a_p, b_p, c_p)_{n \ge 1}$ be an infinite sequence such that $(a_p, b_p, c_p) \in F_p$. Consider $(q_{1,p}, q_{2,p})$ and $(r_{1,p}, r_{2,p})$ the Diophantine quotients and remainders of (a_p, b_p, c_p). We construct a new sequence of rational numbers $\left(\frac{r_1}{a}\right)_p$, $\left(\frac{r_2}{a}\right)_p$. Then, there exist two subsequence's $\left(\frac{r_2}{a}\right)_{p_n}$ and $\left(\frac{r_1}{a}\right)_{p_n}$ converging to the same limit l:

$$(a_p, b_p, c_p) \in F_p \Rightarrow \begin{cases} \exists l > 0, \left(\frac{r_1}{a}\right)_\infty = \left(\frac{r_2}{a}\right)_\infty = l \\ (q_1)_\infty = (q_2)_\infty \end{cases}.$$

Proof. Let $\left(\frac{r_1}{a}\right)_{S(n), n \ge 1}$ be a converging subsequence extracted from the sequence $\left(\frac{r_1}{a}\right)_{p \ge 3}$. Let's simplify notations by adopting the following conventions:

$$\lim_{n \to \infty} \left(\frac{r_1}{a}\right)_{S(n)} = \lim_{n \to \infty} \left(\frac{r_1}{a}\right)_{p_n} = \lim_{n \to \infty} \frac{r_{1,n}}{a_n} = l$$

We'll do the same with $\left(\frac{r_2}{a}\right)_{S(n), n \ge 1}$.

In the one hand

$$\begin{aligned}(a,b,c) \in F_p &\Rightarrow \frac{c-b}{a} = \frac{r_2 - r_1}{a} [\textit{Theorem } 3.10] \\ &\Rightarrow \left(\frac{c-b}{a}\right)_\infty = \left(\frac{r_2 - r_1}{a}\right)_\infty \\ &\Rightarrow \left(\frac{r_2 - r_1}{a}\right)_\infty = 0 [\text{Lemma } 3.9.]\end{aligned}$$

In the other hand,

$$\begin{aligned}(a,b,c)\in F_p &\Rightarrow 0<r_1,r_2<a\\ &\Rightarrow 0<\frac{r_1}{a},\frac{r_2}{a}<1\\ &\Rightarrow \left(\frac{r_1}{a}\right)_p \text{ and } \left(\frac{r_2}{a}\right)_p \text{ are bound.}\end{aligned}$$

So according to Theorem 2.1. There exist convergent subsequences $\left(\frac{r_2}{a}\right)_{S(n)}$, $\left(\frac{r_1}{a}\right)_{S(n)}$. Hence

$$\begin{aligned}(a,b,c)\in F_p &\Rightarrow \lim_{n\to\infty}\left(\frac{r_2}{a}\right)_{p_n}-\lim_{n\to\infty}\left(\frac{r_1}{a}\right)_{p_n}=0\\ &\Rightarrow \lim_{n\to\infty}\left(\frac{r_2}{a}\right)_{p_n}=\lim_{n\to\infty}\left(\frac{r_1}{a}\right)_{p_n}\\ &\Rightarrow \left(\frac{r_1}{a}\right)_\infty=\left(\frac{r_2}{a}\right)_\infty\end{aligned}$$

Furthermore

$$\begin{aligned}(a,b,c)\in F_p &\Rightarrow (q_2-q_1)_\infty=0[\text{Lemma 3.9.}]\\ &\Rightarrow \begin{cases}(q_1)_\infty=(q_2)_\infty \text{ if } q_2<+\infty\\ (q_1)_\infty=(q_2)_\infty=+\infty \text{ otherwise}\end{cases}\\ &\Rightarrow (q_1)_\infty=(q_2)_\infty .\end{aligned}$$

Lemma 3.12. Let $p>2$ be a prime and let $(a_p,b_p,c_p)_{p\geq 1}$ be an infinite sequence such that $(a_p,b_p,c_p)\in F_p$. Consider $(q_{1,p},q_{2,p})$ and $(r_{1,p},r_{2,p})$ the Diophantine quotients and Remainders of (a_p,b_p,c_p) and the convergent subsequence $\left(\frac{r_2}{a}\right)_{p_n}$ and $\left(\frac{r_1}{a}\right)_{p_n}$. Then

$$\lim_{n\to\infty}\left(\frac{r_2}{a}\right)_{p_n}=\lim_{n\to\infty}\left(\frac{r_1}{a}\right)_{p_n}=l\Rightarrow l\in[0,1].$$

Proof.

$$\begin{aligned}(a_p,b_p,c_p) &\Rightarrow 0<r_{1,p},r_{2,p}<a\\ &\Rightarrow 0<\frac{r_{1,p}}{a_p},\frac{r_{2,p}}{a_p}<1\\ &\Rightarrow 0\leq \lim_{n\to\infty}\left(\frac{r_2}{a}\right)_{p_n}\leq 1\\ &\Rightarrow 0\leq l\leq 1.\end{aligned}$$

Remark 3.3. Figure 1 shows the Flowchart of the proof of Lemma 3.11. it's shows that the basic assumption $b=aq_1+r_1$, $c=aq_2+r_2$ has three possible consequences: $\frac{1}{p(q_2+1)^{p-1}}<\frac{c-b}{a}<\frac{1}{p}q_1^{p-1}$, $\frac{c-b}{a}<\frac{1}{p}$ or $\frac{r_1}{a},\frac{r_2}{a}<1$.

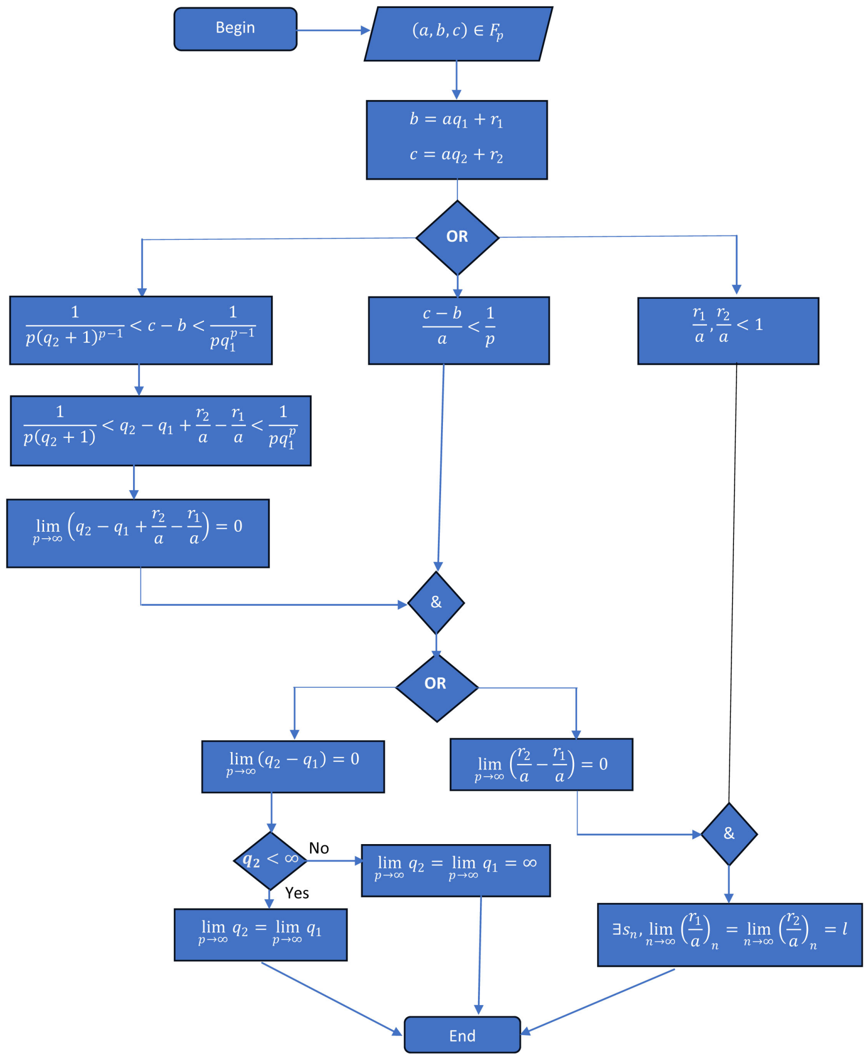

Figure 1. Flowchart of Lemma 3.11.

4. Proof of the Main Results

Proof of Theorem 1

For the proof of this theorem, we will need the following Theorem.

Theorem 4.1. Let $p > 2$ a prime and let $(a_p, b_p, c_p) \in F_p$ such that $a_p < b_p < c_p$. Consider the Diophantine Quotient and Remainders $(q_{1,p}, q_{2,p}, r_{1,p}, r_{2,p})$ of (a_p, b_p, c_p). Then there exist $l \in [0,1]$ such that

$$\begin{cases} \lim_{p\to\infty} \frac{b_p}{a_p} = \lim_{p\to\infty} \frac{c_p}{a_p} = q_\infty + l \text{ if } q_\infty < \infty \\ \lim_{p\to\infty} \frac{b_p}{a_p} = \lim_{p\to\infty} \frac{c_p}{a_p} = +\infty \text{ otherwise} \end{cases}.$$

with $l = \lim_{p\to\infty} \frac{r_{2,p}}{a_p} = \lim_{p\to\infty} \frac{r_{1,p}}{a_p}$.

Proof.

$$\left(a_p,b_p,c_p\right)\in F_p \Rightarrow b_p = q_{1,p}a_p + r_{1,p}, c_p = q_{2,p}a_p + r_{2,p} \text{ where } 0 < r_{1,p}, r_{2,p} < a$$

$$\Rightarrow \frac{b_{p_n}}{a_{p_n}} = q_{1,p} + \frac{r_{1,p}}{a_p}, \frac{c_p}{a_p} = q_{2,p} + \frac{r_{2,p}}{a_p} \text{ where } 0 < r_{1,p}, r_{2,p} < a$$

$$\Rightarrow \exists N, p > N \frac{b_p}{a_p} = q_{1,p} + \frac{r_{1,p}}{a_p}, \frac{c_p}{a_p} = q_{1,p_n} + \frac{r_{2,p}}{a_p}$$

$$\Rightarrow \begin{cases} \lim\limits_{p\to\infty}\left(\frac{b}{a}\right)_p = \lim\limits_{p\to\infty}\left(\frac{c}{a}\right)_p = q_\infty + l \text{ if } q_\infty < \infty \\ \lim\limits_{p\to\infty}\left(\frac{b}{a}\right)_p = \lim\limits_{p\to\infty}\left(\frac{c}{a}\right)_p = l + (+\infty) = +\infty \text{ otherwise} \end{cases}$$

$$\Rightarrow \begin{cases} \lim\limits_{p\to\infty}\left(\frac{b}{a}\right)_p = \lim\limits_{p\to\infty}\left(\frac{c}{a}\right)_p = q_\infty + l \text{ if } q_\infty < \infty \\ \lim\limits_{p\to\infty}\left(\frac{b}{a}\right)_p = \lim\limits_{p\to\infty}\left(\frac{c}{a}\right)_p = +\infty \text{ otherwise} \end{cases}.$$

Proof of Theorem 1. Let $p > 2$ a prime and let $\left(a_p,b_p,c_p\right)\in F_p$ a non-trivial primitive triple of positive integers, $\left(q_{1,p},q_{2,p},r_{1,p},r_{2,p}\right)$ the Diophantine Quotients and Remainders of $\left(a_p,b_p,c_p\right)$ and a positive fix real M.

On the one hand, according to theorem 4.1.

$$\begin{cases} \lim\limits_{p\to\infty}\frac{a_p}{b_p} = \lim\limits_{p\to\infty}\frac{1}{\frac{b_p}{a_p}} = \frac{1}{q_\infty + l} \; if \; q_\infty < \infty \\ \lim\limits_{p\to\infty}\frac{a_p}{b_p} = \lim\limits_{p\to\infty}\frac{1}{\frac{b_p}{a_p}} = 0 \; otherwise \end{cases},$$

we proceed as the same for the sequence $\left(\frac{a}{c}\right)_p$.

In the one hand, we have

$$q_2 \gg 1 \; when \; n \gg 1 \Rightarrow \frac{c}{b} = \frac{aq_2 + r_2}{aq_1 + r_1} = \frac{q_2 + \frac{r_2}{a}}{q_1 + \frac{r_1}{a}} [Lemma 2.4]$$

$$\Rightarrow \frac{c_p}{b_p} \underset{p\to\infty}{\sim} \frac{q_{1,p} + l}{q_{1,p} + l} [Lemma 3.12]$$

$$\Rightarrow \begin{cases} \frac{c_p}{b_p} \underset{p\to\infty}{\sim} 1 \; if \; q_{1,p} \; is \; bound \\ \frac{c_p}{b_p} \underset{p\to\infty}{\sim} \frac{q_{1,p}}{q_{1,p}} \underset{p\to\infty}{\sim} 1 \; otherwise \end{cases}$$

$$\Rightarrow \left(\frac{c}{b}\right)_\infty = 1$$

Remark 4.1. When q1 is bound, we can give another proof of theorem 1.

$$q_2 < M \Rightarrow \frac{c}{b} = \left(\frac{c}{a}\right)\left(\frac{a}{b}\right)$$
$$\Rightarrow \left(\frac{c}{b}\right)_\infty = \left(\frac{c}{a}\right)_\infty \left(\frac{a}{b}\right)_\infty$$
$$\Rightarrow \left(\frac{c}{b}\right)_\infty = \left(q_\infty + l\right)\frac{1}{q_\infty + l}$$
$$\Rightarrow \left(\frac{c}{b}\right)_\infty = 1.$$

Proof of Theorem 2

For the proof of this theorem, we will need the following lemma.

Lemma 4.1. Let $p > 2$ a prime and let $\left(a_p, b_p, c_p\right) \in F_p$ with $a_p < b_p < c_p$. Consider the Main Kimou's Divisors (e, f) of $\left(b_p, c_p\right)$ and q_2 the quotient of Euclidean division of c by a. Then

$$q_2 > 1 \Rightarrow \left(\frac{f}{e}\right)_\infty = 1.$$

Proof. According to the previous lemma hypothesis, we have

$$q_2 > 1 \Rightarrow \begin{cases} e < f < \sqrt[p]{3}e \ if \ abc \not\equiv 0[p], a \equiv 0[p] \\ \dfrac{e}{\sqrt{p}} < f < \sqrt[p]{\dfrac{3}{p}}e \ if \ b \equiv 0[p] \\ e < \dfrac{f}{\sqrt{p}} < \sqrt[p]{3}e \ if \ c \equiv 0[p] \end{cases}.$$

$$\Rightarrow \begin{cases} 1 < \dfrac{f}{e} < \sqrt[p]{3} \ if \ abc \not\equiv 0[p], a \equiv 0[p] \\ \dfrac{1}{\sqrt{p}} < \dfrac{f}{e} < \sqrt[p]{\dfrac{3}{p}} \ if \ b \equiv 0[p] \\ \sqrt[p]{p} < \dfrac{f}{e} < \sqrt[p]{3p} \ if \ c \equiv 0[p] \end{cases}$$

$$\Rightarrow 1 \le \frac{f}{e} \le 1 when p \sim +\infty$$
$$\Rightarrow \left(\frac{f}{e}\right)_\infty = 1.$$

Now let us prove the theorem 2.

Proof of theorem. Consider $(a, b, c) \in F_p$. On the one hand, according to Lemma 4.1., we have

$$q_2 > 1 \Rightarrow \left(\frac{f}{e}\right)_\infty = 1.$$

On the other hand,

$$q_2 > 1 \Rightarrow \left(\frac{c}{b}\right)_\infty = \left(\frac{f}{e}\right)_\infty \left(\frac{\gamma}{\beta}\right)_\infty$$
$$\Rightarrow 1 = \left(\frac{f}{e}\right)_\infty \left(\frac{\gamma}{\beta}\right)_\infty [Theorem 1]$$

$$\Rightarrow 1 = \left(\frac{\gamma}{\beta}\right)_{\infty} \left[Lemma4.1\right].$$

Proof of Theorem 3

For the proof of this theorem, we will need the following lemma.

Lemma 4.2. Let $p > 2$ be a prime, let $(a,b,c) \in F_p$ be a non-trivial primitive triple for Equation (1). Consider the main Kimou's Divisor f of c. We have,

$$\frac{r_1}{a} < \frac{1}{p^2} \Rightarrow \left(1+\frac{r_1}{a}\right)^p \underset{p\to\infty}{\sim} 1.$$

Proof.

$$\frac{r_1}{a} < \frac{1}{p^2} \Rightarrow \left(1+\frac{r_1}{a}\right)^p > 1, \left(1+\frac{r_1}{a}\right)^p = e^{p\ln\left(1+\frac{r_1}{a}\right)}$$

$$\Rightarrow 1 < \left(1+\frac{r_1}{a}\right)^p = e^{p\ln\left(1+\frac{r_1}{a}\right)} < e^{p\frac{r_1}{a}}$$

$$\Rightarrow 1 < \left(1+\frac{r_1}{a}\right)^p < e^{p\frac{1}{p^2}}$$

$$\Rightarrow 1 < \left(1+\frac{r_1}{a}\right)^p < e^{\frac{1}{p}}$$

$$\Rightarrow 1 \le \left(1+\frac{r_1}{a}\right)^p \le 1 \ if\ p \sim +\infty$$

$$\Rightarrow \left(1+\frac{r_1}{a}\right)^p \underset{p\to\infty}{\sim} 1$$

Now let us prove the theorem 3.

Proof. Let $p > 2$ be a prime, let $(a,b,c) \in F_p$ be a non-trivial primitive triple for Equation (1). Consider the Diophantine quotients and remainders of (a,b,c). We have,

$$q_2 = 1, \frac{r_1}{a} < \frac{1}{p^2} \Rightarrow a^p + b^p = c^p$$

$$\Rightarrow a^p + (a+r_1)^p = (a+r_2)^p$$

$$\Rightarrow 1 + \left(1+\frac{r_1}{a}\right)^p = \left(1+\frac{r_2}{a}\right)^p$$

$$\Rightarrow 1+1 \sim e^{p\frac{r_2}{a}}$$

$$\Rightarrow 2 \underset{p\to\infty}{\sim} e^{p\frac{r_2}{a}}$$

$$\Rightarrow \ln 2 \underset{p\to\infty}{\sim} p\frac{r_2}{a}$$

$$\Rightarrow \frac{r_2}{a} \underset{p\to\infty}{\sim} \frac{\ln 2}{p}$$

Remark 4.2. If $b = a+1$ then $\frac{r_1}{a} = \frac{1}{a}$ and $\frac{1}{a} < \frac{1}{p^2}$. Hence

$$b = a + 1 \Rightarrow \frac{r_2}{a} \underset{p\to\infty}{\sim} \frac{\ln 2}{p}.$$

Proof of Theorem 4

For the proof of this theorem, we will need the following lemmas.

Lemma 4.3. Let $p > 2$ be a prime, let $(a,b,c) \in F_p$ be a non-trivial primitive triple for Equation (1). Consider the main Kimou's Divisor f of c. We have,

$$b \equiv 0[p] \Rightarrow e \geq p.$$

Proof.

$$b \equiv 0[p] \Rightarrow e\beta \equiv 0[p][Definition 2.2]$$
$$\Rightarrow e \equiv 0[p][Remark 2.1]$$
$$\Rightarrow e \geq p$$

Lemma 4.4. Let $p > 2$ be a prime, let $(a,b,c) \in F_p$ be a non-trivial primitive triple for Equation (1). Consider the main Kimou's Divisor f of c. We have,

$$b \equiv 0[p], c - b = 1 \Rightarrow e < 2\sqrt[p]{p}(q_1 + 1).$$

Proof.

$$(a,b,c) \in F_p, e \geq 2\sqrt[p]{p}(q_1+1) \Rightarrow \frac{e^p}{p} \geq 2^p (q_1+1)^p$$
$$\Rightarrow c - a \geq 2^p (q_1+1)^p [Lemma 2.1.]$$
$$\Rightarrow a(q_2 - 1) + r_2 \geq 2^p (q_1+1)^p [Lemma 2.3]$$
$$\Rightarrow a(q_1 - 1) + r_2 \geq 2^p (q_1+1)^p [Theorem 2.5]$$
$$\Rightarrow q_1 a \geq 2^p (q_1+1)^p \text{ because } r_1 < a$$
$$\Rightarrow p(1+q_2)^{p-1} q_1 \geq 2^p (q_1+1)^p [Remark 3.2]$$
$$\Rightarrow p(1+q_1)^{p-1} q_1 \geq 2^p (q_1+1)^p [Remark 3.2]$$
$$\Rightarrow pq_1 \geq 2^p (q_1+1)$$
$$\Rightarrow \frac{2^p}{p} \leq \frac{q_1}{q_1+1}$$
$$\Rightarrow \frac{2^p}{p} < 1$$
$$\Rightarrow \text{Absurd.}$$

Hence $e < 2\sqrt[p]{p}(q_1+1)$.

Lemma 4.5. Let $p > 2$ be a prime, let $(a,b,c) \in F_p$ be a non-trivial primitive triple for Equation (1). Consider the main Kimou's Divisor f of c. We have,

$$b \equiv 0[p], f > e \Rightarrow q_2 = 1.$$

Proof. Reciprocal of Remark 3.1.

Now let us prove the theorem 4:

Proof. Let $p > 2$ a prime. Consider that there exist an infinity prime number

$p > 2$ such that the equation $a^p + b^p = c^p$ have solutions. For solutions of each Equation (depending for p), we consider the solution with c is minimal. We obtain an infinity suite of solutions depending on the exponent p: $\left(a_{p_n}, b_{p_n}, c_{p_n}\right)_{n\geq 1}$ that we note simply: $\left(a_{p_n}, b_{p_n}, c_{p_n}\right)_{n\geq 1}$. We have:

$$b_{p_n} \equiv 0\left[p_n\right], c_{p_n} - b_{p_n} = 1 \Rightarrow p_n \leq e_{p_n} < 2\sqrt[p_n]{p_n}\left(q_1 + 1\right)\left[Lemmas 4.3, 4.4\right]$$

$$\Rightarrow p_n \leq e < 4\sqrt[p_n]{p_n}$$

$$\Rightarrow \infty \leq e_\infty \leq 4$$

$$\Rightarrow +\infty \leq 4$$

$$\Rightarrow \text{Absurd.}$$

Hence, $\exists N, \forall n > N, a^{p_n} + b^{p_n} \neq c^{p_n}$ if $c_{p_n} - b_{p_n} = 1$ that we rewrite simply, $\exists N, \forall p > N$, $a^p + b^p \neq c^p$ if $b \equiv 0[p], c - b = 1$, $f > e$.

Remark 4.3. The logigrams of the proofs of theorems 1, 2 and 3 are given in Figures 1 - 4. In each case they provide a schematic summary of the proof, specifying the starting data or hypotheses (rectangle), the intermediate calculations or reasoning (parallelogram), the tests (rhombus) and the final results. In Figure 5, a note links the last result to indicate that it is absurd, and that the opposite hypothesis should be considered.

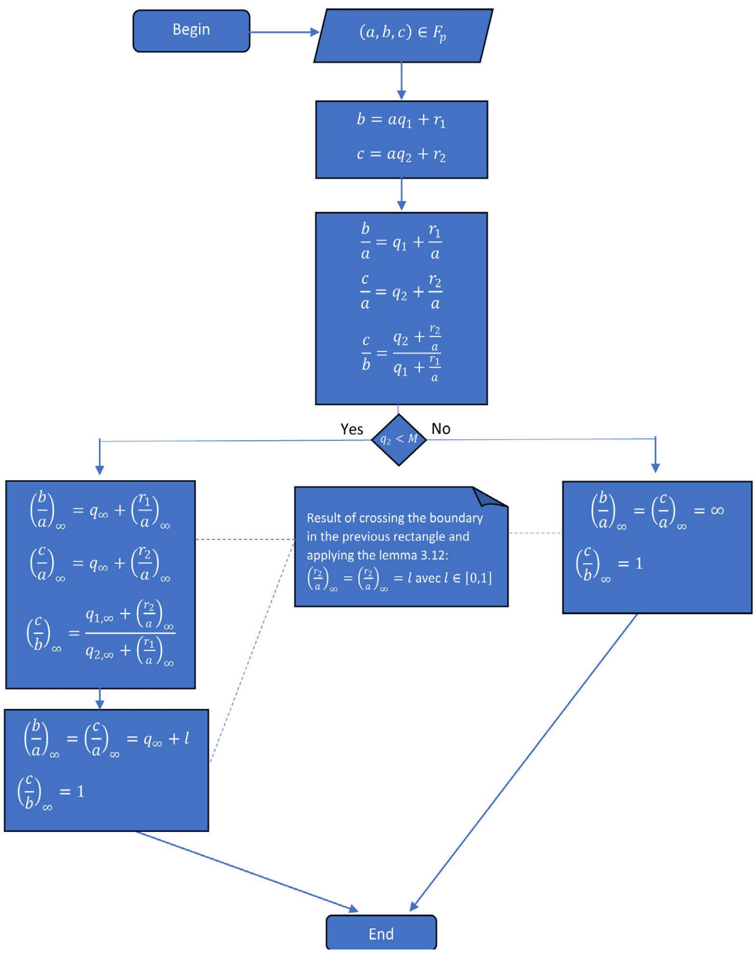

Figure 2. Flowchart of theorem 1.

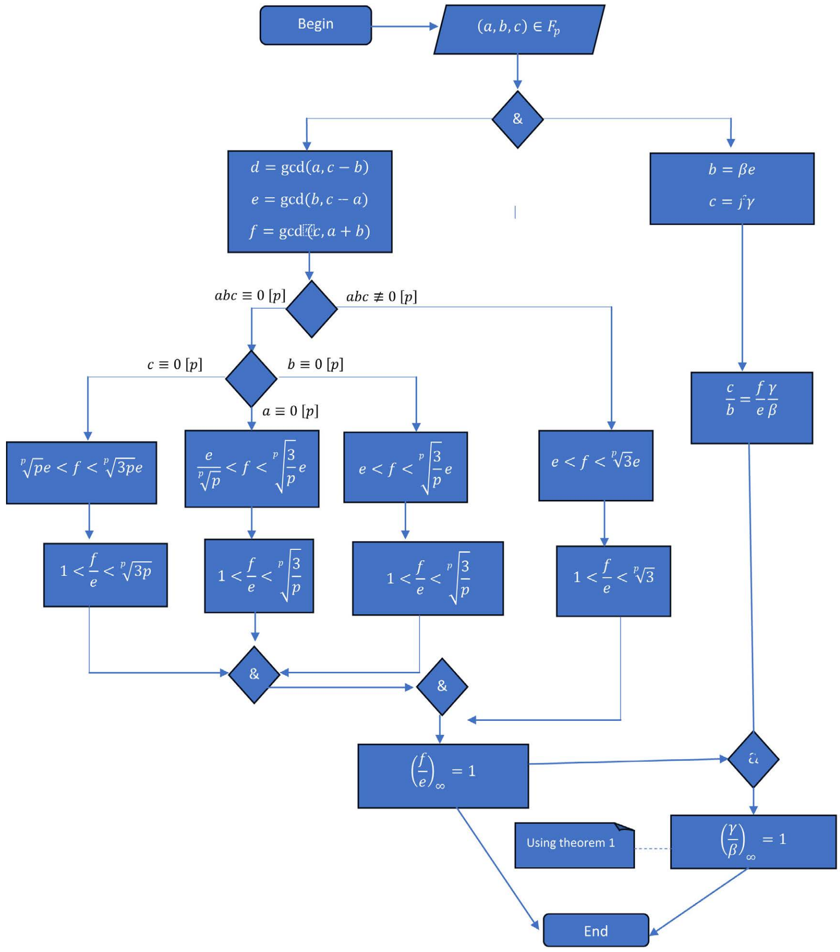

Figure 3. Flowchart of theorem 2.

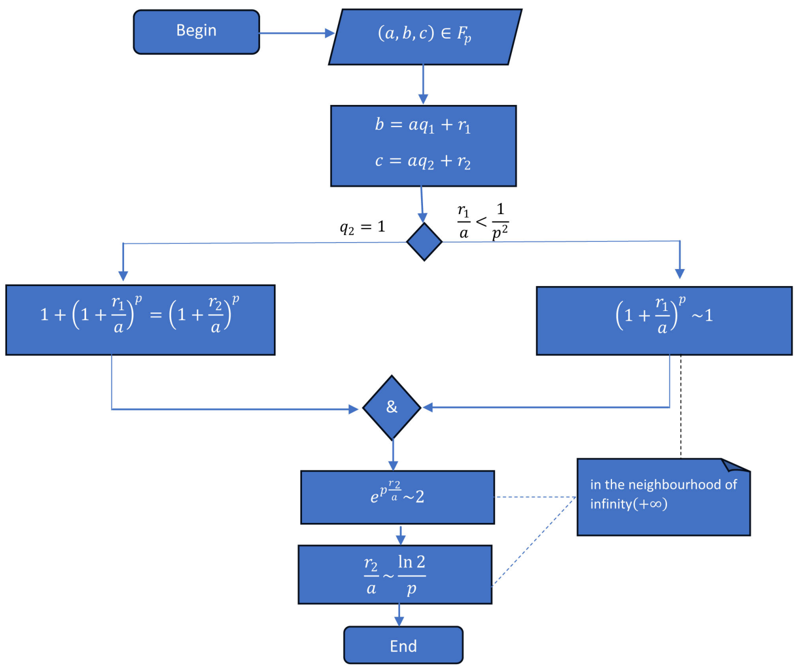

Figure 4. Flowchart of theorem 3.

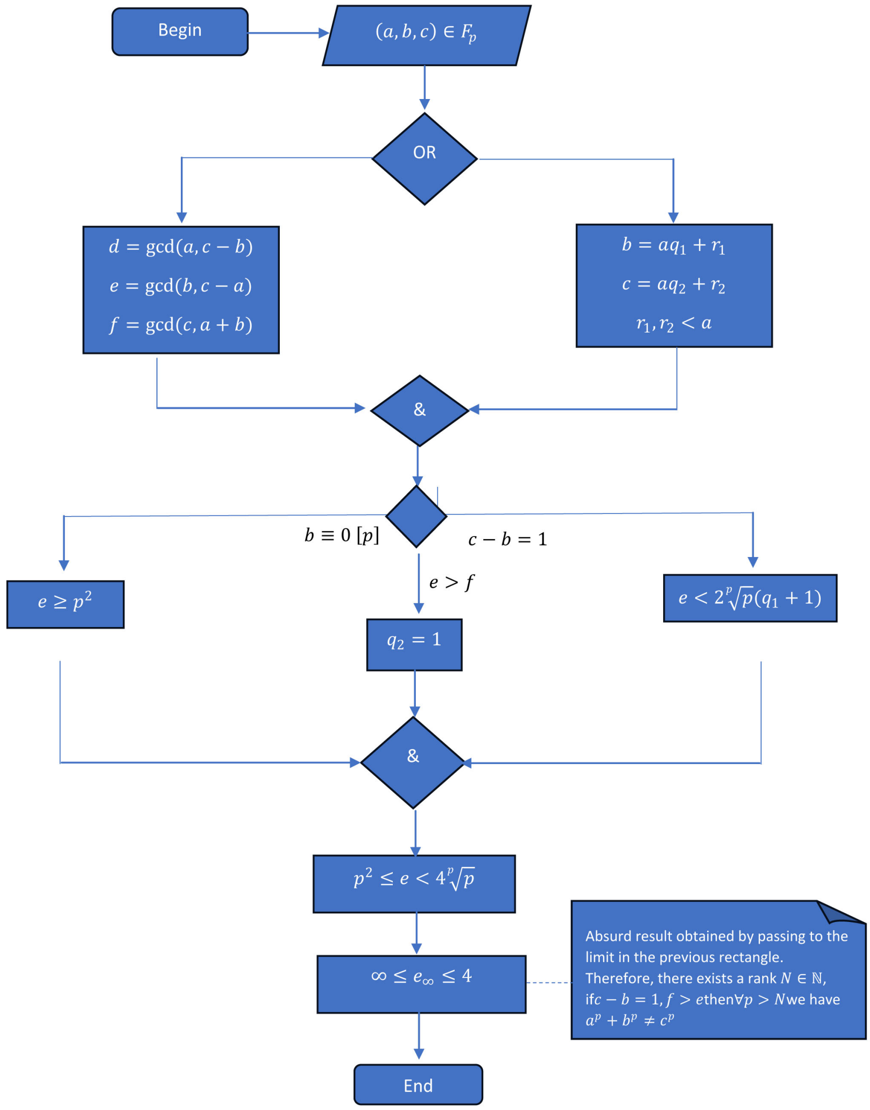

Figure 5. Flowchart of theorem 4.

5. Conclusions

We establish some asymptotic relationships and give an asymptotic direct proof of the secund case of Fermat-Wiles Theorem with $z = y + 1$ and $f > e$. In perspective we intend to prove asymptotically

- the secund case of FLT with $z = y + 1$ and $e > f$ and achieved the case $z = y + 1$;
- the fundamental case $y = x + 1$;
- the general case of FLT.

This study offers a new way to directly compute the Fermat-Wiles Theorem or similar Diophantine equation.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Paulo, R. (1999) Fermat's Last Theorem for Amateurs. Springer-Verlag New-York Inc., New-York.

[2] Paulo, R. (1979) 13 Lecture on Fermat Last Theorem. Springer-Verlag New-York Inc., New-York.

[3] Filaseta, M. (1984) An Application of Faltings' Results to Fermat's Last Theorem. *Mathematical Report of the Academy of Science*, **6**, 31-32.

[4] Granville, A. (1985) The Set of Exponents for Which Fermat's Last Theorem Is True, Has Density One. *Mathematical Report of the Academy of Science*, **7**, 55-60.

[5] Ribenboim, P. (1993) Density Results on Families of Diophantine Equations with Finitely Many Solutions. *L'Enseignement Mathématique*, **39**, 3-23.

[6] Wiles, A. (1995) Modular Elliptic Curves and Fermat's Last Theorem. *Annals of Mathematics*, **141**, 443-551. https://doi.org/10.2307/2118559

[7] Nag, B.B. (2019) On Fermat's Last Theorem. *Journal of Advances in Mathematics and Computer Science*, **34**, 1-4. https://doi.org/10.9734/JAMCS/2019/v34i230211

[8] Nag, B.B. (2021) An Elementary Proof of Fermat's Last Theorem for Epsilons. *Advances in Pure Mathematics*, **11**, 735-740. https://doi.org/10.4236/apm.2021.118048

[9] Mauldin, R.D. (1997) A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem. *Notices of the AMS*, **44**, 1436-1437.

[10] Kimou, P.K. (2023) On Fermat Last Theorem: The New Efficient Expression of a Hypothetical Solution as a Function of Its Fermat Divisors. *American Journal of Computational Mathematics*, **13**, 82-90. https://doi.org/10.4236/ajcm.2023.131002

[11] Kimou, P.K. and Tanoé, F.E. (2023) Diophantine Quotients and Remainders with Applications to Fermat and Pythagorean Equations. *American Journal of Computational Mathematics*, **13**, 199-210. https://doi.org/10.4236/ajcm.2023.131010

[12] Tanoé, F.E. and Kimou, P.K. (2023) Pythagorician Divisors and Applications to Some Diophantine Equations. *Advances in Pure Mathematics*, **13**, 35-70. https://doi.org/10.4236/apm.2023.132003