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The Cumulative Method for Multiplication and Division

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Abstract

This paper provides a method of the process of computation called the cumulative method, it is based upon repeated cumulative process. The cumulative method is being adapted to the purposes of computation, particularly multiplication and division. The operations of multiplication and division are represented by algebraic formulas. An advantage of the method is that the cumulative process can be performed on decimal numbers. The present paper aims to establish a basic and useful formula valid for the two fundamental arithmetic operations of multiplication and division. The new cumulative method proved to be more flexible and made it possible to extend the multiplication and division based on repeated addition/subtraction to decimal numbers.

Keywords

Multiplication and Division, Cumulative Method, Repeated Process, Decimal Numbers

1. Introduction

Multiplication and division is an essential part of mathematics, knowledge and abilities required to solve the mathematical problems related to the concept. Several researches have shown that one of the most common problems in learning mathematics is fundamental knowledge. For instance, lacking knowledge of basic operations makes it difficult for students to comprehend other mathematical concepts, such as algebra and geometry, and causes them to lose interest in the subject (Akhter and Akhter, 2018 [1]). Nowadays, approaches for learning multiplication and division have been widely modelled. Fischbein, Deri, Nello and Marino (1985) [2] stated that the concept of multiplication is intuitively related to a repeated addition model but the multiplier must be an integer. Clark and Kamii (1996) [3] showed that multiplication developed as a transition from additive to a multiplicative way of thinking. However, some researches (Anghileri, 1989 [4]; Kaput, 1985 [5]; Karlsson *et al.*, 2022 [6]; Larsson *et al.*, 2016-2017 [7] [8]) have differed in interpreting multiplication based on repeated addition only. Andrea Maffia and Maria Alessandra Mariotti (2018) [9] investigated the relations between the repeated sum and the array model and between the two models and multiplication properties.

Larsson (2017) [10] showed a robust conceptualisation of multiplication as repeated addition or equal groups. Larsson *et al.*, (2018) [8] demonstrated that the conception of multiplication as repeated addition represents problematic instruction, especially when multiplication is extended to multi-digits and decimals. Peanark *et al.*, (2023) [11] investigated students' learning achievement of decimal multiplication and division. Several researches about generalization and conceptualization in the teaching and learning multiplication and division are described by (Greer, 1992 [12]; Loveridge *et al.*, 2013 [13]; Petit *et al.*, 2023 [14]; Sari *et al.*, 2021 [15]).

This paper is to introduce a new cumulative method. The cumulative method proved procedures and formulas to extend the multiplication and division based on repeated addition/subtraction to decimal numbers.

2. Theoretical Framework

2.1. Conceptual Development

Conceptual and procedural knowledge are two critical aspects for understanding development and individual differences in decimal knowledge. Conceptual knowledge can be built through investigation and exploration based on the basic concepts of decimals (including magnitudes, principles, and notations). Whilst, procedural knowledge involves smoothness with the four decimal arithmetic operations.

Understanding decimals requires learning that many principles that apply to whole numbers do not apply to decimals. The transition from whole numbers conception to rational numbers conception generates misconceptions such as synthetic concepts that represent an intermediate state of knowledge between the elementary perspective of numbers and the intended scientific perspective. Decimal arithmetic does pose some difficulties beyond those of whole number arithmetic including interpreting decimal notation, operating decimals, and comparing and sorting decimal.

2.2. Difficulties with Decimals

It is familiar that decimal multiplication and division are problematic for two reasons: One reason for this is that multiplication and division usually are introduced as repeated addition/subtraction of equally sized groups, which supports neither calculations nor understanding of decimal multiplication. Another reason for the difficulty to conceptualise decimal multiplication is the direction of effect that the operations produce, which implies that the direction of effects of multiplication and division of decimal numbers is the opposite as with whole numbers.

The difficulties in performing decimal multiplication and division operations is the inspiration to propose the cumulative method method for performing decimal multiplication and division operations using the repeated addition technique.

2.3. Procedures and Objectives

The proposed method Investigates in greater depth understanding of decimal multiplication and division procedures and algorithms as well as the numbering decimal positional system structure, and analyzing how the two are related. This method emphasizes facility and absolute accuracy with decimals.

The consensus view is that decimal multiplication and division have inadequate algebraic models of multiplication and division; these operations have limited conceptual content. Furthermore, the building of appropriately flexible algebraic models of decimal multiplication and division is not easy, there for the materials and situations leading to the building of algebraic models must necessarily be developed. The cumulative method is intended to achieve the following objectives:

- To introduce an analytical method that accompanies the algebraic methods to perform multiplication and division with decimal numbers.
- To provide a method depends on the decomposition of numbers allowing to use the repeated addition procedures.
- To derive the algebraic formulas for describing the repeated process in multiplication and division operations with whole and decimal numbers.

3. Cumulative Formulas

3.1. Cumulative Multiplication Formula

Theorem 1. *If* $x, y, z \in \mathbb{R}$ *then* $xy = z$ *If and only if* $z = \sum_{k=0}^{d} \sum_{i=0}^{x_k} y \times 10^{-k}$, *where* $x = \sum_{k=0}^{d} x_k \times 10^{-k}$ *, and d is the number of decimal places.*

PROOF. Let $x = x_0 \cdot x_1 x_2 \cdots x_d$ be in decimal form, where $x_0 \in \mathbb{N}$, and each x_k is an integer with $0 \le x_k \le 9$, then *x* may be represented as the sum of the finite series $x = \sum_{k=0}^{d} x_k \times 10^{-k}$. Hence

$$\begin{aligned} z &= xy \\ &= \left(x_0 \cdot x_1 x_2 \cdots x_d\right) y \\ &= \left(\sum_{k=0}^{d} x_k \times 10^{-k}\right) y \\ &= \sum_{k=0}^{d} y\left(x_k \times 10^{-k}\right) \\ &= \sum_{k=0}^{d} x_k \left(y \times 10^{-k}\right) \\ &= \sum_{k=0}^{d} \sum_{i=0}^{x_k} \left(y \times 10^{-k}\right). \end{aligned}$$

Now,

$$\begin{aligned} z &= \sum_{k=0}^{d}\sum_{i=0}^{x_k} y\times 10^{-k} \\ &= \sum_{i=0}^{x_0} y\times 10^{0} + \sum_{i=0}^{x_1} y\times 10^{-1} + \cdots + \sum_{i=0}^{x_d} y\times 10^{-d} \\ &= x_0\times y + x_1\times 0\cdot y + \cdots + x_d\times 0\cdot \overbrace{0\cdots 0}^{d} y \\ &= y\left(x_0 + 0\cdot x_1 + \cdots + 0\cdot \overbrace{0\cdots 0}^{d} x_d \right) \\ &= y\sum_{k=0}^{d} x_k\times 10^{-k} \\ &= yx. \end{aligned}$$

Example 1. *Let* $y = 5.13, x = 2.17$. *Then*

$$\begin{aligned} xy &= \sum_{k=0}^{d}\sum_{i=0}^{x_k} y\times 10^{-k} \\ &= \sum_{k=0}^{2}\sum_{i=0}^{x_k} 5.13\times 10^{-k} \\ &= \sum_{i=0}^{2} 5.13\times 10^{0} + \sum_{i=0}^{1} 5.13\times 10^{-1} + \sum_{i=0}^{7} 5.13\times 10^{-2} \\ &= (0+5.13+5.13)+(0+0.513)+\left(0+\overbrace{0.0513+\cdots+0.0513}^{7}\right) \\ &= 11.1621 \end{aligned}$$

Example 2. *Let* $y = 45.6, x = 0.0123$. *Then*

$$\begin{aligned} xy &= \sum_{k=0}^{d}\sum_{i=0}^{x_k} y\times 10^{-k} \\ &= \sum_{k=0}^{4}\sum_{i=0}^{x_k} 45.6\times 10^{-k} \\ &= \sum_{i=0}^{0} 45.6 + \sum_{i=0}^{0} 4.56 + \sum_{i=0}^{1} 0.456 + \sum_{i=0}^{2} 0.0456 + \sum_{i=0}^{3} 0.00456 \\ &= 0+0+0.456+0.0912+0.01368 \\ &= 0.56088 \end{aligned}$$

3.2. Cumulative Division Formula

Theorem 2. *If* $x, y, z \in \mathbb{R}$ *then* $\frac{z}{y} = x$ *If and only if* $z - \sum_{k=0}^{d}\sum_{i=0}^{x_k} y\times 10^{-k} = 0$, *where* $y\times 10^{-k} \le z \quad \forall k$ *and* $x = \sum_{k=0}^{d} x_k\times 10^{-k}$.

It suffices to consider $\frac{z}{y} = x$ and $z - xy = 0$; then the proof is contained in Theorem 1 and involves the same kind of argument.

Example 3. *Using the cumulative division formula, divide* 1.23 *by* 2.5.

Let $z = 1.23, y = 2.5$. By Theorem 2, since

$$
\begin{aligned}
& z-\sum_{k=0}^{d}\sum_{i=0}^{x_k} y\times 10^{-k} \\
& =1.23-\overbrace{0.25-\cdots-0.25}^{4}-\overbrace{0.025-\cdots-0.025}^{9}-0.0025-0.0025 \\
& =1.23-\sum_{i=0}^{0}2.5-\sum_{i=0}^{4}0.25-\sum_{i=0}^{9}0.025-\sum_{i=0}^{2}0.0025 \\
& =1.23-0-1-0.225-0.005 \\
& =0
\end{aligned}
$$

it follows that

$$
\begin{aligned}
x &= \sum_{k=0}^{d} x_k \times 10^{-k} \\
&= 0+0.4+0.009+0.0002 \\
&= 0.492
\end{aligned}
$$

Example 4. *if* $z=48,\ y=12.5$, *find* $\frac{z}{y}$.

By Theorem 2,

$$
\begin{aligned}
& z-\sum_{k=0}^{d}\sum_{i=0}^{x_k} y\times 10^{-k} \\
& =48-\overbrace{12.5-\cdots-12.5}^{3}-\overbrace{1.25-\cdots-1.25}^{8}-\overbrace{0.125-\cdots-0.125}^{4} \\
& =48-\sum_{i=0}^{3}12.5-\sum_{i=0}^{8}1.25-\sum_{i=0}^{4}0.125 \\
& =48-37.5-10-0.5 \\
& =0
\end{aligned}
$$

it follows that

$$
\begin{aligned}
x &= \sum_{k=0}^{d} x_k \times 10^{-k} \\
&= 3+0.8+0.04 \\
&= 3.84
\end{aligned}
$$

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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