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Optimization of M/M/s/N Queueing Model with Reneging in a Fuzzy Environment

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Abstract

This paper deals with the study of multi-server queueing model in a fuzzy environment with imposition of reneging of customers. Entry of the customers in the system is assumed to be Poisson process and exponential service time distribution under first-come-first-served basis. Specific of this investigation is to derive the various fuzzy performance measures such as fuzzy queue length, fuzzy waiting time in queue, fuzzy response time and fuzzy optimal number of servers in explicit form for the finite capacity multi-server queueing system by using recursive method. For the validity of the model we have obtained the numerical illustrations in tabular form which shows that fuzzy-queue can be more realistic than crisp queue.

Keywords

Fuzzy Environment, Poisson, Optimal, Reneging

1. Introduction

Since the birth of queueing theory concept in 1909 by the contribution of A. K. Erlang [1], it has been studying in various frameworks, one of which is fuzzy analysis of queueing models. Zadeh [2] introduced fuzzy sets that opened the door for queueing theorists to take extensive study of queueing systems. From time to time several researchers have been attracted to study the queueing system in the form of fuzzy set theory. Some of them are worth noting: Bellman and Zadeh [3], Zadeh [4], Prade [5], Yager [6], Li and Lee [7], Buckley [8], Negi and Lee [9], Kao *et al.* [10], Buckley *et al.* [11]. A comprehensive discussion on fuzzy queueing systems can be found in Zimmermann [12] and Zhang *et al.* [13]. Dubois and Prade [14] organized the legacy of fuzzy sets in an orderly way, highlighting the main ideas, and pointing out what seem to be promising trends and barren areas. Chen [15] proposed a parametric programming approach to address the notion of the time value of delays in the presence of mixed (fuzzy and random) uncertainties that result from unreliable systems.

Several researchers have contributed to the study of finite capacity fuzzy queueing models. It is worth noting to mention some of the contributions. Pardo and Fuente [16] analyzed the design of a fuzzy finite capacity queueing model based on the degree of customer satisfaction. Shahin *et al.* [17] dealt with the optimization in a fuzzy finite capacity queueing system and they provided an alternative approach to determine the optimal number of servers by considering two criteria, including the level of customer satisfaction and the total cost in a queueing system. Cruz and Woensel [18] provided an overview of different modeling issues, the performance evaluation, and optimization behavior of the finite queueing models based on cycle time, work-in-process. Fazlollahtabar and Gholizadeh [19] developed a finite capacity M/M/1/N queueing model using vague numbers and they proposed the corresponding economic analysis through a novel cost model. Recently, Prameela and Kumar [20] analyzed a finite capacity single-server queueing model with triangular, trapezoidal and hexagonal fuzzy numbers using α-cuts and made various estimations of α.

To the best of our knowledge, very rare literatures can be found in the optimization of fuzzy queueing systems so we are motivated to report some of the works done on the line. Lin and Ke [21] constructed the membership functions of the fuzzy objective values of a controllable queueing model with cost elements, arrival rate and service rate as the fuzzy numbers. Pardo and Fuente [22] dealt with the optimization of the functions of fuzzy profit of queueing models and they determined the rate to be paid by every customer for his service and the level of publicity which the manager must utilize to maximize his profit. Azadeh *et al.* [23] considered the parameter optimization of tandem queue systems with finite intermediate buffers and they proposed a fuzzy simulation based method. Zhao *et al.* [24] developed an electric-power system by the means of coupling fuzzy queue theory. Gonzalez-Lopez *et al.* [25] dealt with the optimization of queueing theory based on vague environment and they presented the analytical results for M/M/1 and M/M/s systems. Recently, De and Mahata [26] used a defuzzification method in the inventory control system. Gholizadeh *et al.* [27] handled the optimization of the disposable appliance supply chain network by the combined genetic algorithm and robust optimization. Very recently, Nayeri *et al.* [28] applied the queueing theory and robust fuzzy stochastic optimization to cope with uncertainty.

In real world, many queueing situations arise in which there may be a tendency of customers to be discouraged by a long queue. Consequently, the customers either decide not to join the queue (balking) or depart after joining the queue without getting the service due to impatience (reneging). Queueing systems with balking and reneging have been studied extensively due to their wide applicability in many areas such as communication systems, production and inventory systems, air defense systems, machine repairing systems, ambulance service. An M/M/1 queue with impatient (balking and reneging) customers was first proposed by Haight [29] [30] in the 1950s. Abou-El-Ata and Hariri [31] investigated the finite capacity multi-server M/M/c/N queue with balking and reneging. Wang *et al.* [32] surveyed the queueing systems with impatient customers in accordance with various dimensions. Bouchentouf *et al.* [33] analyzed a finite capacity single server M/M/1/N feedback queueing system with vacation, balking, reneging and retention of reneged customers and they obtained important measures of effectiveness of the model by using the stationary distribution. Bhardwaj *et al.* [34] considered a queueing system with impatient customers under fuzzy environment. They analyzed the queueing system having two queues in series with reneging customers. Very recently, Chen *et al.* [35] investigated the optimal and equilibrium balking strategies in fuzzy queues under two different levels of information.

In this paper, we develop the mathematical model of optimization of multi-server finite capacity Markovian queueing model in the fuzzy-environment under the reneging behavior of the customers. The novelty of our model is that it deals with finite-capacity multi-server queueing system by embedding it into fuzzy-concepts. In the problems with maintenance and inventory which have a large number of states and exact information of a particular state is difficult to know and also, automatic machining systems have been designed in fuzzy-concepts that motivated us to study the model under investigation. This model may be of the first of this kind in which optimization of model under fuzzy environment with imposition of reneging of customers has been taken under study and major objective of this work is to determine the various fuzzy performance measures such as fuzzy queue length, fuzzy waiting time in queue, fuzzy response time and fuzzy optimal number of servers.

2. Mathematical Model

For our model we have used the following notations:

λ: Mean arrival rate

μ: Mean service rate

γ: Mean percentage rate of change

N: System capacity of queueing model

s: Number of servers

r: Mean reneging rate

P_n: Probability that there are n number of units in the system

The state-transition-rate diagram for our model is shown in **Figure 1**.

With the help of above transition diagram, the steady-state equations are:

$$\mu P_1 - \lambda(1+\gamma)P_0 = 0 \tag{1}$$

$$\lambda(1+\gamma)P_{n-1} + (1+n)\mu P_{n+1} - \left(n\mu + \lambda(1+\gamma)\right)P_n = 0, 1 \le n < s \tag{2}$$

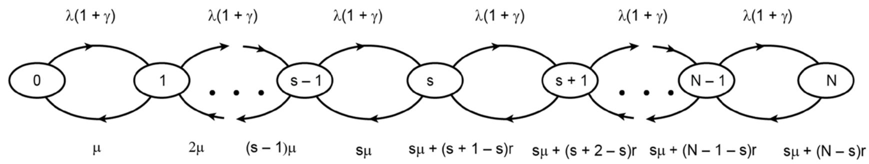

Figure 1. State-transition-rate diagram.

$$\lambda(1+\gamma)P_{n-1}+\{s\mu+(n+1-s)r\}P_{n+1}-\{\lambda(1+\gamma)+s\mu+(n-s)r\}P_n=0,\quad s\le n<N-1 \tag{3}$$

$$\lambda(1+\gamma)P_{N-1}-\{s\mu+(n-s)r\}P_N=0 \tag{4}$$

Solving Equations (1) to (4) recursively, we have

$$P_n=\begin{cases}\dfrac{1}{n!}\left(\dfrac{\lambda(1+\gamma)}{\mu}\right)^n P_0; & 1\le n\le s\\ \dfrac{1}{s!}\left(\dfrac{\lambda(1+\gamma)}{\mu}\right)^s \displaystyle\prod_{i=s+1}^{n}\dfrac{\lambda(1+\gamma)}{s\mu+(i-s)r}P_0; & s<n\le N-1\\ \dfrac{1}{s!}\left(\dfrac{\lambda(1+\gamma)}{\mu}\right)^s \displaystyle\prod_{n=s+1}^{N}\dfrac{\lambda(1+\gamma)}{s\mu+(n-s)r}P_0; & n=N\end{cases} \tag{5}$$

With normalizing condition

$$\sum_{n=0}^{N}P_n=1$$

We obtain the probability that the system is empty which is

$$P_0=\left[\sum_{n=0}^{s}\frac{1}{n!}\left(\frac{\lambda(1+\gamma)}{\mu}\right)^n+\sum_{n=s+1}^{N-1}\left\{\frac{1}{s!}\left(\frac{\lambda(1+\gamma)}{\mu}\right)^s\prod_{i=s+1}^{n}\left(\frac{\lambda(1+\gamma)}{s\mu+(i-s)r}\right)\right\}+\frac{1}{s!}\left(\frac{\lambda(1+\gamma)}{\mu}\right)^s\prod_{n=s+1}^{N}\frac{\lambda(1+\gamma)}{s\mu+(n-s)r}\right]^{-1} \tag{6}$$

Also, the probability that the system is full is given by

$$P_N=\frac{1}{s!}\left(\frac{\lambda(1+\gamma)}{\mu}\right)^s\prod_{n=s+1}^{N}\frac{\lambda(1+\gamma)}{s\mu+(n-s)r}P_0 \tag{7}$$

Other performance measures are

1) Expected number of idle servers is given by

$$E(I)=\sum_{n=0}^{s-1}(s-n)P_n$$

2) Expected number of busy servers is given by

$$E(B)=s-E(I)$$

3) Probability that the servers remain busy is

$$P(B)=\sum_{n=s}^{N}P_n$$

4) Average rate of reneging is

$$R_r = \sum_{n=s}^{N} (n-s) r P_n$$

5) Expected number of customers in the system is

$$L_s = \sum_{n=0}^{N} n P_n$$

$$= P_0 \left[\sum_{n=1}^{s} \frac{1}{(n-1)!} \left(\frac{\lambda(1+\gamma)}{\mu} \right)^n + \sum_{n=s+1}^{N-1} \frac{n}{s!} \left(\frac{\lambda(1+\gamma)}{\mu} \right)^s \prod_{n=s+1}^{N-1} \frac{\lambda(1+\gamma)}{s\mu + (n-s) r} \right. \tag{8}$$

$$\left. + \frac{N}{s!} \left(\frac{\lambda(1+\gamma)}{\mu} \right)^s * \prod_{n=s+1}^{N} \frac{\lambda(1+\gamma)}{s\mu + (n-s) r} \right]$$

6) Expected number of customers waiting in the queue is

$$L_q = \sum_{n=s}^{N} (n-s) P_n$$

$$= P_0 \left[\sum_{n=s+1}^{N-1} (n-s) \left(\frac{\lambda(1+\gamma)}{\mu} \right)^s \prod_{n=s+1}^{N} \frac{\lambda(1+\gamma)}{s\mu + (n-s) r} \right. \tag{9}$$

$$\left. + \frac{(N-s)}{s!} \left(\frac{\lambda(1+\gamma)}{\mu} \right)^s * \prod_{n=s+1}^{N} \frac{\lambda(1+\gamma)}{s\mu + (n-s) r} \right]$$

7) Expected waiting time of customers in the system and queue are

$$W_s = \frac{L_s}{\lambda}, \quad W_q = \frac{L_q}{\lambda} \tag{10}$$

3. Fuzzy Environment

Arrivals of customers, their service and percentage of change of customers in the queueing system remain always uncertain due to the fact that within the set of disjoint time-intervals, the customers may arrive in very slow rate, slow rate, fast arrival rate and very fast arrival rate. The same trait may exist in service rates and in rate of change in customers in the system. Incorporation of such uncertainty characteristics in the queueing model yields more realistic which is possible by the inclusion of the concepts of fuzzy set theory. Fuzzy set theory came into existence only after the fuzzy logic was introduced first by Zadeh [2] which has been used in numerous applications such as facial pattern recognition, air conditioners, washing machines, vacuum cleaners, antiskid braking systems, transmission systems, control of subway systems and unmanned helicopters, knowledge-based systems for multi-objective optimization of power systems, weather forecasting systems, models for new product pricing or project risk assessment, medical diagnosis and treatment plans, and stock trading. Fuzzy logic has been successfully used in numerous fields such as control systems engineering, image processing, power engineering, industrial automation, robotics, consumer electronics, and optimization. This branch of mathematics has instilled new life into scientific fields that have been dormant for a long time.

Let arrival rate $\tilde{\lambda}=(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$, service rate $\tilde{\mu}=(\mu_1,\mu_2,\mu_3,\mu_4)$, percentage change in number of customers $\tilde{\gamma}=(\gamma_1,\gamma_2,\gamma_3,\gamma_4)$, reneging rate $\tilde{r}=(r_1,r_2,r_3,r_4)$ corresponding to attributes very slow, slow, fast, very fast in their respective order of *i*, $1\le i\le 4$ such that

$$\lambda_1\le\lambda_2\le\lambda_3\le\lambda_4,\quad \mu_1\le\mu_2\le\mu_3\le\mu_4,\quad \gamma_1\le\gamma_2\le\gamma_3\le\gamma_4,\quad r_1\le r_2\le r_3\le r_4$$

We define membership of $T_{\tilde{\lambda}}(\lambda)$, $1-F_{\tilde{\lambda}}(\lambda)$, $T_{\tilde{\mu}}(\mu)$, $1-F_{\tilde{\mu}}(\mu)$, $T_{\tilde{\gamma}}(\gamma)$, $1-F_{\tilde{\gamma}}(\gamma)$, $T_{\tilde{r}}(r)$, $1-F_{\tilde{r}}(r)$ as follows:

$$T_{\tilde{\lambda}}(\lambda)=\begin{cases}\dfrac{\lambda-\lambda_1}{w(\lambda_2-\lambda_1)}, & \lambda_1\le\lambda\le\lambda_2\\ 1, & \lambda_2\le\lambda\le\lambda_3\\ \dfrac{\lambda_4-\lambda}{w(\lambda_4-\lambda_3)}, & \lambda_3\le\lambda\le\lambda_4\\ 0, & \text{otherwise}\end{cases}\tag{11}$$

$$1-F_{\tilde{\lambda}}(\lambda)=\begin{cases}\dfrac{\lambda-\lambda_1}{\lambda_2-\lambda_1}, & \lambda_1\le\lambda\le\lambda_2\\ 1, & \lambda_2\le\lambda\le\lambda_3\\ \dfrac{\lambda_4-\lambda}{\lambda_4-\lambda_3}, & \lambda_3\le\lambda\le\lambda_4\\ 0, & \text{otherwise}\end{cases}\tag{12}$$

$$T_{\tilde{\mu}}(\mu)=\begin{cases}\dfrac{\mu-\mu_1}{w(\mu_2-\mu_1)}, & \mu_1\le\mu\le\mu_2\\ 1, & \mu_2\le\mu\le\mu_3\\ \dfrac{\mu_4-\mu}{w(\mu_4-\mu_3)}, & \mu_3\le\mu\le\mu_4\\ 0, & \text{otherwise}\end{cases}\tag{13}$$

$$1-F_{\tilde{\mu}}(\mu)=\begin{cases}\dfrac{\mu-\mu_1}{\mu_2-\mu_1}, & \mu_1\le\mu\le\mu_2\\ 1, & \mu_2\le\mu\le\mu_3\\ \dfrac{\mu_4-\mu}{\mu_4-\mu_3}, & \mu_3\le\mu\le\mu_4\\ 0, & \text{otherwise}\end{cases}\tag{14}$$

$$T_{\tilde{\gamma}}(\gamma)=\begin{cases}\dfrac{\gamma-\gamma_1}{w(\gamma_2-\gamma_1)}, & \gamma_1\le\gamma\le\gamma_2\\ 1, & \gamma_2\le\gamma\le\gamma_3\\ \dfrac{\gamma_4-\gamma}{w(\gamma_4-\gamma_3)}, & \gamma_3\le\gamma\le\gamma_4\\ 0, & \text{otherwise}\end{cases}\tag{15}$$

$$1-F_{\tilde{\gamma}}(\gamma)=\begin{cases}\dfrac{\gamma-\gamma_1}{\gamma_2-\gamma_1}, & \gamma_1\le\gamma\le\gamma_2\\ 1, & \gamma_2\le\gamma\le\gamma_3\\ \dfrac{\gamma_4-\gamma}{\gamma_4-\gamma_3}, & \gamma_3\le\gamma\le\gamma_4\\ 0, & \text{otherwise}\end{cases} \tag{16}$$

$$T_{\tilde{r}}(r)=\begin{cases}\dfrac{r-r_1}{w(r_2-r_1)}, & r_1\le r\le r_2\\ 1, & r_2\le r\le r_3\\ \dfrac{r_4-r}{w(r_4-r_3)}, & r_3\le r\le r_4\\ 0, & \text{otherwise}\end{cases} \tag{17}$$

$$1-F_{\tilde{r}}(r)=\begin{cases}\dfrac{r-r_1}{r_2-r_1}, & r_1\le r\le r_2\\ 1, & r_2\le r\le r_3\\ \dfrac{r_4-r}{r_4-r_3}, & r_3\le r\le r_4\\ 0, & \text{otherwise}\end{cases} \tag{18}$$

where $w\in[1,\infty)$.

By the method of α-cut, we have

$$\frac{\tilde{\lambda}-\lambda_1}{w(\lambda_2-\lambda_1)}=\alpha_T \quad \text{so that} \quad \tilde{\lambda}=w\alpha_T(\lambda_2-\lambda_1)+\lambda_1$$

$$\frac{\lambda_4-\tilde{\lambda}}{w(\lambda_4-\lambda_3)}=\alpha_T \quad \text{so that} \quad \tilde{\lambda}=\lambda_4-w\alpha_T(\lambda_4-\lambda_3)$$

and

$$\tilde{\lambda}_{\alpha_T}=\left[w\alpha_T(\lambda_2-\lambda_1)+\lambda_1,\lambda_4-w\alpha_T(\lambda_4-\lambda_3)\right] \tag{19}$$

$$\frac{\tilde{\lambda}-\lambda_1}{\lambda_2-\lambda_1}=\alpha_F \quad \text{so that} \quad \tilde{\lambda}=\alpha_F(\lambda_2-\lambda_1)+\lambda_1$$

$$\frac{\lambda_4-\tilde{\lambda}}{\lambda_4-\lambda_3}=\alpha_F \quad \text{so that} \quad \tilde{\lambda}=\lambda_4-\alpha_F(\lambda_4-\lambda_3)$$

and

$$\tilde{\lambda}_{\alpha_F}=\left[\alpha_F(\lambda_2-\lambda_1)+\lambda_1,\lambda_4-\alpha_F(\lambda_4-\lambda_3)\right] \tag{20}$$

Similarly

$$\tilde{\mu}_{\alpha_T}=\left[\tilde{w}_{\alpha_T}(\mu_2-\mu_1)+\mu_1,\mu_4-w\alpha_T(\mu_4-\mu_3)\right] \tag{21}$$

$$\tilde{\mu}_{\alpha_F}=\left[\alpha_F(\mu_2-\mu_1)+\mu_1,\mu_4-\alpha_F(\mu_4-\mu_3)\right] \tag{22}$$

$$\tilde{\gamma}_{\alpha_T}=\left[w\alpha_T(\gamma_2-\gamma_1)+\gamma_1,\gamma_4-w\alpha_T(\gamma_4-\gamma_3)\right] \tag{23}$$

$$\tilde{\gamma}_{\alpha_F}=\left[\alpha_F(\gamma_2-\gamma_1)+\gamma_1,\gamma_4-\alpha_F(\gamma_4-\gamma_3)\right] \tag{24}$$

$$\tilde{r}_{\alpha_T}=\left[w\alpha_T(r_2-r_1)+r_1,r_4-w\alpha_T(r_4-r_3)\right] \tag{26}$$

$$\tilde{r}_{\alpha_F}=\left[\alpha_F(r_2-r_1)+r_1,r_4-\alpha_F(r_4-r_3)\right] \tag{26}$$

By the definition of vague number, we have

$$\begin{aligned}\tilde{\lambda}_{\alpha_T} &= \left[\tilde{\lambda}^L_{\alpha_T}, \tilde{\lambda}^U_{\alpha_T}\right],\quad \tilde{\lambda}_{\alpha_F} = \left[\tilde{\lambda}^L_{\alpha_F}, \tilde{\lambda}^U_{\alpha_F}\right]\\ \tilde{\mu}_{\alpha_T} &= \left[\tilde{\mu}^L_{\alpha_T}, \tilde{\mu}^U_{\alpha_T}\right],\quad \tilde{\mu}_{\alpha_F} = \left[\tilde{\mu}^L_{\alpha_F}, \tilde{\mu}^U_{\alpha_F}\right]\\ \tilde{\gamma}_{\alpha_T} &= \left[\tilde{\gamma}^L_{\alpha_T}, \tilde{\gamma}^U_{\alpha_T}\right],\quad \tilde{\gamma}_{\alpha_F} = \left[\tilde{\gamma}^L_{\alpha_F}, \tilde{\gamma}^U_{\alpha_F}\right]\\ \tilde{r}_{\alpha_T} &= \left[\tilde{r}^L_{\alpha_T}, \tilde{r}^U_{\alpha_T}\right],\quad \tilde{r}_{\alpha_F} = \left[\tilde{r}^L_{\alpha_F}, \tilde{r}^U_{\alpha_F}\right]\end{aligned} \tag{27}$$

Fuzzy probability of empty system

$$\begin{aligned}\left(\tilde{P}_0\right)^L_{\alpha_T} = &\left[\sum_{n=0}^{s}\frac{1}{n!}\left(\frac{\left(w\alpha_T\left(\lambda_2-\lambda_1\right)+\lambda_1\right)\left(1+w\alpha_T\left(\gamma_2-\gamma_1\right)+\gamma_1\right)}{w\alpha_T\left(\mu_2-\mu_1\right)+\mu_1}\right)^n\right.\\ &+\sum_{n=s+1}^{N}\left\{\frac{1}{s!}\left(\frac{\left(w\alpha_T\left(\lambda_2-\lambda_1\right)+\lambda_1\right)\left(1+w\alpha_T\left(\gamma_2-\gamma_1\right)+\gamma_1\right)}{w\alpha_T\left(\mu_2-\mu_1\right)+\mu_1}\right)^s\right.\\ &\left.\left.*\prod_{i=s+1}^{n}\left(\frac{\left(w\alpha_T\left(\lambda_2-\lambda_1\right)+\lambda_1\right)\left(1+w\alpha_T\left(\gamma_2-\gamma_1\right)+\gamma_1\right)}{s\left(w\alpha_T\left(\mu_2-\mu_1\right)+\mu_1\right)+\left(i-s\right)\left(w\alpha_T\left(r_2-r_1\right)+r_1\right)}\right)\right\}\right]^{-1}\end{aligned} \tag{28}$$

$$\begin{aligned}\left(\tilde{P}_0\right)^U_{\alpha_T} = &\left[\sum_{n=0}^{s}\frac{1}{n!}\left(\frac{\left(\lambda_4-w\alpha_T\left(\lambda_4-\lambda_3\right)\right)\left(1+\left(\gamma_4-w\alpha_T\left(\gamma_4-\gamma_3\right)\right)\right)}{\mu_4-w\alpha_T\left(\mu_4-\mu_3\right)}\right)^n\right.\\ &+\sum_{n=s+1}^{N}\left\{\frac{1}{s!}\left(\frac{\left(\lambda_4-w\alpha_T\left(\lambda_4-\lambda_3\right)\right)\left(1+\left(\gamma_4-w\alpha_T\left(\gamma_4-\gamma_3\right)\right)\right)}{\mu_4-w\alpha_T\left(\mu_4-\mu_3\right)}\right)^s\right.\\ &\left.\left.*\prod_{i=s+1}^{n}\left(\frac{\left(\lambda_4-w\alpha_T\left(\lambda_4-\lambda_3\right)\right)\left(1+\left(\gamma_4-w\alpha_T\left(\gamma_4-\gamma_3\right)\right)\right)}{s\left(\mu_4-w\alpha_T\left(\mu_4-\mu_3\right)\right)+\left(i-s\right)\left(r_4-w\alpha_T\left(r_4-r_3\right)\right)}\right)\right\}\right]^{-1}\end{aligned} \tag{29}$$

$$\begin{aligned}\left(\tilde{P}_0\right)^L_{\alpha_F} = &\left[\sum_{n=0}^{s}\frac{1}{n!}\left(\frac{\left(\alpha_F\left(\lambda_2-\lambda_1\right)+\lambda_1\right)\left(1+\alpha_F\left(\gamma_2-\gamma_1\right)+\gamma_1\right)}{\alpha_F\left(\mu_2-\mu_1\right)+\mu_1}\right)^n\right.\\ &+\sum_{n=s+1}^{N}\left\{\frac{1}{s!}\left(\frac{\left(\alpha_F\left(\lambda_2-\lambda_1\right)+\lambda_1\right)\left(1+\alpha_F\left(\gamma_2-\gamma_1\right)+\gamma_1\right)}{\alpha_F\left(\mu_2-\mu_1\right)+\mu_1}\right)^s\right.\\ &\left.\left.*\prod_{i=s+1}^{n}\left(\frac{\left(\alpha_F\left(\lambda_2-\lambda_1\right)+\lambda_1\right)\left(1+\alpha_F\left(\gamma_2-\gamma_1\right)+\gamma_1\right)}{s\left(\alpha_F\left(\mu_2-\mu_1\right)+\mu_1\right)+\left(i-s\right)\left(\alpha_F\left(r_2-r_1\right)+r_1\right)}\right)\right\}\right]^{-1}\end{aligned} \tag{30}$$

$$\begin{aligned}\left(\tilde{P}_0\right)^U_{\alpha_F} = &\left[\sum_{n=0}^{s}\frac{1}{n!}\left(\frac{\left(\lambda_4-\alpha_F\left(\lambda_4-\lambda_3\right)\right)\left(1+\left(\gamma_4-\alpha_F\left(\gamma_4-\gamma_3\right)\right)\right)}{\mu_4-\alpha_F\left(\mu_4-\mu_3\right)}\right)^n\right.\\ &+\sum_{n=s+1}^{N}\left\{\frac{1}{s!}\left(\frac{\left(\lambda_4-\alpha_F\left(\lambda_4-\lambda_3\right)\right)\left(1+\left(\gamma_4-\alpha_F\left(\gamma_4-\gamma_3\right)\right)\right)}{\mu_4-\alpha_F\left(\mu_4-\mu_3\right)}\right)^s\right.\\ &\left.\left.*\prod_{i=s+1}^{n}\left(\frac{\left(\lambda_4-\alpha_F\left(\lambda_4-\lambda_3\right)\right)\left(1+\left(\gamma_4-\alpha_F\left(\gamma_4-\gamma_3\right)\right)\right)}{s\left(\mu_4-\alpha_F\left(\mu_4-\mu_3\right)\right)+\left(i-s\right)\left(r_4-\alpha_F\left(r_4-r_3\right)\right)}\right)\right\}\right]^{-1}\end{aligned} \tag{31}$$

Fuzzy probability that system is full is given by

$$\left(\tilde{P}_N\right)_{\alpha_T}^L = \frac{1}{s!}\left(\frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L}\right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{s\mu_{\alpha_T}^L+(i-s)r_{\alpha_T}^L}\left(\tilde{P}_0\right)_{\alpha_T}^L \tag{32}$$

$$\left(\tilde{P}_N\right)_{\alpha_T}^U = \frac{1}{s!}\left(\frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U}\right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{s\mu_{\alpha_T}^U+(i-s)r_{\alpha_T}^U}\left(\tilde{P}_0\right)_{\alpha_T}^U \tag{33}$$

$$\left(\tilde{P}_N\right)_{\alpha_F}^L = \frac{1}{s!}\left(\frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{\mu_{\alpha_F}^L}\right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{s\mu_{\alpha_F}^L+(i-s)r_{\alpha_F}^L}\left(\tilde{P}_0\right)_{\alpha_F}^L \tag{34}$$

$$\left(\tilde{P}_N\right)_{\alpha_F}^U = \frac{1}{s!}\left(\frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{\mu_{\alpha_F}^U}\right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{s\mu_{\alpha_F}^U+(i-s)r_{\alpha_F}^U}\left(\tilde{P}_0\right)_{\alpha_F}^U \tag{35}$$

Fuzzy expected system size

$$\begin{aligned}\left(\tilde{L}_s\right)_{\alpha_T}^L = \left(\tilde{P}_0\right)_{\alpha_T}^L &\left[\sum_{n=1}^{s}\frac{1}{(n-1)!}\left(\frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L}\right)^n + \sum_{n=s+1}^{N-1}\frac{n}{s!}\left(\frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L}\right)^s\right.\\ &\left.*\prod_{i=s+1}^{N-1}\frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{s\mu_{\alpha_T}^L+(i-s)r_{\alpha_T}^L} + \frac{N}{s!}\left(\frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L}\right)^s \prod_{n=s+1}^{N}\frac{\lambda_{\alpha_T}^L\left(1+\gamma_{\alpha_T}^L\right)}{s\mu_{\alpha_T}^L+(n-s)r_{\alpha_T}^L}\right]\end{aligned} \tag{36}$$

$$\begin{aligned}\left(\tilde{L}_s\right)_{\alpha_T}^U = \left(\tilde{P}_0\right)_{\alpha_T}^U &\left[\sum_{n=1}^{s}\frac{1}{(n-1)!}\left(\frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U}\right)^n + \sum_{n=s+1}^{N-1}\frac{n}{s!}\left(\frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U}\right)^s\right.\\ &\left.*\prod_{i=s+1}^{N-1}\frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{s\mu_{\alpha_T}^U+(i-s)r_{\alpha_T}^U} + \frac{N}{s!}\left(\frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U}\right)^s \prod_{n=s+1}^{N}\frac{\lambda_{\alpha_T}^U\left(1+\gamma_{\alpha_T}^U\right)}{s\mu_{\alpha_T}^U+(n-s)r_{\alpha_T}^U}\right]\end{aligned} \tag{37}$$

$$\begin{aligned}\left(\tilde{L}_s\right)_{\alpha_F}^L = \left(\tilde{P}_0\right)_{\alpha_F}^L &\left[\sum_{n=1}^{s}\frac{1}{(n-1)!}\left(\frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{\mu_{\alpha_F}^L}\right)^n + \sum_{n=s+1}^{N-1}\frac{n}{s!}\left(\frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{\mu_{\alpha_F}^L}\right)^s\right.\\ &\left.*\prod_{i=s+1}^{N-1}\frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{s\mu_{\alpha_F}^L+(i-s)r_{\alpha_F}^L} + \frac{N}{s!}\left(\frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{\mu_{\alpha_F}^L}\right)^s \prod_{n=s+1}^{N}\frac{\lambda_{\alpha_F}^L\left(1+\gamma_{\alpha_F}^L\right)}{s\mu_{\alpha_F}^L+(n-s)r_{\alpha_F}^L}\right]\end{aligned} \tag{38}$$

$$\begin{aligned}\left(\tilde{L}_s\right)_{\alpha_F}^U = \left(\tilde{P}_0\right)_{\alpha_F}^U &\left[\sum_{n=1}^{s}\frac{1}{(n-1)!}\left(\frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{\mu_{\alpha_F}^U}\right)^n + \sum_{n=s+1}^{N-1}\frac{n}{s!}\left(\frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{\mu_{\alpha_F}^U}\right)^s\right.\\ &\left.*\prod_{i=s+1}^{N-1}\frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{s\mu_{\alpha_F}^U+(i-s)r_{\alpha_F}^U} + \frac{N}{s!}\left(\frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{\mu_{\alpha_F}^U}\right)^s \prod_{n=s+1}^{N}\frac{\lambda_{\alpha_F}^U\left(1+\gamma_{\alpha_F}^U\right)}{s\mu_{\alpha_F}^U+(n-s)r_{\alpha_F}^U}\right]\end{aligned} \tag{39}$$

Fuzzy expected number of customers waiting in queue is:

$$\left(\tilde{L}_q\right)_{\alpha_T}^L = \left(\tilde{P}_0\right)_{\alpha_T}^L \left[\sum_{n=s+1}^{N-1} (n-s) \left(\frac{\lambda_{\alpha_T}^L \left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L} \right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_T}^L \left(1+\gamma_{\alpha_T}^L\right)}{s\mu_{\alpha_T}^L + (i-s) r_{\alpha_T}^L} + \frac{(N-s)}{s!} \left(\frac{\lambda_{\alpha_T}^L \left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L} \right) * \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_T}^L \left(1+\gamma_{\alpha_T}^L\right)}{s\mu_{\alpha_T}^L + (i-s) r_{\alpha_T}^L} \right] \quad (40)$$

$$\left(\tilde{L}_q\right)_{\alpha_T}^U = \left(\tilde{P}_0\right)_{\alpha_T}^U \left[\sum_{n=s+1}^{N-1} (n-s) \left(\frac{\lambda_{\alpha_T}^U \left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U} \right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_T}^U \left(1+\gamma_{\alpha_T}^U\right)}{s\mu_{\alpha_T}^U + (i-s) r_{\alpha_T}^U} + \frac{(N-s)}{s!} \left(\frac{\lambda_{\alpha_T}^U \left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U} \right) * \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_T}^U \left(1+\gamma_{\alpha_T}^U\right)}{s\mu_{\alpha_T}^U + (i-s) r_{\alpha_T}^U} \right] \quad (41)$$

$$\left(\tilde{L}_q\right)_{\alpha_F}^L = \left(\tilde{P}_0\right)_{\alpha_F}^L \left[\sum_{n=s+1}^{N-1} (n-s) \left(\frac{\lambda_{\alpha_F}^L \left(1+\gamma_{\alpha_F}^L\right)}{\mu_{\alpha_F}^L} \right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_F}^L \left(1+\gamma_{\alpha_F}^L\right)}{s\mu_{\alpha_F}^L + (i-s) r_{\alpha_F}^L} + \frac{(N-s)}{s!} \left(\frac{\lambda_{\alpha_F}^L \left(1+\gamma_{\alpha_F}^L\right)}{\mu_{\alpha_F}^L} \right) * \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_F}^L \left(1+\gamma_{\alpha_F}^L\right)}{s\mu_{\alpha_F}^L + (i-s) r_{\alpha_F}^L} \right] \quad (42)$$

$$\left(\tilde{L}_q\right)_{\alpha_F}^U = \left(\tilde{P}_0\right)_{\alpha_F}^U \left[\sum_{n=s+1}^{N-1} (n-s) \left(\frac{\lambda_{\alpha_F}^U \left(1+\gamma_{\alpha_F}^U\right)}{\mu_{\alpha_F}^U} \right)^s \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_F}^U \left(1+\gamma_{\alpha_F}^U\right)}{s\mu_{\alpha_F}^U + (i-s) r_{\alpha_F}^U} + \frac{(N-s)}{s!} \left(\frac{\lambda_{\alpha_F}^U \left(1+\gamma_{\alpha_F}^U\right)}{\mu_{\alpha_F}^U} \right) * \prod_{i=s+1}^{N} \frac{\lambda_{\alpha_F}^U \left(1+\gamma_{\alpha_F}^U\right)}{s\mu_{\alpha_F}^U + (i-s) r_{\alpha_F}^U} \right] \quad (43)$$

Fuzzy expected time spent in the system is:

$$\left(\tilde{W}_s\right)_{\alpha_T}^L = \frac{\left(\tilde{L}_s\right)_{\alpha_T}^L}{\tilde{\lambda}_{\alpha_T}^L}, \quad \left(\tilde{W}_s\right)_{\alpha_T}^U = \frac{\left(\tilde{L}_s\right)_{\alpha_T}^U}{\tilde{\lambda}_{\alpha_T}^U} \quad (44)$$

$$\left(\tilde{W}_s\right)_{\alpha_F}^L = \frac{\left(\tilde{L}_s\right)_{\alpha_F}^L}{\tilde{\lambda}_{\alpha_F}^L}, \quad \left(\tilde{W}_s\right)_{\alpha_F}^U = \frac{\left(\tilde{L}_s\right)_{\alpha_F}^U}{\tilde{\lambda}_{\alpha_F}^U} \quad (45)$$

Fuzzy expected time waiting in queue is:

$$\left(\tilde{W}_q\right)_{\alpha_T}^L = \frac{\left(\tilde{L}_q\right)_{\alpha_T}^L}{\tilde{\lambda}_{\alpha_T}^L}, \quad \left(\tilde{W}_q\right)_{\alpha_T}^U = \frac{\left(\tilde{L}_q\right)_{\alpha_T}^U}{\tilde{\lambda}_{\alpha_T}^U} \quad (46)$$

$$\left(\tilde{W}_q\right)_{\alpha_F}^L = \frac{\left(\tilde{L}_q\right)_{\alpha_F}^L}{\tilde{\lambda}_{\alpha_F}^L}, \quad \left(\tilde{W}_q\right)_{\alpha_F}^U = \frac{\left(\tilde{L}_q\right)_{\alpha_F}^U}{\tilde{\lambda}_{\alpha_F}^U} \quad (47)$$

Fuzzy number of idle servers per unit time is:

$$\left(\widetilde{E(I)}\right)_{\alpha_T}^L = \sum_{n=0}^{s-1} \frac{(s-n)}{n!} \left(\frac{\lambda_{\alpha_T}^L \left(1+\gamma_{\alpha_T}^L\right)}{\mu_{\alpha_T}^L} \right)^n \left(\tilde{P}_0\right)_{\alpha_T}^L \quad (48)$$

$$\left(\widetilde{E(I)}\right)_{\alpha_T}^U = \sum_{n=0}^{s-1} \frac{(s-n)}{n!} \left(\frac{\lambda_{\alpha_T}^U \left(1+\gamma_{\alpha_T}^U\right)}{\mu_{\alpha_T}^U} \right)^n \left(\tilde{P}_0\right)_{\alpha_T}^U \quad (49)$$

$$\left(\widetilde{E(I)}\right)_{\alpha_F}^{L} = \sum_{n=0}^{s-1}\frac{(s-n)}{n!}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{\mu_{\alpha_F}^{L}}\right)^{n}\left(\tilde{P}_0\right)_{\alpha_F}^{L} \tag{50}$$

$$\left(\widetilde{E(I)}\right)_{\alpha_F}^{U} = \sum_{n=0}^{s-1}\frac{(s-n)}{n!}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{\mu_{\alpha_F}^{U}}\right)^{n}\left(\tilde{P}_0\right)_{\alpha_F}^{U} \tag{51}$$

Fuzzy expected number of busy servers per unit time is:

$$\left(\widetilde{E(B)}\right)_{\alpha_T}^{L} = s-\left(\widetilde{E(I)}\right)_{\alpha_T}^{L} \tag{52}$$

$$\left(\widetilde{E(B)}\right)_{\alpha_T}^{U} = s-\left(\widetilde{E(I)}\right)_{\alpha_T}^{U} \tag{53}$$

$$\left(\widetilde{E(B)}\right)_{\alpha_F}^{L} = s-\left(\widetilde{E(I)}\right)_{\alpha_F}^{L} \tag{54}$$

$$\left(\widetilde{E(B)}\right)_{\alpha_F}^{U} = s-\left(\widetilde{E(I)}\right)_{\alpha_F}^{U} \tag{55}$$

Probability that server remains busy is

$$\begin{aligned}\left(\widetilde{P(B)}\right)_{\alpha_T}^{L} &= \frac{1}{s!}\left(\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{\mu_{\alpha_T}^{L}}\right)^{s} + \sum_{i=s+1}^{N-1}\frac{1}{s!}\left(\lambda_{\alpha_T}^{L}\left(\frac{1+\gamma_{\alpha_T}^{L}}{\mu_{\alpha_T}^{L}}\right)\right)^{s}\\ &\quad * \prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{s\mu_{\alpha_T}^{L}+(i-s)r_{\alpha_T}^{L}}\right)\left(\tilde{P}_0\right)_{\alpha_T}^{L}\\ &\quad + \frac{1}{s!}\left(\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{\mu_{\alpha_T}^{L}}\right)^{s}\prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{s\mu_{\alpha_T}^{L}+(i-s)r_{\alpha_T}^{L}}\right)\left(\tilde{P}_0\right)_{\alpha_T}^{L}\end{aligned} \tag{56}$$

$$\begin{aligned}\left(\widetilde{P(B)}\right)_{\alpha_T}^{U} &= \frac{1}{s!}\left(\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{\mu_{\alpha_T}^{U}}\right)^{s} + \sum_{i=s+1}^{N-1}\frac{1}{s!}\left(\lambda_{\alpha_T}^{U}\left(\frac{1+\gamma_{\alpha_T}^{U}}{\mu_{\alpha_T}^{U}}\right)\right)^{s}\\ &\quad * \prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{s\mu_{\alpha_T}^{U}+(i-s)r_{\alpha_T}^{U}}\right)\left(\tilde{P}_0\right)_{\alpha_T}^{U}\\ &\quad + \frac{1}{s!}\left(\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{\mu_{\alpha_T}^{U}}\right)^{s}\prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{s\mu_{\alpha_T}^{U}+(i-s)r_{\alpha_T}^{U}}\right)\left(\tilde{P}_0\right)_{\alpha_T}^{U}\end{aligned} \tag{57}$$

$$\begin{aligned}\left(\widetilde{P(B)}\right)_{\alpha_F}^{L} &= \frac{1}{s!}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{\mu_{\alpha_F}^{L}}\right)^{s} + \sum_{i=s+1}^{N-1}\frac{1}{s!}\left(\lambda_{\alpha_F}^{L}\left(\frac{1+\gamma_{\alpha_F}^{L}}{\mu_{\alpha_F}^{L}}\right)\right)^{s}\\ &\quad * \prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{s\mu_{\alpha_F}^{L}+(i-s)r_{\alpha_F}^{L}}\right)\left(\tilde{P}_0\right)_{\alpha_F}^{L}\\ &\quad + \frac{1}{s!}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{\mu_{\alpha_F}^{L}}\right)^{s}\prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{s\mu_{\alpha_F}^{L}+(i-s)r_{\alpha_F}^{L}}\right)\left(\tilde{P}_0\right)_{\alpha_F}^{L}\end{aligned} \tag{58}$$

$$\left(\widetilde{P(B)}\right)_{\alpha_F}^{U}=\frac{1}{s!}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{\mu_{\alpha_F}^{U}}\right)^{s}+\sum_{i=s+1}^{N-1}\frac{1}{s!}\left(\lambda_{\alpha_F}^{U}\left(\frac{1+\gamma_{\alpha_F}^{U}}{\mu_{\alpha_F}^{U}}\right)\right)^{s}$$
$$*\prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{s\mu_{\alpha_F}^{U}+(i-s)r_{\alpha_F}^{U}}\right)\left(\tilde{P}_0\right)_{\alpha_F}^{U} \tag{59}$$
$$+\frac{1}{s!}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{\mu_{\alpha_F}^{U}}\right)^{s}\prod_{i=s+1}^{N}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{s\mu_{\alpha_F}^{U}+(i-s)r_{\alpha_F}^{U}}\right)\left(\tilde{P}_0\right)_{\alpha_F}^{U}$$

Fuzzy reneging rate is

$$\left(\tilde{R}_r\right)_{\alpha_T}^{L}=\left[\sum_{n=s+1}^{N-1}\frac{(n-s)}{s!}\left(\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{\mu_{\alpha_T}^{L}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{s\mu_{\alpha_T}^{L}+(i-s)r_{\alpha_T}^{L}}\right.$$
$$\left.+\frac{(N-s)}{s!}\left(\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{\mu_{\alpha_T}^{L}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_T}^{L}\left(1+\gamma_{\alpha_T}^{L}\right)}{s\mu_{\alpha_T}^{L}+(i-s)r_{\alpha_T}^{L}}\right]\left(\tilde{P}_0\right)_{\alpha_T}^{L} \tag{60}$$

$$\left(\tilde{R}_r\right)_{\alpha_T}^{U}=\left[\sum_{n=s+1}^{N-1}\frac{(n-s)}{s!}\left(\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{\mu_{\alpha_T}^{U}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{s\mu_{\alpha_T}^{U}+(i-s)r_{\alpha_T}^{U}}\right.$$
$$\left.+\frac{(N-s)}{s!}\left(\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{\mu_{\alpha_T}^{U}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_T}^{U}\left(1+\gamma_{\alpha_T}^{U}\right)}{s\mu_{\alpha_T}^{U}+(i-s)r_{\alpha_T}^{U}}\right]\left(\tilde{P}_0\right)_{\alpha_T}^{U} \tag{61}$$

$$\left(\tilde{R}_r\right)_{\alpha_F}^{L}=\left[\sum_{n=s+1}^{N-1}\frac{(n-s)}{s!}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{\mu_{\alpha_F}^{L}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{s\mu_{\alpha_F}^{L}+(i-s)r_{\alpha_F}^{L}}\right.$$
$$\left.+\frac{(N-s)}{s!}\left(\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{\mu_{\alpha_F}^{L}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_F}^{L}\left(1+\gamma_{\alpha_F}^{L}\right)}{s\mu_{\alpha_F}^{L}+(i-s)r_{\alpha_F}^{L}}\right]\left(\tilde{P}_0\right)_{\alpha_F}^{L} \tag{62}$$

$$\left(\tilde{R}_r\right)_{\alpha_F}^{U}=\left[\sum_{n=s+1}^{N-1}\frac{(n-s)}{s!}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_U}^{U}\right)}{\mu_{\alpha_U}^{U}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_U}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{s\mu_{\alpha_F}^{U}+(i-s)r_{\alpha_F}^{U}}\right.$$
$$\left.+\frac{(N-s)}{s!}\left(\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{\mu_{\alpha_F}^{U}}\right)^{s}\prod_{i=s+1}^{N}\frac{\lambda_{\alpha_F}^{U}\left(1+\gamma_{\alpha_F}^{U}\right)}{s\mu_{\alpha_F}^{U}+(i-s)r_{\alpha_F}^{U}}\right]\left(\tilde{P}_0\right)_{\alpha_F}^{U} \tag{63}$$

4. Optimal Profiles

The queueing model studied under fuzzy environment yields various fuzzy parameters which are uncertain in nature. Such an uncertainty be resolved to some extent by using fuzzy optimization technique for which the fuzzy objective function has been constructed. The strategy of minimization of the total cost of the operating horizon is termed as the optimal policy. Taking parameter vector of s, N, λ, μ, L_q as decision variables, we develop the steady-state expected total cost function per unit time for M/M/s/N queueing system then convert the optimal policy into fuzzy environment. Our main goal is to find optimal number of servers s* for which following cost parameters are defined in vector form as:

C_q = cost per unit time when one customer is waiting for service.

C_s = cost per unit time when one customer joins the system and is served according to first-come-first-served discipline.

(C_I, C_B) = cost per unit time when one server is (idle, busy).

C_R = cost of reneging per customer per unit time in queue.

Minimize TEC function

$$\begin{aligned} F(s,N) &= C_q L_q + C_s\left(L_s - L_q\right) + C_B E(B) + C_I E(I) + C_R R_r \\ &= C_q * L_q + C_s * \mu + C_B * E(B) + C_I * E(I) + C_R * R_r \end{aligned} \tag{64}$$

Fuzzy total expected cost functions of the system are:

$$\begin{aligned} \left(\widetilde{TEC}\right)_{\alpha_T}^{L} &= C_q * \left(\tilde{L}_q\right)_{\alpha_T}^{L} + C_s * \mu_{\alpha_T}^{L} + C_B * \left(\widetilde{E(B)}\right)_{\alpha_T}^{L} \\ &\quad + C_I * \left(\widetilde{E(I)}\right)_{\alpha_T}^{L} + C_R * \left(\tilde{R}_r\right)_{\alpha_T}^{L} \end{aligned} \tag{65}$$

$$\begin{aligned} \left(\widetilde{TEC}\right)_{\alpha_T}^{U} &= C_q * \left(\tilde{L}_q\right)_{\alpha_T}^{U} + C_s * \mu_{\alpha_T}^{U} + C_B * \left(\widetilde{E(B)}\right)_{\alpha_T}^{U} \\ &\quad + C_I * \left(\widetilde{E(I)}\right)_{\alpha_T}^{U} + C_R * \left(\tilde{R}_r\right)_{\alpha_T}^{U} \end{aligned} \tag{66}$$

$$\begin{aligned} \left(\widetilde{TEC}\right)_{\alpha_F}^{L} &= C_q * \left(\tilde{L}_q\right)_{\alpha_F}^{L} + C_s * \mu_{\alpha_F}^{L} + C_B * \left(\widetilde{E(B)}\right)_{\alpha_F}^{L} \\ &\quad + C_I * \left(\widetilde{E(I)}\right)_{\alpha_F}^{L} + C_R * \left(\tilde{R}_r\right)_{\alpha_F}^{L} \end{aligned} \tag{67}$$

$$\begin{aligned} \left(\widetilde{TEC}\right)_{\alpha_F}^{U} &= C_q * \left(\tilde{L}_q\right)_{\alpha_F}^{U} + C_s * \mu_{\alpha_F}^{U} + C_B * \left(\widetilde{E(B)}\right)_{\alpha_F}^{U} \\ &\quad + C_I * \left(\widetilde{E(I)}\right)_{\alpha_F}^{U} + C_R * \left(\tilde{R}_r\right)_{\alpha_F}^{U} \end{aligned} \tag{68}$$

Above expected cost functions are non-linear due to upper limit of summation sign used in the expressions with the help of which optimal number of servers s* has to be determined.

5. Numerical Results and Interpretations

Let us consider the car-workshop with capacity of $N = 8$ cars at a time which are repaired/inspected by multiple mechanics and we find various following performance measures of the system. Cars rush to the system to get repaired or to get inspection by the work-shop mechanics in the order of their arrival of first-come-first-served basis. We also find the optimal number of mechanics that can be employed so as to get minimum system costs under the smart service in the sense that cars have minimum time to wait and minimum time spent in the work-shop. **Table 1(a)** and **Table 1(b)** explore many ingredients of our fuzzy queueing models that as α_T increases from 0.10 to 0.50 the number of customers

Table 1. (a) Fuzzy expected total cost at $\alpha_T = 0.10$; (b) Fuzzy expected total cost at $\alpha_T = 0.50$.

(a)

S	$(L_s)^L_{\alpha_T}$	$(L_s)^U_{\alpha_T}$	$(L_q)^L_{\alpha_T}$	$(L_q)^U_{\alpha_T}$	$(W_s)^L_{\alpha_T}$	$(W_s)^U_{\alpha_T}$	$(W_q)^L_{\alpha_T}$	$(W_q)^U_{\alpha_T}$	$(E(I))^L_{\alpha_T}$	$(E(I))^U_{\alpha_T}$	$(E(B))^L_{\alpha_T}$	$(E(B))^U_{\alpha_T}$	$(R_r)^L_{\alpha_T}$	$(R_r)^U_{\alpha_T}$	$(TEC)^L_{\alpha_T}$	$(TEC)^U_{\alpha_T}$
2	0.0773	0.6493	0.5404	1.1301	0.1610	1.0472	1.1259	1.8228	0.0064	0.0051	1.9936	1.9949	1338.2	1117.8	402.7329	336.7756
3	0.0862	0.2602	0.7826	2.2677	0.1795	0.4197	1.6304	3.6576	0.0223	0.0201	2.9777	2.9799	2.6043	4.5183	2.5898	3.5170
4	0.1376	0.1958	1.1794	4.2827	0.2867	0.3159	2.4571	6.9076	0.0668	0.0649	3.9332	3.9351	0.0682	0.1624	2.4083	3.1133
5	0.2593	0.2814	1.4900	6.5412	0.5402	0.4539	3.1041	10.5503	0.1796	0.1809	4.8204	4.8191	0.0088	0.0224	2.9526	4.0230
6	0.4973	0.5097	1.1287	6.2090	1.0360	0.8220	2.3514	10.0145	0.4404	0.4484	5.5596	5.5516	0.0016	0.0042	3.3782	4.4511
7	0.8991	0.8944	$2.0469e^{-4}$	$3.9831e^{-5}$	1.8731	0.8220	$4.2643e^{-4}$	$6.4243e^{-5}$	0.9698	0.9717	6.0302	6.0283	$5.8373e^{-4}$	0.0012	3.6522	3.7084

(b)

S	$(L_s)^L_{\alpha_T}$	$(L_s)^U_{\alpha_T}$	$(L_q)^L_{\alpha_T}$	$(L_q)^U_{\alpha_T}$	$(W_s)^L_{\alpha_T}$	$(W_s)^U_{\alpha_T}$	$(W_q)^L_{\alpha_T}$	$(W_q)^U_{\alpha_T}$	$(E(I))^L_{\alpha_T}$	$(E(I))^U_{\alpha_T}$	$(E(B))^L_{\alpha_T}$	$(E(B))^U_{\alpha_T}$	$(R_r)^L_{\alpha_T}$	$(R_r)^U_{\alpha_T}$	$(TEC)^L_{\alpha_T}$	$(TEC)^U_{\alpha_T}$
2	2.4234	0.0728	1.0238	0.7581	3.0293	0.2428	1.2797	2.5271	0.0071	0.0106	1.9929	1.9886	4868.6	1126.0	1462.1	339.0211
3	0.6506	0.0843	2.0321	1.4102	0.8133	0.2809	2.5402	4.7006	0.0300	0.0332	2.9700	2.9668	15.5909	2.1583	6.8637	2.5095
4	0.2984	0.1376	3.8139	2.5868	0.3730	0.4586	4.7673	8.6228	0.1013	0.0941	3.8987	3.9059	0.2884	0.0434	3.1293	2.6104
5	0.3134	0.2617	5.6171	4.0520	0.3917	0.8722	7.0214	13.5065	0.2894	0.2455	4.7106	4.7545	0.0349	0.0052	3.9139	3.3920
6	0.5221	0.5015	4.7386	4.2185	0.6527	1.6718	5.9195	14.0617	0.7154	0.5899	5.2846	5.4101	0.0068	$9.6732e^{-4}$	4.2292	3.9240
7	0.8593	0.8908	$3.3222e^{-4}$	0.2293	1.0741	2.9693	$4.1527e^{-4}$	0.7643	1.4768	1.2635	5.5232	5.7365	0.0020	0.0005	3.7807	3.6260

in the system and in the line decrease whereas the expected time spent in the system and the waiting time in queue increase. Moreover, expected time spent in the system and the waiting time in queue decrease with the increase of number of servers which is up to our expectation. The expected number of idle servers and expected number of busy servers are increasing with the increase of number of servers and value of α_T. Rate of reneging is decreasing with the increase of number of servers which seems quite natural. As α_T increases from 0.10 to 0.50 the expected cost of the system increase but it decreases with increase of the number of servers. Table 2(a) and Table 2(b) show that when α_F increases from 0.12 to 0.50 number of idle servers, number of busy servers, reneging rate of customers and system expected cost decreases significantly. Table 3(a) and Table 3(b) predict that probability of system being empty, probability of system is full and probability of servers being busy are increasing with the increase of α_T from 0.10 to 0.50. The same are increasing when α_F from 0.12 to 0.50 which has been illustrated in Table 4(a) and Table 4(b). Tables 5(a)-(d) are tables for optimal number of servers that can be employed so as to minimize time spent in system, waiting time in queue and the system costs when different values of α_T and α_F have been used.

For $\lambda = 0.5$, $\lambda_1 = 0.4$, $\lambda_2 = 0.5$, $\lambda_3 = 0.6$, $\lambda_4 = 0.7$, $\gamma = 0.5$, $W = 8$, $\mu_1 = 0.3$, $\mu_2 = 0.4$, $\mu_3 = 0.5$, $\mu_4 = 0.6$, $\gamma_1 = 0.3$, $\gamma_2 = 0.4$, $\gamma_3 = 0.5$, $\gamma_4 = 0.6$, $r_1 = 0.1$, $r_2 = 0.2$, $r_3 = 0.4$, $r_4 = 0.5$, $N = 8$.

Table 2. (a) Fuzzy expected total cost at $\alpha_F = 0.12$; (b) Fuzzy expected total cost at $\alpha_F = 0.50$.

(a)

S	$(L_s)^L_{\alpha_F}$	$(L_s)^U_{\alpha_F}$	$(L_q)^L_{\alpha_F}$	$(L_q)^U_{\alpha_F}$	$(W_s)^L_{\alpha_F}$	$(W_s)^U_{\alpha_F}$	$(W_q)^L_{\alpha_F}$	$(W_q)^U_{\alpha_F}$	$(E(I))^L_{\alpha_F}$	$(E(I))^U_{\alpha_F}$	$(E(B))^L_{\alpha_F}$	$(E(B))^U_{\alpha_F}$	$(R_r)^L_{\alpha_F}$	$(R_r)^U_{\alpha_F}$	$(TEC)^L_{\alpha_F}$	$(TEC)^U_{\alpha_F}$
2	0.1258	56.0258	0.8302	1.1139	0.2537	92.7580	1.6738	1.8443	0.0114	0.0087	1.9886	1.9913	1,142,600	9,117,100	342,770	2,735,100
3	0.0748	26.9658	1.5222	2.2127	0.1508	44.6453	3.0689	3.6635	0.0429	0.0339	2.9571	2.9661	7829.7	192950	2350.9	57888
4	0.0410	21.4032	2.7405	4.1460	0.0828	35.4358	5.5252	6.8643	0.1348	0.1086	3.8652	3.8914	68.3530	3039.2	23.2124	914.7909
5	0.0213	15.0145	4.1574	6.3018	0.0429	24.8584	8.3818	10.4335	0.3738	0.3020	4.6262	4.6980	1.3208	57.4999	3.8861	21.2119
6	0.0122	7.0656	4.0766	5.9730	0.0246	11.6980	8.2189	9.8890	0.9345	0.7482	5.0655	5.2518	0.0792	1.7424	3.9975	4.9189
7	0.0031	2.1651	$2.0736e^{-4}$	$2.3283e^{-4}$	0.0063	3.5845	$4.1806e^{-4}$	$3.8548e^{-4}$	2.0809	1.6261	4.9191	5.3739	0.0252	0.1512	3.6660	3.7470

(b)

S	$(L_s)^L_{\alpha_F}$	$(L_s)^U_{\alpha_F}$	$(L_q)^L_{\alpha_F}$	$(L_q)^U_{\alpha_F}$	$(W_s)^L_{\alpha_F}$	$(W_s)^U_{\alpha_F}$	$(W_q)^L_{\alpha_F}$	$(W_q)^U_{\alpha_F}$	$(E(I))^L_{\alpha_F}$	$(E(I))^U_{\alpha_F}$	$(E(B))^L_{\alpha_F}$	$(E(B))^U_{\alpha_F}$	$(R_r)^L_{\alpha_F}$	$(R_r)^U_{\alpha_F}$	$(TEC)^L_{\alpha_F}$	$(TEC)^U_{\alpha_F}$
2	4.8197	282.4809	1.2282	0.6700	6.0246	941.6031	1.5352	2.2332	0.0039	0.0063	1.9961	1.9937	88057	1.6689	26419	1.7147
3	1.2122	68.5848	2.6843	1.2164	1.5153	228.6158	3.3553	4.0447	0.0154	0.0199	2.9846	2.9801	406.6311	0.1742	124.3062	1.8755
4	0.3657	35.9664	5.4781	2.2588	0.4572	119.8880	6.8477	7.5293	0.0497	0.0565	3.9503	3.9435	2.4433	0.0278	4.1083	2.5401
5	0.1142	19.6213	8.9237	3.6263	0.1427	65.4044	11.1546	12.0875	0.1380	0.1473	4.8620	4.8527	0.0720	0.0048	4.5863	3.3067
6	0.0351	8.0860	8.8184	3.8155	0.0439	26.9532	11.0230	12.7182	0.3348	0.3539	5.6652	5.6461	0.0096	$9.2670e^{-4}$	5.0466	3.8434
7	0.0100	2.4849	$3.3222e^{-4}$	$2.3910e^{-4}$	0.0125	8.2830	$4.1527e^{-4}$	$7.9700e^{-4}$	0.6822	0.7581	6.3178	6.2419	0.0043	$2.2004e^{-4}$	3.7814	3.5801

Table 3. (a) Fuzzy probability of system capacity at $\alpha_T = 0.10$; (b) Fuzzy probability of system capacity at $\alpha_T = 0.50$.

(a)

s	$(P_0)^L_{\alpha_T}$	$(P_0)^U_{\alpha_T}$	$(P_N)^L_{\alpha_T}$	$(P_N)^U_{\alpha_T}$	$(P(B))^L_{\alpha_T}$	$(P(B))^U_{\alpha_T}$
2	0.0039	0.0030	$6.2227e^{-16}$	$1.0271e^{-17}$	0.1604	0.1126
3	0.0069	0.0058	$8.5766e^{-14}$	$7.3221e^{-14}$	0.1231	0.0945
4	0.0133	0.0118	$8.8824e^{-12}$	$1.6875e^{-10}$	0.0744	0.0596
5	0.0260	0.0236	$1.0714e^{-9}$	$8.9744e^{-8}$	0.0342	0.0274
6	0.0500	0.0456	$2.5599e^{-7}$	$1.0785e^{-5}$	0.0114	0.0088
7	0.0904	0.0808	$2.0469e^{-4}$	$3.495e^{-4}$	0.0023	0.0203

(b)

s	$(P_0)^L_{\alpha_T}$	$(P_0)^U_{\alpha_T}$	$(P_N)^L_{\alpha_T}$	$(P_N)^U_{\alpha_T}$	$(P(B))^L_{\alpha_T}$	$(P(B))^U_{\alpha_T}$
2	0.0024	0.0041	$7.1980e^{-17}$	$6.2017e^{-19}$	0.2454	0.0657
3	0.0048	0.0068	$5.7800e^{-14}$	$1.0685e^{-15}$	0.2247	0.0503
4	0.0099	0.0126	$1.5664e^{-11}$	$2.1512e^{-12}$	0.1536	0.0314
5	0.0200	0.0243	$2.7290e^{-9}$	$2.0860e^{-9}$	0.0756	0.0152
6	0.0380	0.0463	$6.0886e^{-7}$	$6.9261e^{-7}$	0.0252	0.0055
7	0.0636	0.0820	$3.3222e^{-4}$	$7.1730e^{-5}$	0.0046	0.0208

Table 4. (a) Fuzzy probability of system capacity at $\alpha_F = 0.12$; (b) Fuzzy probability of system capacity at $\alpha_F = 0.50$.

(a)

s	$(P_0)^L_{\alpha_F}$	$(P_0)^U_{\alpha_F}$	$(P_N)^L_{\alpha_F}$	$(P_N)^U_{\alpha_F}$	$(P(B))^L_{\alpha_F}$	$(P(B))^U_{\alpha_F}$
2	0.0039	0.0031	$7.2572e^{-18}$	$5.2839e^{-18}$	0.0819	0.1107
3	0.0068	0.0059	$1.5420e^{-14}$	$3.3246e^{-14}$	0.0632	0.0920
4	0.0132	0.0120	$2.3120e^{-11}$	$7.6837e^{-11}$	0.0381	0.0577
5	0.0258	0.0240	$1.3186e^{-18}$	$4.4998e^{-8}$	0.0174	0.0264
6	0.0496	0.0462	$2.3190e^{-6}$	$6.3901e^{-6}$	0.0059	0.0085
7	0.0896	0.0822	$1.2077e^{-4}$	$2.3283e^{-4}$	0.0220	0.0205

(b)

s	$(P_0)^L_{\alpha_F}$	$(P_0)^U_{\alpha_F}$	$(P_N)^L_{\alpha_F}$	$(P_N)^U_{\alpha_F}$	$(P(B))^L_{\alpha_F}$	$(P(B))^U_{\alpha_F}$
2	0.0024	0.0041	$1.3678e^{-17}$	$1.1447e^{-16}$	0.1221	0.0657
3	0.0048	0.0068	$1.3382e^{-13}$	$1.5213e^{-13}$	0.1116	0.0503
4	0.0099	0.0126	$3.5805e^{-10}$	$1.5628e^{-10}$	0.0762	0.0314
5	0.0200	0.0243	$1.9504e^{-7}$	$6.1529e^{-8}$	0.0374	0.0152
6	0.0380	0.0463	$2.1847e^{-5}$	$7.2001e^{-6}$	0.0125	0.0055
7	0.0636	0.0820	$5.3910e^{-4}$	$2.3910e^{-4}$	0.0182	0.0208

Table 5. (a) Lower optimal number of servers at $\alpha_T = 0.10$ and 0.50; (b) Upper optimal number of servers at $\alpha_T = 0.10$ and 0.50; (c) Lower optimal number of servers at $\alpha_F = 0.50$ and 0.12; (d) Upper optimal number of servers at $\alpha_F = 0.50$ and 0.12.

(a)

α_T	$S^{*L}_{\alpha_T}$	$(W_s)^L_{\alpha_T}$	$(W_q)^L_{\alpha_T}$	$(TEC)^L_{\alpha_T}$
0.10	2	0.1610		
	2		1.1259	
	2			2.4083
0.50	4	0.3730		
	3		2.5402	
	4			3.1293

(b)

α_T	$S^{*U}_{\alpha_T}$	$(W_s)^U_{\alpha_T}$	$(W_q)^U_{\alpha_T}$	$(TEC)^U_{\alpha_T}$
0.10	4	0.3159		
	4		$6.4243e^{-5}$	
	4			3.1133
0.50	2	0.2428		
	7		2.7643	
	3			2.5095

(c)

α_F	$S^{*L}_{\alpha_F}$	$(W_S)^L_{\alpha_F}$	$(W_q)^L_{\alpha_F}$	$(TEC)^L_{\alpha_F}$
	7	0.0125		
0.50	2		1.5352	
	7			3.7814
	7	0.0063		
0.12	7		$4.1806e^{-4}$	
	7			3.6660

(d)

α_F	$S^{*U}_{\alpha_F}$	$(W_S)^U_{\alpha_F}$	$(W_q)^U_{\alpha_F}$	$(TEC)^U_{\alpha_F}$
	7	8.2830		
0.50	7		$7.9700e^{-4}$	
	2			1.7147
	7	3.5845		
0.12	7		$3.8548e^{-4}$	
	7			3.7470

6. Conclusion

We have developed the queueing model under fuzzy environment and made its intensive studies. The various performance measures obtained have been tabular forms. Also, we have made optimization of fuzzy queueing system for its optimal number of servers with respect to minimization of system costs which reveals that minimization of costs exists generally at threshold values of server. The model studied under fuzzy environment may have widespread applications in artificial intelligence, machine design, and robot and robotic and in many business promotion prospects.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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