American Journal of Operations Research, 2024, 14, 87-103
https://www.scirp.org/journal/ajor
ISSN Online: 2160-8849
ISSN Print: 2160-8830

Fuzzy Inventory Model under Selling Price Dependent Demand and Variable Deterioration with Fully Backlogged Shortages

Tanzim S. Shaikh, Santosh P. Gite

Department of Statistics, University of Mumbai, Mumbai, India
Email: tanzimsshaikh@gmail.com

How to cite this paper: Shaikh, T.S. and Gite, S.P. (2024) Fuzzy Inventory Model under Selling Price Dependent Demand and Variable Deterioration with Fully Backlogged Shortages. *American Journal of Operations Research*, **14**, 87-103.
https://doi.org/10.4236/ajor.2024.142005

Received: March 1, 2024
Accepted: March 26, 2024
Published: March 29, 2024

Abstract

The objective is to develop a model considering demand dependent on selling price and deterioration occurs after a certain period of time, which follows two-parameter Weibull distribution. Shortages are allowed and fully backlogged. Fuzzy optimal solution is obtained by considering hexagonal fuzzy numbers and for defuzzification Graded Mean Integration Representation Method. A numerical example is provided for the illustration of crisp and fuzzy, both models. To observe the effect of changes in parameters, sensitivity analysis is carried out.

Keywords

Deterioration, Selling Price Dependent Demand, Fully Backlogged, Hexagonal Fuzzy Numbers, Graded Mean Integration Representation Method

1. Introduction

Inventories are necessary to keep the commodities in balance. Controlling and keeping track of tangible goods supplies is a difficulty that every business, regardless of industry, faces. There are a number of reasons why companies ought to maintain inventory. It is not possible for goods to reach specific systems precisely when they are needed, for either economic or physical reasons. A deficiency in inventory management can also impede the production process and ultimately raise the cost per unit of production. Inventory management is critical because it enables the company to successfully handle two key challenges: keeping enough inventory on hand to facilitate seamless production and sales processes and reducing inventory expenditures to boost profitability.

Inventory control is an essential part which is to be taken care of for smooth and efficient facilities and an increase in profit. The commodities that undergo deterioration as time passes are the common challenge for managing inventories. Some items do not start deteriorating instantaneously like Fruits, Milk, Vegetables, Meat, Medicines, etc. but after a certain period of time, the deterioration starts speedily.

The first inventory model for deteriorating items was developed by Whitin [1]. Exponentially decaying inventory model was studied by Ghare & Scharder [2]. Covert & Philip [3] considered Weibull deterioration rate in inventory model. Datta & Pal [4] studied the order level inventory model with power demand and variable deterioration rate. Giri & Chaudhuri [5] developed a deterministic model for deteriorating items where demand is stock dependent. In the last two decades, economic conditions changed tremendously. Thus, the time value of money cannot be ignored. Ouyang *et al.* [6] studied the inventory model for deteriorating items under the conditions of the time value of money and inflation. Mishra [7] developed an inventory model with a controllable deteriorating rate and time dependent demand. Sharma *et al.* [8] studied the inventory model with stock dependent demand under inflation. Zhao [9] considered Trapezoidal type demand with Weibull distribution deterioration and partial backlogging. Uthayakumar & Karuppasamy [10] introduced an inventory model in healthcare industries with different types of time dependent demand for deteriorating items.

In the theoretical inventory model, the parameters are certain. But, in real life situations, these parameters may not follow any certainty. In such cases, they are treated as fuzzy parameters. Fuzzy set theory was first introduced by Zadeh [11] in 1965. Fuzzy set theory is highly applicable to inventory models involving marketing parameters. Resulting in a large number of researches published using fuzzy approach in inventory control as well as other fields. Shekarian *et al.* [12] developed a literature review of the fuzzy inventory model which identified and classified common characteristics of these models.

K. Jaggi *et al.* [13], and Kumar & Rajput [14] developed a fuzzy inventory model for deteriorating items with time varying demand. Mandal & Islam [15] investigated a fuzzy EOQ model with constant demand and fully backlogged shortages. Mohanty & Tripathy [16] studied an inventory model with exponentially decreasing demand and fuzzified costs. Sahoo *et al.* [17] analyzed three rates of fuzzy inventory model for deteriorating items with shortages. Biswas & Islam [18] developed a production inventory model where demand is dependent on selling price and advertisement. Indrajitsingha *et al.* [19] analyzed inventory model for non-instantaneous deteriorating items of selling price dependent demand during the pandemic Covid-19, where the deteriorating rate is considered to be time dependent.

As seasons change, we always encounter variations in the price of commodities. Therefore, the concept of selling price dependent demand is considered, which indicates the tendency of the change in demand for certain deteriorating items to the change in the selling price. Also, the production process time period may be impacted in certain circumstances by unforeseen and unanticipated events, allowing for the practical achievement of optimality. Consequently, when executing various industrial tasks, the ideal answer is typically not precisely identified. As such, it is quite challenging for decision makers to give a precise figure that would adequately capture the likely and necessary characteristics of production inventory difficulties. Fuzzy numbers enable to get through this challenge.

In this paper, a fuzzy inventory model with selling price dependent demand is considered. The shortages are allowed and fully backlogged. The deteriorating items maintain their quality for a certain period of time, so there is no deterioration initially and then deterioration occurs which follows two-parameter Weibull distribution. For a fuzzy model, parameters like demand, holding cost, deterioration cost, ordering cost, purchase cost, and shortage cost are assumed to be Hexagonal fuzzy numbers. The Graded mean integration representation method is used for defuzzification.

2. Definition and Preliminaries

2.1. Fuzzy Set

A fuzzy set X on the given universal set is a set of order pairs $\tilde{A}=\{x,\mu_{\tilde{A}}(x):x\in X\}$, where, $\mu_{\tilde{A}}:X\to[0,1]$ is called membership function. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

2.2. α-Cut

The α-cut of $\tilde{A}$ is defined by, $A_\alpha=\{x:\mu_{\tilde{A}}(x)=\alpha,\alpha\ge 0\}$.

If R is a real line, then a fuzzy number is a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}}:X\to[0,1]$, having following properties,

1) $\tilde{A}$ is normal i.e., there exists $x\in R$ such that $\mu_{\tilde{A}}(x)=1$
2) $\tilde{A}$ is piecewise continuous
3) $\sup p(\tilde{A})=cl\{x\in R:\mu_{\tilde{A}}(x)>0\}$
4) $\tilde{A}$ is a convex fuzzy set.

2.3. Generalized Fuzzy Number

Generalized fuzzy number any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1]
2) $\mu_{\tilde{A}}(x)=0$, $-\infty<x\le x_1$
3) $\mu_{\tilde{A}}(x)=L(x)$ is strictly increasing on $[x_1, x_2]$
4) $\mu_{\tilde{A}}(x)=1$, $x_2\le x\le x_3$
5) $\mu_{\tilde{A}}(x)=R(x)$ is strictly decreasing on $[x_3, x_4]$
6) $\mu_{\tilde{A}}(x)=0$, $x_4\le x\le\infty$, where x_1, x_2, x_3, x_4 are real numbers.

2.4. Hexagonal Fuzzy Number

The fuzzy set $\tilde{A}=(a,b,c,d,e,f)$ where, $a\le b\le c\le d\le e\le f$ and defined on

R, is called the Hexagonal fuzzy number, if the membership function of $\tilde{A}$ is given by,

$$\mu_{\tilde{A}}(x)=\begin{cases} L_1(x)=\frac{1}{2}\left(\frac{x-a}{b-a}\right), & a\le x\le b \\ L_2(x)=\frac{1}{2}+\frac{1}{2}\left(\frac{x-b}{c-d}\right), & b\le x\le c \\ 1, & c\le x\le d \\ R_1(x)=1-\frac{1}{2}\left(\frac{x-d}{e-d}\right), & d\le x\le e \\ R(x)=\frac{1}{2}\left(\frac{f-x}{f-e}\right), & e\le x\le f \\ 0, & \text{otherwise} \end{cases}$$

2.5. α-Cut Corresponding to Hexagonal Fuzzy Number

The α-cut of $\tilde{A}=(a,b,c,d,e,f)$, $0\le\alpha\le 1$ is $A(\alpha)=\left[A_L(\alpha),A_R(\alpha)\right]$ where,

$$A_{L_1}(\alpha)=a+(b-a)\alpha=L_1^{-1}(\alpha),$$

$$A_{L_2}(\alpha)=b+(c-b)\alpha=L_2^{-1}(\alpha),$$

$$A_{R_1}(\alpha)=e+(e-d)\alpha=R_1^{-1}(\alpha),$$

$$A_{R_2}(\alpha)=f+(f-e)\alpha=R_2^{-1}(\alpha),$$

And,

$$L^{-1}(\alpha)=\frac{L_1^{-1}(\alpha)+L_2^{-1}(\alpha)}{2}=\frac{a+b+(c-a)\alpha}{2}$$

$$R^{-1}(\alpha)=\frac{R_1^{-1}(\alpha)+R_2^{-1}(\alpha)}{2}=\frac{e+f+(d-f)\alpha}{2}$$

2.6. Graded Mean Integration Representation

If $\tilde{A}=(a,b,c,d,e,f)$ is a hexagonal fuzzy number, then the graded mean integration representation of $\tilde{A}$ is defined as,

$$P(\tilde{A})=\frac{\int_0^{W_A}\frac{h}{2}\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)\mathrm{d}h}{\int_0^{W_A}h\mathrm{d}h}, \text{ with } 0\le W_A\le 1.$$

$$P(\tilde{A})=\frac{a+3b+2c+2d+3e+f}{12}$$

3. Notations and Assumptions

The following notations and assumptions are considered to develop the inventory model:

3.1. Notations

η	:	Demand coefficient
β	:	Demand constant, $\beta \geq 1$
s	:	Selling price (in ₹/unit)
t_m	:	Time during which there is no deterioration
t_1	:	Time at which inventory level becomes zero
T	:	Duration of cycle
C_{HC}	:	Holding cost (in ₹/unit)
C_{PC}	:	Purchase cost (in ₹/unit)
C_{DC}	:	Deterioration cost (in ₹/unit)
C_{OC}	:	Ordering cost (in ₹/order)
C_{SC}	:	Shortage cost (in ₹/unit)
$\theta(t)$	=	$\gamma\lambda t^{\lambda-1}$, Two-parameter Weibull distribution deterioration rate per unit per unit time, where γ represents scale parameter and λ represents shape parameter
$I_1(t)$	:	Inventory level, at any time t, during $[0, t_m]$
$I_2(t)$	:	Inventory level, at any time t, during $[t_m, t_1]$
$I_3(t)$	:	Inventory level, at any time t, during $[t_1, T]$
$C(t_1, T)$	:	Total inventory cost (in ₹)
$\tilde{\eta}$	:	Fuzzy demand coefficient
$\tilde{\beta}$	:	Fuzzy demand constant
$\tilde{C}_{HC}$	:	Fuzzy holding cost (in ₹/unit)
$\tilde{C}_{DC}$	:	Fuzzy deterioration cost (in ₹/unit)
$\tilde{C}_{OC}$	:	Fuzzy ordering cost (in ₹/order)
$\tilde{C}_{PC}$	:	Fuzzy purchase cost (in ₹/unit)
$\tilde{C}_{SC}$	:	Fuzzy shortage cost (in ₹/unit)

3.2. Assumptions

1) Replenishment rate is instantaneous.

2) Shortages are allowed and fully backlogged.

3) The demand rate is selling price dependent, and it is given as, $D(s) = \frac{\eta}{s^{\beta}}$ where, $s > 0$, $\beta \geq 1$.

4) The lead time is zero.

4. Model Formulation

Let q_1 be the total quantity at the beginning of each cycle and after fulfilling q_2 units of backorder inventory. The described inventory model is of deteriorating items which starts deteriorating after a certain period of time. Let T be the length of the cycle. In the time interval $[0, t_m]$, there is no deterioration at all. The deterioration starts at $t = t_m$. During the interval $[t_m, t_1]$, inventory level decreases due to the demand as well as deterioration. At $t = t_1$, inventory falls to zero. The time interval $[t_1, T]$ is shortage period, which is fully backlogged. Let $I_1(t)$, $I_2(t)$, $I_3(t)$ be the inventory levels at any time t, in the interval $[0, t_m]$, $[t_m, t_1]$ and $[t_1, T]$ respectively. The model is represented in Figure 1.

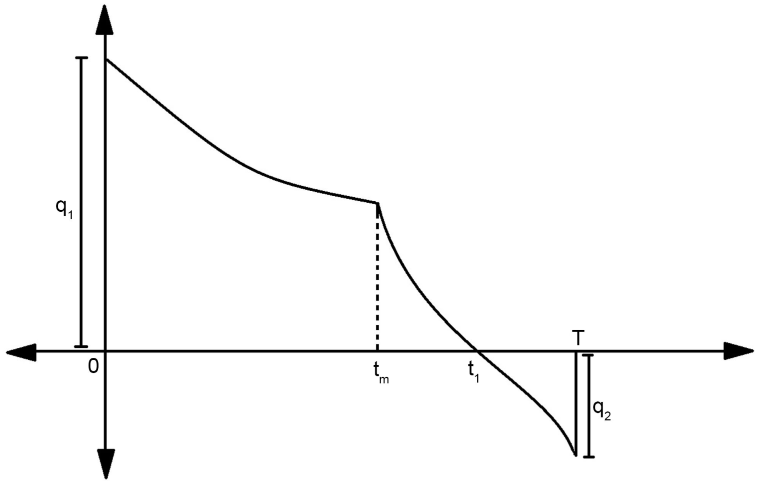

Figure 1. Representation of inventory model.

The differential equations for the inventory model are given as,

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = -\frac{\eta}{s^\beta} \quad 0 \le t \le t_m \tag{1}$$

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} + \theta(t)\cdot I(t) = -\frac{\eta}{s^\beta} \tag{2}$$

$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = -\frac{\eta}{s^\beta} \quad t_1 \le t \le T \tag{3}$$

where, $\theta(t) = \gamma\lambda t^{\lambda-1}$, $0 \le \gamma \le 1$, $\lambda \ge 1$.

The boundary conditions are,

$$I(0) = q_1,\ I(t_1) = 0 \text{ and } I_1(t_m) = I_2(t_m) \tag{4}$$

The solution of Equations (1), (2) and (3) is given by,

$$I_1(t) = \frac{\eta}{s^\beta}(t_m - t) + \frac{\eta}{s^\beta}\left[(t_1 - t_m) + \frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1} - t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^\lambda} \tag{5}$$

$$I_2(t) = \frac{\eta}{s^\beta}\left[(t_1 - t) + \frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1} - t^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t^\lambda} \tag{6}$$

$$I_3(t) = \frac{\eta}{s^\beta}(t_1 - t) \tag{7}$$

Also, using initial boundary condition, $I(0) = q_1$,

We get,

$$q_1 = \frac{\eta}{s^\beta}t_m + \frac{\eta}{s^\beta}\left[(t_1 - t_m) + \frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1} - t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^\lambda} \tag{8}$$

The total ordering quantity Q is the sum of on-hand inventory and back-order inventory which is,

$$Q = q_1 + q_2$$

$$q_2 = \int_{t_1}^{T} I_3(t)\,\mathrm{d}t = -\frac{\eta}{2s^\beta}(T - t_1)^2 \tag{9}$$

Total inventory cost per unit time for the model during a cycle is given by,

$$C(t_1,T)=\frac{1}{T}\left[\text{Purchase Cost}+\text{Holding Cost}+\text{Deterioration Cost}+\text{Shortage Cost}+\text{Ordering Cost}\right]$$

Now,

1)

$$\begin{aligned}\text{Purchase Cost}&=C_{PC}\cdot Q=C_{PC}\cdot(q_1+q_2)\\&=C_{PC}\cdot\left\{\frac{\eta}{s^\beta}t_m+\frac{\eta}{s^\beta}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]e^{-\gamma t_m^\lambda}-\frac{\eta}{2s^\beta}(T-t_1)^2\right\}\end{aligned}\tag{10}$$

2)

$$\begin{aligned}\text{Holding Cost}&=C_{HC}\cdot\left[\int_0^{t_m}I_1(t)\mathrm{d}t+\int_{t_m}^{t_1}I_2(t)\mathrm{d}t\right]\\&=C_{HC}\cdot\left\{\frac{\eta}{2s^\beta}t_m^2+\frac{\eta t_m}{s^\beta}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]e^{-\gamma t_m^\lambda}\right.\\&\left.+\frac{\eta}{s^\beta}\left[\frac{(t_1-t_m)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^\lambda-t_m^\lambda\right)}{\lambda+1}\right]\right\}\end{aligned}\tag{11}$$

3)

$$\begin{aligned}\text{Deterioration Cost}&=C_{DC}\cdot\left[\int_{t_m}^{t_1}\theta(t)\cdot I_2(t)\mathrm{d}t\right]\\&=C_{DC}\cdot\left\{\frac{\eta\gamma\lambda}{s^\beta}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^\lambda}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}\end{aligned}\tag{12}$$

4)

$$\text{Shortage Cost}=-C_{SC}\left[\int_{t_1}^{T}I_3(t)\mathrm{d}t\right]=C_{SC}\left[\frac{\eta}{2s^\beta}(T-t_1)^2\right]\tag{13}$$

5)

$$\text{Ordering Cost}=C_{OC}\tag{14}$$

Hence, Total inventory cost per unit time is,

$$\begin{aligned}&C(t_1,T)\\&=\frac{1}{T}\left[C_{PC}\cdot\left\{\frac{\eta}{s^\beta}t_m+\frac{\eta}{s^\beta}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]e^{-\gamma t_m^\lambda}-\frac{\eta}{2s^\beta}(T-t_1)^2\right\}\right.\\&+C_{HC}\cdot\left\{\frac{\eta}{2s^\beta}t_m^2+\frac{\eta t_m}{s^\beta}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]e^{-\gamma t_m^\lambda}\right.\\&\left.+\frac{\eta}{s^\beta}\left[\frac{(t_1-t_m)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^\lambda-t_m^\lambda\right)}{\lambda+1}\right]\right\}\\&\left.+C_{DC}\cdot\left\{\frac{\eta\gamma\lambda}{s^\beta}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^\lambda}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+C_{SC}\left[\frac{\eta}{2s^\beta}(T-t_1)^2\right]+C_{OC}\right]\end{aligned}\tag{15}$$

For minimization of the total cost $C(t_1, T)$, the optimal value of t_1 and T can be obtained by solving the following differential equation,

$$\frac{\partial C(t_1,T)}{\partial t_1}=0\quad\text{and}\quad\frac{\partial C(t_1,T)}{\partial T}=0,$$

And it should satisfy the condition

$$\left(\frac{\partial^2 C(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 C(t_1,T)}{\partial t_1\cdot\partial T}\right)>0.$$

Fuzzy Model

Due to uncertainty in the market, it is not easy to define all parameters precisely, we assume some of these parameters $\tilde{\eta},\tilde{\beta},\tilde{C}_{HC},\tilde{C}_{DC},\tilde{C}_{OC},\tilde{C}_{PC},\tilde{C}_{SC}$ may change within some limits.

Let $\tilde{\eta}=(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6)$, $\tilde{\beta}=(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6)$,
$\tilde{C}_{SC}=(C_{SC1},C_{SC2},C_{SC3},C_{SC4},C_{SC5},C_{SC6})$,
$\tilde{C}_{HC}=(C_{HC1},C_{HC2},C_{HC3},C_{HC4},C_{HC5},C_{HC6})$,
$\tilde{C}_{DC}=(C_{DC1},C_{DC2},C_{DC3},C_{DC4},C_{DC5},C_{DC6})$,
$\tilde{C}_{OC}=(C_{OC1},C_{OC2},C_{OC3},C_{OC4},C_{OC5},C_{OC6})$,
$\tilde{C}_{PC}=(C_{PC1},C_{PC2},C_{PC3},C_{PC4},C_{PC5},C_{PC6})$ are Hexagonal fuzzy numbers.

The corresponding total inventory cost in fuzzy environment is given by,

$$\begin{aligned}
&\tilde{C}(t_1,T)\\
&=\frac{1}{T}\Bigg[\tilde{C}_{PC}\cdot\left\{\frac{\tilde{\eta}}{s^{\tilde{\beta}}}t_m+\frac{\tilde{\eta}}{s^{\tilde{\beta}}}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}}{2s^{\tilde{\beta}}}(T-t_1)^2\right\}\\
&+\tilde{C}_{HC}\cdot\left\{\frac{\tilde{\eta}}{2s^{\tilde{\beta}}}t_m^2+\frac{\tilde{\eta}t_m}{s^{\tilde{\beta}}}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\right.\\
&\left.+\frac{\tilde{\eta}}{s^{\tilde{\beta}}}\left[\frac{(t_1-t_m)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\right\}\\
&+\tilde{C}_{DC}\cdot\left\{\frac{\tilde{\eta}\gamma\lambda}{s^{\tilde{\beta}}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+\tilde{C}_{SC}\cdot\left[\frac{\tilde{\eta}}{2s^{\tilde{\beta}}}(T-t_1)^2\right]+\tilde{C}_{OC}\Bigg]
\end{aligned}\tag{16}$$

Let $\tilde{C}_i(t_1,T)$ be the corresponding total inventory cost obtained by replacing $\tilde{\eta}_i,\tilde{\beta}_i,\tilde{C}_{HCi},\tilde{C}_{DCi},\tilde{C}_{OCi},\tilde{C}_{PCi},\tilde{C}_{SCi}$ in Equation (16) for i = 1, 2, 3, 4, 5, 6.

The defuzzification of the fuzzy total cost $\tilde{C}(t_1,T)$ by graded mean representation is given by,

$$\begin{aligned}
\widetilde{GC}(t_1,T)=\frac{1}{12}\Big[&\tilde{C}_1(t_1,T)+2\tilde{C}_2(t_1,T)+3\tilde{C}_3(t_1,T)+3\tilde{C}_4(t_1,T)\\
&+2\tilde{C}_5(t_1,T)+\tilde{C}_6(t_1,T)\Big]
\end{aligned}$$

$$\begin{aligned}
&\widetilde{GC}(t_1,T)\\
&=\frac{1}{12T}\Bigg[\tilde{C}_{PC1}\cdot\left\{\frac{\tilde{\eta}_1}{s^{\tilde{\beta}_1}}t_m+\frac{\tilde{\eta}_1}{s^{\tilde{\beta}_1}}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}_1}{2s^{\tilde{\beta}_1}}(T-t_1)^2\right\}\\
&+\tilde{C}_{HC1}\cdot\left\{\frac{\tilde{\eta}_1}{2s^{\tilde{\beta}_1}}t_m^2+\frac{\tilde{\eta}_1t_m}{s^{\tilde{\beta}_1}}\left[(t_1-t_m)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\right.\\
&\left.+\frac{\tilde{\eta}_1}{s^{\tilde{\beta}_1}}\left[\frac{(t_1-t_m)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\right\}\\
&+\tilde{C}_{DC1}\cdot\left\{\frac{\tilde{\eta}_1\gamma\lambda}{s^{\tilde{\beta}_1}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+\tilde{C}_{SC1}\cdot\left[\frac{\tilde{\eta}_1}{2s^{\tilde{\beta}_1}}(T-t_1)^2\right]+\tilde{C}_{OC1}\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{2}{12T}\Bigg[\tilde{C}_{PC2}\cdot\Bigg\{\frac{\tilde{\eta}_2}{s^{\tilde{\beta}_2}}t_m+\frac{\tilde{\eta}_2}{s^{\tilde{\beta}_2}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}_2}{2s^{\tilde{\beta}_2}}\left(T-t_1\right)^2\Bigg\}\\
&+\tilde{C}_{HC2}\cdot\Bigg\{\frac{\tilde{\eta}_2}{2s^{\tilde{\beta}_2}}t_m^2+\frac{\tilde{\eta}_2 t_m}{s^{\tilde{\beta}_2}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\\
&+\frac{\tilde{\eta}}{s^{\tilde{\beta}}}\left[\frac{\left(t_1-t_m\right)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\Bigg\}\\
&+\tilde{C}_{DC2}\cdot\left\{\frac{\tilde{\eta}_2\gamma\lambda}{s^{\tilde{\beta}_2}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+\tilde{C}_{SC2}\cdot\left[\frac{\tilde{\eta}_2}{2s^{\tilde{\beta}_2}}\left(T-t_1\right)^2\right]+\tilde{C}_{OC2}\Bigg]\\
&+\frac{3}{12T}\Bigg[\tilde{C}_{PC3}\cdot\Bigg\{\frac{\tilde{\eta}_3}{s^{\tilde{\beta}_3}}t_m+\frac{\tilde{\eta}_3}{s^{\tilde{\beta}_3}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}_3}{2s^{\tilde{\beta}_3}}\left(T-t_1\right)^2\Bigg\}\\
&+\tilde{C}_{HC3}\cdot\Bigg\{\frac{\tilde{\eta}_3}{2s^{\tilde{\beta}_3}}t_m^2+\frac{\tilde{\eta}_3 t_m}{s^{\tilde{\beta}_3}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\\
&+\frac{\tilde{\eta}_3}{s^{\tilde{\beta}_3}}\left[\frac{\left(t_1-t_m\right)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\Bigg\}\\
&+\tilde{C}_{DC3}\cdot\left\{\frac{\tilde{\eta}_3\gamma\lambda}{s^{\tilde{\beta}_3}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+\tilde{C}_{SC3}\cdot\left[\frac{\tilde{\eta}_3}{2s^{\tilde{\beta}_3}}\left(T-t_1\right)^2\right]+\tilde{C}_{OC3}\Bigg]\\
&+\frac{3}{12T}\Bigg[\tilde{C}_{PC4}\cdot\Bigg\{\frac{\tilde{\eta}_4}{s^{\tilde{\beta}_4}}t_m+\frac{\tilde{\eta}_4}{s^{\tilde{\beta}_4}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}_4}{2s^{\tilde{\beta}_4}}\left(T-t_1\right)^2\Bigg\}\\
&+\tilde{C}_{HC4}\cdot\Bigg\{\frac{\tilde{\eta}_4}{2s^{\tilde{\beta}_4}}t_m^2+\frac{\tilde{\eta}_4 t_m}{s^{\tilde{\beta}_4}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\\
&+\frac{\tilde{\eta}_4}{s^{\tilde{\beta}_4}}\left[\frac{\left(t_1-t_m\right)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\Bigg\}\\
&+\tilde{C}_{DC4}\cdot\left\{\frac{\tilde{\eta}_4\gamma\lambda}{s^{\tilde{\beta}_4}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+\tilde{C}_{SC4}\cdot\left[\frac{\tilde{\eta}_4}{2s^{\tilde{\beta}_4}}\left(T-t_1\right)^2\right]+\tilde{C}_{OC4}\Bigg]\\
&+\frac{2}{12T}\Bigg[\tilde{C}_{PC5}\cdot\Bigg\{\frac{\tilde{\eta}_5}{s^{\tilde{\beta}_5}}t_m+\frac{\tilde{\eta}_5}{s^{\tilde{\beta}_5}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}_5}{2s^{\tilde{\beta}_5}}\left(T-t_1\right)^2\Bigg\}\\
&+\tilde{C}_{HC5}\cdot\Bigg\{\frac{\tilde{\eta}_5}{2s^{\tilde{\beta}_5}}t_m^2+\frac{\tilde{\eta}_5 t_m}{s^{\tilde{\beta}_5}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\\
&+\frac{\tilde{\eta}_5}{s^{\tilde{\beta}_5}}\left[\frac{\left(t_1-t_m\right)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\Bigg\}\\
&+\tilde{C}_{DC5}\cdot\left\{\frac{\tilde{\eta}_5\gamma\lambda}{s^{\tilde{\beta}_5}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}+\tilde{C}_{SC}\cdot\left[\frac{\tilde{\eta}_5}{2s^{\tilde{\beta}_5}}\left(T-t_1\right)^2\right]+\tilde{C}_{OC5}\Bigg]\\
&+\frac{1}{12T}\Bigg[\tilde{C}_{PC6}\cdot\Bigg\{\frac{\tilde{\eta}_6}{s^{\tilde{\beta}_6}}t_m+\frac{\tilde{\eta}_6}{s^{\tilde{\beta}_6}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}-\frac{\tilde{\eta}_6}{2s^{\tilde{\beta}_6}}\left(T-t_1\right)^2\Bigg\}\\
&+\tilde{C}_{HC6}\cdot\Bigg\{\frac{\tilde{\eta}_6}{2s^{\tilde{\beta}_6}}t_m^2+\frac{\tilde{\eta}_6 t_m}{s^{\tilde{\beta}_6}}\left[\left(t_1-t_m\right)+\frac{\gamma}{\lambda+1}\left(t_1^{\lambda+1}-t_m^{\lambda+1}\right)\right]\mathrm{e}^{-\gamma t_m^{\lambda}}\\
&+\frac{\tilde{\eta}_6}{s^{\tilde{\beta}_6}}\left[\frac{\left(t_1-t_m\right)^2}{2}+\frac{\gamma\lambda\left(t_1^{\lambda+2}-t_m^{\lambda+2}\right)}{(\lambda+1)(\lambda+2)}-\frac{\gamma t_1 t_m\left(t_1^{\lambda}-t_m^{\lambda}\right)}{\lambda+1}\right]\Bigg\}
\end{aligned}
$$

$$+\tilde{C}_{DC6}\cdot\left\{\frac{\tilde{\eta}_6\gamma\lambda}{s^{\tilde{\beta}_6}}\left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\frac{t_1t_m^{\lambda}}{\lambda}+\frac{t_m^{\lambda+1}}{\lambda+1}\right]\right\}$$

$$+\tilde{C}_{SC6}\cdot\left[\frac{\tilde{\eta}_6}{2s^{\tilde{\beta}_6}}(T-t_1)^2\right]+\tilde{C}_{OC6}\Bigg]$$

For minimization of the total cost $\widetilde{GC}(t_1,T)$, the optimal value of t_1 and T can be obtained by solving the following differential equation,

$$\frac{\partial\widetilde{GC}(t_1,T)}{\partial t_1}=0 \text{ and } \frac{\partial\widetilde{GC}(t_1,T)}{\partial T}=0,$$

And it should satisfy the condition

$$\left(\frac{\partial^2\widetilde{GC}(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2\widetilde{GC}(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2\widetilde{GC}(t_1,T)}{\partial t_1\cdot\partial T}\right)>0.$$

5. Numerical Example

5.1. Crisp Model

Consider an inventory model with following parametric values.

$\eta = 1500$, $\beta = 2.4$, $s = 4$, $\gamma = 0.02$, $\lambda = 4$, $t_m = 1.25$,

$C_{PC} = 3$/unit, $C_{DC} = 5$/unit, $C_{HC} = 0.2$/unit, $C_{OC} = 200$/order,

$C_{PC} = 7$/unit.

Following the solution procedure, we obtained the optimal solution as,

$t_1 = 0.9680$, $T = 1.7437$, Total cost $C(t_1, T) =$ **245.1534**,

$q_1 = 52.2783$, $q_2 = 16.1996$, $Q =$ **68.4779**.

5.2. Fuzzy Model

The values of different parameters are, $s = 4$, $\gamma = 0.02$, $\lambda = 4$, $t_m = 0.45$, (**Table 1**)

$\tilde{C}_{HC}=(0.05,0.10,0.15,0.25,0.30,0.35)$, $\tilde{C}_{DC}=(2,3,4,6,7,8)$,

$\tilde{C}_{OC}=(50,100,150,250,300,350)$, $\tilde{C}_{SC}=(4,5,6,8,9,10)$,

$\tilde{C}_{PC}=(1.5,2.0,2.5,3.5,4.0,4.5)$, $\tilde{\beta}=(2.1,2.2,2.3,2.5,2.6,2.7)$,

$\tilde{\eta}=(1200,1300,1400,1500,1600,1700,1800)$

The solution of fuzzy model, determined by Graded Mean Representation Method is,

$t_1 = 0.9854$, $T = 1.7527$, Fuzzy total cost $\widetilde{GC}(t_1,T)$ = **239.4447**,

$q_1 = 53.2897$, $q_2 = 15.8680$, $Q =$ **69.1577**.

Table 1. Changes in time and total cost as fuzzy parameters are reduced.

Parameters are Hexagonal fuzzy number	t_1	T	$\widetilde{GC}(t_1,T)$
$\tilde{\eta},\tilde{\beta},\tilde{C}_{HC},\tilde{C}_{DC},\tilde{C}_{OC},\tilde{C}_{PC},\tilde{C}_{SC}$	0.9854	1.7527	239.4447
$\tilde{\eta},\tilde{\beta},\tilde{C}_{DC},\tilde{C}_{OC},\tilde{C}_{PC},\tilde{C}_{SC}$	0.9796	1.7499	239.7344
$\tilde{\eta},\tilde{\beta},\tilde{C}_{DC},\tilde{C}_{OC},\tilde{C}_{SC}$	0.9077	1.7304	242.7754
$\tilde{\eta},\tilde{\beta},\tilde{C}_{OC},\tilde{C}_{SC}$	0.9073	1.7302	242.7987
$\tilde{\eta},\tilde{\beta},\tilde{C}_{OC}$	0.9678	1.7435	245.3036
$\tilde{\eta},\tilde{\beta}$	0.9678	1.7435	245.3036

6. Sensitivity Analysis

Considering the above example for sensitivity analysis to study the effect of change in different parameters involved in the model. (Table 2)

1) As the value of η increases, Figure 2 and Figure 3 indicates that, value of t_1 & T decreases significantly but fuzzy total cost $\widetilde{GC}(t_1,T)$ and Q increases.

2) As the value of β increases, Figure 4 and Figure 5 indicates that, value of t_1 & T increases and fuzzy total cost $\widetilde{GC}(t_1,T)$ and Q decreases drastically.

Table 2. Sensitivity analysis of different parameters.

Graded Mean Representation Method						
η	t_1	T	$\widetilde{GC}(t_1,T)$	q_1	q_2	Q
1300	1.0131	1.7725	221.2310	49.2151	13.9560	63.1711
1400	1.0056	1.7646	229.8855	52.6022	15.0137	67.6159
1500	0.9978	1.7566	238.5196	55.9155	16.0777	71.9931
1600	0.9898	1.7482	247.1468	59.1577	17.1314	76.2891
1700	0.9815	1.7396	255.7565	62.3203	18.1877	80.5080
β	t_1	T	$\widetilde{GC}(t_1,T)$	q_1	q_2	Q
2.2	0.8989	1.6803	295.5123	64.0014	21.6908	85.6922
2.3	0.9284	1.7108	271.5838	57.5650	18.9313	76.4963
2.4	0.9517	1.7348	250.6481	51.3864	16.5102	67.8966
2.5	0.9704	1.7541	232.3465	45.6253	14.3950	60.0202
2.6	0.9856	1.7699	216.3574	40.3502	12.5508	52.9009
γ	t_1	T	$\widetilde{GC}(t_1,T)$	q_1	q_2	Q
0.005	0.9994	1.7599	238.6083	53.9185	15.5880	69.5065
0.01	0.9947	1.7574	238.9000	53.7094	15.6783	69.3877
0.02	0.9854	1.7527	239.4447	53.2897	15.8680	69.1577
0.03	0.9764	1.7480	239.9594	52.8778	16.0464	68.9242
0.04	0.9676	1.7436	240.4329	52.4691	16.2299	68.6990
λ	t_1	T	$\widetilde{GC}(t_1,T)$	q_1	q_2	Q
2	0.9652	1.7418	239.9190	52.2052	16.2550	68.4602
3	0.9760	1.7477	239.6896	52.7919	16.0505	68.8424
4	0.9854	1.7527	239.4447	53.2897	15.8680	69.1577
5	0.9924	1.7563	239.2420	53.6538	15.7277	69.3815
6	0.9971	1.7587	239.0864	53.8932	15.6331	69.5264
t_m	t_1	T	$\widetilde{GC}(t_1,T)$	q_1	q_2	Q
0.25	0.9838	1.7522	239.5591	53.2263	15.9135	69.1398
0.35	0.9842	1.7523	239.5239	53.2401	15.9011	69.1412
0.45	0.9854	1.7527	239.4447	53.2897	15.8680	69.1577
0.55	0.9877	1.7534	239.3150	53.3895	15.8019	69.1914
0.65	0.9917	1.7547	239.1251	53.5722	15.6907	69.2629

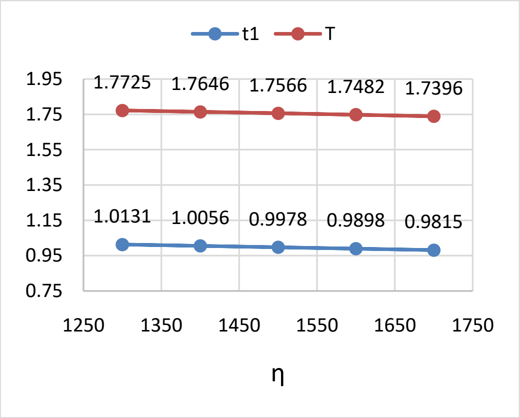

Figure 2. Effect of changes in parameter η on t_1 & T.

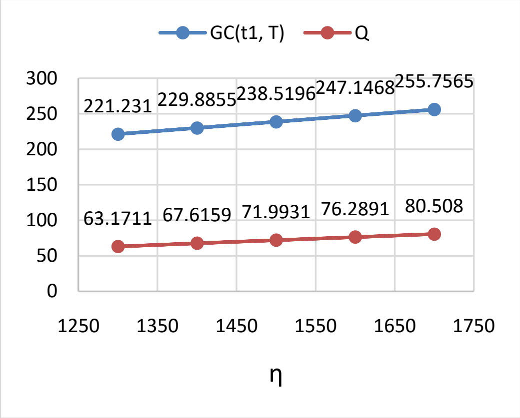

Figure 3. Effect of changes in parameter η on $\widetilde{GC}(t_1,T)$ & Q.

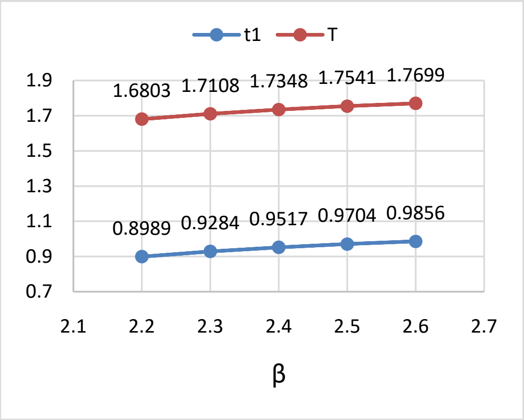

Figure 4. Effect of changes in parameter β on t_1 & T.

3) As the value of γ increases, Figure 6 and Figure 7 indicates that, value of t_1, T and Q decreases significantly and fuzzy total cost $\widetilde{GC}(t_1,T)$ increases.

4) As the value of λ increases, Figure 8 and Figure 9 indicates that, value of t_1, T and Q increases and fuzzy total cost $\widetilde{GC}(t_1,T)$ decreases insignificantly.

5) As the value of t_m increases, Figure 10 and Figure 11 indicates that, value

of t_1, T and Q increases insignificantly and fuzzy total cost $\widetilde{GC}(t_1,T)$ decreases insignificantly.

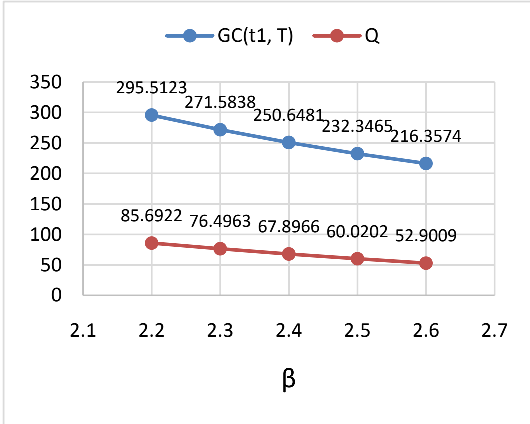

Figure 5. Effect of changes in parameter β on $\widetilde{GC}(t_1,T)$ & Q.

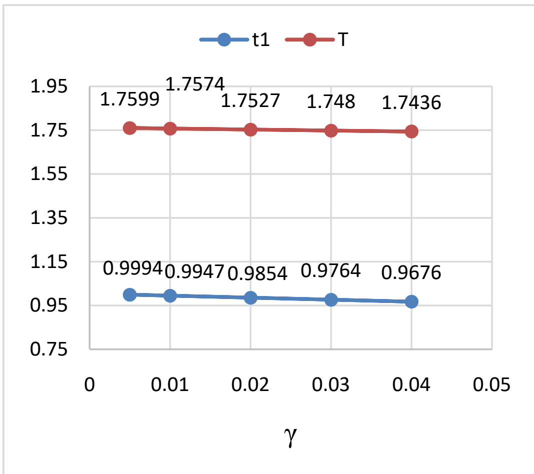

Figure 6. Effect of changes in parameter γ on t_1 & T.

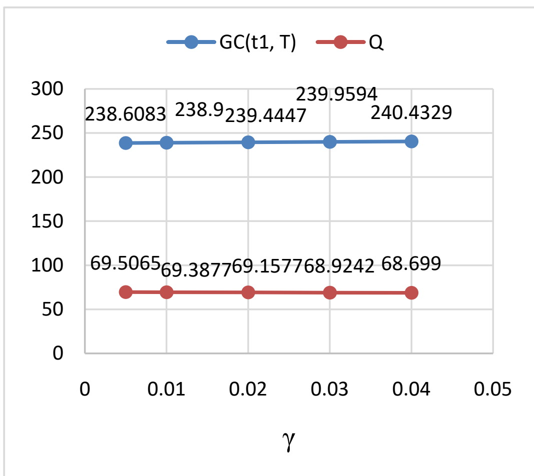

Figure 7. Effect of changes in parameter γ on $\widetilde{GC}(t_1,T)$ & Q.

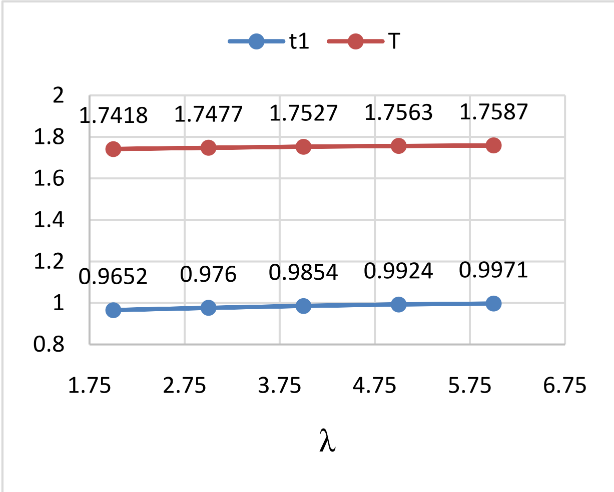

Figure 8. Effect of changes in parameter λ on t_1 & T.

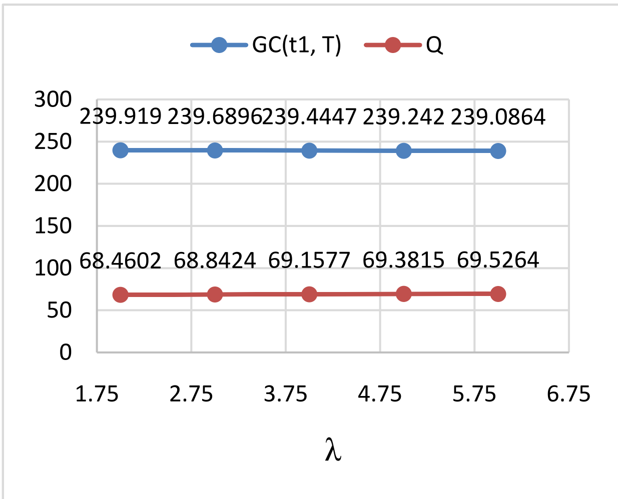

Figure 9. Effect of changes in parameter λ on $\widetilde{GC}(t_1, T)$ & Q.

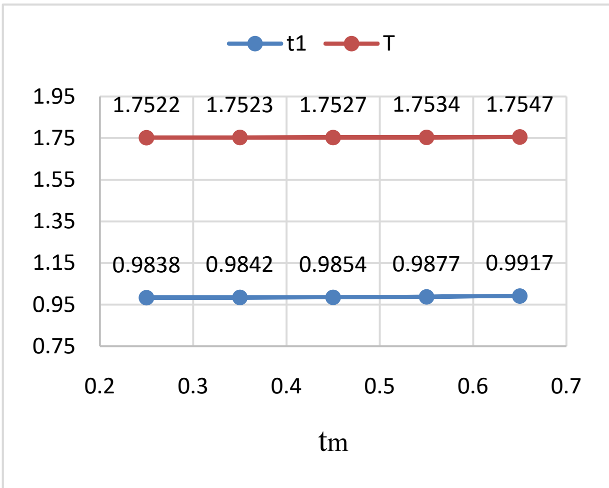

Figure 10. Effect of changes in parameter t_m on t_1 & T.

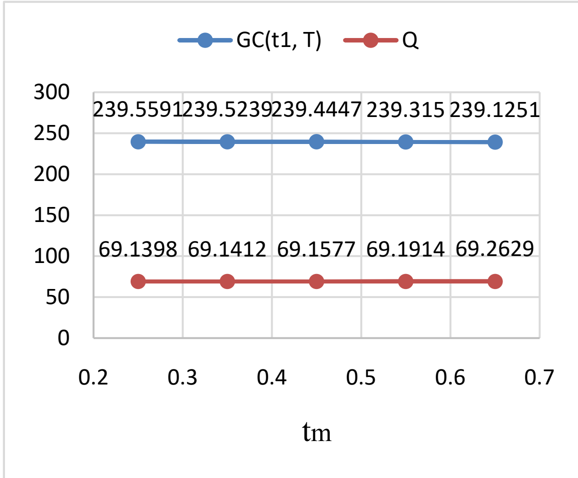

Figure 11. Effect of Changes in Parameter t_m on $\widetilde{GC}(t_1, T)$ & Q.

It can be observed that, economic order quantity and fuzzy total cost is more sensitive towards demand coefficient and demand constant.

7. Conclusions

In this paper, the inventory model for deteriorating items deteriorates after a certain period of time and not instantaneously, where demand is dependent on selling price and shortages are allowed and fully backlogged. The total average cost for both crisp and fuzzy models is calculated. For the fuzzy inventory model, hexagonal fuzzy numbers are used and for defuzzification, the Graded mean integration representation method is used. By comparing the results of both models, the crisp and fuzzy model, it can be seen that a fuzzy model provides the optimum value of the total average cost.

In the modern industrialized era, the study emphasizes how important it is to adopt optimal inventory management procedures. Fuzzy inventory systems have demonstrated encouraging outcomes in terms of order quantity optimization, inventory cost reduction, and customer satisfaction. By taking into account different changes in demand patterns, this research adds a new dimension to the current understanding of inventory systems. It is anticipated that the developed fuzzy inventory systems would find practical usage in applications for businesses looking to maximize revenues while operating in uncertain environments.

In the future aspect, one can extend this paper by taking shortages with partial backlogging.

Acknowledgements

The authors are thankful to the anonymous reviewers for their thoughtful comments and suggestions that helped throughout the submission process. This research work received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Whitin, T.M. (1957) Theory of Inventory Management. Princeton University Press, Princeton.

[2] Ghare, P.M. and Scharder, G.P. (1963) A Model for an Exponentially Decaying Inventory. *Journal of Industrial Engineering*, **14**, 238-243.

[3] Covert, R.P. and Philip, G.C. (1973) An EOQ Model for Items with Weibull Distribution Deterioration. *AIIE Transactions*, **5**, 323-326. https://doi.org/10.1080/05695557308974918

[4] Datta, T.K. and Pal, A.K. (1988) Order Level Inventory System with Power Demand Pattern for Items with Variable Rate of Deterioration. *Indian Journal of Pure and Applied Mathematics*, **19**, 1043-1053.

[5] Giri, B.C. and Chaudhuri, K.S. (1998) Deterministic Models of Perishable Inventory with Stock-Dependent Demand Rate and Nonlinear Holding Cost. *European Journal of Operational Research*, **105**, 467-474. https://doi.org/10.1016/S0377-2217(97)00086-6

[6] Ouyang, L.Y., Hsieh, T.P., Dye, C.Y. and Chang, H.C. (2003) An Inventory Model for Deteriorating Items with Stock-Dependent Demand under the Conditions of Inflation and Time-Value of Money. *The Engineering Economist*, **48**, 52-68. https://doi.org/10.1080/00137910308965051

[7] Mishra, V.K. (2013) An Inventory Model of Instantaneous Deteriorating Items with Controllable Deterioration Rate for Time Dependent Demand and Holding Cost. *Journal of Industrial Engineering and Management*, **6**, 495-506. https://doi.org/10.3926/jiem.530

[8] Sharma, V. and Chaudhary, R.R. (2013) An Inventory Model for Deteriorating Items with Weibull Deterioration with Time Dependent Demand and Shortages. *Research Journal of Management Sciences*, **2**, 28-30.

[9] Zhao, L. (2014) An Inventory Model under Trapezoidal Type Demand, Weibull-Distributed Deterioration, and Partial Backlogging. *Journal of Applied Mathematics*, **2014**, Article ID: 747419. https://doi.org/10.1155/2014/747419

[10] Uthayakumar, R. and Karuppasamy, S.K. (2019) An EOQ Model for Deteriorating Items with Different Types of Time-Varying Demand in Healthcare Industries. *The Journal of Analysis*, **27**, 3-18. https://doi.org/10.1007/s41478-018-0100-y

[11] Zadeh, L.A. (1965) Fuzzy Sets. *Information and Control*, **8**, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[12] Shekarian, E., Kazemi, N., Abdul-Rashid, S.H. and Olugu, E.U. (2017) Fuzzy Inventory Models: A Comprehensive Review. *Applied Soft Computing*, **55**, 588-621. https://doi.org/10.1016/j.asoc.2017.01.013

[13] Jaggi, C., Sharma, A. and Mandeep, M. (2012) A Fuzzy Inventory Model for Deteriorating Items with Initial Inspection and Allowable Shortage under the Condition of Permissible Delay in Payment. *International Journal of Inventory Control and Management*, **2**, 167-200. https://doi.org/10.58517/IJICM.2012.2202

[14] Kumar, S. and Rajput, U.S. (2015) Fuzzy Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging. *Applied Mathematics*, **6**,

496-509. https://doi.org/10.4236/am.2015.63047

[15] Mandal, W.A. and Islam, S. (2016) Fuzzy E.O.Q. Model for Deteriorating Items, with Constant Demand, Shortages, and Fully Backlogging. *Oxford Journal of Intelligent Decision and Data Science*, **2016**, 29-45.

[16] Mohanty, B. and Tripathy, P.K. (2017) Fuzzy Inventory Model for Deteriorating Items with Exponentially Decreasing Demand under Fuzzified Cost and Partial Backlogging. *International Journal of Mathematics Trends and Technology*, **51**, 182-189. https://doi.org/10.14445/22315373/IJMTT-V51P524

[17] Sahoo, S., Acharya, M. and Nayak, M.M. (2019) A Three Rates of EOQ/EPQ Model for Instantaneous Deteriorating Items Involving Fuzzy Parameter under Shortages. *International Journal of Innovative Technology and Exploring Engineering*, **8**, 405-418.

[18] Biswas, A.K. and Islam, S. (2019) A Fuzzy EPQ Model for Non-Instantaneous Deteriorating Items Where Production Depends on Demand Which Is Proportional to Population, Selling Price as Well as Advertisement. *Independent Journal of Management & Production*, **10**, 1679-1703. https://doi.org/10.14807/ijmp.v10i5.897

[19] Indrajitsingha, S.K., Raula, P., Samanta, P., Misra, U. and Raju, L. (2021) An EOQ Model of Selling-Price-Dependent Demand for Non-Instantaneous Deteriorating Items during the Pandemic COVID-19. *Walailak Journal of Science and Technology*, **18**, Article 13398. https://doi.org/10.48048/wjst.2021.13398