American Journal of Computational Mathematics, 2024, 14, 240-247
https://www.scirp.org/journal/ajcm
ISSN Online: 2161-1211
ISSN Print: 2161-1203

Impact of a Bumpy Nonuniform Electric Field on Oscillations of a Massive Point-Like Charged Particle

Haiduke Sarafian

The Pennsylvania State University, University College, York, PA, USA
Email: has2@psu.edu

How to cite this paper: Sarafian, H. (2024) Impact of a Bumpy Nonuniform Electric Field on Oscillations of a Massive Point-Like Charged Particle. *American Journal of Computational Mathematics*, **14**, 240-247.
https://doi.org/10.4236/ajcm.2024.142008

Received: April 12, 2024
Accepted: June 9, 2024
Published: June 12, 2024

Abstract

As a general feature, the electric field of a localized electric charge distribution diminishes as the distance from the distribution increases; there are exceptions to this feature. For instance, the electric field of a charged ring (being a localized charge distribution) along its symmetry axis perpendicular to the ring through its center rather than as expected being a diminishing field encounters a local maximum "bump". It is the objective of this research-oriented study to analyze the impact of this bump on the characteristics of a massive point-like charged particle oscillating along the symmetry axis. Two scenarios with and without gravity along the symmetry axis are considered. In addition to standard kinematic diagrams, various phase diagrams conducive to a better understanding are constructed. Applying Computer Algebra System (CAS), [1] [2] most calculations are carried out symbolically. Finally, by assigning a set of reasonable numeric parameters to the symbolic quantities various 3D animations are crafted. All the CAS codes are included.

Keywords

Nonlinear Oscillations, Nonuniform Electric Field, Computer Algebra System, *Mathematica*

1. Introduction & Analysis

1a. The electric field of a charged ring of radius R with charge Q along the symmetry axis of the ring through the center, perpendicular to the ring is [3] [4]

$$E = kQ\frac{x}{\left(R^2+x^2\right)^{\frac{3}{2}}}, \tag{1}$$

Aside from the electrostatic coupling constant, k = 9.0 × 10^9 Nm^2/C^2, and charge Q the plot of the distance-related term of the field for R = 20 cm is shown,

EfieldRing[*x*_]:=*kQ x*/(*R*²+*x*²)^3/2;

plotEfieldRing=Plot[EfieldRing[*x*]/.{*k*–>1,*Q*–>1,*R*–>20 10^−2},{*x*,−2,2},PlotRange →All,PlotStyle→{Black,Thick},GridLines→Automatic,AxesLabel →{"*x*(m)","E_field"}]

Figure 1 shows that the origin of the field is asymmetric and that at either side of the ring, it encounters a local extremum. As mentioned in the abstract one of the objectives of this investigation is to learn about the impact of this "bump" on the character of the related kinematic quantities, see upcoming paragraphs. Variable x is being used for the distance because, at the first trial, the ring was assumed oriented vertically with its symmetry axis along the x-axis. In this scenario, gravity has no impact on the mobile massive particle. Shortly in the upcoming sections, the ring is oriented horizontally making the gravity effectively active.

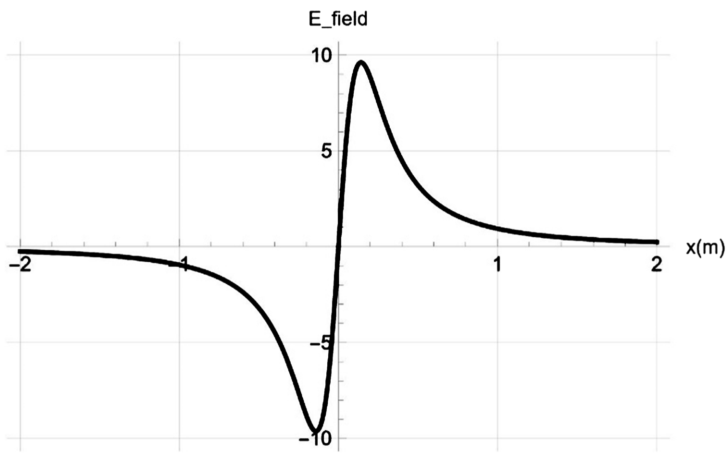


Figure 1. The electric field of a charged ring along the symmetry axis through the center perpendicular to the ring.

To observe the impact of the mentioned bump we consider a situation where a massive point-like charged particle is placed at the center of the ring. Wishing to throw the particle along the horizontal direction, the particle is given an initial speed. By trial and error to fulfill the desired scenario a set of parameters is assigned to the relevant quantities. These are stored in the **values** listing, units are: Q = 10 nC, q = −2 nC, and m = 2 nkg. The Q is the charge on the ring, particle's charge and mass are, respectively.

values={*k*→9 10^9,*g*–>9.8,*R* →20 10^−2,*Q*→10 10^−9,*q*→2 10^−9,m→2 10^−9};

A composite factor is defined,

factor=*kQq*/ *m*/.values;

The equation of the motion of the particle is subject to,

$$\ddot{x}(t)+\text{factor}\frac{x(t)}{\left(R^2+x(t)^2\right)^{3/2}}=0\,, \tag{2}$$

Where over double-dot is the second order derivative w/time.

```
eqn=x''[t]+(factor x[t]/(R^2+x[t]^2)^(3/2))/.values;
```

Solving this equation numerically with the shown initial conditions yields the interested kinematic quantities,

```
soleqn=NDSolve[{eqn==0,x[0.001]==0.0001(*(R/.values)*),x'[0]==25},
x[t],{t,0.001,1}];
{positionx,velocityx,accx}={x[t]/.soleqn,D[x[t]/.soleqn,{t,1}],D[x[t]/.soleqn,
{t,2}]};
```

Plots of these quantities are depicted in Figure 2.

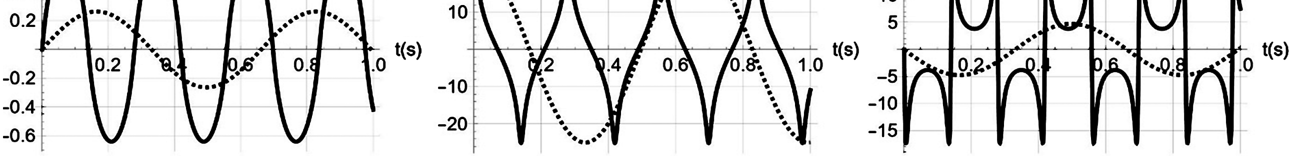
```
plotx=Plot[positionx,{t,0.001,1.0},AxesLabel->{"t(s)","x(m)"},PlotStyle->{T
hick,Black}, GridLines->Automatic];
plotvx=Plot[velocityx,{t,0.001,1.0},AxesLabel->{"t(s)","v(m/s)"},PlotStyle->{
Thick,Black}, GridLines->Automatic];
plotax=Plot[1/50accx,{t,0.001,1.0},AxesLabel->{"t(s)","a(m/s^2)"},PlotStyle->
{Thick,Black}, GridLines->Automatic];
GraphicsGrid[{{Show[{plotx,ploty}],Show[{plotvx,plotvy}],Show[{plotax,
plotay}]}},ImageSize->700]
```

For the sake of size compatibility, some of the plots are scaled.

The inclusion of Figure 2 is essential in showing the impact of the mentioned "bump" of the electric field depicted in Figure 1. Noticing, although curves in Figure 2(a) have the same characters, the impact of the mentioned "bump" is quite distinct shown in Figure 2(b) & Figure 2(c) these are intuitively unpredictable.

In many words for instance Figure 2(a) shows for the chosen parameters under the impact of the electric field the particle within a short period, e.g. 0.3 s returns to the initial position, while in a much longer time, the linear force takes e.g. 0.65 s to do the same. One may easily conclude similar conclusions focusing on Figure 2(b) & Figure 2(c).

As a crude approximation, the curve in Figure 1 may be viewed as two lines joined at the bump one with a positive and the other with a negative slope. Although this appears a rough approximation, nevertheless it sustains the general feature. Based on the mentioned observation an interested individual may try augmenting the scope of this work.



(a) (b) (c)

Figure 2. Each frame depicts a pair of identical kinematic quantities. From left to right, the first one is the positions vs. time. The rest of the frames are self-explained. The solid curves are with the electric force, the dashed ones result from the assumed linear force! Moreover, the description of the linear force is forth shortcoming.

For a deeper understanding, we extend the graphic analyses by including additional plots, namely, a standard, and then extended "phase" diagrams. Here again, each shown frame of Figure 3 embodies identical paired curves similar to the pairs shown in Figure 2.

Notice that the equations of motions conducive to the shown plots in Figure 3 are different. The solid curves depict the impact of electric force, and the dashed curves the linear mechanical force. For the sake of understanding the impact of the geometry terms the same **factor** and initial velocities are considered. Specifically, for the electric force.

factor $[x(t)/(R^2+x(t)^2)^{3/2}]$, and for the linear mechanical force, **factor** $[x(t)]$ is used. Note also while plotting the curves we have applied various magnification factors adjusting the size of the graphics pieces.

Now for instance, if one focuses on one of the frames of Figure 3, e.g. 3C, it reveals some of the kinematic characteristics of the oscillations. E.g. the electric and the mechanical oscillations do begin at the same initial position. Noticing, in the shown period each oscillator ends up at the max shown respective distance. And, the dotted curve is stretched much further than the solid curve, this shows the impact of the geometry *i.e.* distance term of the force. Then the oscillator reverses its motion sliding back to the other end continues repetition. Similar conclusions are applied to Figure 3(b) & Figure 3(c).

1b. In subsection **1a** for the vertically held ring a negatively charged particle was placed on a horizontal axis and was projected with an initial speed. Because of the contrast of the sign of the charges, this scenario was conducive to oscillations. In an alternative counterintuitive situation in this subsection, we consider a scenario in which the ring is held horizontally. Placing a negative charge on the vertical symmetry axis that is thrown downward may be tuned resulting in not a fall but an oscillation. In contrast to the situation described in **1a** gravity effectively contributes to the kinematics of the oscillation. If the parameters are not chosen properly the particle under gravity will fall through the ring. Yet the parameters are adjusted conducive to oscillations. The oscillation's equation of motion is given by (3). Therefore, it is natural to analyze the impact of gravity and compare its impact to case **1a**.

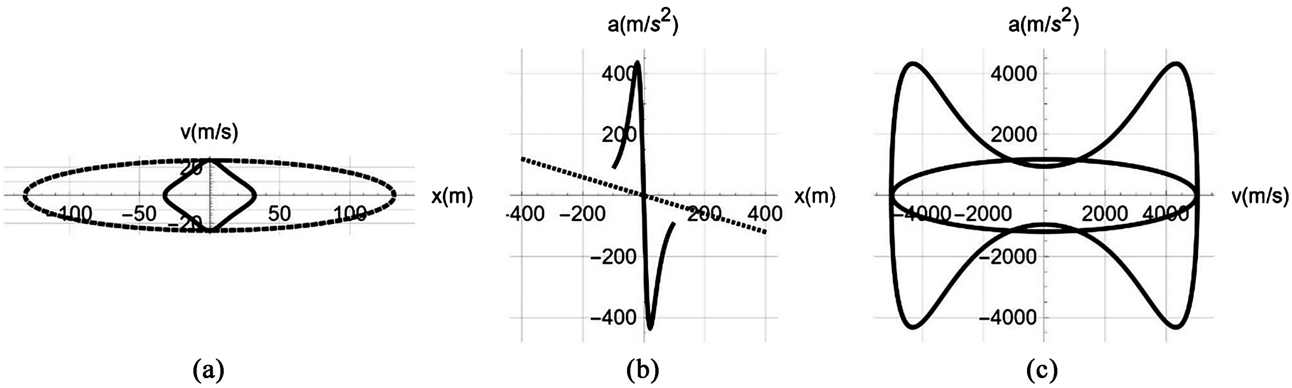


Figure 3. The solid curves are electric and the dashed are the linear force.

The modified version of the equation of motion including gravity is,

$$\ddot{z}(t)+\text{factor}\frac{z(t)}{\left(R^2+z(t)\right)^{3/2}}+g=0, \tag{3}$$

eqz=z"[t]+(factor z[t]/(R^2+z[t]^2)$^{3/2}$)+g/.values;

Solving (3) numerically with the same initial conditions as used in solving (2) yields,

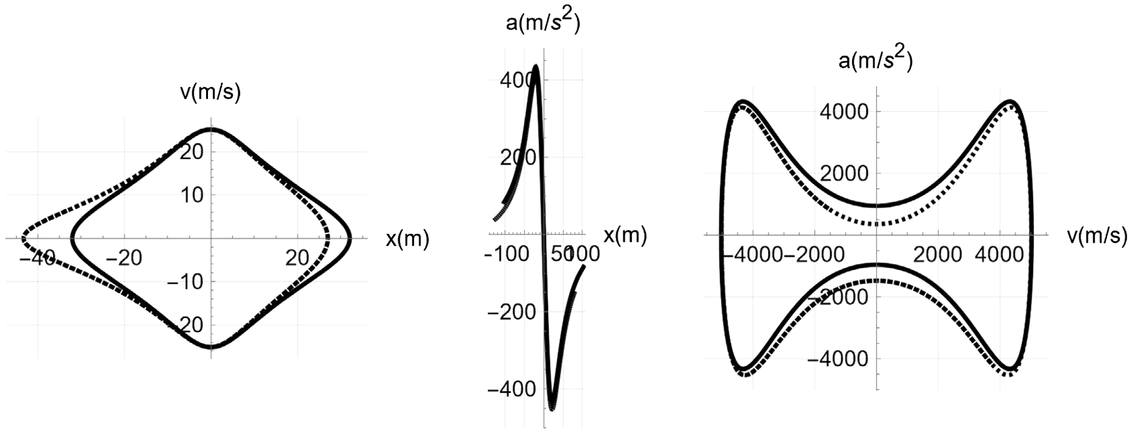
soleqz=NDSolve[{eqz==0,z[0.001]==0.001(*(R/.values)*),z'[0]==25}, z[t],{t,0.001,10}];

Utilizing these solutions the needed kinematic quantities are plotted. These are depicted in Figure 4.

The inclusion of gravity adds an additive term, **g** to the equation of motion. Utilizing the solution, the instantaneous speed and acceleration are calculated and graphed, respectively. These are shown in Figure 4.

```
eqz = Derivative[2][z][t] + factor*(z[t]/(R^2 + z[t]^2)^(3/2)) + 4*g /. values;
soleqz=NDSolve[{eqz==0,z[0.001]==0.001(*(R/.values)*),z'[0]==25},z[t],
{t,0.001,10}];
plotvz=ParametricPlot[Flatten[{50positionz,velocityz}],{t,0.001,2},AxesLab
el->{"z(m)","v(m/s)"},PlotStyle->{Black,Thick},GridLines->Automatic,Plot
Range->All];
plotaz=ParametricPlot[Flatten[{150positionz,0.5accz}],{t,0.001,2}, AxesLa-
bel->{"z(m)","a(m/s2)"},PlotStyle->{Black,Thick},GridLines->Automatic,Pl
otRange->All];
plotvaz=ParametricPlot[Flatten[{200velocityz,5accz}],{t,0.001,2},AxesLabel
->{"v(m/s)","a(m/s2)"},PlotStyle->{Black,Thick},GridLines->Automatic,Plo
tRange->All];
```

As explained earlier, for the case where gravity has the value of **g** the phase diagrams almost are indistinguishable vs. the solid curves of Figure 2. Yet the dashed plots of Figure 4 are different from Figure 2, this shows the impact of a



(a) (b) (c)

Figure 4. Phase diagrams associated with 4 g and g, the dashed and the solid, respectively. Labeled axes are descriptive of the shown frames.

strong gravity **4 g**. A similar observation applied to Figure 4(c) vs. Figure 2(c).

In general, for a regular gravity pull the impact of gravity is miniscule. Nevertheless, as intuitively might have been expected the oscillation of the particle about the center of the ring is not quite symmetric. This is shown in Figure 4(a). The mentioned feature is not quite vivid in Figure 4(b) & Figure 4(c).

Alternatively, the gravity acceleration may be adjusted as if the setup is considered on a different planet. Figure 5 displays the impact of gravity which is **4 g**. The black curve is the regular **g**, while the dashed curve shows the impact of **4 g**. As expected, the **4 g** case pulls the charged particle further downward and, in the meantime, increases the oscillation period.

Having all this detailed information on hand we decided to complete this analysis by including 3D animations of the motion, each case has its features. For the sake of briefing, only two animated frames are included.

GraphicsGrid[{{plot3DVertical,plot3Dhorizontal}},ImageSize->500]

The *Mathematica* codes creating the two frames shown in Figure 6 are lengthy

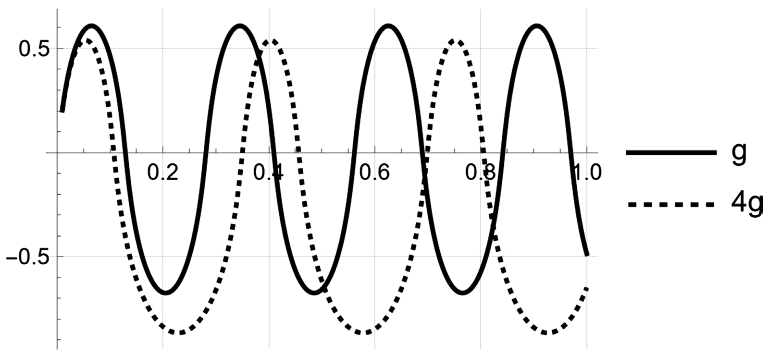


Figure 5. Oscillation characters under two different gravity pulls are compared. The black and the dashed curves are associated with g and 4 g gravities. respectively.

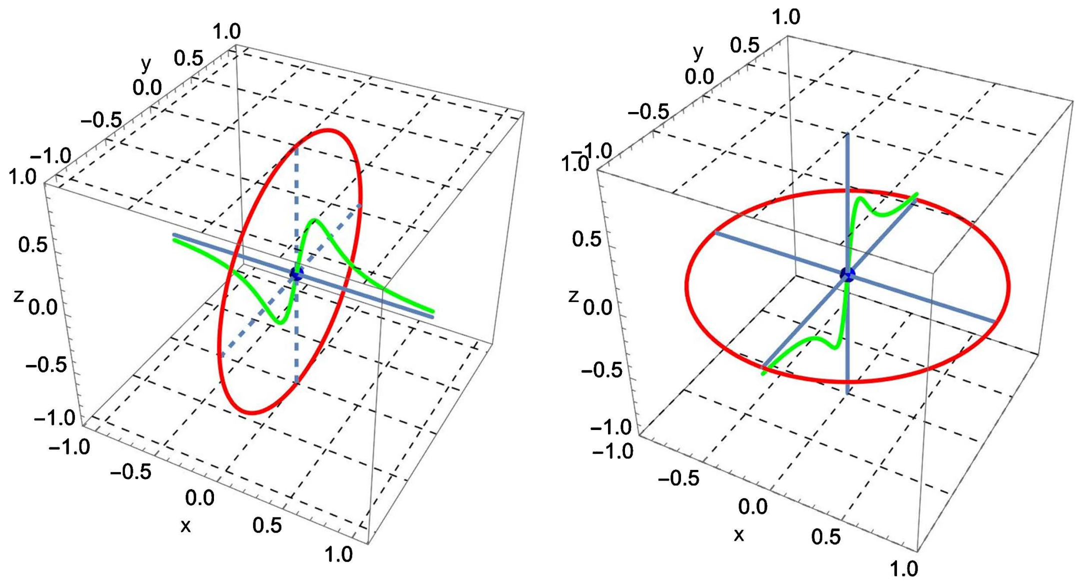


Figure 6. An animation frame depicting a charged horizontal ring, in Red. A negatively charged particle on the vertical symmetry axis, in Blue.

hence are suppressed. The code above Figure 6 has two pieces, **plot3DVertical and plot3Dhorizontal.** The *Mathematica* version of the article is capable of animation but neither the docx nor the pdf versions. The author is willing to forward the *Mathematica* codes of the animation to interested individuals.

2. Conclusions and Remarks

The motivation for crafting this research-oriented article is to justify that counter-balancing gravity with electric force is a plausible scenario conducive to oscillation.

We considered a massive, charged point-like particle moving along the symmetry axis of a horizontally held charged ring such that instead of "falling" through the ring it oscillates.

The entire analysis is done utilizing a Computer Algebra System (CAS), specifically *Mathematica*. Our approach to computation/calculation avoids using the traditional paper and pencil promoting the usefulness and the power of CAS. Most of the CAS codes are included to assist interested readers in practicing. To this end, one project has been suggested as conducive to a related augmented version of the current work. Interested readers may find references [5] [6] and [7] [8] [9] resourceful and insightful for the physics of nonuniform electric fields and *Mathematica* coding, respectively. The narrow scope objective of the previous work [7] in this current work has been expanded focusing on and comparing the impact of the various scenarios of gravity.

Acknowledgments

The author appreciates the financial support of the awarded Faculty Scholarship and Creative Accomplishment of the York Campus, 2023.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Mathematica Version 14.0. http://Wolfram.com

[2] Wolfram, S. (2003) The Mathematica Book. 5th Edition, Cambridge University Publications, Cambridge.

[3] Jackson, J.D. (1999) Classical Electrodynamics. 3rd Edition, John Wiley & Sons Inc., New Jersey.

[4] Halliday, D., Resnick, R. and Walker, J. (2022) Fundamentals of Physics. 12 Edition, John Wiley and Sons Inc. New Jersey.

[5] Strogatz, S. (1994) Nonlinear Dynamics and Chaos. Perseus Publishing, New York.

[6] Ennis, R.H. and McGuire, G.C. (2001) Nonlinear Physics with Mathematica for Scientists and Engineers. Published by Birdhouse, Hard Spring, Peg 605. https://doi.org/10.1007/978-1-4612-0211-0

[7] Sarafian, H. (2010) Static Electric-Spring and Nonlinear Oscillations. *Journal of Electromagnetic Analysis and Applications*, **2**, 75-81.

https://doi.org/10.4236/jemaa.2010.22011

[8] Sarafian, H. (2019) Mathematica Graphics Examples. 2nd Edition, Scientific Research Publishing, Wuhan.

[9] Sarafian, H. (2023) Haiduke Sarafian's Collective Articles 2020-2023. Scientific Research Publishing, Wuhan.