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Application of the Non-Local Physics in the Theory of the Matter Movement in Black Hole

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ABSTRACT

The theory of the matter movement in a black hole in the frame of non-local quantum hydrodynamics (NLQHD) is considered. The theory corresponds to the limit case when the matter density tends to infinity. From calculations follow that NLQHD equations for the black hole space have the traveling wave solutions. The domain of the solution existence is limited by the event horizon where gravity tends to infinity. The simple analytical particular cases and numerical calculations are delivered.

Keywords: The Theory of Traveling Waves; Generalized Hydrodynamic Equations; Foundations of Quantum Mechanics; Matter Movement in Black Hole

1. Introduction

The first ideas about the existence of cosmic objects which gravitation is so big that the escape velocity would be faster than the speed of light, were formulated in 1783 by English geologist named John Mitchell. In 1796, Pierre-Simon Laplace promoted the same idea in his book Exposition du système du Monde. In 1916 Albert Einstein introduced an explanation of gravity called general relativity. According to the general theory of relativity, a black hole is a region of space from which nothing, including light, can escape. It is the result of the denting of spacetime caused by a very compact mass. Around a black hole there is an undetectable surface which marks the point of no return, called an event horizon. It is called "black" because it absorbs all the light that hits it, reflecting nothing, just like a perfect black body in thermodynamics. Black holes possess a temperature (and therefore the internal energy) and emit Hawking radiation through slow dissipation by antiprotons.

In 1930, Subrahmanyan Chandrasekhar predicted that stars heavier than the sun could collapse when they ran out of hydrogen or other nuclear fuels to burn and die. In 1967, John Wheeler gave black holes the name "black hole" for the first time. Astronomers have identified numerous stellar black hole candidates, and have also found evidence of supermassive black holes at the center of every galaxy. In 1970, Stephen Hawking and Roger Penrose proved that black holes must exist.

Let us investigate the possibilities delivered by the unified generalized quantum hydrodynamics [1-4] for investigation of these problems. From position of non-local quantum hydrodynamics (NLQHD) the mentioned theory has two limit cases connected with the density ρ evolution:

1) The density $\rho \to 0$. From the physical point of view this case corresponds to the motion in the Big Bang regime. This regime is considered in my previous paper published in this issue [5];

2) The density $\rho \to \infty$. From the physical point of view this case corresponds to the matter motion in the Black Hole regime.

Here we intend to consider the second limit case on the basement of non-local physics which particular interpretation is the generalized Boltzmann physical kinetics. We need not to deliver here main ideas and deductions of the generalized Boltzmann physical kinetics and non-local physics. The fundamental methodic aspects of the mentioned theory are considered in [5]. A rigorous description can be found, for example, in the monographs [3,4, 6], see also [7-11].

Strict consideration leads to the following system of the generalized hydrodynamic equations (GHE) [3,4,10] written in the generalized Euler form:
continuity equation for species α

$$\frac{\partial}{\partial t}\left\{\rho_\alpha-\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\boldsymbol{v}_0\right)\right]\right\}+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho_\alpha\boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0\right)+\ddot{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}=R_\alpha, \tag{1.1}$$

and continuity equation for mixture

$$\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\boldsymbol{v}_0\right)\right]\right\}+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho\boldsymbol{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial r}\cdot\left(\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0\right)+\ddot{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}=0. \tag{1.2}$$

Momentum equation for species

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_\alpha\boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}-\boldsymbol{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\left(\rho_\alpha\boldsymbol{v}_0\right)\right)\right]\\&-\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}\times\boldsymbol{B}\\&+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+p_\alpha\ddot{\mathrm{I}}-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+p_\alpha\ddot{\mathrm{I}}\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\left(\boldsymbol{v}_0\boldsymbol{v}_0\right)\boldsymbol{v}_0+2\ddot{\mathrm{I}}\left(\frac{\partial}{\partial\mathbf{r}}\cdot\left(p_\alpha\boldsymbol{v}_0\right)\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\ddot{\mathrm{I}}p_\alpha\boldsymbol{v}_0\right)\right.\right.\\&\left.\left.-\boldsymbol{F}_\alpha^{(1)}\rho_\alpha\boldsymbol{v}_0-\rho_\alpha\boldsymbol{v}_0\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\boldsymbol{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right]\right\}=\int m_\alpha\boldsymbol{v}_\alpha J_\alpha^{st,el}\mathrm{d}\boldsymbol{v}_\alpha+\int m_\alpha\boldsymbol{v}_\alpha J_\alpha^{st,inel}\mathrm{d}\boldsymbol{v}_\alpha.\end{aligned} \tag{1.3}$$

Generalized moment equation for mixture

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho\boldsymbol{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}-\sum_\alpha\boldsymbol{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\left(\rho_\alpha\boldsymbol{v}_0\right)\right)\right]\\&-\sum_\alpha\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}\times\boldsymbol{B}\\&+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho\boldsymbol{v}_0\boldsymbol{v}_0+p\ddot{\mathrm{I}}-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+p_\alpha\ddot{\mathrm{I}}\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\left(\boldsymbol{v}_0\boldsymbol{v}_0\right)\boldsymbol{v}_0+2\ddot{\mathrm{I}}\left(\frac{\partial}{\partial\mathbf{r}}\cdot\left(p_\alpha\boldsymbol{v}_0\right)\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\ddot{\mathrm{I}}p_\alpha\boldsymbol{v}_0\right)\right.\right.\\&\left.\left.-\boldsymbol{F}_\alpha^{(1)}\rho_\alpha\boldsymbol{v}_0-\rho_\alpha\boldsymbol{v}_0\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\boldsymbol{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\boldsymbol{v}_0\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right]\right\}=0\end{aligned} \tag{1.4}$$

Energy equation for component

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha\boldsymbol{v}_0+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0\right)-\boldsymbol{F}_\alpha^{(1)}\cdot\rho_\alpha\boldsymbol{v}_0\right]\right\}\\&+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha\boldsymbol{v}_0+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha\boldsymbol{v}_0+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0\boldsymbol{v}_0+\frac{7}{2}p_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{1}{2}p_\alpha v_0^2\ddot{\mathrm{I}}\right.\right.\right.\\&\left.+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\ddot{\mathrm{I}}+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\ddot{\mathrm{I}}\right)-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}\cdot\boldsymbol{v}_0\boldsymbol{v}_0-p_\alpha\boldsymbol{F}_\alpha^{(1)}\cdot\ddot{\mathrm{I}}-\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{F}_\alpha^{(1)}-\frac{3}{2}\boldsymbol{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\\&\left.\left.-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha\boldsymbol{F}_\alpha^{(1)}\right]\right\}-\left\{\rho_\alpha\boldsymbol{F}_\alpha^{(1)}\cdot\boldsymbol{v}_0-\tau_\alpha\left[\boldsymbol{F}_\alpha^{(1)}\cdot\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial}{\partial\mathbf{r}}\cdot p_\alpha\ddot{\mathrm{I}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right]\right\}\\&=\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,el}\mathrm{d}\boldsymbol{v}_\alpha+\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,inel}\mathrm{d}\boldsymbol{v}_\alpha.\end{aligned} \tag{1.5}$$

and after summation the generalized energy equation for mixture (please see Equation (1.6) below).

Here $\boldsymbol{F}_\alpha^{(1)}$ are the forces of the non-magnetic origin, $\boldsymbol{B}$—magnetic induction, $\vec{\mathrm{I}}$—unit tensor, q_α—charge of the α—component particle, q_α—static pressure for α—component, ε_α—internal energy for the particles of α—component, $\boldsymbol{v}_0$—hydrodynamic velocity for mixture. For calculations in the self-consistent electro-magnetic field the system of non-local Maxwell equations should be added.

$$
\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha \varepsilon_\alpha n_\alpha-\sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha\boldsymbol{v}_0+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0\right)-\boldsymbol{F}_\alpha^{(1)}\cdot\rho_\alpha\boldsymbol{v}_0\right]\right\}
$$
$$
+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\boldsymbol{v}_0+\frac{5}{2}p\boldsymbol{v}_0+\boldsymbol{v}_0\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha\boldsymbol{v}_0+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0\right)\right.\right.
$$
$$
+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0\boldsymbol{v}_0+\frac{7}{2}p_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{1}{2}p_\alpha v_0^2\vec{\mathrm{I}}+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\vec{\mathrm{I}}+\varepsilon_\alpha n_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\vec{\mathrm{I}}\right)-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}\cdot\boldsymbol{v}_0\boldsymbol{v}_0-p_\alpha\boldsymbol{F}_\alpha^{(1)}\cdot\vec{\mathrm{I}} \quad (1.6)
$$
$$
\left.\left.-\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{F}_\alpha^{(1)}-\frac{3}{2}\boldsymbol{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha\boldsymbol{F}_\alpha^{(1)}\right]\right\}-\boldsymbol{v}_0\cdot\sum_\alpha\rho_\alpha\boldsymbol{F}_\alpha^{(1)}
$$
$$
+\sum_\alpha\tau_\alpha\boldsymbol{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}\left(\rho_\alpha\boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial}{\partial \boldsymbol{r}}\cdot p_\alpha\vec{\mathrm{I}}-\rho_\alpha\boldsymbol{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right]=0.
$$

2. Propagation of Plane Traveling Waves in Black Hole

Newtonian gravity propagates with the infinite speed. This conclusion is connected only with the description in the frame of local physics. Usual affirmation-general relativity (GR) reduces to Newtonian gravity in the weak-field, low-velocity limit. In literature you can find criticism of this affirmation because the conservation of angular momentum is implicit in the assumptions on which GR rests. Finite propagation speeds and conservation of angular momentum are incompatible in GR. Therefore, GR was forced to claim that gravity is not a force that propagates in any classical sense, and that aberration does not apply. But here I do not intend to join to this widely discussed topic using only unified non-local model.

Let us apply generalized quantum hydrodynamic Equations (1.1)-(1.6) for investigation of the traveling wave propagation inside the black hole using non-stationary 1D Cartesian description. It means that consideration corresponds so to speak to "the black channel".

Call attention to the fact that Equations (1.1)-(1.6) contain two forces of gravitational origin, $\boldsymbol{F}$—the force acting on the unit volume of the space and $\boldsymbol{g}$—the force acting on the unit mass. As result we have from Equations (1.1)-(1.6):

(continuity equation)

$$
\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\rho\boldsymbol{v}_0\right)\right]\right\}+
\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho\boldsymbol{v}_0-\tau\left[\frac{\partial}{\partial t}\left(\rho\boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\rho\boldsymbol{v}_0\boldsymbol{v}_0\right)+\vec{\mathrm{I}}\cdot\frac{\partial p}{\partial \boldsymbol{r}}-\boldsymbol{F}\right]\right\}=0, \quad (2.1)
$$

(continuity equation, 1D case)

$$
\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho v_0\right)\right]\right\}
+\frac{\partial}{\partial x}\left\{\rho v_0-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0\right)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial x}-F\right]\right\}=0, \quad (2.2)
$$

(momentum equation)

$$
\frac{\partial}{\partial t}\left\{\rho\boldsymbol{v}_0-\tau\left[\frac{\partial}{\partial t}\left(\rho\boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho\boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p}{\partial \boldsymbol{r}}-\boldsymbol{F}\right]\right\}
-\boldsymbol{g}\left[\rho-\tau\left(\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\left(\rho\boldsymbol{v}_0\right)\right)\right]
$$
$$
+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho\boldsymbol{v}_0\boldsymbol{v}_0+p\vec{\mathrm{I}}-\tau\left[\frac{\partial}{\partial t}\left(\rho\boldsymbol{v}_0\boldsymbol{v}_0+p\vec{\mathrm{I}}\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho\left(\boldsymbol{v}_0\boldsymbol{v}_0\right)\boldsymbol{v}_0
+2\vec{\mathrm{I}}\left(\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(p\boldsymbol{v}_0\right)\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\vec{\mathrm{I}}p\boldsymbol{v}_0\right)-\boldsymbol{F}\boldsymbol{v}_0-\boldsymbol{v}_0\boldsymbol{F}\right]\right\}=0 \quad (2.3)
$$

(momentum equation, 1D case)

$$
\frac{\partial}{\partial t}\left\{\rho v_0-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0\right)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial x}-F\right]\right\}
-g\left[\rho-\tau\left(\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho v_0\right)\right)\right]
$$
$$
+\frac{\partial}{\partial x}\left\{\rho v_0^2+p-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^2+p\right)
+\frac{\partial}{\partial x}\left(\rho v_0^3+3pv_0\right)-2Fv_0\right]\right\}=0, \quad (2.4)
$$

(energy equation)

$$\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p-\tau\left[\frac{\partial}{\partial t}\left(\frac{\rho v_0^2}{2}+\frac{3}{2}p\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0\right)-\boldsymbol{F}\cdot\mathbf{v}_0\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0-\tau\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}pv_0^2\ddot{\mathrm{I}}+\frac{5}{2}\frac{p^2}{\rho}\ddot{\mathrm{I}}\right)-\boldsymbol{F}\cdot\mathbf{v}_0\mathbf{v}_0-p\boldsymbol{g}\cdot\ddot{\mathrm{I}}-\frac{1}{2}v_0^2\boldsymbol{F}-\frac{3}{2}\boldsymbol{g}p\right]\right\}$$
$$-\left\{\boldsymbol{F}\cdot\mathbf{v}_0-\tau\left[\boldsymbol{g}\cdot\left(\frac{\partial}{\partial t}(\rho\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p\ddot{\mathrm{I}}-\boldsymbol{F}\right)\right]\right\}=0, \tag{2.5}$$

(energy equation, 1D case)

$$\frac{\partial}{\partial t}\left\{\rho v_0^2+3p-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^2+3p\right)+\frac{\partial}{\partial x}\left(\rho v_0^3+5pv_0\right)-2Fv_0\right]\right\}+\frac{\partial}{\partial x}\left\{\rho v_0^3+5pv_0-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^3+5pv_0\right)+\frac{\partial}{\partial x}\left(\rho v_0^4+8pv_0^2\right)\right.\right.$$
$$\left.\left.-2Fv_0^2-v_0^2F\right]\right\}+5\frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho}F-\frac{\partial}{\partial x}\frac{p^2}{\rho}\right)\right\}-2\left\{Fv_0-\tau g\left(\frac{\partial}{\partial t}(\rho v_0)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial x}-F\right)\right\}=0, \tag{2.6}$$

Nonlinear evolution Equations (2.1)-(2.6) contain forces $\boldsymbol{F}$, $\boldsymbol{g}$ acting on space and masses including cross-term (see for example the last line in Equation (2.6)). The relation $\boldsymbol{F}=\rho\boldsymbol{g}$ comes into being only after the mass appearance as result of the Big Bang.

Let us introduce now the main mentioned before assumption leading to the theory of motion inside the black holes: the density $\rho\to\infty$. Derivating the basic system of equation we should take into account two facts:

1) The density can tend to infinity by the arbitrary law;

2) The ratio of pressure to density defines the internal energy of the mass unit $E=p/\rho$ and should be considered as a dependent variable by $\rho\to\infty$.

As result we have the following system of equations:

$$\frac{\partial}{\partial t}\left\{\tau\frac{\partial u}{\partial x}\right\}-\frac{\partial u}{\partial x}+\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial u}{\partial t}+2u\frac{\partial u}{\partial x}+\frac{\partial E}{\partial x}-g\right]\right\}=0, \tag{2.7}$$

$$\frac{\partial}{\partial t}\left\{u-\tau\left[\frac{\partial u}{\partial t}+2u\frac{\partial u}{\partial x}+\frac{\partial E}{\partial x}-g\right]\right\}-g\left[1-\tau\frac{\partial u}{\partial x}\right]$$
$$+\frac{\partial}{\partial x}\left\{u^2+E-\tau\left[\frac{\partial}{\partial t}\left(u^2+E\right)+\frac{\partial}{\partial x}\left(u^3+3Eu\right)-2gu\right]\right\}$$
$$=0. \tag{2.8}$$

$$\frac{\partial}{\partial t}\left\{u^2+3E-\tau\left[\frac{\partial}{\partial t}\left(u^2+3E\right)+\frac{\partial}{\partial x}\left(u^3+5Eu\right)-2gu\right]\right\}+\frac{\partial}{\partial x}\left\{u^3+5Eu-\tau\left[\frac{\partial}{\partial t}\left(u^3+5Eu\right)+\frac{\partial}{\partial x}\left(u^4+8Eu^2\right)-3gu^2\right]\right\}$$
$$+5\frac{\partial}{\partial x}\left\{\tau E\left(g-2\frac{\partial E}{\partial x}\right)\right\}-2gu+2\tau g\left(\frac{\partial u}{\partial t}+2u\frac{\partial u}{\partial x}+\frac{\partial E}{\partial x}-g\right)=0, \tag{2.9}$$

where u is the velocity component along the x direction. Let us introduce the coordinate system moving along the positive direction of x-axis in 1D space with velocity $C=u_0$ equal to phase velocity of considering object

$$\xi=x-Ct. \tag{2.10}$$

Taking into account the De Broglie relation we should wait that the group velocity u_g is equal $2u_0$. In moving coordinate system all dependent hydrodynamic values are function of (ξ,t). We investigate the possibility of the traveling wave formation. For this solution there is no explicit dependence on time for coordinate system moving with the phase velocity u_0. Write down the system of Equations (2.7)-(2.9) in the moving coordinate system using the relation $\xi=x-ut$:

(continuity equation, 1D case)

$$\frac{\partial u}{\partial\xi}-\tau\left(\frac{\partial u}{\partial\xi}\right)^2-\frac{\partial}{\partial\xi}\left\{\tau\left[\frac{\partial E}{\partial\xi}-g\right]\right\}=0, \tag{2.11}$$

(momentum equation, 1D case)

$$\left(\frac{\partial E}{\partial\xi}-g\right)+3\tau g\frac{\partial u}{\partial\xi}-5\tau\frac{\partial u}{\partial\xi}\frac{\partial E}{\partial\xi}-3E\frac{\partial}{\partial\xi}\left\{\tau\frac{\partial u}{\partial\xi}\right\}=0, \tag{2.12}$$

(energy equation, 1D case)

$$2u\left(\frac{\partial E}{\partial \xi}-g\right)+5E\frac{\partial u}{\partial \xi}-10u\tau\frac{\partial u}{\partial \xi}\frac{\partial E}{\partial \xi}-6uE\frac{\partial}{\partial \xi}\left\{\tau\frac{\partial u}{\partial \xi}\right\}$$
$$-11\tau E\left(\frac{\partial u}{\partial \xi}\right)^2-10\frac{\partial}{\partial \xi}\left\{\tau E\frac{\partial E}{\partial \xi}\right\}+5\frac{\partial}{\partial \xi}\left\{\tau Eg\right\}+6\tau gu\frac{\partial u}{\partial \xi}$$
$$+2\tau g\left(\frac{\partial E}{\partial \xi}-g\right)=0, \tag{2.13}$$

Non-local equations are closed system of three differential equations with three dependent variables u, E, g. In this case *no needs* to use the additional Poisson equation leading to the Newton gravitational description.

If the non-locality parameter τ is equal to zero the mentioned system becomes unclosed.

Let us introduce the length scale ξ_0, the velocity scale u_0, time scale $\tau_0 = x_0/u_0$, and scales for the gravitation acceleration $g_0 = u_0/\tau_0 = u_0^2/x_0$ and for the internal energy of the mass unit $E_0 = u_0^2$. Using these scales one obtains

$$\frac{\partial \tilde{u}}{\partial \tilde{\xi}}-\tilde{\tau}\left(\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right)^2-\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\left[\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-\tilde{g}\right]\right\}=0, \tag{2.14}$$

$$\left(\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-\tilde{g}\right)+3\tilde{\tau}\tilde{g}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}-5\tilde{\tau}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-3\tilde{E}\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right\}=0, \tag{2.15}$$

$$2\tilde{u}\left(\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-\tilde{g}\right)+5\tilde{E}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}-10\tilde{u}\tilde{\tau}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-6\tilde{u}\tilde{E}\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right\}$$
$$-11\tilde{\tau}\tilde{E}\left(\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right)^2-10\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\tilde{E}\frac{\partial \tilde{E}}{\partial \tilde{\xi}}\right\}+5\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\tilde{E}\tilde{g}\right\}+6\tilde{\tau}\tilde{g}\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}$$
$$+2\tilde{\tau}\tilde{g}\left(\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-\tilde{g}\right)=0, \tag{2.16}$$

We need also an approximation for the non-local parameter $\tilde{\tau}$. Take this approximation in the form

$$\tilde{\tau} = H/\tilde{u}^2, \tag{2.17}$$

where H is dimensionless value. In the dimension form

$$\tau = u_0 x_0 \frac{H}{u^2}. \tag{2.18}$$

It means that the nonlocal parameter is proportional to the kinematic velocity and inversely with square of the velocity. Relation (2.18) resembles the Heisenberg relation “time-energy”. Remark now that (as follow from the numerical calculations) the choice of the non-local parameter in this case has the small influence on the results of modeling.

3. Results of Mathematical Modeling

Now we are ready to display the results of the mathematical modeling realized with the help of Maple (the versions Maple 9 or higher can be used).

The system of Equations (2.14)-(2.16) has the great possibilities of mathematical modeling as result of changing the parameter H and five Cauchy conditions describing the character features of initial perturbations which lead to the traveling wave formation. Maple program contains Maple’s notations—for example the expression $D(u)(0)=0$ means in the usual notations $\left(\partial\tilde{u}/\partial\tilde{\xi}\right)(0)=0$, independent variable t responds to $\tilde{\xi}$.

We begin with investigation of the problem of principle significance—is it possible after a perturbation (defined by Cauchy conditions) to obtain the traveling wave as result of the self-organization? With this aim let us consider the initial perturbations:

$$u(0)=1,\ E(0)=1,\ g(0)=1,\ D(u)(0)=0,\quad D(E)(0)=1. \tag{3.1}$$

The following Maple notations in figures are used: u—velocity $\tilde{u}$, E—energy $\tilde{E}$, and g—acceleration $\tilde{g}$. Explanations are placed under all following figures. The mentioned calculations are displayed in **Figures 1-4**.

All calculations are realized using the conditions (3.1) but by the different value of the H parameter, namely $H = 0.001; 1; 1000$. **Figure 1** reflects the evolution of the dependent values in the area of the event horizon in details.

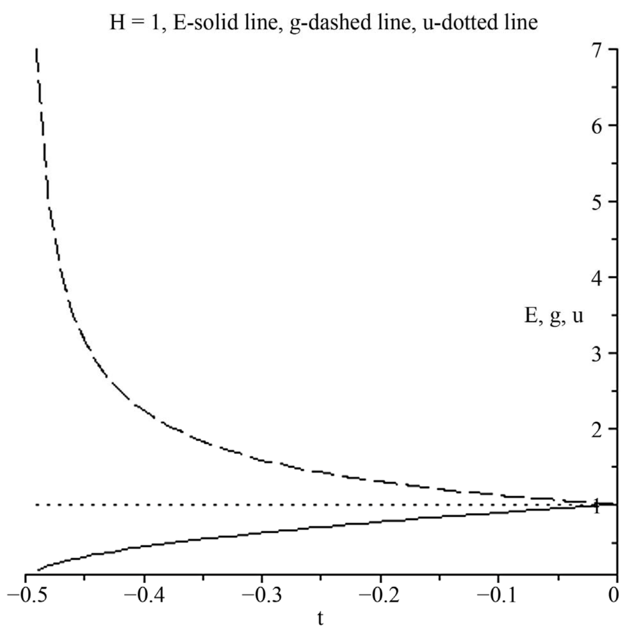

Figure 1. *u*—velocity $\tilde{u}$ (dotted line), *H* = 1, *E*—energy $\tilde{E}$ (solid line), and *g*—acceleration $\tilde{g}$ (dashed line), area of event horizon.

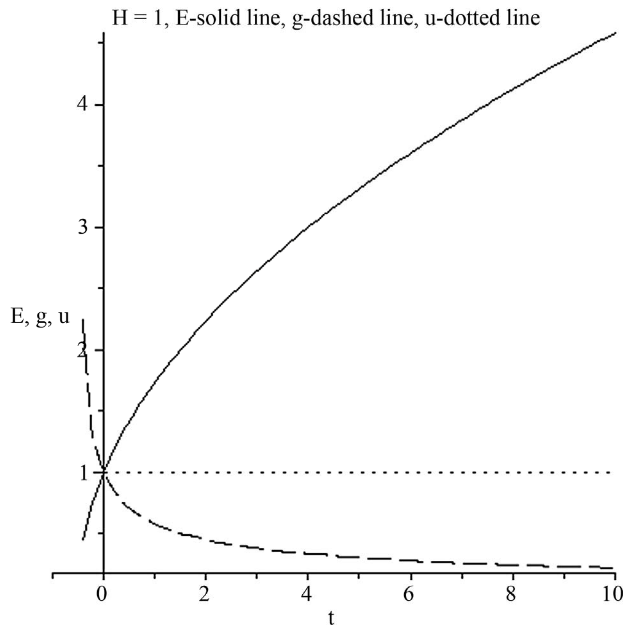

Figure 2. ***u*****—velocity** $\tilde{u}$ **(dotted line),** ***H*** **= 1,** ***E*****—energy** $\tilde{E}$ **(solid line), and** ***g*****—acceleration** $\tilde{g}$ **(dashed line).**

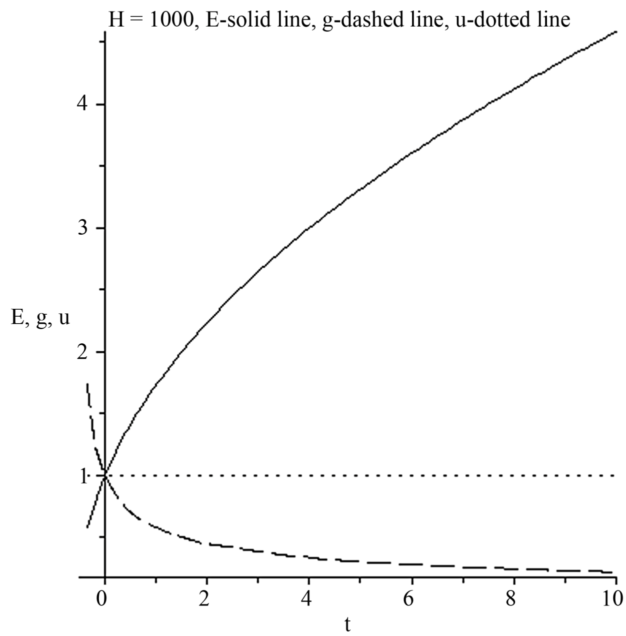

Figure 3. ***u*****—velocity** $\tilde{u}$ **(dotted line),** ***H*** **= 1000,** ***E*****—energy** $\tilde{E}$ **(solid line), and** ***g*****—acceleration** $\tilde{g}$ **(dashed line).**

In all calculations the boundary of the transition area of events is limited by the condition (obtained as the self-consistent result of calculations) $\tilde{\xi}_{\lim} > -0.5$.

As follow from calculations (see **Figures 1-4**) the variation of H-parameter has the weak influence on the numerical results. Let us show also the results obtained for $H = 0.0001$ (see **Figure 5**) and the corresponding numerical results near singularity $\tilde{\xi}_{\lim} = -0.5$; namely:

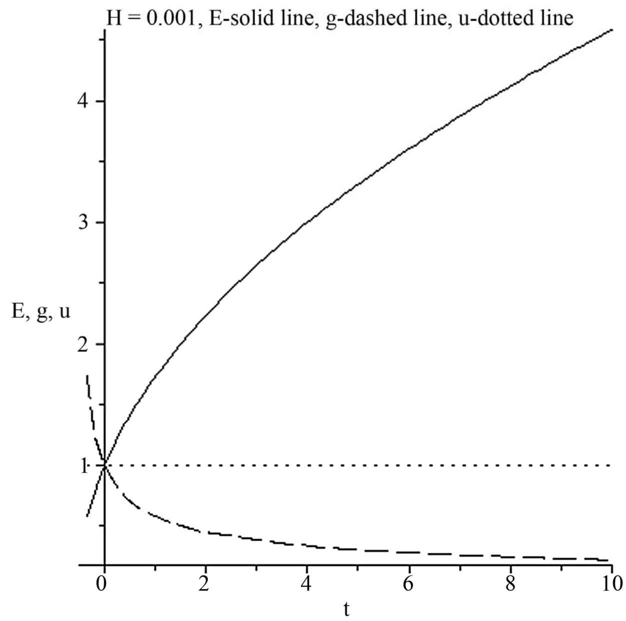

Figure 4. ***u*****—velocity** $\tilde{u}$ **(dotted line),** ***H*** **= 0.001,** ***E*****—energy** $\tilde{E}$ **(solid line), and** ***g*****—acceleration** $\tilde{g}$ **(dashed line).**

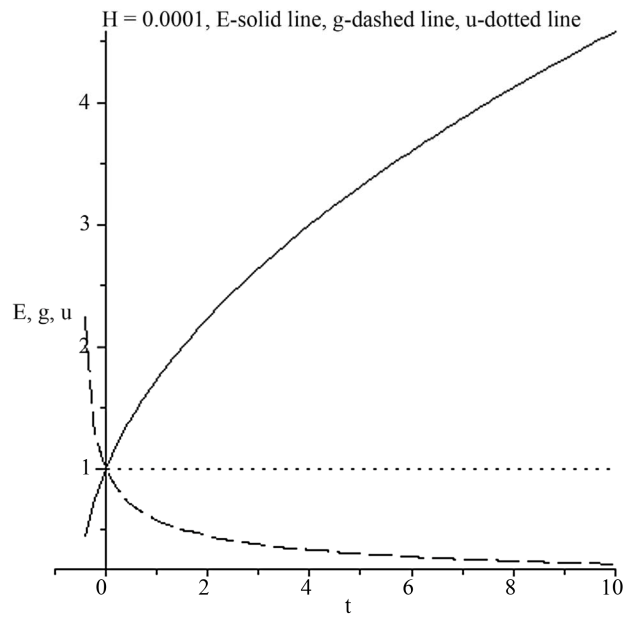

Figure 5. ***u*****—velocity** $\tilde{u}$ **(dotted line),** ***H*** **= 0.0001,** ***E*****—energy** $\tilde{E}$ **(solid line), and** ***g*****—acceleration** $\tilde{g}$ **(dashed line).**

$H = 0.0001$; $\tilde{\xi} = -0.4999999$. We have the following results of calculations $\tilde{E} = 0.382 \times 10^{-3}$; $\tilde{g} = 2615.014$; $\tilde{u} = 1$.

As we see the self-consistent solutions lead with the high accuracy to the relation

$$\tilde{u} = 1. \tag{3.2}$$

Let us use this condition for analytical transformations of the Equations (2.14)-(2.16). We have correspondingly

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$$\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\left[\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-\tilde{g}\right]\right\}=0, \tag{3.3}$$

$$\frac{\partial \tilde{E}}{\partial \tilde{\xi}}-\tilde{g}=0, \tag{3.4}$$

$$2\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\tilde{E}\frac{\partial \tilde{E}}{\partial \tilde{\xi}}\right\}-\frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{\tau}\tilde{E}\tilde{g}\right\}=0. \tag{3.5}$$

From (3.4), (3.5) follow

$$\frac{\partial}{\partial \tilde{\xi}}\left(\tilde{\tau}\tilde{E}\frac{\partial \tilde{E}}{\partial \tilde{\xi}}\right)=0, \tag{3.6}$$

$$\tilde{\tau}\tilde{E}\frac{\partial \tilde{E}}{\partial \tilde{\xi}}=\text{const} \tag{3.7}$$

and for chosen $\tilde{\tau}$ approximation

$$\frac{\partial \tilde{E}^2}{\partial \tilde{\xi}}=C,\ \tilde{E}^2=C\tilde{\xi}+C_1, \tag{3.8}$$

$$\tilde{E}^2=\left[\frac{\partial \tilde{E}^2}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}\tilde{\xi}+\tilde{E}^2(0). \tag{3.9}$$

It means that for large $\tilde{\xi}$

$$\tilde{E}\cong\sqrt{2\tilde{E}(0)\left[\frac{\partial \tilde{E}}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}}\sqrt{\tilde{\xi}} \tag{3.10}$$

or in the dimensional form

$$E\cong\sqrt{2E(0)\left[\frac{\partial E}{\partial \xi}\right]_{\xi=0}}\sqrt{\xi} \tag{3.11}$$

where $\xi=x-ut$. Taking into account (3.4), (3.6) one obtains

$$\tilde{E}\tilde{g}=\text{const}=\tilde{E}(0)\tilde{g}(0) \tag{3.12}$$

$$\tilde{g}\cong\tilde{g}(0)\frac{\tilde{E}(0)}{\sqrt{\left[\frac{\partial \tilde{E}^2}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}\tilde{\xi}+\tilde{E}^2(0)}}=\frac{\tilde{g}(0)}{\sqrt{1+\left[\frac{\partial \ln \tilde{E}^2}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}\tilde{\xi}}} \tag{3.13}$$

and for large $\tilde{\xi}$

$$\tilde{g}\cong\tilde{g}(0)\sqrt{\frac{\tilde{E}(0)}{2\left[\frac{\partial \tilde{E}}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}}}\frac{1}{\sqrt{\tilde{\xi}}}. \tag{3.14}$$

After the penetration through the frontier barrier the external matter is moving in the black channel in the form of the traveling wave. In this 1D Cartesian model the gravitational acceleration decreases as $\tilde{\xi}^{-0.5}$ with the rise of the $\tilde{\xi}$-distance and, on the contrary, the internal energy of the mass unit increases as $\tilde{\xi}^{0.5}$.

The influence of the tidal force on the object in the black channel can be calculated using (3.13), (3.14). From (3.13) follows

$$\mathrm{d}\tilde{g}=-\left[\frac{\partial \tilde{E}}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}\tilde{g}(0)\frac{\tilde{E}^2(0)}{\left[\left[\frac{\partial \tilde{E}^2}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}\tilde{\xi}+\tilde{E}^2(0)\right]^{3/2}}\mathrm{d}\tilde{\xi}. \tag{3.15}$$

Relation (3.15) reflects the change Δg in the tidal force acting at the time moment t across the body element Δx. This change tends to infinity if the point of singularity

$$\tilde{\xi}_s=-\left[\frac{\partial \ln \tilde{E}^2}{\partial \tilde{\xi}}\right]^{-1}_{\tilde{\xi}=0} \tag{3.16}$$

which corresponds to the frontier barrier. For example for Cauchy conditions (3.1) $\tilde{\xi}_s=-0.5$,

$$\Delta\tilde{g}\cong-\frac{1}{\left[2\tilde{\xi}+1\right]^{3/2}}\Delta\tilde{\xi}. \tag{3.17}$$

In this case the change Δg in the tidal force acting at the time moment t across the body element Δx turns into infinity by $\tilde{\xi}=-0.5$. In the following if

$$\left[\frac{\partial \tilde{E}^2}{\partial \tilde{\xi}}\right]_{\tilde{\xi}=0}\tilde{\xi}+\tilde{E}^2(0)\neq 0 \tag{3.18}$$

the Δg change of the tidal force acting at the time moment t across the body element Δx has not the catastrophic character.

4. Discussion and Conclusion

As one can see during all investigation we needn't to use the theory Newtonian gravitation for solution of nonlinear non-local evolution equations (EE). In contrast with the local physics this approach in the frame of quantum non-local hydrodynamics leads to the closed mathematical description for the physical system under consideration.

If the density tends to infinity the matter evolution inside of "the black channel" (1D Cartesian model) is organizing in the form of the traveling waves.

Numerical modeling leads to appearance of the singularity on the left side of domain where the gravitational acceleration turns into infinity. This singularity corresponds to event horizon and the whole neighboring area of the strong gravitational variation can be named as the transition area of events, (see **Figure 1**).

All calculations are realized for the case $\xi=x-ut$, corresponding to the wave traveling along the positive direction of the x-axis. Obviously after the initial per-

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turbations the analogical wave propagates in the opposite direction $\left(-\tilde{\xi}\right)$ after the sign change $x \to -x$, $u \to -u$. In the theory of Black Hole (BH) with the spherical symmetry it leads near the event horizon to the appearance of black body radiation which was predicted by Stephen Hawking. Hawking radiation reduces the mass and the energy of the black hole and is therefore also known as black hole evaporation. The structure of this radiation significantly depends on the topological features of BH.

Usually the appearance of the analogical picture in the left hand half-plane does not lead to information of the principal significance, but not for the case under consideration.

Really, after rotation the right half-plane picture by 180° two domains (see **Figures 1** and **2**) create the joined domain with the width $\tilde{\xi} = 1$ and minimums for $\tilde{E}$ and $\tilde{g}$ in the centre of the infinite square well. On the whole the configuration reminds the known quantum mechanical problem of the particle evolution in a box with the infinite potential barriers of the gravitational origin. It is well known that the solution of the analogical problem in the Schrödinger quantum mechanics leads to the discrete energetic levels. Quantum calculations of oscillators in the arbitrary potential fields can be found in [4].

Finally some words concern the following investigations. Numerical calculations, realized in the spherical coordinate system for the dependent variables (r—radius, t—time) cannot change principal results of the shown calculations in the Cartesian coordinate system. But some other effects (where the real form of the black hole is significant) obviously need in a 3D non-stationary calculation.

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