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The Optimal Sensing Coverage for Road Surveillance

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Abstract

So far path coverage problem has been studied widely to characterize the properties of the coverage of a path or a track in an area induced by a sensor network, in which the path or track is usually treated as a curve and the width of it can be ignored. However, sensor networks often are employed to carry out road surveillance or target tracking, in which the interesting area is only the surface of the road, thus the width of the road must be considered. This paper analyzes the optimal sensing coverage of the road in this kind of applications, assuming that sensor nodes are deployed along both sides of the road determinately. The optimal position of sensor nodes is studied considering the sensing range of sensors and the width of the road, and the purpose is to cover the road surface completely with minimal nodes. The isosceles triangle model is proposed and proved to be the most suitable, that is to say all sensors get the maximal available sensing area if any three nearest sensors located on both sides of the road form an isosceles triangle. Comparing with the equilateral triangle model proposed in other articles, this model increases the coverage rate and supplies complete coverage of the road.

Keywords: Sensor, Network, Optimal, Coverage, Road

1. Introduction

Coverage is an important performance index of a sensor network, because it represents how well the object of interest is monitored by sensors, or how effective a sensor network is in detecting objects intruding the field of watch. Knowing the fundamental coverage property of a sensor network helps to better construct sensor networks. For example, it could tell us how densely sensors should be deployed to detect an intruding object with a given probability or to trace a given fraction of trajectory of the object.

Road surveillance is an important kind of applications, and can be used for traffic surveillance, overspeed early warning, target detection and classification etc. Unattended road surveillance can be realized by wireless sensor network, which is quite economical of manpower and the efficiency of surveillance is also improved, and it is a new direction for application research of wireless sensor network [1,2].

The aim of this work is to analyze the coverage property of a sensor network, which is deployed determinately along both sides of a road. In particular, we focus on the optimal sensing coverage, in order to get complete coverage over the road and connectivity between sensors, using minimal number of sensors. It is fundamental in some applications, such as road surveillance, target tracing and others. Because it not only determines the cost of a network, but also influences the veracity and validity of the results. In those applications, the width of the road and the sensing range of sensors must be taken into account, because they are primary factors which affect the relative position among deployed sensors.

The path coverage problem has been extensively studied in recent years, which is useful for the study of this paper. For example, Junko Harada *et al.* [3] analyzed the path coverage property of a sensor network, characterized the path coverage in terms of three metrics: fraction of coverage, probability of complete coverage, and probability of partial coverage, and derived the expressions for the three coverage metrics as functions of the sensor density, the sensing range of a sensor, and the communication range of a sensor. Pallavi Manohar *et al.* [4] analyzed the statistical properties of the coverage of a one-dimensional path induced by a two dimensional non homogeneous random sensor network. Sundhar Ram *et al.* [5] analyzed the one-dimensional path-coverage induced by an area coverage process in a random sensor network and obtain the trackability measures defined in the literature. *k*-track coverage [6] investigated the problem of finding the configuration of a network with n sensors so that the number of tracks intercepted by k sensors is optimized without providing redundant area coverage over the entire region. These studies considered the coverage property of a path or a track in an area induced by a sensor network. Expressions or formulas derived in these papers are not suitable for the problem proposed in this paper.

Kurlin *et al.* [7] proposed a method to find the minimal number of sensors randomly deployed along a one-dimensional path to make a network connected with a given probability. The paper also described a powerful method for explicitly computing the probability of connectivity of 1-dimensional networks. However, the path coverage probability is not taken into account, which is vital in road surveillance applications.

The optimal configuration of sensor nodes has also been focused on, such as the equilateral triangle model [8], which means that if the region R is large enough as compared to the sensing range of each sensor node, to cover region R completely with minimal number of nodes, sensor nodes should be placed as follow: any three disks composed by the coverage area of a sensor node centered at itself should intersect at one point and form an equilateral triangle with side length $\sqrt{3}r_s$, where r_s is the sensing range of sensors, see **Figure 1**. Since sensor nodes can but be placed along both sides of the road in the kind of road surveillance application, they form an equilateral triangle if and only if the width of the road d objects to (1). However, the width of the road may vary from a couple of meters to dozens of meters, and the sensing range of sensors is also variable in practice, and both of them can not object to (1) in most times.

$$d = \sqrt{3}r_s \times \sin\frac{\pi}{3} = \frac{3}{2}r_s \tag{1}$$

The contribution of this work is to analyze the optimal sensing coverage for road surveillance taking account of the sensing range of sensors and the width of the road. As a result, we find that the maximal available sensing area of the networked sensors can be achieved when the positions of every three closest sensors form an isosceles triangle.

The rest of this article is organized as follows. In Section 2, we present some related background on the road coverage. In Section 3, we analyze the optimal sensing coverage, present and prove our propositions while the sensing range of sensors r_s and the width of the road d subject to $d/2 < r_s \le d$ and $r_s > d$ respectively. In Section 4, we compare our isosceles model with the equilateral model. Some simulation experiments about the number of sensor nodes required to cover a road optimally have been done based on our research results in Section 5. The Section 6 is the conclusion of this paper.

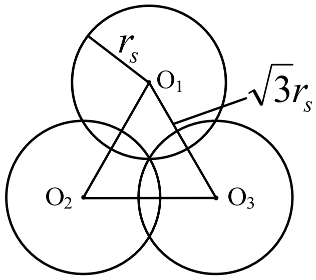

Figure 1. The equilateral triangle model.

2. Background of Road Coverage

Road surveillance and target tracking are common applications of Wireless Sensor Network. The coverage probability varies from different purpose, and we assume that we need to cover the road surface completely in this paper.

2.1. Sensing Model

We assume that each sensor has the same sensing range, r_s. A sensor can detect all events within its sensing range with probability 1, but it cannot detect any events at all outside the sensing range. This simplified sensing model is usually called "Boolean sensing model" [9,10].

We also assume that each sensor has an identical communication range, r_w. A sensor can communicate with all sensors within its communication range.

Zhang *et al.* [8] proved that the condition of $r_w \ge 2r_s$ is both necessary and sufficient to ensure that complete coverage of a convex region implies connectivity in an arbitrary network, assuming the monitored region is a convex set. For clarity of discussion, we assume that communication range is at least twice of sensing range in this paper, and then the set of sensor nodes is connective if it covers the road surface completely.

2.2. Structure of the Network

The sensor nodes may be found or damaged by trucks and people if they are deployed on the road surface in practice. Thus they are usually placed along both sides of the road, even keep away from the road sides. **Figure 2** shows the structure of the network. We can see that all sensor nodes form two parallel lines. We assume that all sensors are deployed along both sides of the road.

2.3. Relationship between Width of the Road and Sensing Range of Sensors

In this article, we denote the width of the road as d. Then the relationship between d and sensing range of sensors r_s may be as follow:

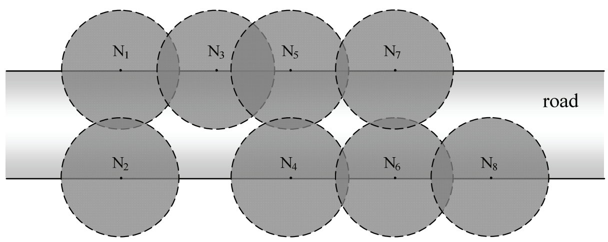

Figure.2. The structure of the network.

1) $r_s \le d/2$. Then, the sensors set can not cover the road surface completely, no matter how many sensors are placed. See **Figure 3(a)**.

2) $d/2 < r_s \le d$. The sensing range of a single senor can not cover the road, but the sensors set may cover the road completely if they are placed properly. See **Figure 3(b)**.

3) $r_s > d$. The sensing range of a single senor could cover the road, and the sensors set can cover the road completely if they are placed properly. See **Figure 3(c)**.

The second and third cases are studied in the next section, in order to get the optimal sensing coverage of the road.

3. Optimal Sensing Coverage of the Road

Optimal sensing coverage means that: first, the subset of sensors should completely cover the road. Given that the coverage area of a sensor node is a disk centered at itself, and the radius is the sensing range. The problem of road coverage is equivalent to cover the road with disks whose centers are along both sides of the road. To achieve complete coverage, there must be seamless between disks. Second, the number of working nodes is minimal, that is to say the overlap of sensing areas of all the working nodes is minimal [8]. The available sensing area of a sensor node in road coverage is the part on the road surface. The relative position of sensor nodes is discussed in this section aiming at the optimal sensing coverage of the road, while $d/2 < r_s \le d$ and $r_s > d$.

Theorem 1 *Three sensors are located on both sides of a road, if the width of the road d and the sensing range of the sensors* r_s *subject to* $d/2 < r_s \le d$, *then they achieve maximal available coverage area if and only if their coverage disks intersect at one point and their centers form an isosceles triangle.*

The maximal available sensing area is the area composed by three half-disks on the road surface, as shown in **Figure 4**. The sufficient condition is proved in the first instance. For clarity of discuss, we regard the road as two parallel lines.

Sufficient condition *Suppose the centers of three disks are located in two parallel lines, if they intersect at one point and their centers form an isosceles triangle, then they achieve the maximal seamless available sensing area.*

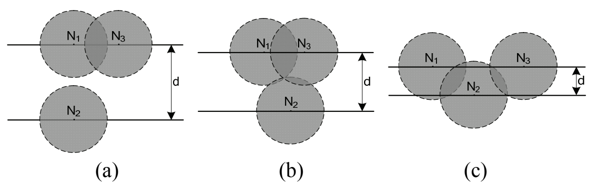

Figure 3. Relationship between *d* and r_s**, (a)** $r_s \le d/2$**; (b)** $d/2 < r_s \le d$**; (c)** $r_s > d$.

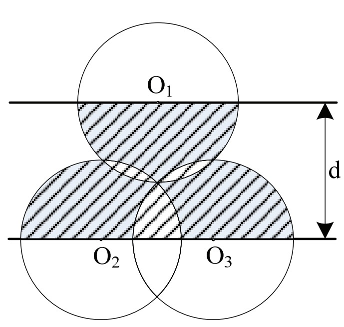

Figure 4. Three disks intersect at one point and their centers form an isosceles triangle.

Proof. All instances that three disks intersect at one point are shown in **Figure 5** ensuring that they are seamless. When they intersect at point P_1, disks O_1 and $O_2^{'}$ are tangent, and disk O_2 reaches the leftmost position. When they intersect at point P_4, their centers (disks O_1, $O_2^{''''}$ and $O_3^{''''}$) form an isosceles triangle. Points P_2 and P_3 denote the in-betweens. The red curve in **Figure 5** is the subset of all the crossing point of three disks. Since the radiuses of three disks are the same, according to symmetry, the crossing points on the right of point P_4 are symmetrical to the left ones. For clarity of study, only the states between P_1 and P_4 are considered.

Maximizing the seamless available sensing area of three disks is equivalent to minimizing the overlap of them, namely, minimizing $S = S_1 + S_2 + S_3$ as shown in **Figure 6**.

Step1 *Count the area of the overlap of two disks*

As shown in **Figure 7**, disk O_1 and disk O_2 intersect at point *A* and *B* with the same radius of *r*, the line $\overline{O_1O_2}$ is perpendicular to $\overline{AB}$ at point *P*. Let $\overline{O_1O_2}=l$, $\angle PO_2A = \alpha$, let $S_{\sec.AO_2B}$ and $S_{\triangle ABO_2}$ denote the area of sector AO_2B and $\triangle AO_2B$ respectively, let *S* denote the area of the overlap of the two disks, Then

$$PA = \sqrt{r^2 - \left(\frac{l}{2}\right)^2}, \alpha = \arccos\left(\frac{l}{2r}\right)$$

$$S_{\sec.AO_2B} = \alpha r^2 = r^2 \arccos\left(\frac{l}{2r}\right)$$

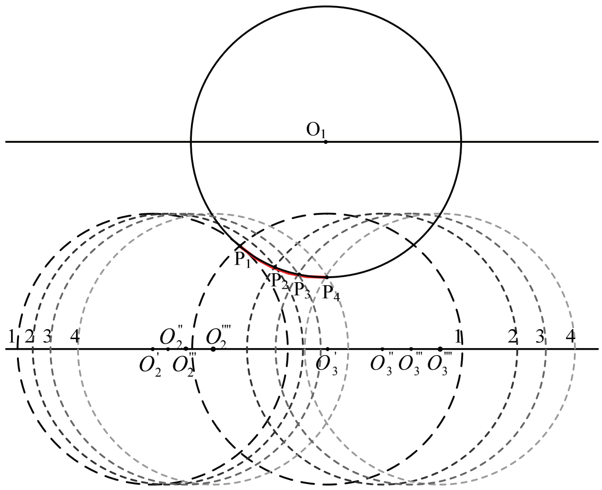

Figure 5. The disks intersect at one point.

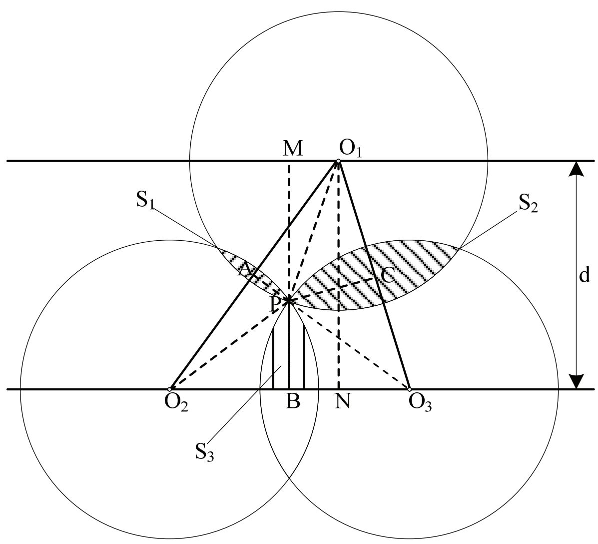

Figure 6. Three disks intersect at one point but their centers do not form an isosceles triangle.

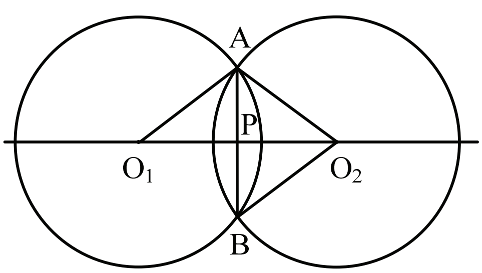

Figure 7. Two disks intersect.

$$S_{\Delta ABO_2} = \frac{l}{2} = \sqrt{r^2 - \left(\frac{l}{2}\right)^2}$$

$$S = 2\times\left(S_{\text{sec}.AO_2B} - S_{\Delta ABO_2}\right)$$

$$= 2r^2 \arccos\left(\frac{l}{2r}\right) - l\sqrt{r^2 - \left(\frac{l}{2}\right)^2} \tag{2}$$

Step2 *Count the area of the overlap of three disks*

As shown in Figure 6, three disks intersect at point P, let $PA\perp O_1O_2$, $PB\perp O_2O_3$, $PC\perp O_1O_3$, $O_1N\perp O_2O_3$, $BM // O_1N$, let $PB=x$, it can be educed from **Figure 5** that $x\in\left[d-r,\ d/2\right]$, then

$$O_1O_2 = \sqrt{O_1N^2 + O_2N^2}$$

$$= \sqrt{d^2 + \left(\sqrt{r^2-x^2} + \sqrt{r^2-(d-x)^2}\right)^2}$$

$$O_1O_3 = \sqrt{O_1N^2 + O_3N^2}$$

$$= \sqrt{d^2 + \left(\sqrt{r^2-x^2} - \sqrt{r^2-(d-x)^2}\right)^2}$$

$$O_2O_3 = 2O_2B = 2\sqrt{r^2-x^2} \tag{3}$$

Let S denote the area of overlap of the three disks, it can be derived from (2) and (3) that

$$S = S_1 + S_2 + S_3$$

$$= 2r^2 \arccos\left(\frac{\sqrt{d^2 + \left(\sqrt{r^2-x^2} + \sqrt{r^2-(d-x)^2}\right)^2}}{2r}\right)$$

$$-\sqrt{d^2 + \left(\sqrt{r^2-x^2} + \sqrt{r^2-(d-x)^2}\right)^2}$$

$$\times\sqrt{r^2 - \frac{d^2 + \left(\sqrt{r^2-x^2} + \sqrt{r^2-(d-x)^2}\right)^2}{4}}$$

$$+ r^2 \arccos\left(\frac{\sqrt{r^2-x^2}}{r}\right) - x\sqrt{r^2-x^2}$$

$$+2r^2 \arccos\left(\frac{\sqrt{d^2 + \left(\sqrt{r^2-x^2} - \sqrt{r^2-(d-x)^2}\right)^2}}{2r}\right)$$

$$-\sqrt{d^2 + \left(\sqrt{r^2-x^2} - \sqrt{r^2-(d-x)^2}\right)^2}$$

$$\times\sqrt{r^2 - \frac{d^2 + \left(\sqrt{r^2-x^2} - \sqrt{r^2-(d-x)^2}\right)^2}{4}} \tag{4}$$

S can be regarded as the function of x, and x objects to the constrain $\left[d-r,\ d/2\right]$ as mentioned before.

Step3 *Count the first derivative of* $S(x)$

We count the first derivative of (4) and simplify the expression, and then the result is

$$S'(x)=\frac{2x\left(d-x\right)}{\sqrt{r^2-x^2}} \tag{5}$$

$\because\ d-r\le x\le d/2$, $\therefore\ d-x>0$, $\therefore\ S'(x)>0$, $\therefore\ S(x)$ is a monotonic increasing function subject to the constrain of $\left[d-r,\ d/2\right]$, and $S(x)$ gets minimum at $x=d-r$, in this instance, we can derive from (3) that

$$\begin{aligned} O_1O_2&=\sqrt{d^2+\left(\sqrt{r^2-x^2}+\sqrt{r^2-(d-x)^2}\right)^2}\\ O_1O_3&=\sqrt{d^2+\left(\sqrt{r^2-x^2}-\sqrt{r^2-(d-x)^2}\right)^2}\end{aligned} \tag{6}$$

It is clear that $\triangle O_1O_2O_3$ is an isosceles triangle according to (6). And then we can summarize that disks O_1, O_2 and O_3 get the maximal seamless available sensing area at this time.

The sufficient condition has been confirmed, and then the necessary condition is proved.

Necessary condition *Suppose the centers of three disks are located in two parallel lines, if they achieve the maximal seamless available sensing area, then they must intersect at one point and their centers form an isosceles triangle.*

Proof. First, we prove that they must intersect at one point, and then their centers form an isosceles triangle.

Step 1 Reduction to absurdity is adopted. Assume that three disks do not intersect at one point and the overlap of them is minimal, then there must exist an area belongs to all of them to ensure seamless, see **Figure 8(a)**. Consider, for example, move disk O_3 and make it intersect disks A and B at one point as shown in **Figure 8(b)**. It is clear that S_1 makes no difference, but both S_2 and S_3 become smaller, so S is smaller than before. Thus, the primary assumption is mistake, and they must intersect at one point as their overlap is minimal.

Step 2 Three disks achieve the maximal seamless available sensing area, that is to say their total overlap is minimal. According to the proving of Lemma 1, $S(x)$ is a monotonic increasing function in $\left[d-r,\ d/2\right]$, and $S(x)$ gets the minimum at $x=d-r$, then O_1O_2 = $O_1O_3=\sqrt{2dr}$, namely, the centers of three disks form an isosceles triangle. Then, lemma 2 has been proved.

It is apparent that Theorem 1 is true based on the forenamed proof. When all sensors get the maximal seamless available sensing area, the optimal sensing coverage of the road is achieved, the optimal network structure is shown as in **Figure 9** at this condition. The distance between two nearest sensors on the same side of the road can be expressed as follow, in compliance with (3).

$$N_1N_3=2\sqrt{2dr-d^2} \tag{7}$$

The optimal sensing coverage of the road has been discussed in Theorem 1 while $d/2<r_s\le d$, Theorem 2 is aiming at another state while $d<r_s$.

Theorem 2 *Two sensor nodes are located on both sides of a road respectively, the width of the road d and the sensing range of the sensors r_s object to the constrain $d<r_s$, then they achieve maximal available sensing area if and only if the intersect points of their coverage disks lie exactly on both sides of the road and the distance between their centers is maximal.*

The proof of Theorem 2 is referred to Theorem 1. We just explain it intuitively with graph here. All instances of two disks intersecting are shown in **Figure 10** ensuring that they are seamless. Since both radiuses of two disks are the same, according to symmetry, the positions of disk O_2 on the left of disk O_1 are symmetrical as shown in **Figure 10**. For clarity of study, only the states shown in **Figure 10** are considered. When the center of disk O_2 locates at $O_2^{'}$, the line connecting two centers $O_1O_2^{'}$ is perpendicular to the road, and the distance between two centers is minimal. When the center of disk O_2 locates at $O_2^{'''}$, the two points of intersection lie exactly on both sides of the road, then the distance between two centers is maximal and disk O_2 reaches the rightmost position. Also, the two points of intersection lie exactly on both sides of the road while position $O_2^{''}$, but the distance between two centers is not the maximum.

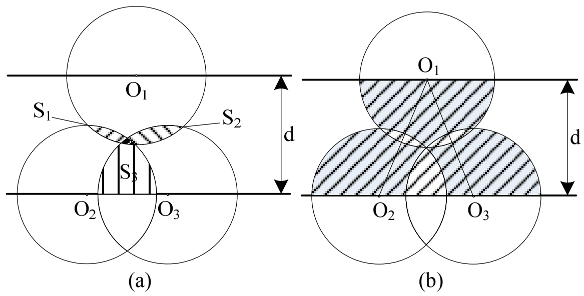

Figure 8. Three disks intersect seamlessly, (a) Three disks do not intersect at one point; (b) Three disks intersect at one point.

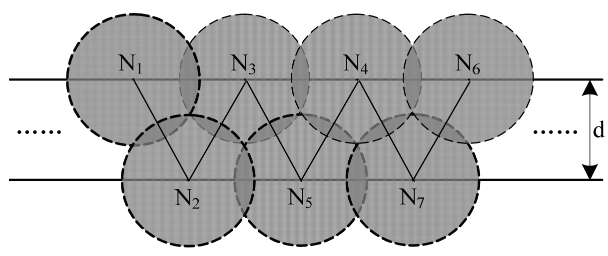

Figure 9. The optimal network structure as $d/2<r_s\le d$.

The red line in **Figure 10** is the subset of all the positions of the centers of disk O_2. It is apparent from **Figure 10** that the overlap of the available sensing area of two disks is minimal while position O_2''', that is to say the two disks achieve maximal available sensing area when their points of intersection lie exactly on both sides of the road and the distance between their centers is maximal.

When all sensors get the maximal seamless available sensing area, the optimal sensing coverage of the road can be achieved, and then the optimal network structure is shown as in **Figure 11**. It is clear that the nearest three sensors form an isosceles triangle also, according to symmetry, as shown in **Figure 11**. The distance between two nearest sensors on the same side of the road can be expressed as follow according to **Figure 11**.

$$N_1N_3 = 2\left(r_s + \sqrt{r_s^2 - d^2}\right) \tag{8}$$

Theorem 3 can be concluded integrating Theorems 1 and 2.
Theorem 3 *To ensure complete coverage, the optimal sensing coverage of the road is that the three nearest sensors located on both sides of the road form an isosceles triangle and the distance between two adjacent sensors on the same side subject to* (9)*, if the sensing range of sensors* r_s *is at least half of the road width d*

$$l = \begin{cases} \sqrt{2dr_s - r_s^2} & d/2 < r_s \le d \\ 2\left(r_s + \sqrt{r_s^2 - d^2}\right) & r_s > d \end{cases} \tag{9}$$

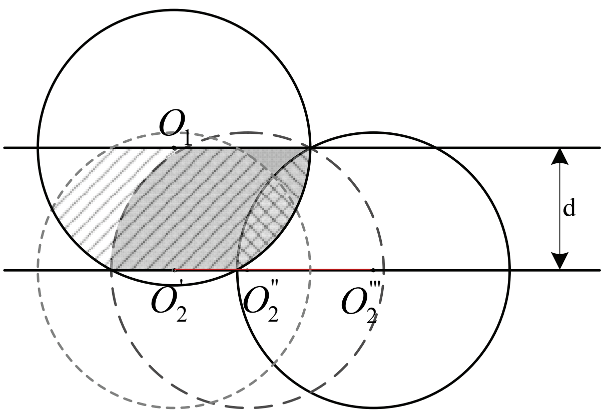

Figure 10. All instances of two disks intersecting while $d < r_s$ **.**

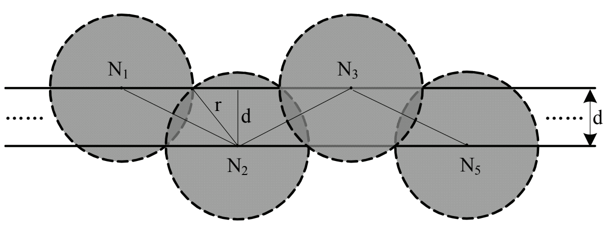

Figure 11. The optimal network structure as $d < r_s$ **.**

Let L denote the length of the road which needs to be covered, the number of sensor nodes n can be derived from **Figure 9** and **Figure 10** as expressed in (10)

$$n = \left\lceil \frac{2L}{l} \right\rceil + 1 \tag{10}$$

4. Comparing with the Equilateral Triangle Model

In this section, the optimization of the isosceles triangle model is validated by compared with the equilateral triangle model, however, the limiting factor of three coverage disks intersecting at one point is released. Actually, the released edition is commonly used in the deployment of wireless sensor network. As known in Section 1, the three nearest sensor nodes form an equilateral triangle if and only if the width of road d and the sensing range r_s object to (1), then the isosceles triangle is equivalent to the equilateral triangle. The optimization is discussed as $d/2 < r_s < 2d/3$, $2d/3 < r_s \le d$ and $r_s > d$ respectively.

4.1. $d/2 < r_s < 2d/3$

The coverage is not seamless if the equilateral triangle model is adopted on this condition, see **Figure 12**. However, the isosceles triangle also works on this condition, see **Figure 13**.

4.2. $2d/3 \le r_s \le d$

Then the coverage is seamless if we adopt the equilateral triangle, but the intersection of the sensing range of three sensor nodes is not one point but an area, see **Figure 14**. We define the coverage efficiency E as the standard to measure them.

$$E = \frac{S_\Delta}{\sum_{i=1}^{3} S_i} \tag{11}$$

S_Δ denotes the area of the triangle formed by three sensor nodes, S_i denotes the available coverage area of the sensor i in the triangle. Since the sum of the internal angels of a triangle is π, and the sensing range of all sensors is r_s, then $\sum_{i=1}^{3} S_i = \frac{\pi}{2} r_s^2$.

It can be educed from **Figure 14** that the area of the equilateral triangle $S_{\Delta E} = \frac{1}{2} \times d \times 2\frac{d}{tan\frac{\pi}{3}} = \frac{\sqrt{3}}{3} d^2$.

Based on (11), the coverage efficiency of it is

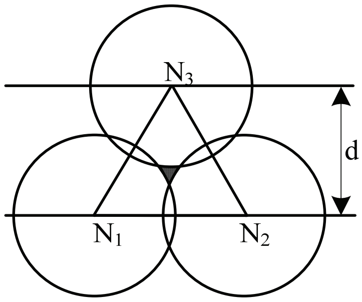

Figure 12. $d/2 < r_s < 2d/3$.

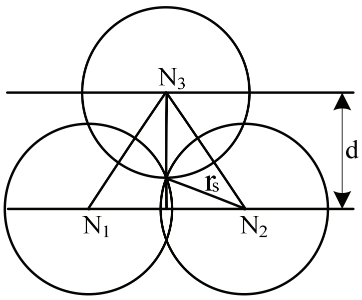

Figure 13. The isosceles triangle model.

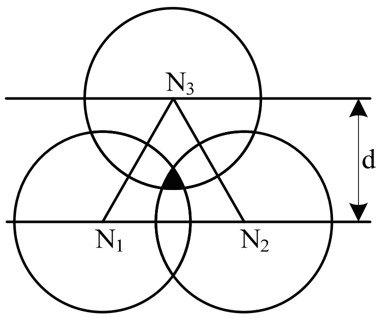

Figure 14. $2d/3 < r_s \le d$.

$$E_E = \frac{2\sqrt{3}d^2}{3\pi r_s^2} \tag{12}$$

And it can be educed from **Figure 13** that the area of the isosceles triangle

$$S_{\Delta I} = \frac{1}{2} \times d \times 2\sqrt{r_s^2 - (d - r_s)^2} = d\sqrt{d(2r_s - d)}$$

Since $2d/3 < r_s$, then the coverage efficiency of the isosceles triangle

$$E_I = \frac{d\sqrt{d(2r_s - d)}}{\frac{\pi}{2} r_s^2} > \frac{d\sqrt{d\left(2 \times \frac{2}{3}d - d\right)}}{\frac{\pi}{2} r_s^2} = \frac{2\sqrt{3}d^2}{3\pi r_s^2} = E_E$$

That is to say the coverage efficiency of the isosceles triangle model is larger than the equilateral triangle's. E_E and E_I can be regarded as functions of r_s, then we can validate our conclusion intuitively from the figures of them with the help of MATLAB. As shown in **Figure 15**, let the road width d equals 5 m, 15 m and 30 m, and $2d/3 < r_s \le d$. It can be concluded form **Figure 15** that, E_E equals E_I only when $r_s = 2d/3$, and E_E is larger than E_I in other times.

4.3. $r_s > d$

As shown in **Figure 11**, the available coverage area of three nodes in the isosceles triangle formed by themselves is equal to the coverage area of a sole node on the road surface, and can be expressed as follow

$$\sum_{i=1}^{3} S_i = \frac{\pi}{2} r_s^2 - \left[r_s^2 \arccos\left(\frac{d}{r_s}\right) - d\sqrt{r_s^2 - d^2} \right]$$

The area of the isosceles triangle

$$S_{\Delta I} = d \times (r_s + \sqrt{r_s^2 - d^2})$$

Then the efficiency is

$$E_I = \frac{d \times (r_s + \sqrt{r_s^2 - d^2})}{\frac{\pi}{2} r_s^2 - \left[r_s^2 \arccos\left(\frac{d}{r_s}\right) - d\sqrt{r_s^2 - d^2} \right]} \tag{13}$$

As to the equilateral triangle, its coverage efficiency is 1/3 constantly while $r_s \ge 2\sqrt{3}d/3$, as shown in **Figure 16**.

When $d < r_s < 2\sqrt{3}d/3$, the available coverage area of a sole node in the equilateral triangle is the shadow in **Figure 17**, then

$$\sum_{i=1}^{3} S_i = 3\left\{ \frac{\pi}{6} r_s^2 - \left[r_s^2 \arccos\left(\frac{d}{r_s}\right) - d\sqrt{r_s^2 - d^2} \right] \right\}$$

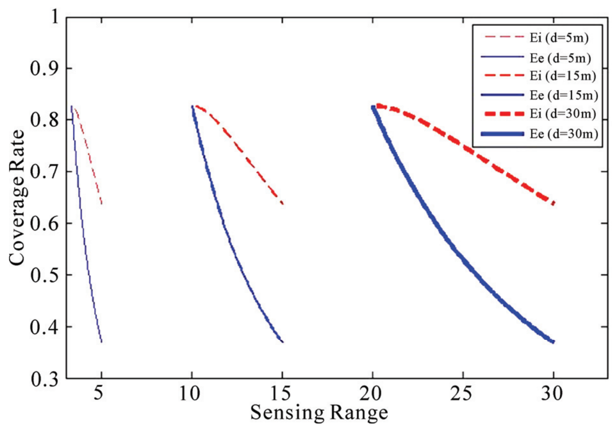

Figure 15. Comparing of model coverage efficiency while $2d/3 < r_s \le d$.

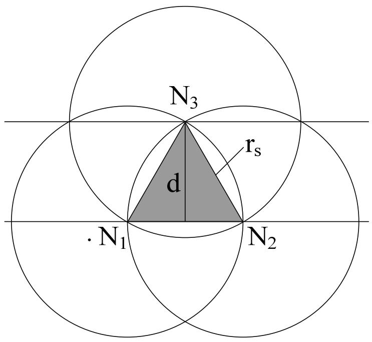

Figure 16. The equilateral triangle model while $r_s = 2\sqrt{3}d/3$**.**

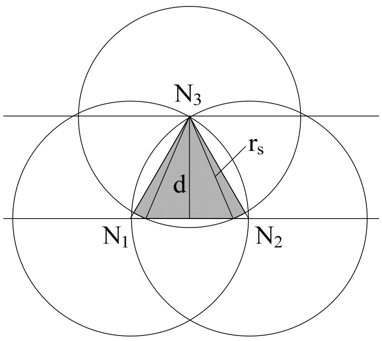

Figure 17. The equilateral triangle model while $d < r_s < 2\sqrt{3}d/3$.

$$= \frac{\pi}{2} r_s^2 - 3r_s^2 \arccos\left(\frac{d}{r_s}\right) + 3d\sqrt{r_s^2 - d^2}$$

And the efficiency is

$$E_E = \frac{\frac{\sqrt{3}}{3} d^2}{\frac{\pi}{2} r_s^2 - 3r_s^2 \arccos\left(\frac{d}{r_s}\right) + 3d\sqrt{r_s^2 - d^2}} \tag{14}$$

To compare them with each other, we also regard them as functions of r_s, and plot them with the help of MATLAB. As shown in **Figure 18**, let the road width d equals 5m, 15m and 30m, and $d < r_s < 3d$. It can be concluded that E_I is larger than E_E, and E_I approximates to 1 as r_s increases, while E_E approximates to 1/3.

5. Simulation Results

According to (9) and (10), it is clear that if the road width d, sensor sensing range r_s and the road length L are given, then the number of sensor nodes required to cover the road optimally can be calculated in determinately deployment instance. In this section, we carry out some simulated experiments based on our conclusions with the help of MATLAB and C++, while d, L and r_s are varying. The calculating of n is based on (9) and (10).

5.1. The Variance of *n* While One of *d*, *L* and r_s Changes

Figure 19 shows the variance of n while L increases from 100m to 300m, in which d and r_s is given. According to (10), the two of them is linear, and the figure confirms that. The slope of the line is $l/2$, it is determined by (9).

Figure 20 shows the change of n while the sensing range of the sensor nodes increasing, in which d and L is given. It can be made out from the figure that n is decreasing as r_s increasing, and n approximates to 0 when r_s is much larger than d.

Figure 21 shows the variance of n with d. We can clearly see that n goes to infinite when d approximates to double of r_s. According to (9) and (10), n is a subsection function and the inflexion comes while $d = r$, as the figure reveals.

5.2. The Variance of *n* as Any Two of *d*, *L* and r_s Change

Figure 22 demonstrates the graph of n as L and r_s vary, in which d = 20m. We can see that n is increasing while r_s decreasing and L increasing, and n approximates to infinite when r_s approximates to 10m that is half of the road width.

Figure 23 shows the graph of n varies with d and r_s, while L=300m. It is clearly that n approximates to 0

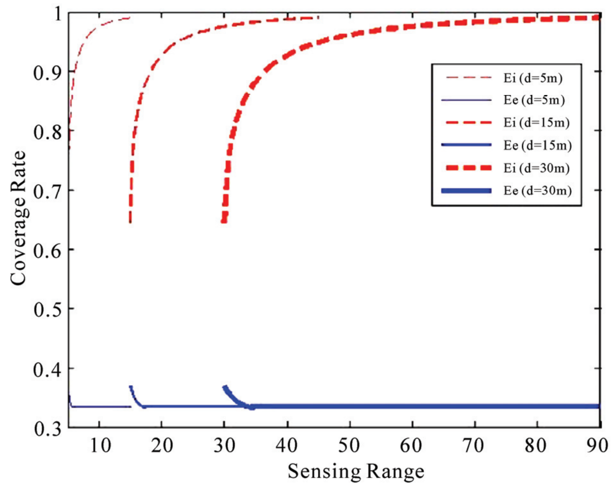

Figure 18. Comparing of model coverage efficiency while $d < r_s < 3d$.

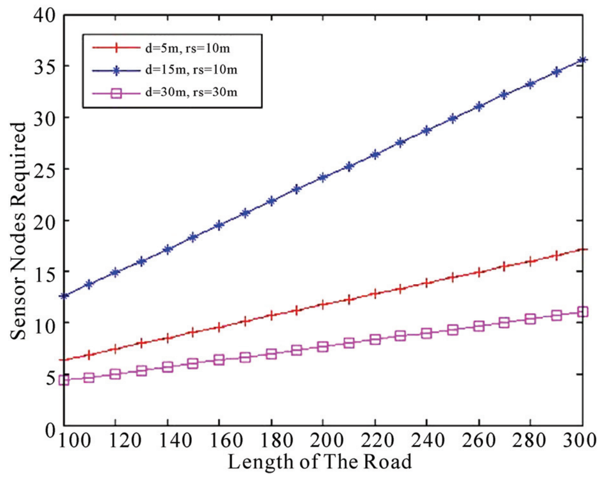

Figure 19. The number of sensor nodes required *n* as a function of the road length *L*.

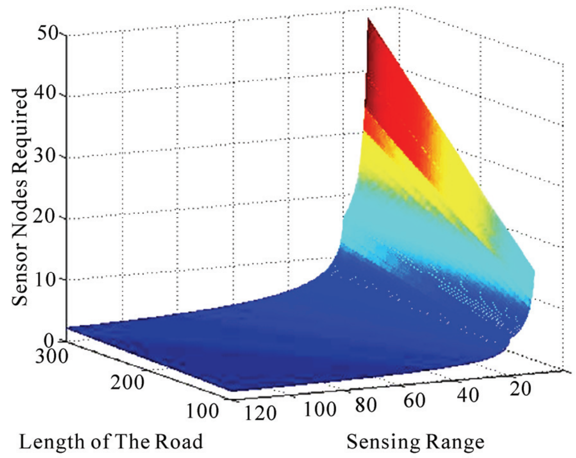

Figure 22. The number of sensor nodes required *n* as a function of the sensing range r_s and the road length *L*.

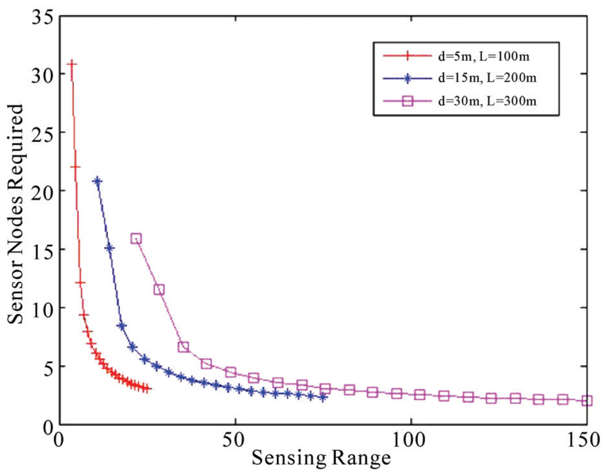

Figure 20. The number of sensor nodes required *n* as a function of the sensing range r_s.

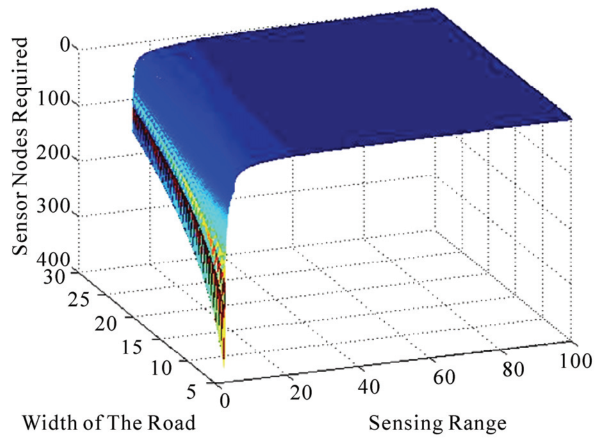

Figure 23. The number of sensor nodes required *n* as a function of the sensing range r_s and the road width *d*.

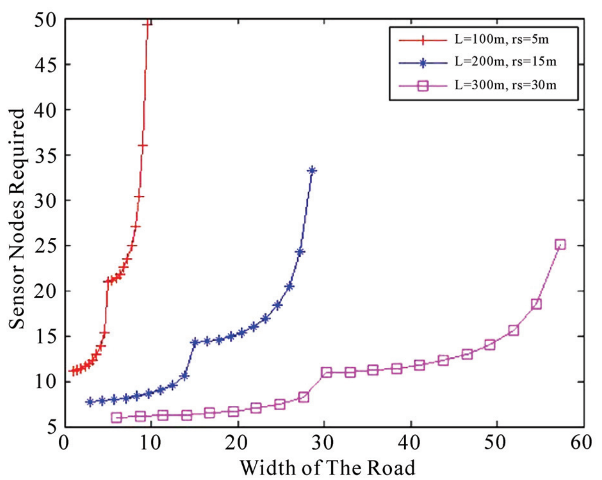

Figure 21. The number of sensor nodes required *n* as a function of the road length *d*.

when r_s increases as $r_s > d/2$, and $n \to +\infty$ while $r_s \to d/2$.

Figure 24 is the graph of *n* varies with *L* and *d*, and r_s =10m. It can be drawn from the figure that *n* is increasing as *d* and *L* increase, and *n* approximates to infinite while *d* approximates to twice of r_s.

6. Conclusions

In this work, we analyze the optimal sensing coverage of the road taking account of the different size of the sensing range of sensors and the width of the road. In practice, we focus on the optimal position of sensors which cover the whole road surface completely. We propose and prove that the optimal position of sensors is that the three nearest sensors on the different sides of the road form an isosceles triangle, if the sensing range of sensors r_s is at least half of the road width *d*. For clarity of

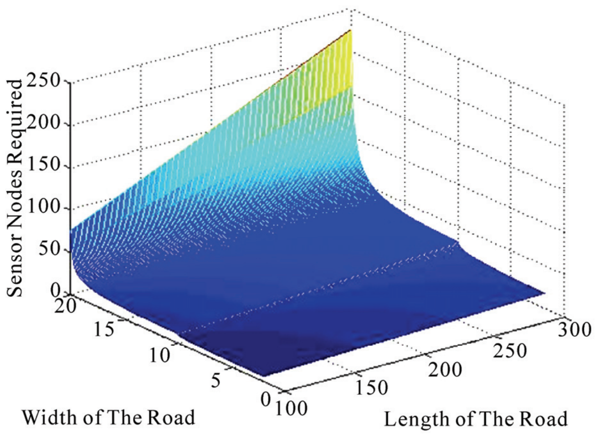

Figure 24. The number of sensor nodes required n **as a function of the road width** d **and length** L.

study, we assume that the communication range r_w is larger than twice of the sensing range r_s to ensure communicative in this article. The optimal position of sensors when $r_w < 2r_s$ should also be studied. We also assume that all sensor nodes are homogeneous, but several kinds of nodes maybe used in practice, the problem of the optimal sensing coverage in this case is our future work.

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8. References

[1] A. Haoui, R. Kavaler and P. Varaiya, “Wireless Magnetic Sensors for Traffic Surveillance,” *Transportation Research Part C*, Vol. 16, 2008, pp. 294-306.

[2] R.V. Field Jr. and M. Grigoriu, “Optimal Design of Sen-sor Networks for Vehicle Detection, Classifcation, and Monitoring,” *Probabilistic Engineering Mechanics*, Vol. 21, 2006, pp. 305-316.

[3] J. Harada, S. Shioda and H. Saito, “Path Coverage Properties of Randomly Deployed Sensors with Finite Data-transmission Ranges,” *Computer Networks*, Vol. 53, 2009, pp. 1014-1026.

[4] P. Manohar, S. S. Ram and D. Manjunath, “On the Path-Coverage by a Non Homogeneous sensor Field,” *IEEE GLOBECOM*, 2006.

[5] S. S. Ram, D. Manjunath, S. K. Iyer and D. Yogeshwaran, “On the Path Coverage Properties of Random Sensor Networks,” *IEEE Transactions on Mobile Computing*, Vol. 6, No. 5, May 2007.

[6] S. Ferrari, “Track Coverage in Sensor Networks,” *Proceedings of the* 2006 *American Control Conference*, Minneapolis, 14-16 June 2006.

[7] V. Kurlin and L. Mihaylova, “How Many Randomly Distributed Wireless Sensors are Enough to Make a 1-Dimensional Network Connected With a Given Probability?” *IEEE Transactions on Information Theory*, Vol. 10, No. 10, 2008.

[8] H. H. Zhang and J. C. Hou, “Maintaining Sensing Coverage and Connectivity in Large Sensor Networks,” *Journal of Ad Hoc and Sensor Wireless Networks*, Vol. 1, No. 1-2, 2005, pp. 89-124.

[9] B. Liu and D. Towsley, “A Study on the Coverage of Large-scale Sensor Networks,” 1*st IEEE International Conference on Mobile Ad-Hoc and Sensor Systems*, 2004.

[10] D. Tian and N. D. Georganas, “A coverage-preserving node scheduling scheme for large wireless sensor networks,” 1*st ACM International Workshop on Wireless Sensor Networks and Applications*, 2002, pp. 32-41.

