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Theoretical Analysis and Experimental Verification of Crack Initiation Characteristics of Compression-Shear Plane Crack with Hydraulic Pressure

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Abstract

In this paper, the crack initiation characteristics of compression-shear plane crack with hydraulic pressure were studied by using theoretical analysis and experimental verification methods. The formula derivation process of stress intensity factor of crack tip and open-type crack initiation angle and initiation strength was expounded in detail. Cement mortar specimens prefabricated with open-type crack were made for biaxial compression test. The results show that the mode I stress intensity factor is inversely proportional to the dip angle of pre-exciting crack, water pressure and crack width. The fracture toughness is most easily achieved when the dip angle of pre-exciting crack is 60˚. The mode II stress intensity factor is symmetrically distributed with the dip angle and independent of the water pressure and crack width. For open-type crack, the crack initiation angle decreases with the increase of the dip angle of pre-exciting crack, water pressure and crack width; the crack initiation strength is inversely proportional to the water pressure and proportional to the lateral pressure. The research results can provide ideas for the study of crack initiation under the coupling of ground stress and osmotic pressure in tunnel engineering.

Keywords

Compression-Shear Plane Crack, Stress Intensity Factor, Experimental Verification, Crack Initiation Characteristics

1. Introduction

As we all know, there are a lot of fissures in the rock mass, and the existence of fissures will reduce the mechanical properties of rock mass. In geotechnical engineering, the complex stress state of the coupling of ground stress and osmotic pressure promotes the expansion of the fissure, and the engineering failure caused by the crack expansion in rock is of common occurrence. In recent years, the crack initiation and expansion of the pre-existing fissure with hydraulic pressure has attracted the attention of researchers, and it is an important research direction and interesting in rock mechanics research. Liu S *et al.* [1] studied the fracture characteristics of fractured rock mass under different confining pressures and seepage water pressures based on Realistic Failure Process Analysis-Seepage. And derived the initial crack strength and the effect of seepage water pressure and confining pressure on the crack propagation of fractured rock mass. Kanaun [2] studied the evolution of the crack boundaries in the process of fluid injection, time dependence of pressure distributions and crack openings. Zhang D F *et al.* [3] took 2-D sidelong crack propagation as an example, giving out crack propagation steps by using the modified crack propagation criterion. Zhao Y L *et al.* [4] [5] have studied the crack initiation law of frictional crack under seepage pressure. They developed a damage fracture mechanics model of rock cracks under jointed action of compressive-shear stress field and seepage field. The evolution equations of additional stress intensity factor at the branch crack cusp were obtained considering rock bridge damage. And the mathematical model of seepage-fracture coupling of rock masses cracks propagation was established. XB LI *et al.* [6] studied the mechanical model of fracture and damage and the evolution equation of stress intensity factor at crack tip under the action of compression-shear field and seepage field, and proposed the crack initiation criterion by using the stress intensity factor.

In this paper, the crack initiation characteristics of pre-exciting plane crack with hydraulic pressure were studied. Based on the theoretical analysis, the variation of the stress intensity factor at the crack tip with the dip angle, water pressure and crack width was discussed. The variation of crack initiation angle with the dip angle of crack, water pressure and crack width was verified by experiment. And the variation of crack initiation strength with water pressure and lateral pressure was also verified.

2. Theoretical Analysis of the Initiation Characteristics of Pre-Exciting Plane Crack with Hydraulic Pressure

2.1. The Mode I Stress Intensity Factor (SIF) of Open-Type Crack

The compression-shear mixed crack model with hydraulic pressure is shown in Figure 1. σ_1 and σ_3 are far field compression stress and $\sigma_3 = \lambda\sigma_1$ ($0 \le \lambda \le 1$); α is the angle between the crack and the σ_3 direction; p is the internal water pressure of the crack; the crack tip is reduced to two curvature radii of ρ semicircle; the crack length is $2a$ and θ is the crack initiation angle.

According to the fracture mechanics, stress concentration exists at crack tip. Before the crack closure, it is generally judged whether it is cracked by the

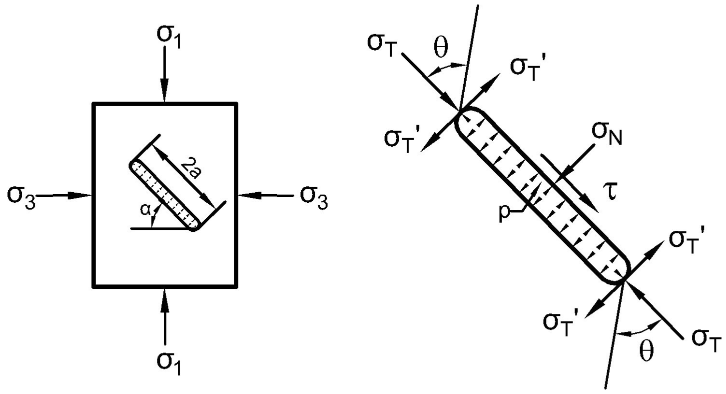

Figure 1. Model of open-type crack with seepage pressure.

stress distribution at the edge of the crack tip. The stress component of any section can be obtained by the unit body balance equation as the far field stress (σ_1, σ_3) is known [7].

Surface stress of crack surface:

$$\sigma_N = \frac{1}{2}\left[(\sigma_1+\sigma_3)+(\sigma_1-\sigma_3)\cos(2\alpha)\right]-p\,. \tag{1}$$

$$\sigma_T = \frac{1}{2}\left[(\sigma_1+\sigma_3)-(\sigma_1-\sigma_3)\cos(2\alpha)\right]-p\,. \tag{2}$$

$$\tau = \frac{1}{2}(\sigma_1-\sigma_3)\sin(2\alpha)\,. \tag{3}$$

σ_N is the normal stress perpendicular to the crack surface; τ is the shear stress on the crack surface, and σ_T is the vertical direction stress at the crack tip. According to the fracture mechanics [8], the mode I stress intensity factor produced by the normal compression stress σ_N on the fissure surface is:

$$K_I^N = -\sigma_N\sqrt{\pi a} = -\left\{\frac{1}{2}\left[(\sigma_1+\sigma_3)+(\sigma_1-\sigma_3)\cos(2\alpha)\right]-p\right\}\sqrt{\pi a}\,. \tag{4}$$

σ_T' has a tension effect on the edge of the crack, the maximum tensile stress appears in the crack tip and the size is equal to σ_T [9], as shown in **Figure 1**. Then the mode I stress intensity factor produced by the maximum tensile stress σ_T' at the crack tip is:

$$K_{\rm I}^T = \sigma_T\sqrt{\rho/a}\sqrt{\pi a} = \left\{\frac{1}{2}\left[(\sigma_1+\sigma_3)-(\sigma_1-\sigma_3)\cos(2\alpha)\right]-p\right\}\sqrt{\rho/a}\sqrt{\pi a}\,. \tag{5}$$

According to the superposition principle of linear elastic fracture mechanics, the mode I stress intensity factor of crack is: $K_{\rm I} = K_{\rm I}^T + K_{\rm I}^N$

It can be seen that the initiation and expansion of compression-shear mode I cracks depend on the normal stress of two vertical directions, the radius of curvature of crack tip and the circumference angle. The normal compressive stress σ_N inhibits the expansion of compression-shear fracture, and σ_T promotes the expansion. The seepage water pressure p has a promotion effect on the initiation and expansion of compression-shear fracture.

2.2. The Mode II Stress Intensity Factor of Open-Type Crack

The mode II fracture of I-II mixed mode crack under uniform shear stress is independent of the internal osmotic pressure of the crack. The analytic solution for the mode II crack stress intensity factor of composite crack is given [6]:

$$K_{\mathrm{II}} = -\tau\sqrt{\pi a} = -\frac{1}{2}\sqrt{\pi a}\left(\sigma_1 - \sigma_3\right)\sin\left(2\alpha\right). \tag{6}$$

2.3. Crack Initiation Characteristics with Hydraulic Pressure

2.3.1. Crack Initiation Angle of Open-Type Crack with Hydraulic Pressure

I-II type mixed mode fracture occurs under compression-shear condition. The maximum hoop stress theory is generally used in rock mechanics to study the crack initiation angle [10]. According to the fracture mechanics, the expression of stress component that only retains the singular term of crack tip in polar coordinate system is:

$$\begin{cases} \sigma_{rr} = \dfrac{1}{2\sqrt{2\pi r}}\left[K_{\mathrm{I}}(3-\cos\theta)\cdot\cos\dfrac{\theta}{2} + K_{\mathrm{II}}\left(3\cos\theta - 1\right)\cdot\sin\dfrac{\theta}{2}\right] \\ \sigma_{\theta\theta} = \dfrac{1}{2\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left[K_{\mathrm{I}}\left(1+\cos\theta\right) - 3K_{\mathrm{II}}\sin\theta\right] \\ \tau_{r\theta} = \dfrac{1}{2\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left[K_{\mathrm{I}}\sin\theta + K_{\mathrm{II}}\left(3\cos\theta - 1\right)\right] \end{cases} \tag{7}$$

The maximum hoop stress theory holds that the crack will initiate along the θ direction corresponding to the maximum value of $\sigma_{\theta\theta}$. According to the advanced mathematics, the maximum value of $\sigma_{\theta\theta}$ need satisfy the following conditions:

$$\partial\sigma_{\theta\theta}/\partial\theta = 0,\ \partial^2\sigma_{\theta\theta}/\partial^2\theta < 0. \tag{8}$$

By substitution of Equation (7) into Equation (8):

$$\begin{aligned} &\frac{\partial\sigma_{\theta\theta}}{\partial\theta} = -\frac{1}{\sqrt{2\pi r}}\cdot\frac{3}{2}\cos\frac{\theta}{2}\left[K_{\mathrm{I}}\sin\theta + K_{\mathrm{II}}\left(3\cos\theta - 1\right)\right] = 0 \\ &\frac{\partial^2\sigma_{\theta\theta}}{\partial^2\theta} = \frac{3}{8\sqrt{2\pi r}}K_{\mathrm{I}}\left(\sin\frac{\theta}{2}\sin\theta - 2\cos\frac{\theta}{2}\cos\theta\right) \\ &\qquad + \frac{3}{8\sqrt{2\pi r}}K_{\mathrm{II}}\left(3\cos\theta + 6\cos\frac{\theta}{2}\sin\theta - 1\right) < 0 \end{aligned} \tag{9}$$

It can be obtained by formula (9) that $\cos\frac{\theta}{2} = 0$ or $K_{\mathrm{I}}\sin\theta + K_{\mathrm{II}}\left(3\cos\theta - 1\right) = 0$. If $\cos\frac{\theta}{2} = 0$, the solution is $\theta = \pm\pi$ which means that the cracking surface coincides with the original crack surface. That's meaningless. Therefore, the solution of crack initiation angle is the root of $K_{\mathrm{I}}\sin\theta + K_{\mathrm{II}}\left(3\cos\theta - 1\right) = 0$.

2.3.2. Crack Initiation Strength of Open-Type Crack with Hydraulic Pressure

In this paper, the following compress shear fracture criterion of rock, which is proposed in the literature [11] [12] [13], is used to study the I-II type mixed mode crack initiation strength under compression-shear condition.

$$\lambda_{12}K_{I}+\left|K_{II}\right|=\bar{K}_{IIC}. \tag{10}$$

λ_{12} is the compression-shear coefficient, which is related to the material properties. $\bar{K}_{IIC}$ is the shear fracture toughness under compression.

The initiation strength of open-type crack with hydraulic pressure is obtained by substituting the Equations ((4)-(6) into (10)) [6]:

$$\sigma_1'=\frac{2\bar{K}_{IIC}/\sqrt{\pi a}+A\sigma_3-2p\lambda_{12}\left(1-\sqrt{\rho/a}\right)}{B}$$

$$A=\sin(2\alpha)+\lambda_{12}\left[1-\sqrt{\rho/a}-\left(1+\sqrt{\rho/a}\right)\cos(2\alpha)\right]$$

$$B=\sin(2\alpha)-\lambda_{12}\left[1-\sqrt{\rho/a}+\left(1+\sqrt{\rho/a}\right)\cos(2\alpha)\right]$$

3. Comparative Analysis of Experimental and Theoretical Results of Compression-Shear Plane Crack Initiation and Expansion with Hydraulic Pressure

The cement mortar specimens prefabricated with open-type through crack were fabricated. Specimen width 2*b* = 70 mm, height 2*h* = 140 mm, thickness 2*t* = 40 mm. There is an oblique crack with a length of 2*a* = 12 mm in the middle of the specimen. Material parameter *e* = 22.09 gpa, *υ* = 0.142, internal friction angle $\varphi=50^{\circ}$. The compression-shear fracture parameters of the material have been measured by the method presented in the literature [12].
$K_{IIC}=0.582\ \text{MPa}\cdot\text{m}^{1/2}, \lambda_{12}=0.92$. In the experiment, the crack is sealed by clamping the seal ring and the tempered glass with a clamp.

3.1. Effect of the Dip Angle of Pre-Exciting Crack on Stress Intensity Factor and Crack Initiation Angle

In the experiment, the vertical pressure, lateral pressure, water pressure and crack thickness were controlled as follows: $\sigma_1=20\ \text{MPa}$, $\sigma_3=0\ \text{MPa}$, $p=1\ \text{MPa}$, $\rho=1.5\ \text{mm}$. The dip angle of pre-exciting crack was changed and set the dip angle to 15˚, 30˚, 45˚, 60˚, 75˚. The analytical solution of stress intensity factor and crack initiation angle was calculated by substituting the experimental parameters into the above derivation formulas. The results are as follows (**Table 1**).

It is intuitive to see the variation of the stress intensity factor and the crack initiation angle with the dip angle of pre-exciting crack. The mode I stress intensity factor decreases with the increase of the dip angle, and the minimum value is obtained when the dip angle is 60 degrees. The mode II stress intensity factor is symmetrically distributed with the dip angle of pre-exciting crack, and the maximum value is obtained when the dip angle is 45 degrees.

As shown in **Figure 2**, it was observed in the experiment that the crack initiation angle decreases with the increase of the dip angle of pre-exciting crack which is consistent with the theoretical derivation.

3.2. Effect of Water Pressure on Stress Intensity Factor and Crack Initiation Angle

In the experiment, the dip angle of pre-exciting crack was controlled to $\beta = 45^\circ$. The water pressure was changed and set the water pressure to 0 MPa, 0.5 MPa, 1 MPa. The analytical solution of stress intensity factor and crack initiation angle was calculated by substituting the experimental parameters into the above derivation formulas. The results are as follows:

It can be seen from the table (Table 2) that the mode I stress intensity factor decreases with the increase of water pressure. And mode II stress intensity factor is independent of water pressure. The crack initiation angle decreases with the increase of water pressure.

Table 1. The results of SIF and initial angle with different pre-exciting crack dip angles.

The calculation results	Different dip angles of pre-exciting crack(˚)				
	15	30	45	60	75
τ	5	8.66	10	8.66	5
σ_N	17.66	14	9	4	0.34
σ_T	0.34	4	9	14	17.66
K_I	−2.40	−1.65	−0.62	0.41	1.17
K_{II}	−0.69	−1.19	−1.37	−1.19	−0.69
θ	126.6	97.2	79.3	64.1	43.7

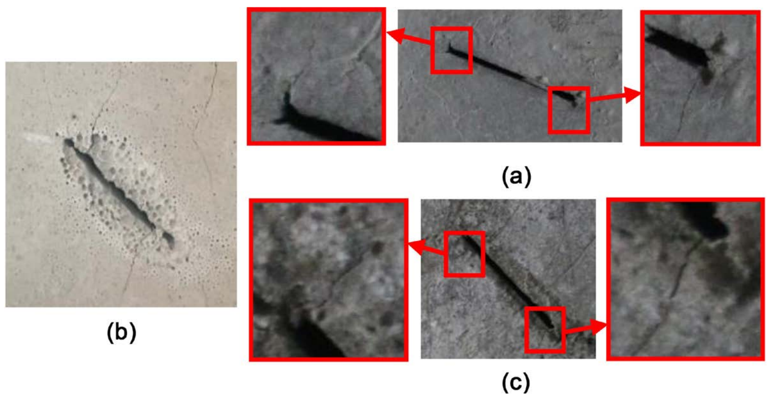

Figure 2. Crack initiation with different pre-exciting crack dip angle. Crack initiation at the dip angle of 30˚; Crack initiation at the dip angle of 45˚; Crack initiation at the dip angle of 60˚.

Table 2. The results of SIF and initial angle with different seepage pressure.

The calculation results	Different seepage pressure(MPa)		
	0	0.5	1
K_I	−0.69	−0.65	−0.62
K_{II}	−1.37	−1.37	−1.37
θ	80.34	79.77	79.33

3.3. Effect of Crack Width s on Stress Intensity Factor and Crack Initiation Angle

In the experiment, the dip angle of pre-exciting crack and water pressure were controlled as follows: The crack width was changed and set the crack width to 1 mm, 1.5 mm, and 2 mm. The analytical solution of stress intensity factor and crack initiation angle was calculated by substituting the experimental parameters into the above derivation formulas. The results are as follows:

It can be seen from the table (Table 3) that the mode I stress intensity factor decreases with the increase of crack width. And mode II stress intensity factor is independent of crack width. The crack initiation angle decreases with the increase of crack width.

3.4. The Experimental Verification of Pre-Exciting Plane Crack Initiation Strength with Hydraulic Pressure

In the experiment, the crack initiation strength was recorded when the lateral pressure was controlled to $\sigma_3 = 0\ \text{MPa}$ and set the water pressure to $p = 0$ MPa, 0.5 MPa and 1 MPa; the crack initiation strength was also recorded when the water pressure was controlled to $p = 0$ MPa and set the lateral pressure to $\sigma_3 = 0\ \text{MPa}, 1\ \text{MPa}, 5\ \text{MPa}, 10\ \text{MPa}$.

The analytical solution of crack initiation strength was calculated by substituting the experimental parameters into the above derivation formulas. By comparing the analytical values with the experimental values, we can see the variation of crack initiation strength with the water pressure and lateral pressure. The results are as follows:

It can be seen from the tables (Table 4 and Table 5) that the crack initiation

Table 3. The results of SIF and initial angle with different crack widths.

The calculation results	Different crack widths (mm)		
	1	1.5	2
K_{I}	−0.73	−0.62	−0.52
K_{II}	−1.37	−1.37	−1.37
θ	80.91	79.33	77.89

Table 4. Crack initiation strength with different seepage pressure.

The values of crack initiation strength	Different seepage pressure (MPa)		
	0	0.5	1
Analytical values	−0.69	−0.65	−0.62
Experimental values	−1.37	−1.37	−1.37

Table 5. Crack initiation strength with different lateral pressure.

The values of crack initiation strength	Different lateral pressure (MPa)			
	0	1	5	10
Analytical values	14.0	16.7	27.51	41.03
Experimental values	15.3	17.0	29.0	42.2

strength decreases with the increase of water pressure and increases with the increase of lateral pressure. The experimental results were consistent with the theoretical results.

4. Conclusions

In this paper, the stress intensity factor (SIF), crack initiation angle and crack initiation strength of open-type crack with hydraulic pressure were derived. The variation of the stress intensity factor at the crack tip with the dip angle, water pressure and crack width was discussed. The crack initiation strength formula of compression-shear open-type crack with hydraulic pressure was verified by comparing the experimental and theoretical results.

The following conclusions are got:

1) For the stress intensity factor of pre-exciting open-type crack with hydraulic pressure, the mode I stress intensity factor decreases with the increase of pre-exciting crack dip angle, and the minimum value is obtained when the dip angle is 60 degrees, which means that the fracture toughness is most easily achieved when the dip angle is 60˚. The water pressure and crack width are inversely proportional to mode I stress intensity factor and promote the expansion of pre-exciting crack. The mode II stress intensity factor is symmetrically distributed with pre-exciting crack dip angle, and the maximum value is obtained when the dip angle is 45˚. The mode II stress intensity factor is independent of the water pressure and the crack width.

2) For the crack initiation angle of pre-exciting open-type cracks with hydraulic pressure, it is proportional to pre-exciting crack dip angle, water pressure and crack width. The theoretical derivation of the variation of the crack initiation angle is consistent with the trend of crack initiation angle was observed during the experiment.

3) The relationship between crack initiation strength and water pressure and lateral pressure was discussed. The crack initiation strength is inversely proportional to the water pressure and proportional to the lateral pressure.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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