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Turbulence Mechanics in Progress—From Classical to Postclassical*

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ABSTRACT

This paper explains the basic steps form the classical turbulence mechanics (CTM) to the postclassical turbulence mechanics (PCTM). When the CTM stems from the characterization of the motion states in the infinitesimal surroundings of the flow-field points by the flow velocity at these points then the PCTM complements this characterization by the curvature of the velocity fluctuation streamlines passing these points. The complementation is formalized by the inclusion of the curvature of the velocity fluctuation streamlines to the arguments of the probability distribution of the motion states in the infinitesimal surroundings of the flow field points. The most radical physical outcome of the realized formalism is the characterization of the turbulence viscosity properties by two types of turbulence viscosity against only one shear viscosity within the CTM.

Keywords: Fluids Mechanics; Turbulence; Mathematical Modeling

1. Introduction

The classical turbulence mechanics (CTM) originates from J. Boussinesq [1] and O. Reynolds [2]. It identifies the turbulence with a chaotic form of fluids motion and sets the turbulence description to the Reynolds-averaged Navier-Stokes equation (RANS, called also the Reynolds equation), gathering the turbulence effects into the symmetric turbulent stress tensor. The applied to this tensor closure assumption reduces its specification to the determination of turbulent shear viscosity coefficient, which turns the modeling (parameterization) of this coefficient to the synonym of the CTM. Different parameterization models of this coefficient from the semi-empirical models [3,4] to more contemporary turbulence models [5,6] have been proposed to solve this task.

Despite the similarity of the setup of the CTM to the setup of the classical fluid mechanics of viscous fluids (CFM)—both are formalized within the law of momentum with the symmetric stress tensor parameterized (for incompressible fluids) by just one (shear) viscosity coefficient—there is still a substantial difference between the two. While the CFM grounds the symmetry of the molecular stress tensor on the constituted absence of the energy-carrying internal rotational degrees of freedom of the medium, then the CTM constitutes the symmetry of the (Reynolds) stress tensor thus ruling out the energy-carrying internal rotational degrees of freedom of turbulent media. Insofar as this kind of the medium rotational degrees of freedom in turbulent media are foreseen by another classical conception of turbulence originating from L. F. Richardson [7] and A. N. Kolmogorov [8] (henceforth, the RK conception which stresses the turbulence order reflected in its hierarchic eddy structure uphold by the cascading energy transfer through the system of eddies of different scales with the large-scale eddies obtaining their energy immediately from the average flow) the grounding statements of CTM and the RK conception prove contradicting.

Unlike the CTM, the postclassical turbulence mechanics (PCTM) [9,10] treats the problem of turbulence in the context of physical doctrine of turbulence (PDT) [11]. The PDT sets the formulation of turbulence mechanics (TM) into a systemic context [12,13], esteems the RK conception and mandates the formulation of the TM within the principles of statistical physics and continuum mechanics [14,15]. The PCTM meets this mandate starting from modifying the very origin of the turbulence description setup. The modification consists in distinguishing the states of motion in infinitesimal surroundings of the flow-field points by the curvature of the velocity

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fluctuation streamlines passing these points, formalized by the inclusion of the curvature of the velocity fluctuation streamlines to the set of arguments of the probability distribution of the motion states in the infinitesimal surroundings of the flow field points. Due to the modification introduced the PCTM appears substantially different from the CTM in its outcome. The most prominent difference is the asymmetry of the turbulent stress tensor providing the turbulent media with two types of viscosity against just one viscosity within the CTM. The difference ends up in not only diverging formalisms but also demonstrates the insufficiency of the boundary conditions imposed within the CTM to determine the flow situation uniquely. Moreover, as shown in the listed in [10] applications of the theory of rotationally anisotropic turbulence, the complementation of the setup of the PCTM with the appropriate closure assumptions, the additional introduced viscosity of turbulent media appears to be much more essential to the description of the related physical processes than the turbulent shear viscosity.

Being grounded on the enlarged physical background the PCTM proves comprising the CTM as its particular case. By comprising the CTM it comprises also the Large-Scale-Eddy (LES) turbulence modeling [16,17], diverging from the conventional setup of the TM by applying the averaging procedure to the small-scale turbulence constituent only. The PCTM pays respect also to several former ideas which have been remained outside the general trends of formulation of the TM. In addition to the RK conception it refreshes the idea of G. Mattioli [18], who first suggested the inclusion of the equation of moment-of-momentum to the setup of turbulence description, as well as some recent ideas like the relation of the turbulent media to the class of micropolar fluids [19-21] and the ideas applied in the structure-based turbulence models [22,23].

The current paper is aimed to explain the PCTM in simple terms and graphs avoiding complicated mathematics and to call up all interested parties, including those who see their mission in defending the turbulence description standards comprised in the CTM, to critically analyze the new situation in the TM altered by the formulation of the PCTM. The discussion starts (Section 2) from decomposition of a velocity fluctuation to its constituents correlating and not correlating with the curvature of the velocity fluctuation streamline. Section 3 discusses the respective situation in terms of energy. Section 4 comments on some problems related to the declared in the PCTM asymmetry of the turbulent stress tensor. Sections 5 and 6 sum up the main points of the novelty introduced by the PCTM into the turbulence description and address the inferences drawn from available data confirming experimentally the grounding assumptions of the PCTM.

2. Adjusted Representation of the Turbulent Velocity Field

The PCTM starts its formalism from the classical representation of the turbulent flow velocity $\boldsymbol{v}$ in the form

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{v}', \tag{1}$$

where

$$\boldsymbol{u} = \langle \boldsymbol{v} \rangle \tag{2}$$

in which the angular brackets denote statistical averaging and $\boldsymbol{v}'$ denotes the fluctuating constituent of velocity. Let now the probability density, specifying the averaging in (2), be detailed as $f(\boldsymbol{v},\boldsymbol{k})$, where $\boldsymbol{k}$ is the curvature of the velocity fluctuation streamline passing a flow field point. By the definition $\boldsymbol{k} = \partial \boldsymbol{e}/\partial s$ in which $\boldsymbol{e} = \boldsymbol{v}'/|\boldsymbol{v}'|$ and s is the length of the curve of $\boldsymbol{v}'$ streamline passing the flow field point. The specification distinguishes the flow situations in the infinitesimal surroundings of the turbulent flow field points by the curvature of the velocity fluctuation streamlines passing these points (**Figure 1**).

Representing $f(\boldsymbol{v},\boldsymbol{k})$ as

$$f(\boldsymbol{v},\boldsymbol{k}) = f_1(\boldsymbol{v}|\boldsymbol{k})\, f_2(\boldsymbol{k}), \tag{3}$$

where

$$f(\boldsymbol{v}|\boldsymbol{k}) = f_1(\boldsymbol{v},\boldsymbol{k}) / f_2(\boldsymbol{k})$$

and

$$f_2(\boldsymbol{k}) = \int f(\boldsymbol{v},\boldsymbol{k})\,\mathrm{d}\boldsymbol{v},$$

we have (**Figure 2**)

$$\boldsymbol{v}' = \overline{\boldsymbol{v}'} + \boldsymbol{v}'', \tag{4}$$

where (and henceforth) the over-bar denotes the averaging by $f_1(\boldsymbol{v}|\boldsymbol{k})$. It is evident that $\boldsymbol{v}''$ in (4) is statistically independent from $\overline{\boldsymbol{v}'}$ and $\boldsymbol{k}$. Notice, that the CTM constitutes the probability density of the averaging procedure in (2) specified as $f(\boldsymbol{v})$, which declares $\boldsymbol{v}' \equiv \overline{\boldsymbol{v}'}$ and $\boldsymbol{v}'' \equiv 0$.

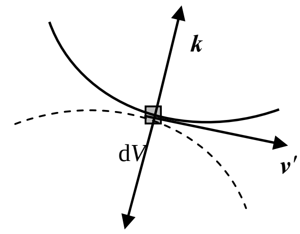

Figure 1. Illustration of distinguishing the flow situations in infinitesimal surrounding of a flow field point (d*V*) by the curvature *k* of the velocity fluctuation streamline passing this point: the flow situations with the same $\boldsymbol{v}'$ but opposite *k* prove different.

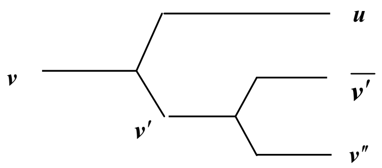

Figure 2. Decomposition of the flow velocity in turbulent flow field accounting for the curvature of the velocity fluctuation streamlines passing the flow-field points.

3. Decomposition of Turbulence Energy

3.1. Primary Decomposition

In terms of energy the decomposition of velocity fluctuation in (4) reads as

$$K = K_1 + K_2 , \tag{5}$$

where $K = \frac{1}{2}\langle v'^2 \rangle$ is the (total) turbulence energy, while $K_1 = \frac{1}{2}\langle v''^2 \rangle$ and $K_2 = \frac{1}{2}\langle \overline{v'}^2 \rangle$ are natural to interpret as the energies of the small-scale and the large-scale turbulence constituents.

The energy graph in **Figure 3** displays the energy situation assumed within the CTM and the situation corresponding to (5), for the cascading energy transfer from the average flow with energy $K^u = \frac{1}{2}u^2$ to the thermal energy U through the turbulent phase of motion represented by the energies K_1 and K_2. Vertical arrows in **Figure 3** mark the levels of energy ceding and receiving, $Q = \sigma_{ij} u_{i,j}$ denotes the work realizing the energy transfer from the average flow either to K or to K_2, Q^* denotes the work realizing the energy transfer from K_2 to K_1 and ε denotes the dissipation rate of turbulence energies K or K_1.

3.2. Secondary Decomposition

The secondary decomposition of the turbulence energy stems from the definition of the kinematical-dynamical pair of the Eulerian flow-field characteristics [10]

$$\boldsymbol{\Omega} = \langle \boldsymbol{v}' \times \boldsymbol{k} \rangle \quad \text{and} \quad \boldsymbol{M} = \langle \boldsymbol{v}' \times \boldsymbol{R} \rangle \tag{6}$$

where $\boldsymbol{R} = |\boldsymbol{k}|^{-2}\boldsymbol{k}$ is the curvature radius-vector corresponding to $\boldsymbol{k}$. The defined $\boldsymbol{\Omega}$ (henceforth, the gyrocity) has the sense of average angular velocity of rotation of medium particles at a flow-field point in respect to the random curvature centres of the velocity fluctuation streamlines passing this point, and $\boldsymbol{M}$ (henceforth, spin) has the sense of average density (per unit mass) of the moment of fluctuating constituent of momentum with $\boldsymbol{R}$ standing for the (random) arm of the moment. Let us note, that so as $\boldsymbol{v}''$ is statistically independent from $\boldsymbol{k}$ (and from $\boldsymbol{R}$), the expressions (6) can be written also as

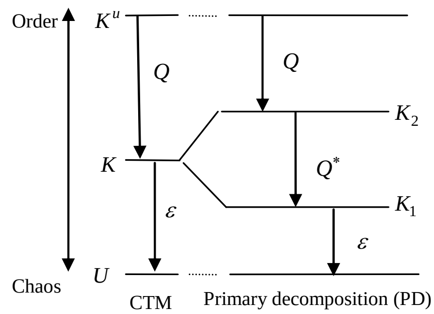

Figure 3. Primary decomposition (PD) of the turbulence energy: the energy graphs of turbulent flow field for the situation assumed within the CTM and for the situation corresponding to (5).

$$\boldsymbol{\Omega} = \langle \overline{\boldsymbol{v}'} \times \boldsymbol{k} \rangle \quad \text{and} \quad \boldsymbol{M} = \langle \overline{\boldsymbol{v}'} \times \boldsymbol{R} \rangle , \tag{7}$$

explaining the gyrocity and the spin as the characteristics of velocity fluctuation constituent $\overline{\boldsymbol{v}'}$ only.

Using (1), (4)-(7) we have for K and K_2

$$K = K^{\Omega} + K^0 , \tag{8}$$

$$K_2 = K^{\Omega} + K_2^0 \tag{9}$$

and

$$K_2^0 + K_1 = K^0 , \tag{10}$$

where

$$K^{\Omega} = \frac{1}{2}\boldsymbol{M} \cdot \boldsymbol{\Omega} , \tag{11}$$

$$K_2^0 = \frac{1}{2}\left\langle \left(\overline{\boldsymbol{v}'} \times \boldsymbol{R}\right)' \cdot \left(\overline{\boldsymbol{v}'} \times \boldsymbol{k}\right)' \right\rangle \tag{12}$$

and

$$K^0 = \frac{1}{2}\left\langle \left(\boldsymbol{v}' \times \boldsymbol{R}\right)' \cdot \left(\boldsymbol{v}' \times \boldsymbol{k}\right)' \right\rangle . \tag{13}$$

Figure 4 outlines the situation corresponding to (5), (8)-(10) as an extension of the situation shown in **Figure 3** for the energy of average flow representing just one source of turbulence energy. Notice that:

(a) the energies K_1 and K_2^0 characterize the motions of different scales of the same order while K^{Ω} and K_2^0 characterize the motions of the same scale of different order;

(b) the decomposition of Q into the sum of Q_1 and Q_2, realizing the energy transfer from the average flow to the turbulence constituents of different order, evidence about the presence of two types of turbulent viscosity against just one viscosity within the CTM;

(c) the nature of the work Q_1, connecting the translatory degrees of freedom of medium motion (realised in the form of the average flow) and its rotational degrees of freedom (realized in the form of rotation characterized by the gyrocity and the spin), specifies an additional viscosity as related to the antisymmetric constituent of the

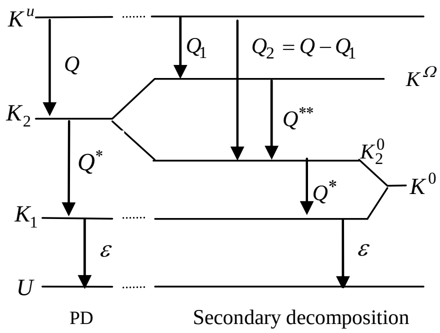

Figure 4. Secondary decomposition of turbulence energy: the energy graphs corresponding to the PD and to the secondary decomposition.

turbulent stress tensor, which abolishes the notion of "evident symmetry" of the turbulent stress tensor holding within the CTM;

(d) for the cascading character of energy transfer the work Q_2 vanishes and **Figure 4** explains the role of the work Q^{**} in transforming the motion organization quality to the form allowing its reception on the level of small-scale turbulence constituent. (For a more detailed discussion of the specifics of the cascading process consider [24-26].)

(e) finally, all representations (4), (8)-(10) are direct corollaries of the adopted specification of the applied averaging and of definitions (6) (or (7)) while the energy transfer directions indicated in **Figure 4** correspond to a typical but not to the only possible situation.

4. The Setup of Description of Turbulent Flows

The fundamental inference from definitions (6) (or (7)) is that the turbulent media is related to the class of micropolar fluids [27-33]. The relation suggests the interconnection of the gyrocity $(\boldsymbol{\Omega})$ and the spin $(\boldsymbol{M})$ by

$$\boldsymbol{M} = \ell^2 \boldsymbol{\Omega}, \qquad (14)$$

where ℓ defines the characteristic average scale of motion, and delegates the description of motion to the system of two equations—the Reynolds equation (with the asymmetric tensor of turbulent stresses) and the equation for the spin $\boldsymbol{M}$.

Referring for the details of the setup of description of turbulent media with the non-vanishing spin to [9,10], we first accent here on the inherent to this description asymmetry of turbulent stresses—the most conflicting point in the relation between of the CTM and the PCTM—starting from the expression for the dual vector to the antisymmetric constituent of turbulent stresses $\sigma_k = e_{kij}\sigma_{ij}$, where e_{kij} denote the components of the Levi-Civita tensor, expressed as [9]

$$\sigma_k = -\rho e_{kij} \left\langle v_s' v_i' R_{j,s} \right\rangle \qquad (15)$$

Denoting the components of the velocity fluctuation constituent along $\boldsymbol{R}$ as $v'_{\mathbf{R}j} = v'_s R_{j,s}$ from (15) we have

$$\sigma_k = -\rho \left\{ e_{kij} \left\langle v'_{\mathbf{Rj}} v'_i \right\rangle \right\}, \qquad (16)$$

or in the vector form

$$\boldsymbol{\sigma} = \rho \left\langle \boldsymbol{v}'_{\boldsymbol{R}} \times \boldsymbol{v}' \right\rangle. \qquad (17)$$

Accounting for (4) and the perpendicularity of $\overline{\boldsymbol{v}'}$ and $\boldsymbol{R}$ we have from (17), that

$$\boldsymbol{\sigma} = \rho \left\langle \boldsymbol{v}''_{\boldsymbol{R}} \times \boldsymbol{v}'' \right\rangle. \qquad (18)$$

Expression (18) explains the antisymmetric constituent of stresses describing the average momentum flux in direction of $\boldsymbol{R}$ either accelerating of decelerating the eddy rotation. Notice, that the velocity fluctuation constituent $\overline{\boldsymbol{v}'}$, playing a crucial role in definitions of $\boldsymbol{\Omega}$ and $\boldsymbol{M}$, does not contribute to $\boldsymbol{\sigma}$. The second accent concerns the work Q_1 in **Figure 4**, represented in terms of $\boldsymbol{\sigma}$ as $Q_1 = -\boldsymbol{\sigma} \cdot \boldsymbol{\omega}$, where $\boldsymbol{\omega} = \frac{1}{2} \nabla \times \boldsymbol{u}$ $\boldsymbol{\omega} = \frac{1}{2} \nabla \times \boldsymbol{u}$ is the vorticity. For the relation of closure for $\boldsymbol{\sigma}$ in the form $\boldsymbol{\sigma} = 4\gamma\ (\boldsymbol{\Omega} - \boldsymbol{\omega})$ [9,10], where γ denotes the coefficient of turbulence rotational viscosity, it is evident that, dependent on the relative values of $\boldsymbol{\Omega}$ and $\boldsymbol{\omega}$, the work Q_1 may be either positive or negative, *i.e.* the medium rotational viscosity manages to explain the eddy-to-mean energy transfer without introduction of notion of "negative viscosity" [34] or without ascribing the actual 3D nature of turbulence with 2D properties. The third accent is related to the effect of rotation of frame on the medium turbulence, which, though not influencing $\boldsymbol{\sigma}$, influences Q_1. The latter explains the frame rotation as a potential cause of the eddy-to-mean energy conversion.

5. Discussion

The PCTM realizes a modification of the TM setup originating from complementation of characterization of the motion states in the infinitesimal surroundings of the flow-field points by the curvature of the velocity fluctuation streamlines passing these points. The modification is undertaken to distinguish the flow field states in the infinitesimal surroundings of the flow field points dependent on the curvature of the velocity fluctuation streamlines passing these points. The necessity for the complementation is one implication of critical analysis of the situation in the TM from the point of view of the widened physical-historical background of the turbulence problem specified as the PDT [11]. The analysis embraces the CTM together with some ideas incompatible with the CTM. Within these ideas the leading positions belong to the RK conception about the cascading eddy structure of turbulence and to the idea about the turbulent media pertaining to the class of micropolar fluids [19-21] (as well as to the related to it earlier idea of G. Mattioli [18], first suggesting the inclusion of the equation of moment-of-momentum to the setup of description of turbulent flows). The analysis displays the statement of the CTM about the evident symmetry of the turbulent stress tensor erring against the principles of continuum mechanics [15], relating the solution of the symmetry properties of the stress tensor to the context of specification of the medium internal rotational degrees of freedom. Within the shortcomings of the CTM note also its inability to propose physically correct explanation to the eddy-to-mean energy conversion, explained within the PCTM as the act of turbulence rotational viscosity neglected within the CTM. The CTM also does not distinguish the turbulence properties in rotating and non-rotating frames which by now is verified observationally.

Let us underline that the PCTM does not "discover" the additional (rotational) viscosity of turbulence—this type of turbulence viscosity has been foreseen and described in a sufficiently complete form by the RK conception—but merely removes the obstacle from the explicit inclusion of this fundamental property of turbulence into the setup of the TM. Let us highlight also, that applying all classical motion integrals—of momentum, of moment-of-momentum and of energy—the PCTM formulates the turbulence description in mechanically closed form and as such completes the formulation of the TM.

We conclude the comments on the PCTM referring to the paper [35], which utilized the data available within the Global Drifter Program to estimate the gyrocity and the spin immediately from observations. The estimated gyrocity and spin provide the grounding declaration of the PCTM about the non-vanishing gyrocity and spin of the turbulent flow with the sense of experimental fact.

Finally, the PDT [11] postulates the conditions of formation of probability distribution properties of momentary states of motion determined as the state of motion fixed in terms of the TM. These conditions are specified within the CTM and within the PCTM differently. The commented role of the TM is emphasized here to stress the scientific merit of the TM wider than a particular turbulence description in average terms.

6. Conclusion

The PCTM mandates a critical analysis of the results following from the CTM. It also mandates the planning of new tasks and research projects from the position of the PCTM. The latter mandate is addressed to the initiators of new projects rather than huge number of scientists participating in turbulence-related applied projects and following firmly established standards. This mandate is also addressed to the people educating new generation of specialists in the field of fluid mechanics.

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