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Pressure-Driven Demand and Leakage Simulation for Pipe Networks Using Differential Evolution

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ABSTRACT

Traditional techniques for hydraulic analysis of water distribution networks, which are referred to as demand-driven simulation method (DDSM), are normally analyzed under the assumption that nodal demands are known and satisfied. In many cases, such as pump outage or pipe burst, the demands at nodes affected by low pressures will decrease. Therefore, hydraulic analysis of pipe networks under deficient pressure conditions using conventional DDSM may cause large deviation from actual situations. In this paper, an optimization model is introduced for hydraulic analysis of water distribution networks using a meta-heuristic method called Differential Evolution (DE) algorithm. In this methodology, there is no need to solve linear systems of equations, there is a simple way to handle pressure-driven demand and leakage simulation, and it does not require an initial solution vector which is sometimes critical to the convergence. Also, the proposed model does not require any complicated mathematical expression and operation.

Keywords: Hydraulic Analysis; Differential Evolution; Optimization Model

1. Introduction

In the recent past, several packages originally developed for steady state analysis of looped water distribution systems. For instance, EPANET2 has been extended to include the possibility of "extended period simulations" (EPS), namely the possibility of simulating long periods of time by means of a succession of steady states, only accounting for the change in storage of reservoirs occurring from one time step to the next [1].

This model, which is used in current engineering practice, is based on the conventional Demand Driven Simulation Method (DDSM). It assumes that nodal outflows are fixed and are satisfied regardless of network pressures. The assumption simplifies the mathematical solution of the problem but is not always appropriate because it is clear that the amount of outflow at nodal outlets depends on network pressures. If the pressure falls below a minimum required level (due to some critical events such as mechanical and hydraulic failures or excess demand), the flow will be significantly reduced. Although some nodes may be able to satisfy their demands, others may meet the demand partially while the rest may fail and may not provide any water at all. The assumption of fixed nodal consumptions is therefore valid only under normal conditions when the pressures can be expected to be adequate to satisfy the stipulated demands. If the operation of the system is simulated under pressure-critical conditions, the relationship between pressure and outflow should, therefore, be taken into account if the simulation results are to be realistic [2-8]. Furthermore, water loss via leakage constitutes a major challenge to the effective operation of municipal distribution networks since it represents not only diminished revenue for utilities, but also undermined service quality [9] and wasted energy resources [10]. A typical leakage control program usually starts with a water audit based on available flow measurements. Although this is an important first step, most practical studies do not go beyond it. In order to assist in leakage reduction and conduct more accurate analysis, a hydraulic model capable of accounting for pressure-driven (also known as head-driven) demand and leakage flow at pipe level is introduced by Giustolisi *et al.* [11]. Meanwhile, there is still a chance to develop a new method for pressure-driven demand and leakage simulation in water distribution networks. In this paper, an optimization model is introduced for hydraulic analysis of water distribution networks using a meta-heuristic algorithm called Differential Evolution (DE). Analysis of hydraulic networks can be achieved by treating it as an optimization problem as shown by Arora [12], Hall [13], and Collins *et al.* [14]. Arora considered a simple two-piped loop while Collins *et al.* have based their approach on rigorous theoretical background and developed nonlinear optimization models, solutions of which yield the hydraulic network analysis [15]. Collins's model can be minimized by application of differential evolution algorithm. In this methodology, there is no need to solve linear systems of equations, there is a simple way to handle pressure-driven demand and leakage simulation, and it does not require an initial solution vector which is sometimes critical to the convergence. Also, the proposed model does not require any complicated mathematical expression and operation. In the next part, Collins's model is described.

2. Co-Content Model Approach

Arora [12] is the first researcher who suggested an approach based on the principle of conservation of energy. This principle states: "Flow in the pipes of a hydraulic network adjust so that the expenditure of the system energy is minimum." Next, Collins *et al.* [14] proposed a model termed the co-content model, that is based on equations having the unknown nodal heads as the basic unknowns, *i.e.*, based on *H* equations. The unknown pipe flows are expressed in terms of the nodal heads and the known pipe resistances, so that the energy loss in pipe $x\left(E_x\right)$ is given by [15]

$$E_x = Q_x h_x = \frac{\left[H_i - H_j\right]^{(1/n)+1}}{R_x^{(1/n)}} \tag{1}$$

In which R_x is the hydraulic resistance function, h_x is head loss in pipe *x*, H_i and H_j are pressure heads in node *i* and node *j*.

Now consider the network of **Figure 1**, with the known and unknown parameters as shown therein. Let the unknown nodal heads at nodes 3, 4, and 5 be H_3, H_4, and H_5, respectively. Herein also consider a ground node G with fixed known level H_{0G}, as shown in **Figure 1**. The nodes 3, 4, and 5 are connected to the ground node G with pseudo pipes, carrying the known nodal outflows q_3, q_4, and q_5 as shown in **Figure 1**.

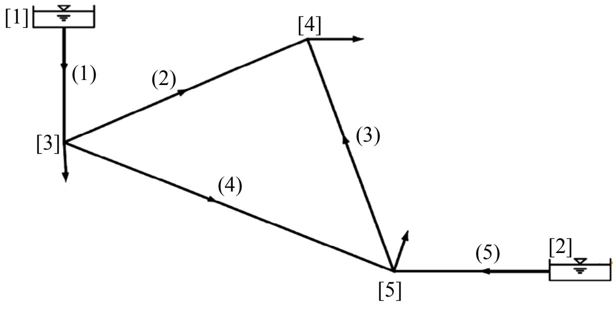

Figure 1. Schematic representation of the looped pipe network with 5 pipes.

The co-content optimization model is expressed as

$$\begin{aligned} Min.C\left(H\right) = & \frac{\left|H_{01}-H_3\right|^{(1/n)+1}}{R_1^{1/n}} + \frac{\left|H_3-H_4\right|^{(1/n)+1}}{R_2^{1/n}} \\ & + \frac{\left|H_5-H_4\right|^{(1/n)+1}}{R_3^{1/n}} + \frac{\left|H_3-H_5\right|^{(1/n)+1}}{R_4^{1/n}} \\ & + \frac{\left|H_{02}-H_5\right|^{(1/n)+1}}{R_5^{1/n}} + \left(\frac{1}{n}+1\right) q_3\left(H_3-H_{0G}\right) \\ & + \left(\frac{1}{n}+1\right) q_4\left(H_4-H_{0G}\right) \\ & + \left(\frac{1}{n}+1\right) q_5\left(H_5-H_{0G}\right) \end{aligned} \tag{2}$$

H_{01} and H_{02} are known pressure heads for source nodes. The first five terms of the objective function represent the energy loss in real pipes $1,\cdots,5$ of the network, respectively, and the last three terms show $\left(1/n+1\right)$ times the energy loss in the pseudo pipes [15]. It should be noted that there are no constraints and therefore an unconstrained model in three decision variables is made. For minimization of optimization model, which are partially differentiating in unknown heads, the node-flow continuity equations are created. Therefore, the solution of the co-content model gives the values of the unknown heads such that the node-flow continuity relationships are satisfied [15]. For simplicity, H_{oG} can be taken as zero, so that the General co-content model can be expressed as

$$\text{Min.}C\left(H\right) = \sum_x \frac{\left|H_i - H_j\right|^{(1/n)+1}}{R_x^{1/n}} + \left(\frac{1}{n}+1\right)\sum_j q_j\left(H_j\right) \tag{3}$$

Collins *et al.* [14] suggested the solution of the NLP optimization of the model. Their method were 1) the Frank-Wolfe method; 2) a piece-wise linear approximation; and 3) the convex simplex method. These methods are highly depends on initial guesses and in some cases they converged to an incorrect solution [14].

3. Head Dependent Analysis

In the common approaches, it is presumed that the nodal demands are always satisfied at all demand nodes, irrespective of the available HGL values at demand nodes [15]. But in practice, if the head at a node is insufficient, a reduction in the water flowing from the tap is expected and, in the worst case, the discharge that can be drafted will be zero, regardless to the actual demand [1]. There are several solutions in the literature for these conditions.

Wagner *et al.* [16] and Chandapillai [17] suggested a parabolic relationship between required nodal head and minimum head. Their relationships are

$$q_j = \begin{cases} 0 & H_j < H_{\min} \\ q_j \left(\dfrac{H_j - H_{\min}}{H^* - H_{\min}} \right)^{\frac{1}{p}} & H_{\min} \le H_j < H^* \\ q_j & H^* \le H_j \end{cases} \tag{4}$$

H^* is the required nodal head. This formulation is easily handled to co-content model without any mathematical complexity.

4. Leakage Simulation

Water losses via leakages constitute a major challenge to the effective operation of municipal WDN since they represent not only diminished revenue for utilities, but also undermined service quality [9] and wasted energy resources [10]. In order to conduct more accurate analysis of a WDN, such as a better estimate of flow through the network (with respect to both satisfied demand and losses through leakage), a hydraulic analysis based on capable of accounting for pressure-driven (also known as head-driven) demand and leakage flow at the pipe level should prove invaluable. To reach this goal, a leakage model is expressed as follows [3]

$$q_{k\text{-}leak} = \begin{cases} \beta_k l_k \left(P_k\right)^{\alpha_k} & \text{if } P_k > 0 \\ 0 & \text{if } P_k \le 0 \end{cases} \tag{5}$$

Where P_k = average pressure in the pipe computed as the mean of the pressure values at the end nodes I and j of the kth pipe; and l_k = length of that pipe. Variables α_k and β_k = two leakage model parameters [11]. The allocation of leakage to the two end nodes can be performed in a number of ways [18]. Here the nodal leakage flow q_{j-leak} is computed as the sum of q_{k-leak} flows of all pipes connected to node j as follows:

$$q_{j\text{-}leak} = \sum_k \frac{1}{2} q_{k\text{-}leak} = \sum_k \begin{cases} \dfrac{1}{2} \beta_k l_k \left(P_k\right)^{\alpha_k} & \text{if } P_k > 0 \\ 0 & \text{if } P_k \le 0 \end{cases} \tag{6}$$

where $P_k = \left(P_i + P_j\right)/2$. This formulation is also easily handled to co-content model without any mathematical complexity.

5. Application of Differential Evolution Algorithm for Minimizing Co-Content Model

For the hydraulic analysis, this study introduces Differential Evolution (DE) algorithm. Because the algorithm was originally developed for solving optimization problems, the hydraulic network analysis wasintroduced into an optimization problem (co-content model). One advantage of the DE algorithm is the fact that it does not require an initial solution vector which is sometimes critical to the convergence. Also, application of DE algorithm in co-content model does not require any complicated mathematical expression and operation. In this model, pressure-driven demand and leakage can be simulated.

5.1. Differential Evolution (DE)

Differential evolution (DE) is a simple powerful and population-based stochastic optimization algorithm that outperforms many meta-heuristic algorithms on numerical single objective optimization problems. In DE each decision variable is represented in the chromosome by a real number. The DE algorithm requires only three control parameters: weight factor (F), crossover rates (CR), and population size (NP). The initial population is randomly generated by uniformly distributed random numbers using the maximum and minimum limitation of each decision variable. Then the fitness values of all the individuals of population are calculated to find out the best individual $x_{best,G}$ of current generation, where G is the index of generation. Three main steps of DE, mutation, crossover, and selection were performed sequentially and were repeated during the optimization cycle [19].

The steps in the procedure of DE are shown in **Figure 2**. They are as follows:

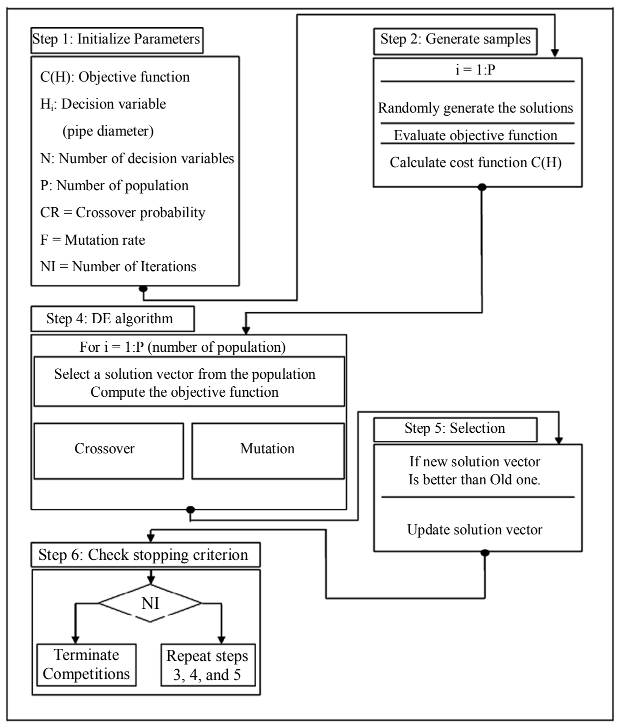

Figure 2. DE procedure for minimization of co-content model.

Step 1. Initialize problem and algorithm parameters
Step 2. Samples Generation
Step 3. Start Iterative Process
Step 3.1. Mutation Operator
Step 3.2. Cross-Over Operator
Step 4. Selection
Step 5. Check The Stopping Criterion.

5.2. Step 1. Initialize the Problem and Algorithm Parameters

In Step 1, the optimization problem is specified as follows:

$$\text{Min.}C(H) = \sum_x \frac{\left|H_i - H_j\right|^{(1/n)+1}}{R_x^{1/n}} + \left(\frac{1}{n}+1\right)\sum_j q_j\left(H_j\right) \quad (7)$$

Where $C(H)$ is an objective function; H is the set of each decision variable. In this paper, the objective function is the co-content model; the unknown heads are the decision variables.

5.3. Step 2. Samples Generation

The initial population, initial values of the mutation factor, F and initial values of the crossover rate, CR for the DE is created arbitrarily by following formula:

$$\begin{aligned} &H(i,j) = H_{\min}(i,j) + \tau_1\left(H_{\max}(i,j) - H_{\min}(i,j)\right) \\ &F(j,0) = F_{\min} + \tau_2\left(F_{\max} - F_{\min}\right) \\ &CR = CR_{\min} + \tau_3\left(CR_{\max} - CR_{\min}\right) \end{aligned} \quad (8)$$

where $\tau_1, \tau_2, \tau_3 =$ independently generated random numbers in the range of $[0,1]$. $H_{\min}(i,j)$ and $H_{\max}(i,j)$ are maximum and minimum limits of variable j and node i. $F_{\min}$ and $F_{\max}$ are maximum and minimum limits of mutation factor. $CR_{\min}$ and $CR_{\max}$ are maximum and minimum limits of crossover rate. Then, the fitness values $C(H)$ of all the individuals of population are calculated. The position matrix of the population of generation G can be represented as:

$$P^{(G)} = \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_{nPop} \end{bmatrix} = \begin{bmatrix} H_1^1 & H_2^1 & \cdots & H_N^1 \\ H_1^2 & H_2^2 & \cdots & H_N^2 \\ \vdots & \vdots & \ddots & \vdots \\ H_1^{nPop} & \cdots & \cdots & H_N^{nPop} \end{bmatrix} \quad (9)$$

N is the number of unknown nodes.

5.4. Step 3. Start Iterative Process

In this step, two main steps of DE, mutation, and crossover, are performed sequentially and new solution vectors are created.

5.4.1. Mutation Operator

In this step, mutation operator is used, for each solution vector in the population, to create new solutions in DE according to the following formula:

$$H^{\text{new}}(i,j) = H(i,C) + F\left(H(i,A) - H(i,B)\right) \quad (10)$$

A, B, and C are random solution vectors.

5.4.2. Cross-Over Operator

In the crossover operator, the new vector is generated by choosing some parts of mutation vector, and other parts come from the target vector. The crossover operator of DE is shown as follows:

$$H^{\text{new}}(i,j) = \begin{cases} H^{\text{new}}(i,j) & \text{if rand} < CR \\ H(i,j) & \text{otherwise} \end{cases} \quad (11)$$

where CR represents the crossover probability. If random number rand is larger than CR value, the component of mutation vector will be chose to the trial vector. Otherwise, the component of target vector is selected to the trial vectors. The mutation and crossover operators are used to diversify the search area of optimization problems [19].

5.5. Step 4. Selection

The trial vector is carried to the next generation only if it yields a reduction in the value of the objective function in the case of the minimization problem. Otherwise, the target vector will be selected for the next generation.

The population of the next generation is selected as follows:

$$H^{\text{new}}(j) = \begin{cases} H^{\text{new}}(j) & \text{if } C\left(H^{\text{new}}(j)\right) < C\left(H(j)\right) \\ H(j) & \text{otherwise} \end{cases} \quad (12)$$

where $C\left(H(j)\right)$ represents the cost of the jth individual in the current generation. The F selections for the next generation is given by

$$F(j, G+1) = F_{\min} + \tau_2\left(F_{\max} - F_{\min}\right) \quad (13)$$

where G is the generation number. It should be noted that $G = 0$ in the initial generation.

5.5. Step 5. Check the Stopping Criterion

In this section, Steps 3, 4 and 5 are repeated until the termination criterion is satisfied.

6. Numerical Examples

In this section, the hydraulic analyses for several conditions in some water distribution networks are done. All of computations were executed in MATLAB programming language environment with an Intel(R) Core(TM) 2 Duo CPU P8700 @ 2.53 GHz and 4.00 GB RAM. In this study proposes the use of mass balance and energy balance in the network for demonstrating the effectiveness of DE in comparison with other methods.

The average of mass and energy balance is shown by δ and is calculated by following formula:

$$\delta = \text{mean}\left(abs\left(\sum_{\substack{i \text{ connected to} \\ j \text{ through } k}} \frac{\left|H_i - H_j\right|^{(1/n)}}{R_k^{1/n}} - q_j \right)\right), \quad (14)$$

$$j = 2, \cdots, N$$

In all numerical examples $CR_{\min} = CR_{\max} = 0.2$, $F_{\min} = 0.2$ and $F_{\max} = 0.8$. To check the performance of the DE for the minimization of co-content model, ten optimization runs were performed using different random initial solutions in all examples.

6.1. Numerical Example 1

In order to demonstrate the advantages of the proposed model in pressure-driven demand condition, the simplified water distribution network shown in **Figure 3**, was used. For the sake of simplicity, the same Hazen-Williams roughness coefficient $C = 130$ was assumed for all the 14 pipes of identical length of 1000 m, while no minor losses have been added. The following diameters have been used in the example: 500 mm (P-2); 400 mm (P-1); 300 mm (P-4, P-7); 250 mm (P-10); 200 mm (P-3, P-5, P-6, P-13); 150 mm (P-8, P-9, P-11, P-12, P-14). The nodal demands are $q_2 = 1$, $q_3 = 1$, $q_4 = 2$, $q_5 = 15$, $q_6 = 15$, $q_7 = 10$, $q_8 = 5$ (m^3/min). Without loss of generality, in this example, the minimum head requirement H_i^* has been assumed equal to the ground elevation Z_i [1]. So the relationship between required nodal head and minimum head is:

$$q_j = \begin{cases} 0 & H_j < Z_j \\ q_j & Z_j \le H_j \end{cases} \quad (15)$$

Todini [1] proposed a three steps approach for solving this network and its solution is reported in the 4th column of **Table 1**. In proposed methodology, pressure-driven model can be applied in hydraulic analysis without any mathematical formulation. In this situation, an if-then rule is added to co-content model and optimization process is conducted. The DE technique is applied to solve this problem in three cases. DE model parameters selected are as follows: number of decision variables = 7; number of population for case 1 = 10, case 2 = 20, case 3 = 20; number of iteration for case 1 = 1000, case 2 = 1000 and case 3 = 5000. The bound variables were set between 50 and 140. The best, worst and average solutions of DE algorithm in three cases are shown in **Table 2**. This table compares the average of mass and energy balance of the three cases with those obtained using Todini algorithm. As it can be seen in **Table 2**, DE found the optimal solution more accurately than Todini method in all cases. Results of the best performance of DE and convergence history are reported in **Table 1** and **Figure 4**, respectively.

As you can see in the **Figure 4**, after about 400 iterations the parameter δ becomes convergent and then it doesn't change. The minimum value of δ calculated by DE algorithm is 2.07E-02, while the value obtained for this parameter, by the method introduced by Todini equals to 2.76E-02. Values of δ at each node are compared in the seventh and eighth columns of **Table 1**, using the two proposed methods and the method of Todini.

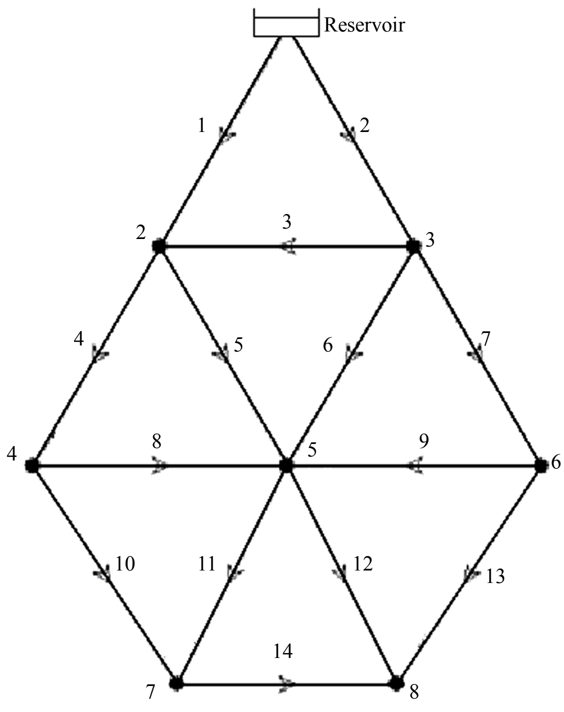

Figure 3. Schematic representation of the looped pipe network used in the numerical example 1.

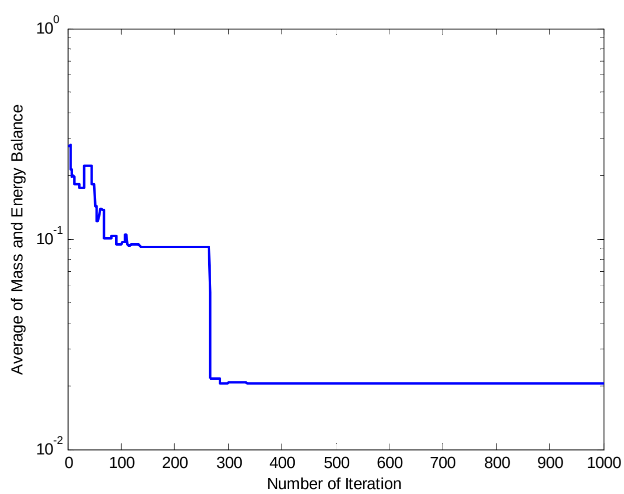

Figure 4. Convergence history of numerical example 1.

Table 1. Head and parameter δ in numerical example 1.

Node	Z(m)	DE	3 steps	DE	3 steps	DE	3 steps
		H(m)	H(m) [2]	H-Z	H-Z [1]	δ	δ [1]
1	140	140	140	0	0	0	0
2	80	129.304	130.07	49.304	50.07	**7.93e-10**	0.0003
3	90	132.288	132.76	42.288	42.76	**7.24e-09**	0.004
4	70	109.587	110.96	39.587	40.96	**5.56e-10**	0.0021
5	80	80.000	88.54	0.000	8.54	**0.0576**	0.034
6	90	90.000	91.45	0.000	1.45	**0.0069**	0.0173
7	90	90.000	90.00	0.000	0.00	**0.0803**	0.106
8	100	88.922	90.43	−11.078	−9.57	**2.54e-09**	0.0439

Table 2. Average of mass and energy balance for numerical example 1.

DE		Mass and Energy Balance (δ)			
		best	worst	mean	std
Number of population	10	2.07E-02	9.01E-02	4.14E-02	2.37E-02
Number of iteration	1000				
Number of population	20	2.07E-02	2.08E-02	2.07E-02	3.38E-05
Number of iteration	1000				
Number of population	20	2.07E-02	2.07E-02	2.07E-02	6.20E-06
Number of iteration	5000				
Three Steps Approach [1]		Maximum Accuracy		2.76E-02	

As you can see at all the nodes, in calculating the minimum value of δ, the proposed method works better than the Todini method.

6.2. Numerical Example 2

The second considered network is a real planned network designed for an industrial area in Apulian town (Southern Italy). The network layout is shown in **Figure 5** and the corresponding data are provided in **Table 3**. With respect to the leakages, they have been assumed as pressure-driven (see Equation (5)) since they are implemented in the pressure-driven network simulation model as above described [11]. The parameter $\beta = 1.0632 \times 10 - 7$ and $\alpha = 1.2$, as reported in Giustolisi *et al.* [11] for this network. Giustolisi *et al.* [11] proposed a hydraulic simulation model, which fully integrates a classic hydraulic simulation algorithm, such as that of Todini and Pilati [20] found in EPANET 2, with a pressure-driven model that entails a more realistic representation of leakage. They applied their model in this network and results are demonstrated in **Table 4**. The DE technique is applied to solve this problem and DE model parameters selected are as follows: number of decision variables = 23; number of population for all case 1 = 50, case 2 = 50, case 3 = 20,

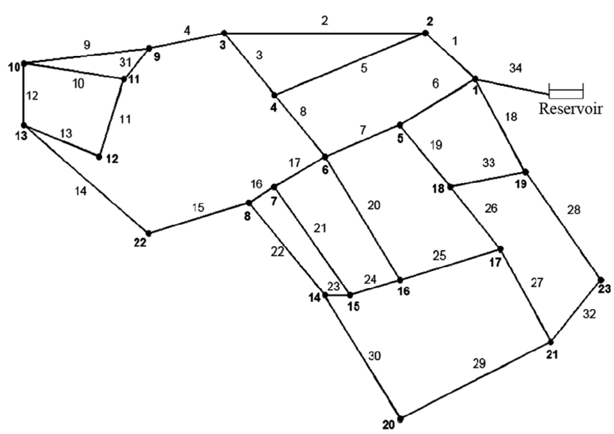

Figure 5. Schematic representation of the looped pipe network used in the numerical example 4.

Table 3. Hydraulic data relevant to the numerical example 2.

Pipe	L (m)	D (mm)	Pipe	L (m)	D (mm)	Pipe	L (m)	D (mm)
1	348.5	327	12	428.4	184	23	165.5	100
2	955.7	290	13	419	100	24	252.1	100
3	483	100	14	1023.1	100	25	331.5	100
4	400.7	290	15	455.1	164	26	500	204
5	791.9	100	16	182.6	290	27	579.9	164
6	404.4	368	17	221.3	290	28	842.8	100
7	390.6	327	18	583.9	164	29	792.6	100
8	482.3	100	19	452	229	30	846.3	184
9	934.4	100	20	794.7	100	31	164	258
10	431.3	184	21	717.7	100	32	427.9	100
11	513.1	100	22	655.6	258	33	379.2	100
						34	158.2	368

case 4 = 20 and case 5 = 50; number of iteration for case 1 = 500, case 2 = 1000, case 3 = 10,000, case 4 = 5000 and case 4 = 2500. In proposed method, there is no need to change mathematical formulation for hydraulic analysis. An if-then rule is added to co-content model and optimization process is done easily. In this example, the bound variables were set between 0 and 36.4.

Table 5 compares different cases of algorithm DE for minimization of model. The best result is related to the case in which the number of population is equal to 50 and the number of iterations equal to 2500. After performing 10 different runs, the best value of δ is obtained equal to 0.0026, while the best result is obtained equal to 0.0181 in Giustolis method. The results of two mentioned methods are compared in the fifth and sixth columns of **Table 4**. In this table, the best result is shown in bold, and it is considered the method DE has calculated the best value of δ at 17 nodes and Gistulishi method has calculated it at 5 nodes. The convergence process of algorithm DE has been shown in two forms in **Figures 6** and **7**. The absolute value of δ is calculated for each iteration in **Figure 6** and the amount of objective function $C(H)$ is calculated for each iteration in **Figure 7**. The algorithm becomes convergent after 2500 iterations.

6.3. Numerical Example 3

Figure 8 shows the network from Mallick *et al.* [21]. The network consists of 2 reservoirs, 13 nodes and 21 pipes. The detailed properties are shown in **Tables 6** and **7**. It is supposed that the desired pressure for each node (H^*) is 30 m, and the minimum pressure ($H_{\min}$) is 10 m [22]. The pipe leakage coefficients are $\beta = 5 \times 10 - 7$ and

Table 4. Head and parameter δ in numerical example 2.

Node number	q (l/s)	H (m) [1]	H (m)	δ [11]	δ
1	10.863	26.9	33.29	0.1547426	**−0.0046**
2	17.034	24.81	31.83	0.02131049	**−0.00648**
3	14.947	21.3	27.39	−0.0477137	**−0.00498**
4	14.28	17.22	25.34	−0.0220368	**−0.00504**
5	10.133	23.54	30.89	−0.0261836	**−0.00378**
6	15.35	20.1	29.02	0.04038949	**−0.00517**
7	9.114	18.91	27.94	−0.0171474	**−0.00275**
8	10.51	17.9	27.34	**−0.0022701**	−0.00351
9	12.182	17.85	26.35	**0.0029365**	−0.00378
10	14.579	12.66	23.24	−0.008277	**−0.0043**
11	9.007	16.23	25.95	0.03155407	**−0.00258**
12	7.575	10.12	22.05	−0.0027315	**−0.00213**
Node number	q (l/s)	H (m) [11]	H (m)	δ [11]	δ
13	15.2	10.03	22.45	0.01259978	**−0.00418**
14	13.55	15.41	25.95	0.06347619	**−0.00418**
15	9.226	14	24.17	−0.0091379	**−0.00287**
16	11.2	14.36	24.05	−0.0070886	**−0.00357**
17	11.469	15.3	25.42	**−0.0001028**	−0.00354
18	10.818	18.83	28.38	0.01188886	**−0.00384**
19	14.675	19.35	28.39	**−5.43E-05**	−0.00505
20	13.318	10.01	23.79	−0.0377624	**−0.00398**
21	14.631	11.48	22.35	**0.00274996**	−0.00411
22	12.012	14	25.46	0.00390141	**−0.0036**
23	10.326	10.45	20.11	−0.0085767	**−0.00296**
24		36.45	36.45		

$\alpha = 1.18$, for this network. The DE technique is applied to solve this problem and DE model parameters selected are as follows: number of decision variables = 13; number of population for all cases = 20; number of iteration for case 1 = 1000, case 2 = 2500, case 3 = 5000 and case 4 = 10,000. The bound variables were set between 0 and 60.96.

In this example, the network has been analyzed according to two cases. In the first case, the dependence of pressure on the demand is not included, but in the latter case, the analysis is performed by taking into account the dependence of pressure on the demand. **Table 8** shows

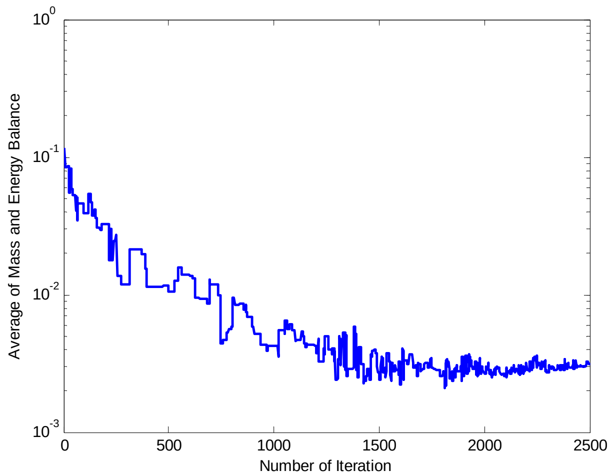

Figure 6. Convergence history of numerical example 2 (case 1).

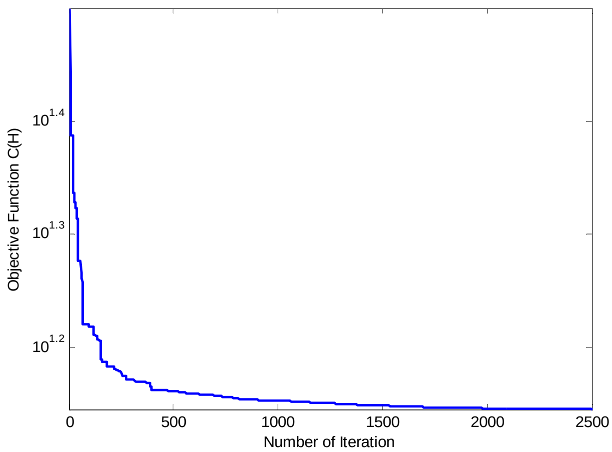

Figure 7. Convergence history of numerical example 2 (case 2).

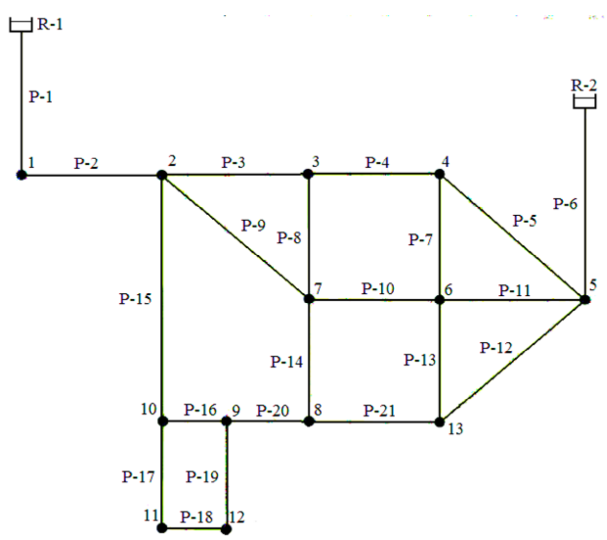

Figure 8. Schematic representation of the looped pipe network used in the numerical example 3.

Table 5. Average of mass and energy balance for numerical example 2.

DE		Mass and Energy Balance (δ)			
		best	worst	mean	std
Number of population	50	6.00E-03	1.76E-02	1.14E-02	3.60E-03
Number of iteration	500				
Number of population	50	2.60E-03	1.01E-02	5.90E-03	2.20E-03
Number of iteration	1000				
Number of population	20	3.90E-03	4.00E-03	4.00E-03	4.08E-05
Number of iteration	10,000				
Number of population	20	3.20E-03	3.80E-03	3.60E-03	1.88E-04
Number of iteration	5000				
Number of population	50	2.60E-03	5.50E-03	3.00E-03	5.88E-04
Number of iteration	2500				
Giustolisi Algorithm [11]		Maximum Accuracy			1.81E-02

Table 6. Pipe characteristics of Sample network from Mallick *et al*. [21].

Pipe Number	L (m)	D (mm)	C
1	609.6	762	130
2	243.8	762	128
3	1524	609	126
4	1127.76	609	124
5	1188.72	406	122
6	640.08	406	120
7	762	254	118
8	944.88	254	116
9	1676.4	381	114
10	883.92	305	112
11	883.92	305	110
12	1371.6	381	108
13	762	254	106
14	822.96	254	104
15	944.88	305	102
16	579	305	100
17	487.68	203	98
18	457.2	152	96
19	502.92	203	94
20	883.92	203	92
21	944.88	305	90

Table 7. Nodes properties of Sample network from Mallick *et al.* [21].

Node Number	q (L/S)	Elevation (m)
1	0	27.43
2	59	33.53
3	59	28.96
4	178	32
5	59	30.48
6	190	31.39
7	178	29.56
8	91	31.39
9	0	32.61
10	0	34.14
11	30	35.05
12	30	36.58
13	0	33.53

Table 8. Average of mass and energy balance for numerical example 3.

DE		Mass and Energy Balance (δ)			
		best	worst	mean	std
Number of population	20	1.90E-03	1.11E-02	5.30E-03	2.60E-03
Number of iteration	1000				
Number of population	20	1.46E-04	7.03E-04	4.55E-04	1.72E-04
Number of iteration	2500				
Number of population	20	1.06E-06	1.37E-06	5.00E-06	4.32E-06
Number of iteration	5000				
Number of population	20	1.18E-08	2.90E-08	1.75E-08	6.29E-09
Number of iteration	10,000				

how the accuracy of parameter δ depends on the iteration number of convergence in the first case. As it can be seen, the value of δ reaches the accuracy 1e-2 after 1000 iterations, the accuracy 1e-4 after 2500 iterations, 1e-6 after 5000 iterations and 1e-8 after 10,000 iterations. **Figure 9** compares the nodal pressures in the mentioned two cases. As it can be observed, if the dependence of pressure on the demand is not included, a negative pressure is made at the nodes 6, 8, 11 and 12, but all pressures in the second case are greater than 5 meters and the minimum pressure of 10 meters has been partly supplied in most of the nodes. **Figure 10** shows the process of changing the

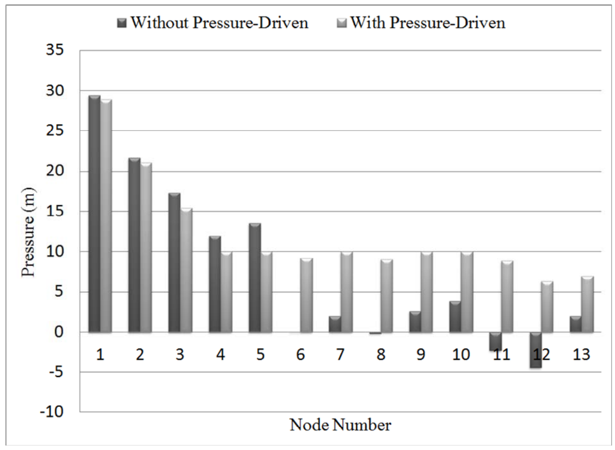

Figure 9. Simulation results with and without pressure-driven demand.

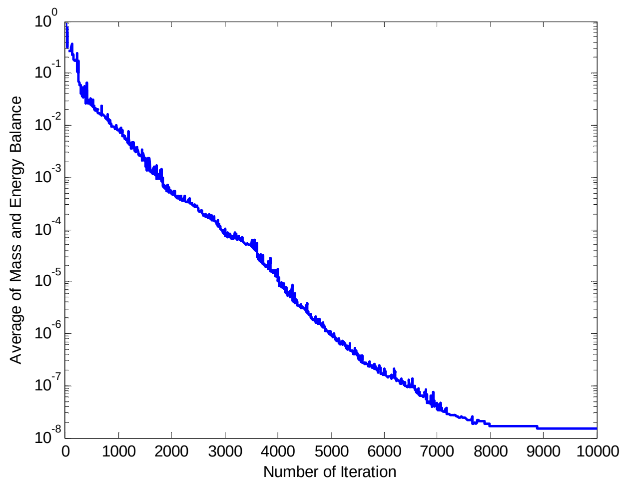

Figure 10. Convergence history of numerical example 3.

parameter δ towards the number of iterations.

7. Conclusions

The purpose of this paper has been to introduce a novel methodology for hydraulic analysis of water distribution systems under deficient pressure conditions considering the pressure-driven demand and leakage. The methodology is illustrated using three networks with different layouts.

The overall results indicate that the proposed method has the capability to handle various pipe networks problems without changing in model and mathematical formulation. Application of DE to co-content model can solve pressure-driven demand and leakage simulation with applying if-then rules in co-content model. The advantage of the proposed methodology is its flexibility in employing different formulation and specifying parameters related to pressure-driven demand. Another advantage of this method is that it can be easily developed for common users to undertake deficient pressure conditions.

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