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A Resolution of the Black Hole Information Paradox via Transfinite Set Theory

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Abstract

A black hole is essentially a relativistic as well as a quantum object. Therefore the information paradox of black holes is a consequence of the clash between these two most fundamental theories of modern physics. It is logical to conclude that a resolution of the problem requires some form of a quantum gravity theory. The present work proposes such a resolution using set theory and pointless spacetime geometry.

Keywords

Information Paradox, Black Holes, S. Hawking, G. 't Hooft, L. Susskind, Transfinite Set Theory, Noncommutative Geometry, Measure Concentration, Dvoretzky's Theorem, Dark Energy, Casimir Effect, Nano Casimir Reactor

1. Introduction

Physically empty space with no matter, virtual particles or radiation what so ever is still far from being nothing [1] [2]. Empty spacetime in the above sense is a sophisticated sloika, *i.e*. Mille-Feuill of empty sets [1]-[20]. A point in such space is by no means the classical point defined somewhat naively as the intersection of two lines [7]. In fact a proper spacetime, which satisfies the physical quantum reality, is a points-less spacetime with geometry and topology akin to that of von Neumann's continuous geometry [76] [119] [132] as well as A. Conne's noncommutative geometry [76] [132] apart from the arch typical pointless space of E-infinity Cantorian spacetime [11]-[132]. In such Cantorian-fractal geometry what appears to be a point is in actuality an entire Cantor set when magnified by sharpening the resolution of observation [33]-[130]. Combining the preceding fundamental insights gained from applying transfinite set theory and fractals to spacetime geometry with a fundamental theorem due to Aryeh Dvoretzky about measure concentration [65]-[67], we can argue that only 4.5 percent of the information inside a black hole could be regarded as inaccessible while the rest of the 95.5 percent of the infor-

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mation remains on the surface of the black hole no matter how much it shrinks because in the end analysis, spacetime has no ordinary points and no matter how small such points are, they are not zero nor can they vanish into nothingness because empty spacetime is anything but nothing. In fact empty spacetime is a multi-fractal made of infinitely many empty sets with varying degrees of emptiness [1] [2] [12]. In this sense we can reconcile what appeared for a considerable time irreconcilable, namely the view point of S. Hawking [7] with all that speaks for it and speaks against it, with the clearly opposing views of L. Susskind and G. 't Hooft which is understandable, logical and correct, but nevertheless does not propose an alternative waterproof answer to replace that of S. Hawking's famous but by no means entirely correct one [7].

The paper is subdivided into three reasonably short parts. First, we introduce the required background information. Second, we sketch our basic analysis and finally we summarize our conclusions. Last but not least, it should be acknowledged from the outset that the recent revival of black hole physics and the information paradox at the prominent conference in Stockholm [133]-[135] as well as the highly influential writing of L. Susskind and G. 't Hooft [135]-[139] are the prime movers and motivation for the present work.

2. Physical, Mathematical and Cosmological Measure Concentration Phenomena

There are at least three well known types of measure concentration phenomena [64]-[67] [140] relevant to the present work. The first is what we could call physical measure concentration. We could give two examples for that. The first is the Faraday cage demonstrating that for a metal sphere the total electric charge is concentrated on the surface with zero change inside the sphere [7]. This fact is behind the fortunate situation that an aeroplane flying in a thunder and lightening storm can protect the passengers from being electrocuted as is the case with a car with closed doors and windows. The second example is the energy of the quantum particle-wave where 4.5% of the energy is inside the quantum wave concentrated in the quantum particle as measurable ordinary energy while the quantum wave surrounding the quantum particles possesses most of the energy density, namely 95.5% as dark energy [14]-[61]. The best example for mathematical measure concentration is the marvellous Dvoretzky's theorem which states that in sufficiently high dimensionality, 96% of the volume of a sphere is concentrated near to its surface while the "bulk" contains only 4% of the volume [140]. Finally cosmological concentration phenomena may be found theoretically in the fact that all information of a black hole is proportional to the surface and not to the volume of the black hole as demonstrated by the Bekenstein formula [7] [138]. As a second example we could cite the work of the present author showing that the 4.5% ordinary energy of the universe can be measure "inside" the universe while the rest, the 95.5% dark energy exists mainly near to the boundary of the holographic boundary of the universe [7] [11]. It is the fact that our universe could be viewed as a giant black hole that we may argue that black holes, if they exist, will have most of its information, *i.e.* about 96% of the information on the surface of the black hole while 4% of the information could remain locked inside the black hole to which an outside observer will have no access.

3. The Transfinite Theory of Spacetime

To make a long, in fact very long story short, we start our journey to the exact picture of our physico-mathematical spacetime from the bijection formula [74] [75].

$$d_c^{(n)} = (1/\phi)^{n-1} \tag{1}$$

corresponding to von Neumann-Conne's dimensional function [74] [75] of Penrose foliated universe, *i.e.* Penrose tiling. This is necessarily a fractal universe resembling a compactified holographic boundary, *i.e.* compactified Klein modular curve for a transfinite E8E8 Lie exceptional symmetry groups constituting our bulk [74] [75]. Here $d_c^{(n)}$ is the Hausdorff dimension corresponding to n Menger-Urysohn dimension and $\phi = 2/(1+\sqrt{5})$ is the Hausdorff dimension of the zero set, *i.e.* $n = 0$. To see that we set $n = 0$ in $d_c^{(n)}$ and find [74] [75]

$$d_c^{(0)} = (1/\phi)^{0-1} = (1/\phi)^{-1} = \phi. \tag{2}$$

The corresponding von Neumann-Conne formula is [76]

$$D = a + b\phi \tag{3}$$

where $a, b \in \mathbb{Z}$ and $\phi = (\sqrt{5}-1)/2$. Setting $a = 0$ and $b = 1$ one finds the same value, namely

$$D = 0 + (1)(\phi) = \phi = d_c^{(0)}. \quad (4)$$

This is our zero set which models the pre-quantum particle [129]

$$D(QP) \equiv (0, \phi) \quad (5)$$

for the pre-quantum wave, *i.e*. the surface or cobordism of the quantum particle we just need to insert $n = -1$ which is the dimension of the neighbourhood of a point. That way we find [74] [75]

$$d_c^{(-1)} = (1/\phi)^{-1-1} = (1/\phi)^{-2} = \phi^2 \quad (6)$$

or equivalently by setting $a = 1$ and $b = -1$ one finds [74]-[76]

$$D(QW) = 1 - \phi = \phi^2 = d_c^{(-1)}. \quad (7)$$

This is the empty set which models the quantum particle. We see that the zero set $d_c^{(0)}$ separates the sets $d_c^{(n)}$ from the empty sets $d_c^{(-n)}$ and we are thus justified to speak of the degree of emptiness of an empty set [74]-[76] as we move from $n = 1$ to $n = -2$ until we reach the truly insubstantial nothingness for $n = -\infty$ which leads to $d_c^{(-\infty)} = (1/\phi)^{-\infty} = \phi^{\infty} = \text{zero}$.

4. The Menger-Urysohn Dimensionality and the Dimensions of the Empty Set *d*(Menger) = −1

The discussion of the section on the transfinite theory of spacetime depended crucially upon the extension of the notion of topological dimensions into the negative regime. In this section we show how simple, intuitive and easily grasped this extension of the deductive topological dimension theory which goes back to the Russian Paul Urysohn and the Austrian Karl Menger [74]-[76]. Take a 3D cube. The dimension of the cube is 3 but the dimension of the six sides of the surface of the cube is $D = 3 - 2 = 1$ which is a trivial result. Applying the same argument to the two dimensional surface, the borders are lines and therefore we have $D = 2 - 1 = 1$ which is equally trivial. Continuing this for the line we find that the dimensions of the edges of the line must follow the same formula, namely $D = n - 1$. For a line $n = 1$ we find edge points $D = 1 - 1 = 0$, again a trivial result. Here triviality stops because continuing for the point $n = 0$ one finds a non-trivial result, namely $D = 0 - 1 = -1$. Earlier on we called this the empty neighbourhood of a point or the empty set [1] [2]. Continuing for $D = -\infty$ one finds then the truly completely empty set with the Hausdorff dimension $D^{(-\infty)} = (\phi)^{\infty} = \text{zero}$ and the bidimension representation [12] [74]-[76]

$$D(-\infty) \equiv (-\infty, 0). \quad (8)$$

5. An Exact Picture of Quantum Spacetime

We clearly live in a 3 + 1 = 4 dimensional world. These are three special dimensions plus a temporal dimension preventing things from happening all at once in some folkloristic philosophy. Einstein's special and general relativity takes the time dimension far more seriously as a dimension which one can put on equal footing as the space dimension if not even more [7]. Inserting $n = 4$ in our bijection formula one finds [1] [2] [74]-[76]

$$d_c^{(4)} = (1/\phi)^{4-1} = (1/\phi)^3 = 4 + \phi^3. \quad (9)$$

Consequently we have [12]

$$D(4) \equiv (4; 4 + \phi^3). \quad (10)$$

This means the fractal Hausdorff dimension is larger than the corresponding Menger-Urysohn topological dimension by the amount ϕ^3 which we will see later on that it is equal to the intrinsic so called latent topological Casimir pressure of empty spacetime [1] [2]. Now having found $4 + \phi^3$ is an important result but we would like to scrutinize this result and analyse it in a far deeper way.

Let us look back at our spacetime as being made of an infinite mixture of all possible Cantor sets. That means starting from the unit set, *i.e*. classical line $d_c^{(1)} = 1$ and the zero set $d_c^{(0)} = \phi$ until we reach the totally empty set. The sum of all these sets is clearly [1] [2].

$$\sum_{1}^{-\infty} d_c^{(n)} = d_c^{(1)} + d_c^{(0)} + d_c^{(-1)} + \cdots = 1 + \phi + \phi^2 + \phi^3 + \cdots = \frac{1}{1-\phi} = \frac{1}{\phi^2} = 2 + \phi \quad (11)$$

Now since $d_c^{(0)} = \phi$ represents for us a random Mauldin-Williams triadic Cantor set [1] [2] living in one dimension, then gaging the sum $2+\phi$ in terms of this Cantor set, we can say that the dimension of our mixture of Cantor sets is simply [1] [2] [74]-[76].

$$D = (2+\phi)/\phi = (2+\phi)(1/\phi) = (2+\phi)(1+\phi) = \left(1/\phi^2\right)(1/\phi) = (1/\phi)^3 = 4 + \phi^3 \quad (12)$$

which is the Hausdorff dimension of our spacetime. However there is a still more profound interpretation of these results because $1+\phi = 1/\phi$ is the inversion of the Hausdorff dimension ϕ^2 of a quantum particle while $(1/\phi)^2 = 2+\phi$ is the inversion of the Hausdorff dimension of the quantum wave ϕ^2. In other words $4+\phi^3$ is both the intersection between the particle like behaviour given by $(1/\phi) = 1+\phi$ and the wave like behaviour dimension $(1/\phi)^2 = 2+\phi$

$$(1+\phi)(2+\phi) = 4+\phi^3 \quad (13)$$

as well as the union of the two, namely

$$(1+\phi)+(2+\phi) = 4+\phi^3 \quad (14)$$

where $(1+\phi)$ and $(2+\phi)$ are the un-normed probability of particle and wave respectively. In other words our space is blind to the union and intersection and cannot tell them apart which is the deepest explanation possible for the particle-wave duality of quantum mechanics [1] [2]. Armed with all the preceding results let us see if we can construct a simple exact picture of micro spacetime.

We start with the zero set pre-particle. This is

$$D(0) = (0,\phi). \quad (15)$$

Surrounding $D(0)$ is its cobordism or the guiding quantum wave which means the empty set [1] [2]

$$D(-1) = \left(-1,\phi^2\right). \quad (16)$$

The cobordism, or surface of the guiding Bohm-Einstein quantum wave, is clearly $D(-2)$ which is given by [1] [2]

$$D(-2) = \left(-2,\phi^3\right). \quad (17)$$

Now something quite remarkable happens at this point. The particle wave entity is floating in spacetime with an average dimension $4+\phi^3$ which means a normed average dimension $1/\left(4+\phi^3\right) = \phi^3$. That means spacetime is the cobordism, *i.e.* the surface of the guiding Bohm-Einstein, ergo ghost quantum wave and consequently the surface of the quantum wave is given by the expectation value [2]

$$\langle D(-2)\rangle = \left\langle (-2),\phi^3\right\rangle. \quad (18)$$

In other words our quantum spacetime is nothing but a zero set pre-quantum particle surrounded by an empty set pre-quantum wave floating in the surface of the quantum wave given by the expectation value $\left\langle -2,\phi^3\right\rangle$. The ghost is no ghost at all unless we consider spacetime to be a ghost. In the next section we will see how all of that relates to Casimir energy and dark energy [2].

6. The Casimir Local Topological Pressure

If two plates which are conducting but uncharged are put very close vis-à-vis each other, it is an experimental fact that they are pulled together by what is known as the Casimir effect [94] [120] [121]. The E-infinity set theoretical explanation advanced some time ago is as follows: The nano distance between the two plates is as near as we can come to create an empty set. That means inside the plates we have a topological pressure ϕ^2 stemming from the state of the entropy-like disorder measured by the Hausdorff dimension ϕ^2 of the empty set. Outside, adjacent to the plates, we have the zero set of the quantum particles. Consequently the net topological pressure is the difference between the empty set quantum wave ϕ^2 and the zero set quantum particle which means $\phi - \phi^2 = \phi^3$ giving us the intrinsic latent topological pressure of empty spacetime which by inversion, leads to the Hausdorff dimension $4 + \phi^3$. In other words the Casimir topological pressure is the same at the counter factual part in the general formula for Hardy's quantum entanglement, namely [58] [68]-[71]

$$P = \phi^{n+3} \tag{19}$$

and setting the number of particles $n = 0$ we are left with the global part [1]-[34]

$$P = \phi^3. \tag{20}$$

Thus we could think of ϕ^3 as the excess in the fluctuation of spacetime dimension $4 + \phi^3$ over the average topological dimension 4 causing the extra pressure of ϕ^3 which manifests itself locally as the Casimir effect. A pedestrian way to understand that is the following: The un-normalized probability due to quantum particles is $1/\phi = 1 + \phi$ and for the quantum wave is $1/\phi^2 = 2 + \phi$. The minimal integer approximation of the sum of both is clearly 1 + 2 = 3 and the maximal is 3 + 2 = 5. Consequently the average integer value is $(3+5)/2 = 4$. On the other hand D is not 4 but $4 + \phi^3$ showing that ϕ^3 is indeed a measure for topological dimensional or entropic fluctuation in full agreement with all previous conclusions which we made earlier on [7] [11]-[20].

7. Dark Energy Is Global Dvoretzky Concentration of Casimir Energy

The two Casimir plates of the classical Casimir effect experiment, plays obviously the role of a boundary condition. What happens then when this boundary is pushed to infinity? The answer is it becomes the boundary of the holographic boundary of the universe [7]. This boundary is obviously a one sided boundary with nothing outside to push back to create a statical balance. It is a Möbius-like multi-dimensional boundary and consequently the universe must expand into the insubstantial nothingness surrounding our universe which is a clopen (which means open and closed topologically) and a multiverse of its own self. Considering that this E-infinity multiverse universe has formally infinite dimensions, we see that the Dvoretzky theorem [140] will apply and as a result 95.5% of the energy of this universe will be concentrated at the edge of the universe. Consequently the difference between the Casimir energy and dark energy is the difference between local and global as well as two sided and one sided boundary conditions [120]-[127].

8. How to Build a Nano Casimir-Dark Energy Reactor

From the preceding discussion it is an inescapable conclusion that we could build a nano universe using nano technology and that such a universe will constitute a unit cell of a nano Casimir-dark energy reactor [94] [96] [120] from which we can extract an infinite amount of clean energy using empty spacetime as our fuel.

To start we can use a large amount of C_{60} Fullerene [7] nano particles which are the smallest dodecahedron [7] known to exist at present. There are many reasons to suspect that the universe geometry and topology resembles that of a dodecahedron so that each C_{60} particle is an approximately universe prototype geometrically and topologically speaking [7]. Each two faces of the C_{60} powder will act as a Casimir cell and the complexity of the heap will produce an artificial local high dimensionality for which Dvoretzky's theorem applies [140]. Adding a large number of these artificial nano universes we end up with a macroscopic multiverse from which energy can in principle be extracted from its most outer surface. The rest is technological details. However we are more than aware that the devil lies in the detail. Nevertheless all that we can say at this point is that we will use in principle the same technology used to minimize the usually harmful effects of the Casimir effect on nano devices only this time in a reversed fashion, *i.e.* trying to maximise these effects rather than minimizing it [120]-[127].

9. A Set Theoretical Resolution of the Black Hole Information Paradox

We started by discussing black holes, then moved from there to Casimir energy, dark energy and nano reactors [140]. With the benefit of hindsight let us see what the connection is between these seemingly very different problems. Maybe it is good for a deep understanding of the problem at hand to ask oneself can we really solve a mind teaser like the information paradox without knowing really what space is, what time is and worse still, what the word nothingness means. The present author admits that he thought that this is not possible and that was exactly his point of departure. First of all totally empty space is not nothing but something substantial even without any fluctuation or pair creation and annihilation. Empty spacetime is a multidimensional empty set. The empty set is far from being nothing. As soon as you mention the word set then the word nothing is not there. Insubstantial nothingness is not even a set. The border line is the totally empty set given by (ϕ) to the power of (infinity). The empty set on the other hand is given by two dimensions, namely minus one and ϕ^2 both of which are not zero or nothing. Consequently by shrinking a spherical black hole indefinitely it can never become a zero point with a zero surface area because quantum space has no ordinary (naïve) classical point exactly as the pointless spaces of von Neumann's noncommutative geometry and E-infinity Cantorian spacetime [7]. The information density becomes extremely large but not infinity as the black hole becomes extremely small but never zero so that at the end about 95.5% of the information encoded on the surface of the black hole is never lost and only 4.5% of the information inside the black hole becomes inaccessible to us. Paradoxically for us living inside this giant black hole we call the universe, the situation is reversed for something related to information, via entropy which we call energy [120]-[127]. Only 4.5% of the energy of the universe is accessible to us while 95.5% of the energy of the universe cannot be measured directly and we known that it is there only because of observing its effect manifested via the accelerated cosmic expansion [140].

10. Conclusion

Actual empty spacetime is far from being nothing and is a multi-dimensional empty set, which is a substantial something. This spacetime is pointless and what appears to be a point is, upon magnification, a complete random Cantor set. In such spacetime a black hole will always have a non-zero surface area no matter how small it shrinks and will never vanish and have a zero surface area. In such a situation, and by a well known brilliant theorem due to the great late A. Dvoretzky who was at a time the President of the Wiseman Institute, at least 95.5% of the information of a black hole will not be lost. The situation is analogous to that of the ordinary and the dark energy of the universe and could be used as a guiding principle in the design of a nano Casimir-dark energy reactor. In the end analysis we think that our inability to give a waterproof definition to the words point, line and so on coupled with mildly ignoring the impact of nonlinear dynamics, chaos and fractals on fundamental quantum physics contributes to a delay of the development of black hole research and related subjects. We hope the present work has at a minimum helped positively in this direction.

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