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Portfolio Selection by Maximizing Omega Function using Differential Evolution

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ABSTRACT

Paper presents alternative solution seeking approach for portfolio selection problem with Omega function performance measure which allows determining capital allocation over the number of assets. Omega function computability is difficult due to substandard structures and therefore the use of standard techniques seems to be relatively complicated. Differential evolution from the group of evolutionary algorithms was selected as an alternative computing procedure. Alternative approach is analyzed on the Down Jones Industrial Index data. Presented approach enables to determine good real-time solution and the quality of results is comparable with results obtained by professional software.

Keywords: differential evolution, Omega function, problem of portfolio selection

1. Introduction

In general, portfolio theory deals with the selection of an appropriate mix of assets in a portfolio in order to meet predetermined properties. Various mathematical models, which measure the portfolio performance measurement can be used to support the decision making process of selection of the portfolio assets. The aim is to determine the allocation of the available resources in the selected group of assets that results in maximization of portfolio performance. Follow this idea, various measurements of performance can be used. The performance measurement techniques are e.g. Sharpe ratio, Treynor ratio, Jensen's alpha, Information Ratio, Sortino ratio, Omega function and the Sharpe Omega ratio ([1], [2], [3], [4], [5], [6], [7], [8]). The paper deals with Omega function, which computability is difficult due to substandard structures of performance level and therefore the use of standard techniques seems to be relatively complicated. The alternative computing procedures includes variety of different approaches. Nowadays a lot of research attention is focused on evolutionary algorithms. The paper presents the algorithm of differential evolution that is able to deal with nonlinear objective function with success (enables quick achievement of suboptimal solutions) and thus demonstrates suitability of evolutionary algorithms for financial modeling. Above mentioned approach is analyzed on assets included in the Down Jones Industrial Index and its historical data[1] published from July 1st 2001 to April 1st 2012 on weekly basis were used.

[1] http://finance.yahoo.com/ (2012)

2. Portfolio selection models based on Omega function

Omega function is measure which incorporates all the distributional, characteristics of a return series. The measure is a function of the returns leveled and requires no parametric assumption on the distribution. Precisely, it considers the returns below and above a specific loss threshold and provides a ratio of total probability weighted losses and gains that fully describes the risk reward properties of the distribution [8]:

$$\Omega(MAR) = \frac{\int_{MAR}^{b} (1 - F(x))dx}{\int_{a}^{MAR} F(x)dx}$$

where:
MAR denotes the return level regarded as a loss threshold,
(a, b) denotes yields range,
$F(x)$ the cumulative distribution function of asset returns.

In the next part we formulate the problem of portfolio selection based on omega function performance measure. As it is mentioned, the Omega function involves consideration of all the information contained in the time series of returns. The aim of portfolio selection problem is to maximize the level of Omega performance measure, where the variables $w_1, w_2, \ldots w_d$ (where d represents the number of assets) represent the weights of each asset in the portfolio. Corresponding problem can be formulated as follows [9]:

$$\max\ \Omega(\mathbf{w}) = \frac{\sum_{t=1}^{T} \max\left(\mathbf{w}^{\mathrm{T}}\mathbf{r}_t - MAR, 0\right)}{\sum_{t=1}^{T} \max\left(MAR - \mathbf{w}^{\mathrm{T}}\mathbf{r}_t, 0\right)}$$

Subject to:

$$\mathbf{w}^{\mathrm{T}}\mathbf{e} = 1$$

$$\mathbf{w} \geq 0$$

Where:

T represents the number of periods,

$\mathbf{r}_t$ represents returns vector of portfolio assets in the period $t = 1,2,\ldots T$,

$\mathbf{w}$ denotes a vector of variables $w_1, w_2, \ldots w_d$ representing the weight of each asset in the portfolio.

The computational complexity of the presented problem arises from its non-standard structure. Therefore, evolutionary algorithms seem to be a suitable alternative to standard techniques, due to its ability to achieve the suboptimal solutions in relatively short time. The differential evolution is one of the popular and well known techniques.

3. Differential Evolution

Differential evolution (introduced by Price and Storn [10]) belongs to the class of evolutionary techniques, comprise a large number of nontraditional computing techniques whose common characteristic is that they are inspired by the observation of the nature processes (genetic algorithms, ant colony optimization, differential evolution, etc.). Nowadays evolutionary algorithms are considered to be effective tools that can be used to search for solutions of optimization problems (napr. [11], [12], [13], [14]). The big advantage over traditional methods is that they are designed to find global extremes (with built-in stochastic component) and that their use does not require a priori knowledge of optimized function (convexity, differential etc.), and in that way they work well to solve continuous non-linear problems, where is hard to use traditional mathematic methods. The principle of basic version of differential evolution can be described by following pseudocode:

BEGIN
SETTING of control parameters;
INITALIZATION of population;
  EVALUATION of each individual;
  WHILE (*STOPPING CRITERION* is not satisfied) **DO**
    FOR (each individual of the population) **DO** (*REPRODUCTIVE CYCLE*):
      CREATE differential vector;
      CREATE trial vector;
      CREATE test vector;
      IF (*EVALUATION* of test vector) > (*EVALUATION* of current selected individual) **THEN** (*SUBSTITUDE* the selected individual with the test vector) ;
      ENDIF
    ENDFOR
  ENDWHILE
  EVALUATE process of calculating;
END

Evolutionary algorithms differ from more traditional optimization techniques in such a way that they involve a search from a "population" of individuals, not from a single one. Each individual represents one candidate solution for the given problem that is represented by parameters of individual. Associated with each individual is also the *fitness*, which represents the relevant value of objective function. A population can be viewed as $np.d$ matrix (np – number of individuals in the population, d – number of parameters of individual). Every step involves a competitive selection that carried out poor solutions. Consider the problem of portfolio selection it is possible to summarize the steps of the algorithm as follows:

Setting of the control parameters. Differential evolution is controlled by a special set of parameters. Recommended values for the parameters are usually derived empirically from experiments ([15], [16], [17]):

d – dimensionality. Number of parameters of individual is equal to number of assets.

np – population size. Number of individuals in population. recommended setting is 5d to 30d, respectively 100d, in case the optimized function is multimodal ([15], [16]).

g – generations. Represent the maximum number of iteration (g is also stopping criterion).

cr – crossover constant, $cr \in \langle 0,1 \rangle$. The value of cr was set on the base of experiments.

f – mutation constant, $f \in \langle 0,1 \rangle$. The value of f was set on the base of experiments.

Initialization. The population $\mathrm{P}^{(0)}$ was randomly initialized at the beginning of evolutionary process according to the rule:

$$\mathrm{P}_{(i)}^{(0)} = w_{i,j}^{(0)} = \frac{rnd\langle 0,1 \rangle}{\sum_{j=1}^{d} w_{i,j}^{(0)}} \quad i = 1,2,\ldots np \;\; j = 1,2,\ldots d$$

that ensure that the total weights of portfolio is equal to one. Each individual is then evaluated with the *fitness* (given by function $\Omega(\mathbf{w})$).

The test of stopping condition. In its canonical form, the only stopping criterion is to reach the maximal number of iterations (represent by parameter g).

Reproductive cycle. This cycle comprise the crossing and mutation to create individuals for the next generation. For each individual wig, i =1, 2,...np, from the population another three different individuals are chosen (vectors r1, r2, r3). The difference of the first two vectors (r1 and r2) gives the differential vector, which is multiplied by mutation constant f and added to vector r3. Thus, we get trial vector v. Formally:

$$\mathbf{v}_j^t = w_{r3,j}^t + f.\left(w_{r1,j}^t - w_{r2,j}^t\right) \quad j = 1,2,...d\,, \quad t = 1,2,...g$$

After the mutation process comes the formation of a new individual, which is also called test vector wtest so that one element after another is selected from the currently selected individual wig and from the trial vector v and for every pair is generated a random number from the interval $\langle 0,1 \rangle$, which is compared with the crossing constant cr. If the generated random number is less than or equal to cr, to the relevant position of wtest comes the element of trial vector v, otherwise of current selected individual wig. Formally:

$$w_j^{test} = \begin{cases} w_{r3j}^g + f\left(w_{r1j}^g - w_{r2j}^g\right), & \text{if } rand_j \langle 0,1 \rangle \le cr \vee j = k \\ w_{ij}^g, & \text{otherwise} \end{cases}$$

where

$i = 1,2,...np\,, \quad j = 1,2,...d\,, \quad k \in \{1,2,...d\}$, for

$\mathbf{r1}, \mathbf{r2}, \mathbf{r3} \in \{1,2,...np\}\,, \quad \mathbf{r1} \ne \mathbf{r2} \ne \mathbf{r3} \ne i$

To ensure the feasibility of solution we use the following rule: if $w_j^{test} < 0$, then $w_j^{test} = rnd\langle 0,1 \rangle$ and

$$\mathrm{P}_{(i)}^{test} = \mathrm{w}_{i,j}^{test} = \frac{w_j^{test}}{\sum_{j=1}^{d} w_j^{test}} \qquad i = 1,2,...np \;\; j = 1,2,...d\,.$$

where k is a random index, which always ensures a change of at least one parameter in the test vector. The value of the objective function for the test vector is compared to the value of objective function of the current selected individual and to the next generation is selected the vector with the better objective value.

$$\mathbf{w}_i^{g+1} = \begin{cases} \mathbf{w}^{test}, & \text{ak } f_{cost}(\mathbf{w}^{test}) \le f_{cost}(\mathbf{w}_i^g) \\ \mathbf{w}_i^g, & \text{otherwise} \end{cases}$$

So that process continues in each generation for all individuals. The result is a new generation with the same number of individuals.

Evaluation. The whole process of reproduction continues until the last (users specified) number of generations is reached. The value of the best individual from each generation is reflected to history vector, which shows the progression of an evolutionary process.

4. Empirical Results

The portfolio analyze was based on index Dow Jones Industrial, which is one of the major market indexes, as well as one of the most popular indicators of the U.S. market. It is a stock market index, and one of several indices created by Wall Street Journal editor and Dow Jones & Company co-founder Charles Dow. It was founded on May 26, 1896. It is an index that shows how 30 large publicly owned companies based in the United States have traded during a standard trading session in the stock market. (so called Large-Cap companies – companies with market capitalization above 10 mild USD). Data[2] are processed weekly for the period July 1st 2001 to April 1st 2012. A total of 559 data were analyzed.

Table 1 : Company overview DJI.

Company Name	Ticker
4M Co.	MMM
Alcoa Inc.	AA
American Express Co.	AXP
AT&T Inc.	T
Bank of America Corp.	BAC
Boeing Co.	BA
Caterpillar Inc.	CAT
Chevron Corp.	CVX
Cisco Systems Inc.	CSCO
Coca-Cola Co.	KO
DuPont	DD
Exxon Mobil Corp.	XOM
General Electric Co.	GE
Hewlett-Packard	HPQ
Home Depot Inc.	HD
IBM	IBM
Intel	INTC
Johnson & Johnson	JNJ
JPMorgan Chase & Co.	JPM
Kraft Foods Inc. Cl A	KFT
McDonald's Corp.	MCD

[2] http://finance.yahoo.com/ (2012)

Merck & Co. Inc.	MRK
Microsoft Corp.	MSFT
Pfizer Inc.	PFE
Procter & Gamble Co.	PG
Travelers Cos. Inc.	TRV
United Technologies Corp.	UTX
Verizon Communications Inc.	VZ
Wal-Mart Stores Inc.	WMT
Walt Disney Co.	DIS

The input parameter of threshold (*MAR*) was set to 0.055. A disadvantage of algorithm of differential evolution, as well as of other evolutionary approaches, is that it has a dependence on the control parameter setting. Due to this fact, our effort was to determine effective settings of the parameters *f* and *cr*. The tests were done on above mentioned data, with the simultaneous use of the set $np = 300$ a $g = 500$. The tested values of parameters *f* and *cr* were from the interval $\langle 0,1 \rangle$ as sequence of levels 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. The interval limits were not considered during testing (purely deterministic and purely stochastic nature of the algorithm). For each combination of pairs, five experiments were conducted. The average value of Omega functions for each combination of pairs is shown in Figure 1. The control parameters were set on the base of the article [18], which describes the possibility of setting the parameters with the help of some statistical methods e.g. Kruskal-Wallis test, Bartlett's test, Cochran-Hartley's test.

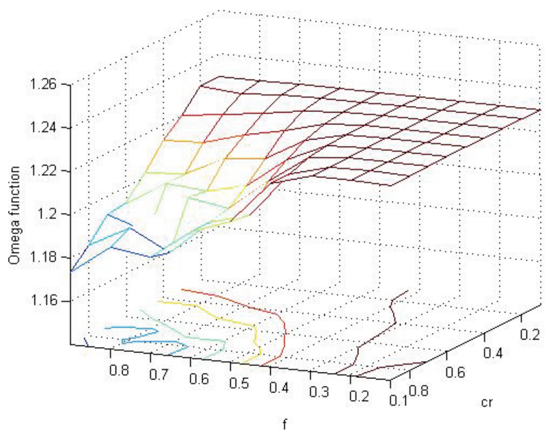

Figure 1

The algorithms were implemented in MATLAB 7.1. Two functions were created: Differential evolution adapted for solving portfolio selection problem and the function for calculation of objective function (Omega function) value. All the experiments were run on PC INTEL(R) Core(TM) 2 CPU, E8500 @ 3.16 GHz, 3.25 GB RAM under Windows XP. The best result was the value 1.230054969 of Omega function.

Based on the testing parameters problem was ten times re-solved (Table 2) with parameter settings $f = 0.1$, $cr = 0.2$, $np = 3000$ a $g = 2000$, and best obtained value of the Omega function was 1.230055034. Convergence of the solution can be seen in Figure 2. The rapid convergence till 200 generations is evidently seen from the Figure 2. Values obtained after 200 generations are close enough to the final solution.

Based on model results, recommended allocation is to invest assets in McDonald's companies at the rate 82.230 %, Caterpillar Inc. at the rate 13.998 % and IBM 3.772 %. According to result achieved, it is not recommended to invest to other companies since values of weights (variables) are equal to 0 %.

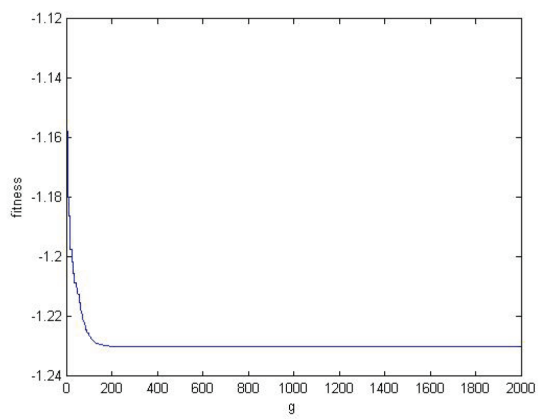

Figure 2

Table 2 : Solutions values

	Value of Omega function
Solution 1	1.23005496457
Solution 2	1.23005497790
Solution 3	1.23005498325
Solution 4	1.23005495551
Solution 5	1.23005496369
Solution 6	1.23005498844
Solution 7	1.23005498579
Solution 8	1.23005499256
Solution 9	1.23005504187
Solution 10	1.23005500989
MAX	1.23005504187
MIN	1.23005495551
Average	1.23005498635

Mentioned problem was also solved using the Risk Solver Platform V.12.0, with the result equal to 1.23004199, which is a smaller value compared to the best value computed of Omega function (the difference is 0.000013). The relevance of presented approach is demonstrated also by the fact even lower values of control parameters (*np* = 300 and *g* = 500) provided the solution on the level 1.230054969. Based on showed results, it can be stated the suitability of presented approach, which enables to determine the good real time solution.

5. Conclusion

The portfolio selection problem is one of the basic problems of allocating capital over the number of assets. From different sets of performance measurement tools to assist us with our portfolio evaluations, authors chose portfolio performance measure - Omega function, which computability is difficult due to substandard structures and therefore the use of standard techniques seems to be relatively complicated. Based on it, we use one of the evolutionary algorithms that allow solving various types of optimization problems (differential evolution). Presented approach enables to determine good real-time solution. The quality of results is comparable with results obtained by professional software.

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