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Estimation of Unobserved Inflation Expectations in India Using State-Space Model

Siddhartha Chattopadhyay[1]*, Sohini Sahu[2], Saakshi[2]

[1]Department of Humanities and Social Sciences, Indian Institute of Technology, Kharagpur, India
[2]Department of Economic Sciences, Indian Institute of Technology, Kanpur, India
Email: *siddhartha@hss.iitkgp.ernet.in, ssahu@iitk.ac.in, saaki@iitk.ac.in

Abstract

This paper estimates the unobserved inflation expectations in India between 1993: Q1 to 2017: Q1 from the Fisher equation relation based on the state space approach using Kalman Filter. We find inflation forecast obtained from Fischer equation by applying Kalman Filter match well with the inflation forecasts made by the Survey of Professional Forecasters and Inflation Expectations Survey of Household conducted by the Reserve Bank of India and the International Monetary Fund for the Indian economy.

Keywords

Fisher Equation, Kalman Filter, Expected Inflation, India

1. Introduction

Inflation expectation is an important marker for monetary policy makers. Both theoretically and empirically, it has been established that current inflation is influenced by expectations about future inflation and vice-versa. Especially for economies where inflation targeting is an explicit policy goal, knowledge of inflation expectations is required to attain inflation anchoring. Recently RBI has undertaken inflation targeting as one of the major monetary policy objectives for India. However, data of inflation expectation given by RBI in Inflation Expectation Survey of Household (IESH) and Survey of Professional Forecasters (SPF) are very limited. RBI started publishing quarterly IESH from third quarter of 2006 and SPF from fourth quarter of 2007. Hence, obtaining alternative measure of inflation forecast for a reasonably longer time period is required. This paper calculates the inflation forecast of India by applying unobserved component modelling based on Kalman Filter (KF) to the Fisher equation.

The inflation dynamics, as captured by the New Keynesian Phillips Curve

(NKPC), has inflation expectations as one of its explanatory variables. In this context, various authors have used different specifications of inflation expectations, a comprehensive survey of which can be found in Mavroeidis *et al.*[1] [1]

In the Indian context, authors have modelled the unobserved inflation expectations following techniques like perfect foresight [3] [4] [5] [6], moving average [7], single and double exponential smoothing [7] etc. Our work adds to the existing literature by using another procedure for estimating expected inflation of India.

This paper estimates the unobserved inflation expectations in India based on the Fisher equation relation using the state space approach or the KF method following Mishkin [8], Fama and Gibbons [9], Burmeister *et al.* [10] and Hamilton [11]. The novelty of this methodology when compared to the other techniques is that expected inflation is estimated from a well-established economic relationship which is the Fisher equation. Looking at it from another perspective, this estimation also verifies if the Fisher equation holds for the Indian economy. We find that between 1993: Q1 and 2017: Q1 the unobserved inflation expectations estimated from the Fisher equation match well with inflation forecasts made by the Survey of Professional Forecasters and Inflation Expectation Survey of Household conducted by the Reserve Bank of India (RBI) and the forecasts by the International Monetary Fund (IMF) for the Indian economy.

Given that the survey-based inflation expectations data for India are relatively recent and hence the data points are limited, our methodology ensures that the expected inflation estimated from the Fisher Equation can go back in time, even before the availability of survey-based inflation expectations data.

The paper proceeds as follows. We present the methodology in Section 2, followed by the data description in Section 3. Section 4 concludes the paper.

2. Methodology

Our starting point is the Fisher equation that relates the nominal interest rate and the real interest rate. It is given by the following equation:

$$rr_t^a = i_t - \pi_t^e \tag{1}$$

where, rr_t^a is the ex-ante real interest rate, i_t is the nominal interest rate captured by the policy rate and $\pi_t^e = E_t\left(\pi_{t+1}\right)$ is the expected inflation rate. Note that ex-ante real interest rate is unobserved since the expected inflation is unobserved. The ex-post real interest rate is $rr_t = i_t - \pi_t$.

Our variable of interest here is the one period ahead expected inflation. We apply the state space approach based on KF to the Fisher equation to estimate inflation expectation[2]. To do so, we decompose *ex-post* real interest rate rr_t, into *ex-ante* real interest rate, rr_t^a and the forecast error, $\mu_t = \pi_t^e - \pi_t$ as shown in Equation (2). We further assume that ex-ante real interest rate follows

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[1]A more general treatment of modelling unobserved components in structural time series models can be found in Harvey [2].

[2]See **Appendix** for a brief outline of Kalman Filter methodology.

an AR(1) process as given in Equation (3)[3].

$$rr_t = rr_t^a + \mu_t \qquad (2)$$

$$rr_t^a = \rho rr_{t-1}^a + \varepsilon_t \qquad (3)$$

where, $\rho \in (0,1)$ is the persistence. We assume, $\mu_t \sim IN(0,1)$ and $\varepsilon_t \sim IN(0,1)$ and independent of each other for all t.

Note, Equation (2) is our signal equation and Equation (3) is our state equation. We apply KF on Equation (2) and Equation (3) and estimate the parameters numerically through MLE using following initial conditions, $\rho = 0.83, \pi_0^f = 8, \sigma_u^2 = 1.87$ for our estimation[4]. These initial conditions yield $rr_{1|0} = 4$. However, we do not have any prior information about the variance of the state variable, ex-ante real interest rate. Hence, we have used diffuse prior for obtaining an initial guess of the variance of state variable, $P_{1|0}$[5]. Once convergence is achieved after 7 iterations, expected inflation is calculated as, $\hat{\pi}_t^e = i_t - \hat{\pi}_{t|t-1}^a$.

3. Data

Quarterly data on CPI for India between 1992: Q1 to 2017: Q1 has been collected from the FRED database provided by the Federal Reserve Bank of St. Louis. For quarterly nominal interest rate for India, we have used 91 days Treasury bill rate collected from different volumes of RBI bulletin. Inflation forecasts made by the International Monetary Fund are collected from World Economic Outlook Database[6]. The data on Survey of Professional Forecasters (SPF) and Inflation Expectations Survey of Households (IESH) are collected from different issues of RBI bulletin. We have used data of inflation rate based on WPI and CPI and 91 days Treasury bill rate to calculate expected inflation from Fisher equation by applying KF. There is no missing date in either of these series.

4. Results

Figure 1 plots the one period ahead expected quarterly inflation for India between 1993: Q2 and 2017: Q1 generated by applying the KF to the Fisher equation along with the actual inflation based on WPI. Actual inflation and its forecast are measured in percentage. Figure 2 on the other hand plots the inflation forecast of SPF with the same based on CPI after applying KF. Both figures show that inflation forecast obtained by using KF to Fisher equation do a very good job.

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[3]We have checked that ex-post real interest rate is AR(1). See **Appendix** for detail.

[4]A simple correlogram shows that, ex-post real interest rate is a process with first autocorrelation coefficient 0.89 and variance, 1.89. As a result, initial value of $\rho = 0.89$. We have decomposed the inflation rate into trend and cycle through HP filter. We set, $\pi_{10} = 8$ which is the inflation rate for 1992: 4. **Appendix** shows the plot of inflation and its trend obtained from HP filter. We have achieved convergence after 7 iterations. See **Appendix** for the correlogram and descriptive statistics of ex-post real interest rate.

[5]For details see, Harvey [12] and Koopman, Shephard and Doornik [13].

[6]IMF publishes biannual inflation forecast for Fall and Spring from 1990 onwards. It publishes inflation forecast for one year and two year ahead. We have used only one year ahead forecast in our paper.
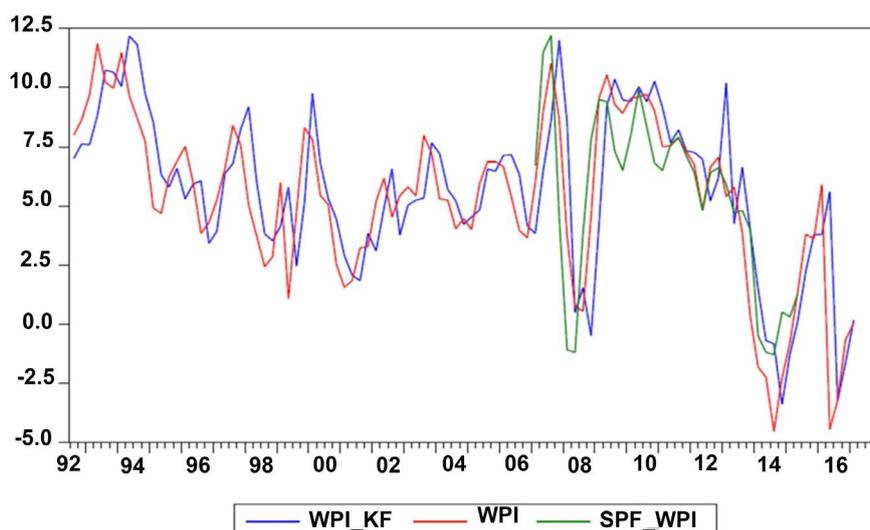
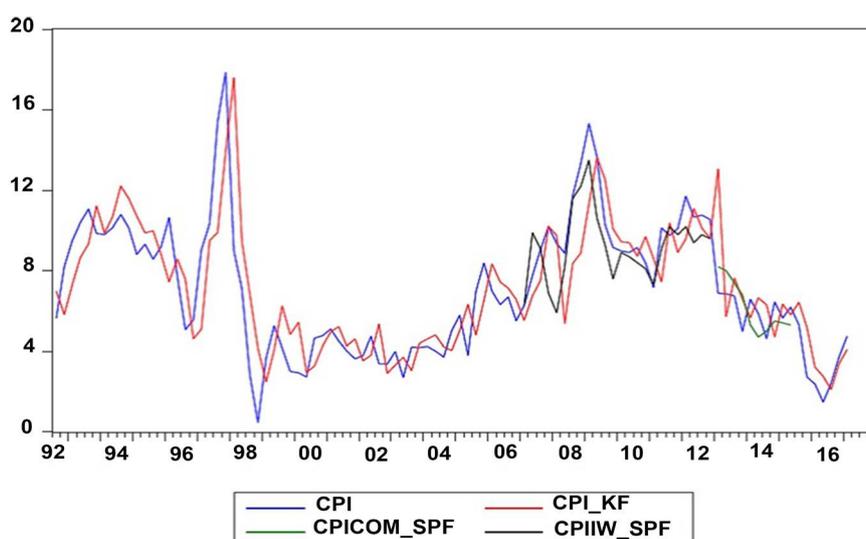**Figure 1.** Inflation based on WPI and its forecast.

Figure 2. Inflation based on CPI and its forecast.

Note, initial forecast obtained from KF is sensitive to initial guess but it quickly converges to SPF inflation forecast of IMF as shown in **Figure 2**. **Figure 2** shows that, forecast of KF quickly converges to IMF forecast from 1994: Q1 when the estimation starts from 1993: Q1.

Figure 3 and **Figure 4** compare the inflation forecast from KF with the same given by IMF and IESH of RBI. Both figures also show evidence in favour of Fisher equation holding for India as the inflation forecast obtained from it through KF matches the data quite well.

Table 1 gives us the descriptive statistics of inflation based on WPI and CPI along with their forecast based on KF, as well as the inflation forecasts made by the IMF for the Indian economy. Results indicate that mean, median and variance of forecast are very close to our actual inflation.
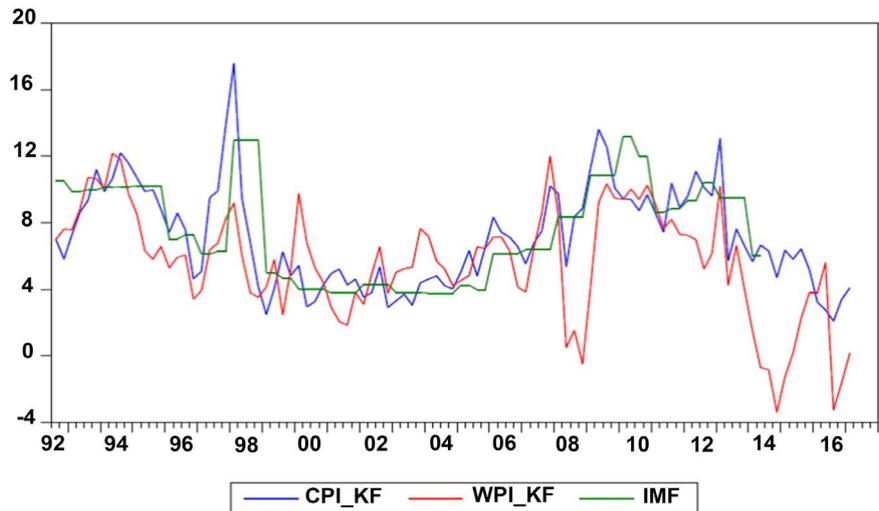
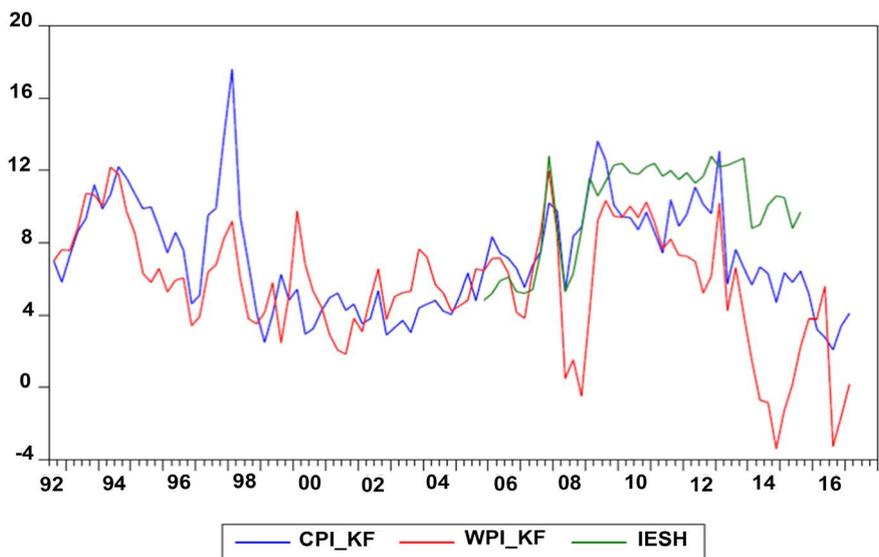**Figure 3.** Inflation forecasts of IMF and Kalman filter.

Figure 4. Inflation Forecasts of IESH and Kalman Filter.

Table 1. Descriptive statistics.

	CPI	WPI	IMF Forecast	CPI_KF	WPI_KF
Mean	7.10	5.34	7.58	7.16	5.65
Median	6.76	5.44	7.25	6.65	5.92
Maximum	17.86	11.86	13.18	17.59	12.17
Minimum	0.46	−4.55	3.76	2.08	−3.40
Standard Deviation	3.32	3.50	2.95	3.02	3.34

5. Conclusions

This paper estimates the unobserved quarterly inflation expectations in India between 1993: Q1 and 2017: Q1 from the Fisher equation by using the state-space

model. Our estimates match well with the inflation forecasts made by the SPF and IESH conducted by the RBI and the inflation forecasts of India made by the IMF. Our analysis also shows that Fisher equation does hold for India as the inflation forecast obtained from it by applying KF matches the data quite well.

Recently India has undertaken inflation targeting as one of the major monetary policy objectives. Achieving this objective requires information about inflation forecast to anchor inflationary expectations. However, data of inflation expectation given by RBI in IESH and SPF are very limited. RBI started publishing quarterly IESH from third quarter of 2006 and SPF from fourth quarter of 2007. Hence, obtaining alternative measure of inflation forecast for a reasonably longer time period is required. The uniqueness of our paper is that, it provides an alternative methodology to estimate inflation expectations of India for a reasonable longer period of time. Our methodology uses Fisher equation to estimate inflation forecast of India and hence has a strong theoretical foundation. Note, the inflation expectation obtained using our methodology nicely matches the survey data published by RBI. Moreover, long time series of monthly inflation and 91-days treasury bill rate are available for India. Therefore, our methodology of forecasting inflation expectations by KF can be used to get a longer time series of inflation expectations for India. Hence, data of inflation expectations obtained by our methodology are expected to facilitate the research on inflation forecast and monetary policy of India significantly.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

The Kalman Filter

KF is an Unobserved Component Modelling (UCM) which extracts an unobserved state recursively form an observed signal equation. To explain KF assume a signal equation,

$$y_t = A + Bs_t + \mu_t \qquad \text{(Signal)}$$

Here, the observed signal is $y_t : (N \times 1)$, unobserved state variable is $s_t : (K \times 1)$, the coefficient matrix $A : (N \times 1)$, represents intercept and $B : (N \times K)$. $\mu_t : (N \times 1)$ is random error. We assume, $\mu_t \sim N\left(0, \sum_\mu\right)$, where $\sum_\mu : (N \times N)$.

We assume the state equation is,

$$s_t = \Phi s_{t-1} + v_t \qquad \text{(State)}$$

Here, $\Phi : (K \times K)$ and $\mu_t \sim N\left(0, \sum_v\right)$, where $\sum_v : (K \times K)$. The random error term is signal (μ) and state (v) are assumed to be independent of each other.

The objective of KF is to estimate the unobserved state variable s_t. The unobserved state variable is estimated recursively from equation (ref: prediction) to (ref: update) as given below.

$$s_{t|t-1} = \Phi s_{t-1|t-1}$$

$$P_{t|t-1} = \Phi P_{t-1|t-1} \Phi' + \sum_v \qquad \text{(Prediction)}$$

$$y_{t|t-1} = A + Bs_{t|t-1}$$

$$V_{t|t-1} = BP_{t|t-1}B' + \sum_v \qquad \text{(Observation)}$$

and,

$$s_{t|t} = s_{t|t-1} + P_{t|t-1} + B'V_{t|t-1}^{-1}\left(y_t - y_{t-1}\right)$$

$$P_{t|t} = P_{t|t-1} + P_{t|t-1} + B'V_{t|t-1}^{-1}BP_{t|t-1} \qquad \text{(Update)}$$

Here, the $y_{t|t-1} = E_{t-1}\left(y_t\right)$ is the conditional mean of y_t given the information available till (t–1). The conditional variance of y_t is $V_{t|t-1}$. The best estimator of state is, $s_{t|t-1} = E_{t-1}\left(s_t\right)$ with variance $P_{t|t-1}$. Here, the Kalman gain is,

$$K_t = P_{t|t-1}B'V_{t|t-1}^{-1}$$

The Kalman gain is determined as a factor that minimizes the variance of state update $s_{t|t}$. The variance of state update is, $P_{t|t}$.

The parameter of the model is $\Theta = \left(A, B, \Phi, \sum_\mu, \sum_v\right)$. Note,

$$y_t \sim N\left(y_{t|t-1}, V_{t|t-1}\right)$$

Here, $y_{t|t-1}$ and $V_{t|t-1}$ and non-linear functions of Θ. As a result Θ is estimated numerically by Maximum Likelihood Estimation as follows,

$$\underset{\Theta}{\text{minimize}}\, l = \frac{1}{T}\sum_{t=1}^{T} l_t$$

$$l_t = -\frac{N}{2}\log(2\pi) - \frac{1}{2}\log(V_{t|t-1}) - \frac{1}{2}(y_t - y_{t-1})' V_{t|t-1}^{-1}(y_t - y_{t-1})$$

given the initial condition, $s_{1|0}, P_{1|0}$ and Θ_0. We can accordingly draw statistical inferences from,

$$\hat{\Theta}_{MLE} \sim AN\left[\Theta, \left(I(\Theta)_{\Theta=\hat{\Theta}_{MLE}}\right)^{-1}\right]$$

where, $I(\Theta)$ is the Fisher Information Matrix.