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Conditional CAPM Using Expected Returns of Brazilian Sustainability Companies

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Abstract

In the last decades, CAPM model has been of great interest in the scientific scene. Despite all the criticism, the improvement of the static CAPM, which has generated new dynamic models, provided investors with stronger guarantee through financial transactions. The CAPM and its static version were and are still very important in the financial scene. Nowadays, more sophisticated adaptations of the CAPM are found, which allow us to explain some matters in finance that had remained unqualified for a couple of time. Considering such discussion about the CAPM validity, this study aims to create a basis for reflection upon the conditional model, comparing it with the static one. In order to verify such facts, tests of conditional models are examined (with beta varying throughout the exercise), something uncommonly studied in the literature. Such tests are suitable to incorporate variances and covariance that change at long run. Methodological wise, the study tested the conditional CAPM model borrowing a leaf from Jagannathan and Wang [1] using macroeconomics and financial variables from the Brazilian sustainability Companies. Based on our findings, there is evidence that the conditional CAPM of Jagannathan and Wang [1] for the North American market is perfectly applicable to the Brazilian sustainability Companies.

Keywords

Conditional CAPM, Financial Markets, Portfolio

1. Introduction

The last two decades witnessed a growth in numbers of empirical studies that examined the product capacity of the static version of Capital Asset Pricing Models (CAPM). Conclusions from these studies demonstrated that static CAPM were unable to give a reasonable explanation to *cross-sectional* variation

of the average returns of the analyzed portfolios.

Costa Jr. [2] emphasized this idea when he mentioned that an original version of CAPM of absolute simplicity, recognized information of a greater relevance and applied it in a comprehensible manner. What happens is that the hypothesis that surrounds this original version requires a market of a perfect competition, which makes one to fear for lack of realism. Answers to this skeptism could be found in the empirical test done in the current study, that is, what is important is not the realism of the hypothesis of startup, but, to know if it is capable of concluding for the adjustment of the models to reality.

The ferrous critics of CAPM performed multivariate tests (multiple regression) and found two variables that explain the greater part of *cross-section* variation of medium returns: Book Value/Market Value index has a positive correlation with the returns of stocks while the variable as a whole is negative and significantly correlated and the beta appeared insignificant in this test [3].

Fama and French [4] found in their model three statistically associated factors that are significant as different from zero. This result suggests that the *proxy* of the factors associate's risks to returns of the human capital and the betas are unstable. Notwithstanding, this model was able to explain the *cross-sectional* of the expected returns.

The CAPM and it static version were and are of great importance in finance. Therefore, in today's applications, we find complex adaptations of CAPM that enable one to envisage results for questions which are yet to be resolved in finance.

Based on this panorama therefore, and considering the whole scope of discussion that surrounds the validity of CAPM, this study aims to present the advantages of the conditional or dynamic model (models that incorporate variances and co-variances which change during a space of time), in relation to a static model.

Therefore, we study the tests of conditional models (*beta* variance during a period) that are not commonly studied in literature. These tests are convenient in order to incorporate variances and co-variances and changes in a future period. In the conditional model test, we highlight the studies of Jagannathan and Wang [1], and Ferson and Harvey [5].

Bonomo [6] mentioned yet, important studies about conditional CAPM among these, we cite [7] where a *beta* of a group of assets is defined as a conditional covariance of error committed upon forecast of the returns on assets and the error on forecasted market returns. These models have various beta coefficients while the standard CAPM have only one.

Finally, this study is structured in five sections, firstly, being contemplation of introductory aspects of the study; the second section has the background of Conditional Capital Asset Pricing Model, thirdly, about the methodological approach of Fama and MacBeth [8].

2. Background of Conditional Capital Asset Pricing Model

CAPM is defined as a model which relates an expected profitability of an asset in

a certain market and equilibrium with its undiversified risks, also known as *beta*. Besides Sharpe, other authors also formulate CAPM, in its static version. Among these authors are Lintner [9], Mossin [10] and Treynor. This version of static CAPM or conditional has some consistent results when we perform empirical tests in order to verify the adherence capacity of the model to the reality of some economies.

In all tests of non-conditional CAPM such as that of Fama and MacBeth [8], Black, Jensen and Scholes [11] it was supposed that beta would be static, that is, the assets systematic risk would not change.

Haugen [12] shows that Black, Jensen and Scholes consider that there is a positive linear relationship between beta and the expected return. As a consequence of this fact, Black, Jensen and Scholes [11] encounter in their test of CAPM a positive relationship between profitability and the beta.

Merton [13] shows that the Consumption Capital Asset Pricing Model (ICAPM) had as an objective, generalize the CAPM model of Sharpe [14] for an intertemporal context. The original ICAPM takes the hypothesis that the investors consumed all the reaches after a period, such that the said reaches and the consumptions are confused.

The static CAPM of Sharpe-Lintner-Black, given as R_i, which denotes the returns on, shares I and R_m the portfolio market returns for all shares of the economy. The version of Black [15] is:

$$E[R_i] = \gamma_0 + \gamma_1 \beta_i \tag{2.1}$$

where γ_0 and γ_1 are defined as expected market returns and risk Premium expected from the market respectively, and where β_i is defined as:

$$\beta_i = Cov(R_i, R_m) / Var[R_m]. \tag{2.2}$$

Fama and French [3] followed Black [15] and examined empirically the static CAPM, arriving at a conclusion that, there is a weak relationship between medium return and the *beta*, and finding strong evidence against static CAPM.

Thus, Jagannathan and Wang [1] [16] developed a study which partially contradicts these evidences. In these same studies they observed that, upon application of CRSP index as a base for market portfolio, they found in their non-conditional model, implicit in the conditional CAPM, an explanation close to 30% of *cross-sectional variation of the medium returns of* 100 market portfolios, similar to that used by Fama and French [3]. For the implementation of CAPM therefore, is commonly used as *proxy* all the shares that are enlisted in the *New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX)*, which could be considered as a reasonable *proxy* for the market returns on portfolio of all assets. However, Fama and French [3] found that, upon usage of that *proxy*, the same was not sufficient for a satisfactory analysis of the performance of CAPM.

As a result of this fact and in order to ameliorate the *proxy*, Jagannathan and Wang [1] followed Mayers [17] and included in their models returns on human

capital. When human capital is also included in the portfolio of the market, the non-conditional model implicit in conditional CAPM conditional is then capable of explaining more than 50% of the *cross-sectional* variation of the medium return. Besides this, the statistics tests where unable to give answers as they reject the model.

3. Methodology of Fama and Macbeth

Haugen [12] Shows that Fama and MacBeth [8] methodology introduced a significant difference as related to the former tests, since they arrived at coherent results concerning fundamental forecasts of CAPM [15].

Fama and MacBeth [8] Constituted 20 portfolios which contain shares enlisted in *NYSE* for the period of 1926 through 1929. Latter, they estimated the beta of each of the portfolios and highlighting the monthly returns of the market index for the period of 1930 through 1934. They used the betas of each of the portfolios of the prior periods to forecast the monthly returns of the portfolios for the periods subsequent to 1935 through 1938. The process estimating the market beta was repeated nine times until 360 estimations were ascertained which was in the January 1935 through June of 1968.

Haugen [12] showed that in this case, Fama and MacBeth adopted betas and returns from different periods. The estimated beta in a period is used to estimate interest rate of returns for a future period. The results of these tests were very comforting, in that, CAPM gained the supports of scientists after the publication of this study.

Even though the critics of the model are yet to find in various studies that takes it as literary support, amongst these, one would observe the model produced by Jagannathan and Wang [1] through Fama and MacBeth [8] that utilizes the same methodology.

4. The Conditional CAPM Model for Brazil Using Returns of Brazilian New Market

The selected variables (in the first place) are consisted of integral part of the Conditional *CAPM Model* for Brazil. It refers to the portfolios constructed through the monthly share returns negotiated at the Stock Market of São Paulo (New Market Ibovespa), GDP of the market and, for the premium, the spread between Inter-finances Operation Deposit Index (DI), reported by the Central of Custody and Liquidation of Private Sector Papers (CETIP) and the interest rate (Selic), that is aimed to serve as a forecast for the variations of the business cycle.

Using the approach cited above, seven portfolios were created for the Brazilian market, containing five shares of Ibovespa New Market during the period of Jan./2008 through Dec./2013.

The data were collected from the Central Bank of Brazil, and the Económatica databases.

Following the steps above, Jagannathan and Wang [1], used the returns of all

the shares of NYSE and AMEX and constituted 100 portfolios in function of size variable with monthly returns from July of 1963 to December of 1990, summing 330 observations. For each portfolio one calculates a regression between shares that compose the portfolio and the market indexes (NYSE and AMEX).

We created a time series of the monthly returns for each of the five portfolios (Brazil). The model for the moment is estimated using the method of generalized moment.

Also, we used the average value of each of the coefficients to determine their significance, and thus, the portfolios were gradually re-balanced annually.

According to Fama and MacBeth [8] these portfolios were rebalanced period by period, before the estimation of the beta attains the total of the estimation of the analyzed period. All the shares were attributed the same weight in each portfolio.

An observation that confronts the literature review with the research deals with the prior decision as to selection of the Brazilian index, as gearing the regional markets of the region. This implies an implicitly assumption that the market is segmented.

5. Analysis of Results Based on Brazilian (Sustainability Companies)

The regressions of the models are estimated using Fama & MacBeth [8] methodology. The model was estimated using the generalized model of the moments. Through the correction of the errors we verified that if the residual variance has an effect on the price of the assets or the expected rate of returns and, base on the results, there is no indication that the assets with residual variance greater than the average, produces rate of return higher than the weighted average during the future period. Five portfolios were constructed with five shares in each one. The tested period ranged from January, 2008 through december 2013. For the Brazilian market the premium is represented by *spread* between the interest rate of CETIP and that of SELIC. While in the human capital it is represented by the market Gross Domestic Product (*GDP*), the market *proxy* will be Ibovespa. Thus, the equation that is being estimated for the Brazilian market would be as follows: $E[R_{it}] = c_0 + c_{\text{size}} \log(ME_i) + c_{\text{ibov}}\beta + c_{\text{premio}}\beta + c_{\text{pib.mer}}\beta$.

Results available in **Table 1** show that *t value* for C_{ibov} is 0.24. The R^2 of the regression is only 7.50%. This means to say that the *cross-sectional* variance of the average returns is yet to be fully applied when we use a static CAPM without the inclusion of the market *GDP* in the case of Brazil.

The model [18] for the correction of errors as per estimation, is not significant. Thus, after correction of errors, that treat the error of the model so that one would use this term to reflect on the behavior of the variables in short run with its value a long run, that is, it is a means of reconciliation of the behavior in a short run of a variable with its behavior for a future period. The C_{ibov} is not significantly different from zero. When the size variable is introduced into the

Table 1. Static CAPM without human capital (Brazil).

Coeficients	C_0	C_{ibov}	C_{premio}	$C_{\text{pib mer}}$	C_{size}	R-square
Estimate	−3.47	1.25				7.50
t-value	−0.87	0.24				
p-value	0.00	0.00				
Correction-t	−0.22	0.08				
Correction-p	0.00	0.00				
Estimate	−0.64	0.44			0.68	44.20
t-value	−0.56	0.06			4.20	
p-value	0.00	0.00			0.00	
Correction-t	−0.45	0.87			2.56	
Correction-p	0.00	0.00			0.00	

model, we found for C_{size} a *t-value* of 4.20 and the R^2 rose to 44.20%.

Notwithstanding the increase of R^2 and the fact that the model did not present any significant changes after the correction of the errors, the model appears inconsistent (because even after inclusion of the size variable, for the Brazilian market, it does appears to not have been influenced as a result of the static model not absorb the effects of this variable). Analysis of the Brazilian market appears to be in the same direction as conclusions reached for the, the North American market.

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$$E[R_{it}] = c_0 + c_{\text{size}} \log(ME_i) + c_{\text{ibov}}\beta + c_{\text{premio}}\beta + c_{\text{pib.mer}}\beta.$$

Results in **Table 2** show that the estimated value for C_{premim}, is not significantly different from zero. The *t-value* for C_{premim} is −0.70. The R^2 is only 10.20%. Note that the R^2 is similar to the result encountered in the previous model.

When the model for the correction of errors is introduced the *t-value* for C_{premio} becomes −0.45. When the variable *size* is added to the model the *t-value* for C_{size} comes to 3.86. And when one introduce the model for correction of errors, the *t-value* for C_{size} declines to 2.10, and R^2 grows to 40.50%.

Table 2. Static CAPM with human capital (Brazil).

Coeficientes	C_0	C_{ibov}	C_{premio}	$C_{\text{pib mer}}$	C_{size}	R-square
Estimate	−3.10	−1.09	−4.74			10.20
t-value	−0.89	−0.05	−0.70			
p-value	0.00	0.00	0.00			
Correção-t	−0.24	−0.12	−0.45			
Correção-p	0.00	0.01	0.00			
Estimate	−0.69	0.56	0.53		0.61	40.50
t-value	−0.30	0.12	0.33		3.86	
p-value	0.00	0.08	0.32		0.00	
Correção-t	−0.14	0.01	0.16		2.10	
Correção-p	0.00	0.05	0.45		0.00	

The value of R^2 for the Brazilian market remained 40.50% (a value close to that found in the static CAPM) and the estimated value for C_{premim}, and after the correction of the errors, it became significantly different from zero. This fact could be explained by non-inclusion of market GDP. In this regards, the conditional model appears to be more effective for the explanation of the *cross-sectional* variances average of the market returns for Brazilian market.

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$$E[R_{it}] = c_0 + c_{\text{size}} \log(ME_i) + c_{\text{ibov}}\beta + c_{\text{premio}}\beta + c_{\text{pib.mer}}\beta.$$

Results showed by **Table 3** show that the estimated value for $C_{\text{pib.mer}}$, using Fama-MacBeth methodology, is not significantly different from zero. The *t-value is* −0.41 and R^2 is 12.10%.

While in the Brazilian market, when one introduces a model for the correction of the errors *t-value for* $C_{\text{pib.merr}}$ drops to −0.13, the *p-value* goes to 0.21 and the coefficient C_{premio} becomes significant. When the *size* is added to the model, the *t-value for* C_{size} becomes 3.10, and o R^2 rises to 52.30%.

The conditional CAPM with the inclusion of *GDP* of the Brazilian market appears to be closer in results at to that of the United States. Besides that the C_{premio} and $C_{\text{pib.mer}}$ variables have become significantly different from zero after

Table 3. Conditional CAPM with human capital (Brazil).

Coeficientes	C_0	C_{ibov}	C_{premio}	$C_{pib\ mer}$	C_{size}	R-square
Estimate	−2.01	3.65	2.60	−0.59		12.10
t-value	−0.85	0.15	−0.18	−0.41		
p-value	0.00	0.00	0.00	0.00		
Correção-t	−0.18	0.02	−0.19	−0.03		
Correção-p	0.00	0.03	0.16	0.02		
Estimate	−0.56	10.75	1.86	−1.28	0.71	52.30
t-value	−0.34	0.31	0.18	−0.43	3.10	
p-value	0.00	0.00	0.00	0.00	0.00	
Correção-t	−0.11	0.02	0.10	−0.13	0.21	
Correção-p	0.23	0.12	0.56	0.01	0.00	

Table 4. Conditional CAPM without human capital (Brazil).

$$E[R_{it}] = c_0 + c_{size} \log(ME_i) + c_{ibov}\beta + c_{premio}\beta + c_{pib.mer}\beta.$$

Coeficientes	C_0	C_{ibov}	C_{premio}	$C_{pib\ mer}$	C_{size}	R-square
Estimate	−1.33	3.87		−0.52		10.20
t-value	−0.79	0.70		−0.35		
p-value	0.00	0.00		0.00		
Correção-t	−0.19	0.13		−0.03		
Correção-p	0.00	0.01		0.04		
Estimate	−0.88	12.58		−1.35	0.78	51.30
t-value	−0.36	0.60		−0.66	4.10	
p-value	0.00	0.00		0.00	0.00	
Correção-t	−0.02	0.02		−0.13	0.38	
Correção-p	0.01	0.02		0.15	0.00	

the correction of the errors, the consistence of the model does not seem to have been touched.

Results found in Table 4 show that the estimated value of $C_{pib.mer}$, using Fama-MacBeth methodology is not significantly different from zero. The *t-value is* −0.35 and R^2 is only 10.20%. However, after the correction of the errors, we conclude that $C_{lpib.mer}$ becomes significantly different from zero as against the North American market.

When we introduce the *size* variable, the *t-value* becomes 4.10 and R^2 grows to 51.30%. Besides the rise of R^2 the model is not consistent. It is necessary to permit that beta varies at long run so that the expected *cross-sectional* returns of the market would be explained.

6. Final Comments

The static CAPM, without the inclusion of the human capital variable do not

appear to satisfactorily explain the expected *cross-sectional* returns of the analyzed markets.

After inclusion of variable "*size*", the R^2 of all the models had an abrupt change. And besides this fact that the finding is being coherent with what is found in literature, we conclude that the models for the analyzed countries appear inconsistent for they did not present any changes in the parameters at long run.

The model did not appear to present satisfactorily the reality of the various economies. Firstly, because we know that business cycle is dynamic in most economy and as per model analyzed above this variable was not contemplated and secondly, because the market *proxy* would not just be enough to represent any economy.

The model needs to be ameliorated with the inclusion of new variables that better represent each market.

Therefore, we must not discard static CAPM, because it is capable of explaining the market for a determined space of time.

As different from the North American market, the Brazilian sustainability companies have an increasing relation between the average returns of the portfolios and the size, thereby showing a substantially high return for a bigger sized portfolio.

In relation to the conditional CAPM, without the inclusion of human capital variable we observed in the Brazilian case, the estimated value of C_{premim} is not significantly different than zero for the new market shares.

However, when we introduce the model for the correction of errors variable C_{premim} becomes significantly different from zero for the case of Brazil. In case of North America and even after adoption of the model for the correction of errors, the variable C_{premim} continues to be significantly different from zero. This signifies that the risk premium drastically influenced the market analyzed.

When the *size* variable is incremented to the model, the R^2 rises proportionately for the Brazilian.

When the *size* variable is added to the model, the R^2 suffers a considerable increase, even though the variable *size* presents some effects on the model. This means that the conditional CAPM, even without the inclusion of human capital, are able to explain the efficacy of the *cross-sectional* variance medium returns of the analyzed portfolios. This happens in that the *size* variable or size effect aggregately influenced the Brazilian New Market.

In relation to the conditional model using New Market Ibovespa Portfolios, we may conclude with no doubt that the power of explanation of the model increases reasonably for each one of the cases analyzed.

The model appears to be able to capture the effects of the dynamics of the economy. By introducing the *size* variable, the models have a considerable increase in their R^2, but note that this variable appears to be more significant in the Brazilian sustainability companies as probably as a result of differences found in

the composition of new market shares of these markets.

Finally, there is evidence that the conditional CAPM of Jagannathan and Wang [1] for the North American market are perfectly applicable to the Brazilian sustainability companies. Our finding in this study permits us to differentiate and also identify an important tool for the potential investor of these countries.

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